Robert C. Merton
Massachusetts Institute of Technology

Myron S. Scholes
University of Chicago

Mathew L. Gladstein
Donaldson, Lufkin & Jenrette Securities Corporation

The Returns and Risks of Alternative Put-Option Portfolio Investment Strategies*

As in our earlier paper on call-option strategies, we find in this study that investors can use uncovered put-option-writing and protective put-option-buying strategies to produce patterns of returns for investment portfolios, not reproducible by any simple strategy of combining stock with fixed-income securities. Put options are viewed as term insurance, insuring against a loss in value of underlying stock: investors sell insurance by using the uncovered put-writing strategies and investors buy insurance protecting the value of stocks within a portfolio by using protective put-option-buying strategies. No put strategy or call strategy can dominate any other strategy if options are priced correctly. Although there is no single best strategy for all investors, they are not indifferent among strategies because the patterns of returns differ. We, therefore, show these important patterns of returns on put-option strategies not only using single stocks but also using portfolios of stocks.

I. Introduction

This paper is the second in a series of analyses we have prepared based on simulations of a variety of portfolio strategies over a significant period of time. The first of these analyses[1] examined the "fully covered" call-option-writing strategy and the buying strategy which combines purchase of call options and commercial paper. In the study presented here, we examine the "uncovered" put-writing strategy and the buying strategy which combines the purchase of put options and the underlying stocks. As a preface to the simulations, we analyze the quantitative discrepancies between the parity model of put pricing and a model which takes into account the possibility of early exercise of the put. In connection with this analysis, we simulate the returns to the "conversion" strategy which combines the purchase of put options and the underlying stocks with the simultaneous sale of the corresponding call options.

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1. Merton, Scholes, and Gladstein (1978). In the present paper, all references to "our earlier paper" are to this study.

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As discussed in our earlier paper, the introduction of options markets provides a significant expansion in the patterns of portfolio returns that were heretofore unavailable in combinations of equities and fixed-income instruments. To benefit from the additional investment opportunities provided by options, investors must be informed as to the return characteristics of various option strategies. However, there has been only limited first-hand experience in trading of options in organized markets because the option exchanges have been in existence for such a short period of time. Indeed, organized trading in call options began in April 1973 and trading in put options began only recently, in June 1977. Our simulations are designed to provide the needed and missing empirical background which might otherwise have been available if these markets had existed over a number of market cycles.

In a similar fashion to our study of call-option strategies, the analyses presented here illustrate the return characteristics of these put-option strategies over a period of varying market environments and demonstrate how put-option strategies can be used to modify the risk-return patterns of the underlying stocks. To provide continuity with our earlier study, we have chosen the same format for presentation and have used the same two samples of underlying stocks.

The stocks used in the first sample are all of the 136 stocks on which listed options were available as of December 1975.[2] As discussed in detail in our earlier paper, this sample is subject to selection bias in the sense that a portfolio of these stocks can be expected to outperform a randomly chosen group of stocks during the simulation period.[3] Hence, we have prepared a parallel set of simulations on a second group of stocks that are not subject to this bias. This second sample contains the 30 stocks in the Dow Jones Industrial Average as of the beginning date of the simulation period.[4] To identify which sample of stocks is associated with the various simulations, the prefix "136" is used for simulations based on the large sample and the prefix "DJ" is used for simulations based on the 30 Dow Jones stocks.

The simulation period for the earlier study was the 12½-year interval from July 1, 1963 to December 31, 1975. For the present study, it has been extended to the 14-year interval from July 1, 1963 to June 30,

2. Listed price date were not available for all 136 stocks for the entire simulation period. See Merton et al. (1978) for a breakdown of the number of stocks by period from July 1963 to June 1972. After June 1972, price data were available for all 136 stocks.

3. Although we did not choose this group of stocks on the basis of their past performance, past performance was almost surely a consideration in its selection by the options exchanges. There is no selection bias for this group in the January 1976–June 1977 interval of the simulation period.

4. Because Anaconda was merged, the DJ portfolio contains only 29 stocks in the last 6-month period.

1977. As with the earlier study, all the strategies examined have semiannual holding periods between portfolio revisions, and there are therefore a total of 28 basic observation subperiods in the simulations. To allow for comparisons of the return patterns among the strategies examined in both studies, summary return statistics on the previously examined call-option strategies are provided for the extended period through June 1977.

As was pointed out in our earlier call-option study, the return statistics for the various option strategies are sensitive to the choice of the simulation period (1963–77) and to the length of the holding period between portfolio revisions (6 months). While we believe that the simulations provide a reasonable indication of the differences in risk patterns (i.e., standard deviation, skewness, and extreme values) among various option strategies, we warn the reader against attaching great significance to the differences in average returns. Just as with the average returns on stock-investment strategies, the average returns from option strategies are especially sensitive to the periods chosen, and hence these averages, which are based on (ex post) historical returns, provide unreliable indicators of the (ex ante) expected returns. If, as is assumed in our simulations, stocks and options are "fairly priced," the lower-risk strategies will have lower expected returns. Therefore, if a lower-risk strategy happens to have a higher average return than a higher-risk strategy over the simulation period, the reader should not use these historical results to project future performance from these strategies. We deal with this sensitivity of the average returns to the choice of simulation period, at least in part, by providing in the Appendixes tables which display the average returns for every subperiod of the simulation period.

The sensitivity of the return statistics to the simulation and holding periods is, of course, not unique to option-investment strategies. The same concerns would apply to any study of investment strategies based on past history. However, unlike stock and bond investment strategies, the performance of option strategies is also significantly affected by the choice of premium levels. We believe that the premium levels used here are reasonable estimates of the ones that would have been paid or received during the simulation period. However, as in our earlier call-option study, sensitivity tables are provided to show the effect of higher or lower premiums on the pattern of returns for these strategies. These tables permit the reader who believes that our premiums are biased to "correct" our results to fit his own beliefs about premium levels. For those who agree with our fair-value estimates for premiums, the tables show the effect on performance if options are not "fairly priced." Finally, these tables will show that the principal effect of different premium levels is to change the average returns, and therefore they serve to reinforce our assertion that the differences in risk patterns among the strategies observed here are robust with respect to our assumptions.

For the effective use of our results, a basic knowledge of the functional characteristics of options is essential. A detailed nontechnical discussion explaining the basic features of both call and put options is provided in our earlier paper. For a more technical discussion, there is an excellent survey article by Smith (1976).

II. On Put-Option Pricing

To generate a time series of returns for put-option strategies, it is, of course, necessary to have put-option prices. As was the case in our earlier paper, the strategy simulations employ premiums derived from an option-pricing model rather than actual market option prices. While the reasons for this choice are essentially the same as in our call-option study, they bear repeating here.

To generate a representative pattern of returns requires a long time period covering a variety of market environments, but before 1973 there was no organized market for call options, and trading in put options has only just started. Prior to the organized markets, all option transactions took place through options dealers on an individual trade basis and there was no standardization of the option contracts with respect to either the exercise price or the expiration date. Short of obtaining a dealer's book,[5] the two systematic sources for OTC option prices are dealers' newspaper advertisements and weekly submissions to the Security and Exchange Commission by the Put and Call Brokers and Dealers Association. However, the prices advertised were not always the transactions price, and many of the advertised "specials" were only available in limited quantities. The SEC submissions were for only 10 companies, and these companies varied from week to week. Moreover, as Gould and Galai (1974, p. 111) point out, "These prices are nominal quotations and, since the price on any newly written put or call is negotiated, there are surely some observational and judgmental errors in these data." Even if these data "errors" are neglected, to simulate a specific strategy requires price data for options with the same exercise price/stock price ratio and the same expiration date for a large number of stocks. Such price data are not available for put options. Finally, the terms of options bought and sold through options dealers provided for adjustments to the exercise price when cash dividends were paid on the stock. No such adjustments are made for

5. Black and Sholes (1973) did test their model using actual transactions recorded in a dealer's diary from 1966 to 1969. Gould and Galai (1974) also use these data. However, the time period spanned by these data is still rather short.

listed options. Therefore, the premiums paid for OTC options are not comparable to the listed option prices for dividend-paying stocks.

The call-option pricing formula used in our earlier study was the well-known "Black-Scholes" formula adjusted for dividends.[6] However, no such simple formula has been derived for put prices. If options were "European" (i.e., not exercisable prior to the expiration date), then, to avoid arbitrage, the price of a put on one share must satisfy

$$P_e = C - S + Ee^{-rT} + \sum_{i=1}^{m} d_i e^{-rt_i}, \tag{1}$$

where

$P_e \equiv$ price of a European put with exercise price E and time until expiration T;
$C \equiv$ price of a European call option with the same exercise price and time until expiration;
$S \equiv$ price of the underlying stock;
$r \equiv$ (continuously compounded) market rate of interest;
$d_i \equiv$ cash dividend per share on the stock payable at time t_i ($0 \leq t_i \leq T$; $i = 1, 2, \ldots, m$).

Formula (1) is sometimes called the "parity-model" formula.[7] If the assumptions required for its derivation were satisfied, then the pricing of puts would be no more involved than the pricing of calls. Indeed, given any formula for pricing calls (e.g., the Black-Scholes formula), then (1) could be used to price the corresponding put. Unfortunately, as has been shown elsewhere (see Merton 1973*a*, 1973*b*, pp. 159–60), there always exists a stock price such that for a given maturity put option, the put is worth more if exercised immediately than if not. Hence, the fundamental assumption of the parity model is violated, and the put-option price given by formula (1) will be systematically biased below the correct put-option price. Figure 1 illustrates the price relationship between an "American" put that can be exercised at any time, P, and its European counterpart, P_e, where $\bar{S}(T)$ denotes that stock price where it is optimal to exercise a put with time until expiration T.

Although its derivation is beyond the scope of this paper, a put-pricing model that takes into account the possibility of early exercise has been derived along the methodological lines of Black and Scholes

6. Black and Scholes (1973). For a discussion of the dividend adjustment, see Merton et al. (1978) and Appendix A of the present paper.

7. For a derivation of this formula, see Merton (1973*b*, p. 157). Stoll (1969) and Gould and Galai (1974) derive and test it. In addition to the "European" assumption, the formula derivation requires: (1) the borrowing rate equal to the lending rate, (2) use of the proceeds from short sales, (3) a known dividend policy over the life of the options, (4) no transactions costs or differential tax rates.

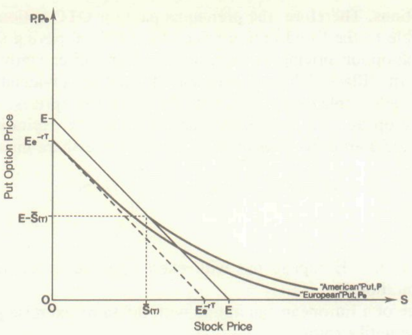

FIG. 1.—American versus European put prices

(see Appendix A and Merton [1973*b*, pp. 173–74]). The model simultaneously provides the put price and the schedule of stock prices, $\bar{S}(T)$, where it is optimal to exercise the put. Like the Black-Scholes call-pricing model, the derived put-pricing model uses for inputs: (1) the current stock price per share, S; (2) the length of time until expiration of the option, T; (3) the exercise price, E; (4) the market rate of interest for riskless, fixed-income securities, r; (5) the variance rate for the logarithmic returns on the underlying stock, σ^2; (6) the cash dividend payouts on the stock during the life of the option, $\{d_i\}$. All the inputs except the variance rate and the dividends require no estimation. Moreover, given the relatively short duration of the options, the dividend policy can be reasonably well estimated, and to a lesser extent so can the variance rate.[8] Unfortunately, unlike the Black-Scholes call-price case, there is no closed-form solution to the underlying partial differential equations for the put price. Hence, the put-option formula must be computed by numerical methods. Fortunately, recent research[9] has produced numerical procedures which make such computations economically feasible.

In our simulations, we used the sample variance of the previous 6 months of daily logarithmic price changes to estimate the variance rate for each stock during each of the 28 6-month holding periods. The dividends used were the actual dividends paid on the stock during the

8. For further discussion of this point, see Latane and Rendleman (1976) and Merton et al. (1978).

9. Such procedures are developed explicitly for put options in Brennan and Schwartz (1977) and Parkinson (1977). See also Schwartz (1977). The procedure used in our paper is an adaptation of Parkinson's method.

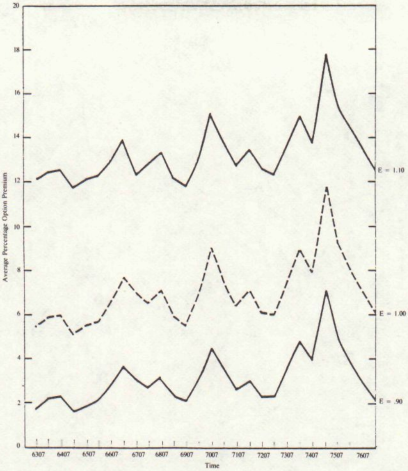

FIG. 2.—Average percentage put-option premium versus time (136-stock sample).

life of the option. While the dividend estimate, unlike the variance-rate estimate, uses future information not actually available at the time that the position is established, the bias of this procedure is trivial in its impact on the option premiums and the strategy returns. The interest rate used throughout the study is the 6-month prime commercial paper rate.

With respect to the inputs into the model, the put-option premium will be larger for larger values of E, σ^2, T and $\{d_i\}$ and will be smaller for larger values of S and r. Figure 2 plots the average of the percentage put-option premiums (i.e., the put price divided by the stock price) for 6-month puts on each of the underlying stocks in the 136-stock sample for the 28 6-month periods from July 1963 to January 1977. The three time series of average percentage premiums are for different option series with exercise price/initial stock price ratios of 0.9, 1.0, and 1.1, respectively. Figure 3 plots the corresponding average percentage premiums for the DJ stock sample.

As was the case for call premiums examined in our earlier paper, the average percentage premiums vary substantially. The changes in the premiums reflect the changes in interest rates, variance rates, and

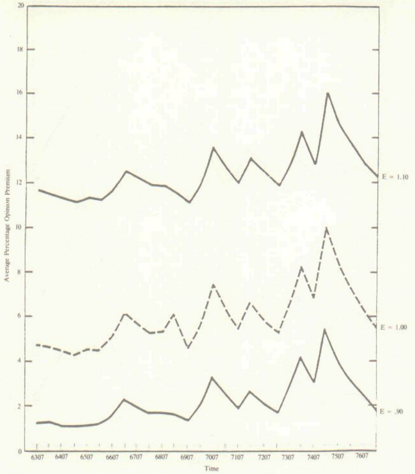

FIG. 3.—Average percentage put-option premium versus time (DJ stock sample).

dividend yields over time. Inspection of figures 2 and 3 reinforces the conclusion reached in our earlier paper on call options that option premiums will vary substantially through time, and therefore any study of the returns from option strategies must take such variations into account.

To put the levels of put-option premiums in perspective, we relate them to the more familiar call-option premiums by computing the average put/call price ratios (i.e., the put price divided by the call price). To compute the call prices, the Black-Scholes formula adjusted for dividends (as described in our earlier paper and in Appendix A here) was used. With respect to the model inputs, the put/call price ratio is larger for larger values of E and $\{d_i\}$ and is smaller for larger values of S, σ^2, and r. Figures 4 and 5 plot the average of the price ratios for 6-month puts and calls with the same exercise price on each of the underlying stocks in the 136 and DJ stock samples, respectively. For the entire simulation period the average price ratio for the option series with an exercise price/initial stock price ratio of 0.9 was 19.9% for the 136-stock sample and 16.3% for the DJ sample. The corresponding average ratios for the other option series were 84.9% and 90.2% for the

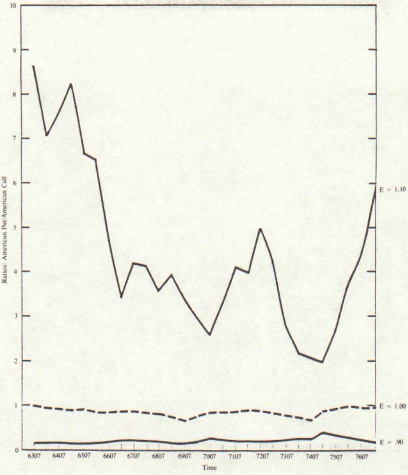

FIG. 4.—Average put/call price ratio versus time (136-stock sample)

exercise price/initial stock price ratio of 1.0,[10] and 442.2% and 612.2% for the exercise Price/initial stock price ratio of 1.1. As inspection of figures 4 and 5 demonstrates, the average price ratios for each stock and option series varied significantly among the 28 6-month holding periods.

To avoid confusion with the procedure used in the strategy simulations in this paper, note that the average percentage premiums and average put/call price ratios presented here are only representative of the levels used in the simulations. In the simulations, a separate premium is estimated for each stock and for each 6-month period.

We close this section by returning to the comparison between the parity model of put-option pricing and the derived put-option pricing model. As previously discussed, there is no theoretical issue: the parity model will systematically understate the value of a put. However, there is an important empirical issue, namely, the magnitude of this bias. At least two empirical papers have used the parity model to evaluate the efficiency of the OTC options market (see Stoll [1969] and Gould and

10. Note that the average at-the-market put/call price ratio for the DJ sample exceeds one in some periods. This is induced by the combination of a large dividend yield on the stocks and a relatively low interest rate.

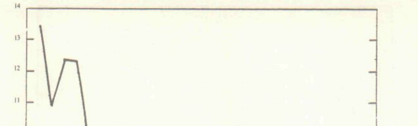

FIG. 5.—Average put/call price ratio versus time (DJ stock sample)

Galai [1974]), and the magnitude of the bias will affect, at least in part, the validity of their conclusions. Because the source of the differences between the two models is early exercise of the put, the magnitude of the bias measures the value of the right to exercise early. If this right has virtually no value, then the parity model would be an attractive approximation to the exact solution because of its relative computational simplicity and because it can be used together with any theory of call pricing to generate put prices.

While a complete analysis of the bias for all parameter values is outside the scope of this study, we can shed some light on this issue by comparing the option premiums generated by the two models for the sample of stocks and observations period chosen for our strategy simulations.

The absolute dollar bias, measured by taking the correct put price minus the parity model put price, will always be nonnegative. With respect to the inputs into the models, the magnitude of the bias will be larger for larger values of E and r, and it will be smaller for larger values of S, σ^2, and $\{d_i\}$. The percentage bias, measured by dividing the absolute dollar bias by the parity model put price, is also nonnegative. The percentage bias will be larger for larger values of r, and smaller for

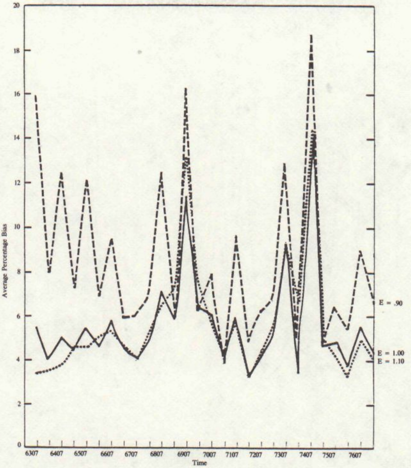

FIG. 6.—Average percentage bias versus time; put/parity model price ratio (136-stock sample).

larger values of σ^2 and $\{d_i\}$. However, unlike the absolute dollar bias, the change in the percentage bias with respect to a change in either S or E is not always in the same direction. Figure 6 plots the average of the percentage bias for 6-month puts on each of the underlying stocks in the 136-stock sample for the 28 6-month periods from July 1963 to January 1977. The three time series of average percentage bias are for the different option series with exercise price/initial stock price ratios of 0.9, 1.0, and 1.1, respectively. Figure 7 plots the corresponding average percentage premiums for the DJ stock sample.

As inspection of figures 6 and 7 illustrates, even on an average basis, the percentage bias is significant for all three option series. As expected, the bias is largest in periods of high interest rates such as in 1969 and 1974. Hence, it is our conclusion that for all but the "roughest" of evaluations, the parity model is not an appropriate put-pricing model.

Because the only difference between the parity model and the correct model is the early-exercise provision, our results demonstrate that the right of early exercise has significant value. Hence, the frequency and time pattern of early exercise as well as the levels of the underlying stock prices where early exercise is likely to take place are integral

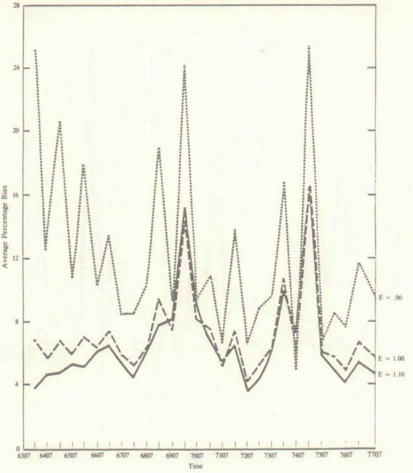

FIG. 7.—Average percentage bias versus time; put/parity model price ratio (DJ stock sample).

parts of the analysis of put-option investment strategies. Again, since organized trading in put options has only just started, there is, at the moment, no market experience with these patterns. Therefore, we have computed these statistics as part of our simulation analysis.

For puts on non-dividend-paying stocks, the early-exercise stock price/exercise price ratio, $[\bar{S}(T)/E]$, will be a monotonically decreasing function of the length of time until expiration, T.[11] However, if the underlying stock pays cash dividends during the life of the put, then $[\bar{S}(\mathrm{T})/E]$ need no longer be monotonic. A large anticipated future dividend on the stock will cause the optimal exercise points prior to its ex-date to be much lower than they would be otherwise. In particular, as an ex-dividend date is approached, the ratio will become a locally increasing function if T. This more complicated time pattern associated with dividend-paying stocks is illustrated in figures 8 and 9, where the average $[\bar{S}(T)/E]$ is plotted as a function of the time until expiration for put options on the underlying stocks in each of the representative stock

11. For further exposure to the time pattern of early exercise, see the tables at the end of the article by Parkinson (1977).

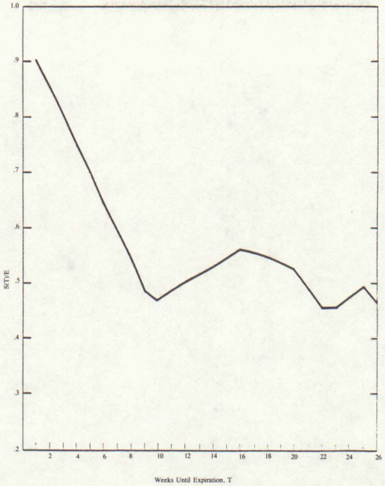

FIG. 8.—Average early-exercise stock price/exercise price ratio versus time until expiration of the put option (136-stock sample).

samples. The ratios plotted are the averages for all stocks in each sample averaged across the 28 6-month observation periods between July 1, 1963 and June 30, 1977.[12] Inspection of figures 8 and 9 shows the "normal" decreasing time pattern for T between 1 and 10 weeks and between 18 and 24 weeks. However, for T between 11 and 18 weeks, the average early-exercise ratio is a distinctly increasing function of T. Because this latter pattern is caused by dividend payments, it might appear surprising that it would be so pronounced since the plotted ratios are averaged across all stocks and over time. The reason is that the ex-dividend dates for different companies tend to cluster around specific months of the year, and hence the time pattern for the average is qualitatively similar to the time pattern for a single stock. Moreover, because during the simulation period the same months are used for

12. Formally, the ratios were computed for each stock sample by the formula

$$\left[\sum_{t=1}^{28}\sum_{j=1}^{n}\bar{S}_{tj}(T)/E_{tj}\right]/28n,$$

where $\bar{S}_{tj}(T)$ is the early-exercise stock price on stock j in time period t when the put has exercise price E_{tj} and time T to go until expiration.

FIG. 9.—Average early-exercise stock price/exercise price ratio versus time until expiration of the put option (DJ stock sample).

rebalancing the portfolios, the patterns will be stable over time. The periodicity of dividend ex-dates induces a "seasonal" effect which makes the time patterns displayed sensitive to the January and July rebalancing dates chosen for the simulations. Therefore, the reader is warned that the values plotted in the figures are only representative. For a given stock in a particular 6-month period, the $\bar{S}(T)/E$ ratios can be quite different.

Table 1 summarizes the frequencies and time patterns of the early-exercise experiences for 6-month put options on each of the underlying stocks in the two stock samples during the 14-year stimulation period. For each sample, there are three different option series with exercise price/initial stock price ratios of 0.9, 1.0, and 1.1, respectively. The frequency percentages in the table are based on daily observations which were stratified into 30 (calendar)-day time intervals. Although the specific frequencies recorded in table 1 depend on the return experience of the underlying stocks during the simulation period,[13]

13. As discussed in the text, the dates of dividend payments will influence the patterns of earlier exercise. Hence, the chosen rebalancing dates for the simulations will induce seasonal effects in the measured frequency rates.

TABLE 1 **Time Patterns for Early Exercise of Put Options, Percentage of Put Options Exercised, July 1963–June 1977**

Percentage of Puts Exercised from Date of Purchase	Out-of-the-Money Exercise Price = .9 Stock Price		At-the-Money Exercise Price = 1.0 Stock Price		In-the-Money Exercise Price = 1.1 Stock Price	
	Marginal	Cumulative	Marginal	Cumulative	Marginal	Cumulative
136-stock sample:						
1–30 days	.1	.1	.8	.8	7.4	7.4
30–60 days	.9	1.0	4.7	5.5	14.3	21.7
60–90 days	1.8	2.8	4.5	10.0	6.6	28.3
90–120 days	2.4	5.2	5.6	15.6	6.3	34.6
120–150 days	7.1	12.3	12.4	28.0	15.2	49.8
150–180 days	12.4	24.7	15.7	43.7	15.6	65.4
At expiration	3.0	27.7	3.1	46.8	3.2	68.6
Not exercised	72.3	100.0	53.2	100.0	31.4	100.0
DJ stock sample:						
1–30 days	.0	.0	.4	.4	5.5	5.5
30–60 days	.5	.5	4.2	4.6	20.6	26.1
60–90 days	1.0	1.5	3.3	7.9	3.3	29.4
90–120 days	.7	2.2	3.9	11.8	5.2	34.6
120–150 days	8.8	11.0	17.2	29.0	20.1	54.7
150–180 days	12.6	23.6	15.5	44.5	16.9	71.6
At expiration	2.7	26.3	3.2	47.7	3.2	74.8
Not exercised	73.7	100.0	52.3	100.0	25.2	100.0

these results lead us to expect that a significant fraction of all put options exercised will be exercised prior to the expiration date. With this, we conclude this section on put-option pricing and proceed with the analysis and simulations of the various put-option investment strategies.

III. Put/Call Conversion Strategies

All put/call conversion strategies have in common that the investor or "converter" purchases a specific number of shares of stock, purchases put options on the same number of shares of the same stock, and simultaneously sells short (i.e., "writes") the same number of call options with the same exercise price and expiration date as the puts.[14] while all such strategies have this feature, their patterns of returns will differ depending on the choice of maturity and exercise price for the options.

There is a common misbelief that the returns on such conversion strategies are riskless. Indeed, the parity model put-pricing formula (1) is derived from the assumption that the conversion returns are riskless. The basis of this misbelief comes from examining the payoff to a conversion position on the expiration date of the options. Assuming that the position was established at time zero and the expiration date of the options is at time T, then the payoff on the expiration date to a N-share converter's position[15] is

$$N\left[E + \sum_{i=1}^{m} d_i e^{r(T-t_i)}\right], \tag{2}$$

where dividends paid on the stock during the investment period are assumed to be reinvested in riskless fixed-income securities. Note that the payoff to the converter is the same, independent of the price of the stock on the expiration date. Hence, with the exception of the intrainvestment-period variations in the interest and dividend rates, the payoff is totally predictable, and therefore the return is (virtually) riskless. If the payoff to an investment is riskless, then to avoid arbitrage,[16] it must be priced to yield a return equal to the interest rate on a riskless investment. The converter's investment, I, is given by

$$I = N(S + P - C), \tag{3}$$

14. Under current rules, the writer of a call receives the proceeds at the time of sale. The "reverse" or call/put conversion strategy which writes a put, shorts the stock, and buys a call is more complicated to analyze because the converter will not in general have the use of the proceeds from the stock short sale.

15. This assumes no transactions costs and no differential tax rates.

16. If institutional restrictions rule out complete arbitrage, then a similar argument can be made using "dominance" (see Merton 1973*b*, p. 143).

where S, P, and C are the stock, put, and call prices, respectively, at the time that the position is established. The parity model formula (1) was derived by setting the return (i.e., the payoff in [2] divided by I in [3]) equal to $\exp(rT)$.

However, the riskless nature of the investment is derived from the assumption that the options are not exercised prior to expiration, and as was demonstrated in the previous section, such an assumption is neither a good theoretical nor a good empirical approximation. Indeed, as illustrated in figures 6 and 7, the converter should, in general, have to pay significantly more than the parity model price to buy a put. Hence, the converter's initial investment will be larger than derived in the parity model, and therefore the converter will earn a return lower than r on those positions where the ex post pattern of the price movements of the stock are such that the put is not exercised prior to its expiration date. To offset, at least in part, these lower return patterns, the converter will earn a return higher than r on those positions where the ex post pattern of stock price movements is such that the put is (optimally) exercised significantly prior to its expiration date. Although the volatility of the returns on conversion strategies will be very much lower than for the underlying stocks, the conversion returns are uncertain because they depend on the time path of stock prices. The expected return on a conversion position will depend on the expected return of the underlying stock. Because the converter will earn lower returns when the put is not exercised prior to expiration and will earn higher returns when it is, the risk position of a put/call converter is slightly "bearish" on the underlying stock.[17] That is, the returns to conversion will be higher in periods of stock price declines and lower in periods of price rises. Therefore, if the underlying stock has an expected return larger than r, then the expected return on the conversion position will be smaller than r.[18]

To develop the empirical return patterns, we have simulated the put/call conversion strategies for both samples of stocks over the period from July 1963 to June 1977. For each sample of stocks, we simulated three strategies that differ only with respect to the exercise price/initial stock price relationship. All strategies examined have in common that the option maturities are 6 months. The three strategies use options with exercise price/initial stock price ratios of 0.9, 1.0, and 1.1, respectively. Hence, if the market price of the stock were $100 per

17. A "bearish" position is one where the returns depend on the stock price and are nonincreasing functions of the stock price.

18. If, e.g., securities were priced so as to satisfy the security market line of the capital asset pricing model, then an expected return on the underlying stock greater than r implies it has a positive "beta." A "bearish" position on the underlying stock will therefore have a negative beta, and hence an expected return below r.

share at the time the position is established, then the first strategy would buy a ("out-of-the-money") 6-month put option with a \$90 exercise price and would write a ("in-the-money") 6-month call option with the same exercise price. The second strategy would buy a ("at-the-money") put the option with a \$100 exercise price and would write a corresponding ("at-the-money") call option. The third strategy would buy a ("in-the-money") put option with a \$110 exercise price and write a corresponding ("out-of-the-money") call option.

For each sample of stocks, we constructed three portfolios of conversion positions to represent the three strategies. All options had 6-month maturities, and within each portfolio, all options had the same exercise price/initial stock price ratio. In each portfolio, an equal number of dollars was invested in conversion positions for each stock in the sample.

Explicitly, if I_j denotes the net investment required for a 100-share conversion position in stock j, then, in a fashion analogous to formula (3),

$$I_j = 100S_{oj} + P_j - C_j, \tag{4}$$

where S_{oj} is the initial price per share of stock j and P_j and C_j are the respective put and call prices on 100 shares of stock j. If there are n stocks in the sample and if the total value of the portfolio is \$$W_o$, then for an equal dollar investment in conversion positions for each stock, the number of puts purchased and calls written on stock, j, N_j, is given by

$$N_j = \frac{W_o}{nI_j}, j = 1, 2, \ldots, n, \tag{5}$$

and the number of shares of stock j held in the portfolio is $100N_j$. Since the magnitude of N_j depends on the value of the portfolio, W_o, we allowed N_j to be a noninteger for purposes of the simulations.

With the following two exceptions, the portfolios are not revised until the end of the 6-month investment period when the options expire. The exceptions are: first, all dividends received on the stocks during the investment period are reinvested in commercial paper until the end of the period. Second, if during the investment period the stock price on a given position reaches a level where (according to our model) it is optimal to exercise the put, then that conversion position is closed. When a position is closed, the put is exercised, the stock held is delivered, and the corresponding calls are repurchased. The call price used for repurchase is computed using the Black-Scholes formula described in Appendix A. The net proceeds from closing out a position are reinvested in commercial paper until the end of the investment period.

All positions not closed prior to expiration are closed at the end of the investment period, and along with the commercial paper investments, the proceeds are added together to give the end-of-period value of the portfolio, $\$W$. Each portfolio is then rebalanced to have an equal dollar investment in conversion positions for each stock by using formulas (4) and (5) where now the total value of the portfolio is $\$W$. Each time a specific portfolio is rebalanced, the options in that portfolio will have the same exercise price/initial stock price ratio. However, the premiums paid and received for options each period will generally be different. The semiannual rate of return for the portfolio is given by $(W - W_o)/W_o$, where W_o is the initial value of the portfolio.

Table 2 summarizes the semiannual rate-of-return experience of the three strategies over the entire simulation period for each of the two stock samples. For comparison with the riskless strategy, we have included the corresponding return experience for "rolling over" 6-month commercial paper. As discussed, the return experience of the conversion strategy will depend on the return experience of the underlying stocks. Therefore, we also have included the return experience on an equally-dollar-weighted portfolio of the underlying stocks for each stock sample. The stock returns assume reinvestment of all dividends, and as with all our simulations, we assume no taxes or transactions costs.

For both stock samples, the volatilities of the conversion strategies, measured either by standard deviation or total range of observed returns, are quite low in comparison with the underlying stocks, but higher than from rolling over commercial paper. For the most part, the volatility increases as one moves in the direction of conversion strategies with higher exercise price/initial stock price ratios where the likelihood of early exercise of the put is greater. The average returns on the conversion strategies were all lower than the average returns from rolling over commercial paper. Because both stock portfolios had, on average, positive excess returns during the simulation period, this finding is consistent with the slightly bearish position of these strategies.

The simulations of the conversion strategies can be used to further investigate the empirical validity of the parity model. However, the statistics presented in table 2 are not in the most useful form to do so. Even if the parity model held exactly (and we know it does not) and the realized returns from conversion strategies exactly equaled the commercial paper rate on a period-by-period basis, the volatility of the returns over the simulation period would not have been zero because the commercial paper rate varied during the period. Hence, the important time series for this purpose are the excess returns of the strategies measured by taking the difference each period between the realized return of the strategies and the commercial paper rate. Table 3 summarizes these excess return experiences. Inspection of table 3 shows

TABLE 2 Summary Statistics for Semiannual Rate-of-Return Simulations, Put/Call Conversion Strategies, July 1963–June 1977

	Exercise Price = .9 Stock Price	Exercise Price = 1.0 Stock Price	Exercise Price = 1.1 Stock Price	Commercial Paper	Stock Portfolio
136-stock sample:					
Average rate of return (%)	3.18	3.17	3.13	3.23	7.65
Standard deviation (%)	1.34	1.41	1.46	1.07	16.11
Highest return (%)	7.99	8.25	8.32	6.17	54.64
Lowest return (%)	1.66	1.59	1.60	1.99	−20.99
Average compound return (%)	3.12	3.11	3.08	3.17	6.34
Growth of $1,000 ($)	2,396	2,391	2,366	2,431	5,901
Coefficient of skewness	1.74	1.75	1.66	1.14	.75
DJ stock sample:					
Average rate of return (%)	3.17	3.11	3.02	3.23	4.58
Standard deviation (%)	1.31	1.39	1.37	1.07	13.67
Highest return (%)	7.75	7.97	7.86	6.17	49.13
Lowest return (%)	1.69	1.63	1.61	1.99	−16.37
Average compound return (%)	3.11	3.06	2.97	3.17	3.71
Growth of $1,000 ($)	2,389	2,352	2,296	2,431	2,829
Coefficient of skewness	1.65	1.68	1.68	1.14	1.16

TABLE 3 **Summary Statistics for Semiannual Rate-of-Return Simulations, Put/Call Conversion Strategies, Excess Returns, July 1963–June 1977 (%)**

	Exercise Price = .9 Stock Price	Exercise Price = 1.0 Stock Price	Exercise Price = 1.1 Stock Price
136-stock sample:			
Average excess rate of return	−.051	−.059	−.095
Standard deviation	.531	.552	.583
Highest excess return	1.819	2.082	2.145
Lowest excess return	−.654	−.691	−.874
Average compound excess return	−.052	−.060	−.097
DJ stock sample:			
Average excess rate of return	−.061	−.119	−.209
Standard deviation	.499	.522	.507
Highest excess return	1.579	1.798	1.866
Lowest excess return	−.646	−.787	−.688
Average compound excess return	−.100	−.120	−.209

that these excess returns are far from predictable even for a large portfolio of conversion positions. Therefore, returns to conversion strategies are uncertain, and the assumptions of the parity model are empirically invalid.

To emphasize the importance of optimal early exercise for put-option strategies, we conclude our analysis of conversion strategies by simulating the returns to a converter who never exercises his put positions prior to the expiration date. Because all positions are held until expiration, the payoff to a position is given by formula (2), which is independent of the terminal stock price. However, the rates of return will vary because the interest rate and premiums for options change from period to period. Table 4 records the return experiences over the simulation period. For all three conversion strategies, the average returns are lower than for the corresponding strategies with optimal early exercise. As expected, the differences are largest for the strategy with the highest exercise price/initial stock price ratio and smallest for the strategy with the lowest ratio. For the out-of-the-money conversion strategy, the differences in average returns between the optimal early-exercise strategy and the one which maintains the position until expiration is only about seven basis points. However, the spread in semiannual average returns for the at-the-money conversion strategy is about 20 basis points, which is about 7% of the total average returns earned. For the in-the-money strategy, the spread doubles to more than 40 basis points or about 14% of the average returns.[19]

In summary, put/call conversion strategies are not riskless. Because

19. The spreads would have been even larger if it were not for the seasonal effect discussed in n. 13 and the text. In general, the spreads will also be larger on non-dividend-paying stocks.

TABLE 4 Summary Statistics for Semiannual Rate-of-Return Simulations, Put/Call Conversion Strategies with No Early Exercise, July 1963–June 1977

	Exercise Price = .9 Stock Price	Exercise Price = 1.0 Stock Price	Exercise Price = 1.1 Stock Price	Commercial Paper
136-stock sample:				
Average rate of return (%)	3.11	2.96	2.67	3.23
Standard deviation (%)	1.13	1.07	.96	1.07
Highest return (%)	6.59	6.22	5.56	6.17
Lowest return (%)	1.68	1.60	1.49	1.99
Average compound return (%)	3.06	2.91	2.63	3.17
Growth of $1,000 ($)	2,355	2,259	2,088	2,431
Coefficient of skewness	1.25	1.23	1.24	1.14
DJ stock sample:				
Average rate of return (%)	3.10	2.93	2.62	3.23
Standard deviation (%)	1.13	1.07	.95	1.07
Highest return (%)	6.61	6.22	5.49	6.17
Lowest return (%)	1.69	1.58	1.47	1.99
Average compound return (%)	3.05	2.88	2.58	3.17
Growth of $1,000 ($)	2,349	2,242	2,062	2,431
Coefficient of skewness	1.26	1.24	1.25	1.14

they are slightly bearish positions, such strategies will in general have expected returns somewhat below the riskless interest rate. Because of the many transactions involved, it is unlikely that conversion strategies would be appropriate investments for any but the lowest-transactions-cost investor. However, by examining these strategies, we have demonstrated some of the properties common to all put-option strategies. In particular, it was shown that an important part of any put-option strategy is the optimal early exercise of the put. Hence, any simulation of put-option strategies that assumes all put positions are held until the expiration date will systematically understate the returns to the buyers of puts and overstate the returns to the writers of puts.

IV. Uncovered Put-Option-Writing Strategies

All uncovered or "naked" put-option-writing strategies have in common that the investor writes a put option and holds a specific amount of investment in fixed-income securities as assurance that he can meet his obligations should the put option be exercised. The patterns of returns for these strategies will differ substantially depending on the maturity and exercise price/initial stock price ratio of the put and the amount of assurance money allocated to the position. Moreover, if the amount of assurance money is less than the exercise price of the option, then the strategy must specify a price level ("filter point") for the underlying stock where the position is liquidated to reflect the investor's inability to meet margin calls.

While each variation applied to the same underlying stocks will generate a different pattern of returns, the number of variations is so large that an exhaustive examination is not feasible. Hence, for each sample of stocks, the study of these writing strategies is limited to three strategies that differ only with respect to the exercise price/initial stock price ratio. For each of these strategies, the amount of assurance money allocated to a position is equal to the number of 100-share put options written times $100E$, where E is the exercise price per share,[20] and this money is assumed to be invested in prime commercial paper. In addition, all the strategies have in common that both the option and commercial paper maturities are 6 months. The three variations in the exercise price/initial stock price ratio used in the simulations are the same as the ones used for the conversion strategies, namely, an out-of-the-money strategy with a ratio of 0.9, an at-the-money strategy with a ratio of 1.0, and an in-the-money strategy with a ratio of 1.1.

20. This requirement ensures that the writer will have a sufficient reserve to meet his put obligations for all possible stock prices. This "maximum reserve" condition far exceeds initial margin requirements. If a smaller reserve base were used, the resulting return figures would be the same as "levering" the portfolio strategies in the text by borrowing at the commercial paper rate.

For each of these three strategies, the qualitative characteristics of the return patterns are quite similar to those of a fully covered call-writing strategy where the call option's maturity and exercise price are the same as for the corresponding put. Indeed, if the conditions for the parity model held, then the return patterns would be identical except for some minor differences in leverage. Therefore, because the qualitative characteristics of fully covered call-writing strategies are discussed in detail in our earlier study (Merton et al. 1978), we will only discuss in detail the one major difference between the uncovered put-writing and corresponding fully covered call-writing strategies. In our earlier study, we made the reasonable assumption that the call options would not be exercised prior to the expiration date.[21] However, as has been amply demonstrated here, such a "no early exercise" assumption is not reasonable for put options. Therefore, the incidence of (optimal) early exercise will cause the return patterns of uncovered put-option-writing strategies to differ from those of the corresponding fully covered call-writing strategies. Of course, if, as a general rule, put buyers do not exercise their puts until the expiration date, then put writers will earn "windfall" higher returns. However, such nonoptimal behavior on the part of put buyers is not a reasonable assumption for simulating put-writing strategies.

Having established the principal differences between the two types of strategies, we now turn to the simulations of their return experiences over the 14-year period from July 1963 to June 1977. For each sample of stocks, three portfolios of uncovered put-writing positions were constructed to represent the three strategies. Within each portfolio, all options had the same exercise price/initial stock price ratio. In each portfolio, an equal number of dollars was invested in uncovered put-writing positions for each stock in the sample. We assume that the option premium received is used to reduce the investment required in the position. Hence, if I_j is the net investment required for a 100-share uncovered put-writing position in stock j, then

$$I_j = 100E_j - P_j, \tag{6}$$

where P_j is the put price on 100 shares of stock j and E_j is the exercise price per share on the put. If there are n stocks in the sample and if the total value of the portfolio is $\$W_o$, then for an equal dollar investment in uncovered put positions for each stock, the number of puts written on stock j, N_j, is given by

$$N_j = \frac{W_o}{nI_j}, j = 1, 2, \ldots n. \tag{7}$$

21. In Merton (1973*b*, pp. 144–45), it is proved that for non-dividend-paying stocks, it is never optimal to exercise a call early. Even for dividend-paying stocks, the errors in simulated returns from neglecting early exercise are small. See Merton et al. (1978) for further discussion of this point.

After the put premiums are received, the portfolio will have

$$100 \sum_{j=1}^{n} N_j E_j$$

dollars invested in prime commercial paper.

If during the investment period the stock price on a given put drops to a level where (according to our model) it is optimal to exercise the put, then the put is repurchased for its in-the-money value by selling off the appropriate amount of commercial paper. Otherwise, the portfolios are not revised until the end of the 6-month investment period when the options expire.

At the end of the investment period, the commercial paper investments mature and all puts not already repurchased will either expire worthless or are repurchased for their in-the-money value. The net of these investments is the end-of-period value of the portfolio, \$$W$. Each portfolio is then rebalanced to have an equal dollar investment in new uncovered put-writing positions for each stock by using formulas (6) and (7), where now the total value of the portfolio is \$$W$. Each time a specific portfolio is rebalanced, the options in that portfolio will have the same exercise price/initial stock price ratio. However, the premiums received for puts and the interest rate earned on the commercial paper will in general be different each period.

Table 5 summarizes the semiannual rate-of-return experience of the three strategies over the entire simulation period for each of the two stock samples. Because the two types of strategies are quite similar, we have included the return experience for the corresponding fully covered call-writing strategies to allow direct comparisons. Because the uncovered put-writing strategies are "bullish" on the underlying stocks,[22] their returns will depend on the returns of the underlying stocks. Hence, we have also included the return data for each stock sample on an equally dollar-weighted portfolio of the underlying stocks. For both stock samples, detailed tables of average compound return and growth of a \$1,000 investment for the at-the-money uncovered put-writing strategy are presented in Appendix B. To make comparisons, the returns on the underlying stocks in each sample and the returns on commercial paper are presented in Appendix D. Figure 10 plots the values for hypothetical initial investments of \$1,000 in each of the three uncovered strategies and the underlying stock portfolio for the 136-stock sample. To facilitate comparisons, a logarithmic scale is used for the portfolio values. Figure 11 provides similar plots for the DJ stock sample.

For both stock samples, the volatility of the uncovered put- and fully covered call-writing strategies increases markedly as one moves in the

22. A "bullish" position is one where the returns depend on the stock price and are nondecreasing functions of the stock price.

TABLE 5 Summary Statistics for Semiannual Rate-of-Return Simulations, Uncovered Put-Writing Strategies, Fully Covered Call-Writing Strategies, July 1963–June 1977

	Exercise Price = .9 Stock Price		Exercise Price = 1.0 Stock Price		Exercise Price = 1.1 Stock Price		
	Uncovered Put	Fully Covered Call	Uncovered Put	Fully Covered Call	Uncovered Put	Fully Covered Call	Stock Portfolio
136-stock sample:							
Average rate of return (%)	3.5	3.4	4.1	3.8	5.0	4.6	7.7
Standard deviation (%)	3.9	4.7	5.6	6.9	7.2	9.1	16.1
Highest return (%)	14.4	14.6	18.8	19.3	23.6	24.7	54.6
Lowest return (%)	−6.0	−9.9	−6.4	−14.4	−5.6	−17.4	−21.0
Average compound return (%)	3.4	3.3	3.8	3.5	4.6	4.1	6.3
Growth of $1,000 ($)	2,571	2,486	2,930	2,691	3,640	3,155	5,901
Coefficient of skewness	.08	−.71	.44	−.50	.58	−.25	.75
DJ stock sample:							
Average rate of return (%)	3.3	3.0	4.0	3.1	5.1	3.6	4.6
Standard deviation (%)	3.1	3.7	4.8	6.1	6.2	8.5	13.7
Highest return (%)	12.1	12.3	16.9	16.9	23.1	22.9	49.1
Lowest return (%)	−2.2	−5.4	−4.0	−9.2	−2.8	−11.9	−16.4
Average compound return (%)	3.2	2.9	3.8	2.9	4.8	3.2	3.7
Growth of $1,000 ($)	2,454	2,233	2,925	2,260	2,819	2,458	2,829
Coefficient of skewness	.35	−.27	.38	−.26	.91	.00	1.16

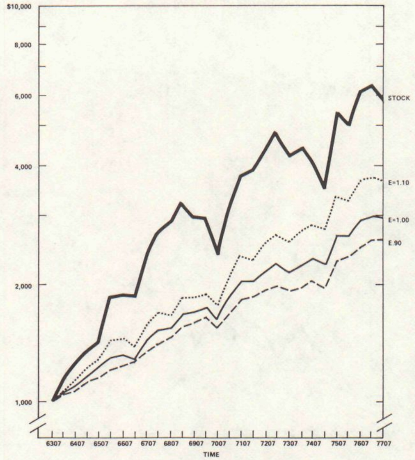

FIG. 10.—Growth of $1,000 invested in each of three uncovered put strategies and stock (136-stock sample).

direction of writing options with higher exercise price/initial stock price ratios. For the out-of-the-money uncovered put strategy, the range of observed returns was about 2,000 basis points (−6.0 to 14.4%) for the 136-stock sample and about 1,400 basis points (−2.2 to 12.1%) for the DJ stock sample. For the in-the-money uncovered put strategy, the range of observed returns is about 2,900 basis points (−5.6 to 23.6%) and 2,600 basis points (−2.8 to 23.1%) for the respective samples. The standard deviation for the out-of-the-money put strategy was less than 55% of the standard deviation of the in-the-money put strategy.

All three of the uncovered put strategies had significantly smaller return volatilities than the underlying stock portfolio. By switching from the stock portfolio to one of the uncovered put strategies, an investor would have reduced the standard deviation of his returns by about 77% for the out-of-the-money strategy, by about 65% for the at-the-money strategy, and by about 55% for the in-the-money strategy.

While the observed range of returns is also reduced by switching from the underlying stock portfolio to an uncovered put-writing strategy, the reduction in the range of returns is not symmetric around gains and losses. For example, the largest loss for the at-the-money

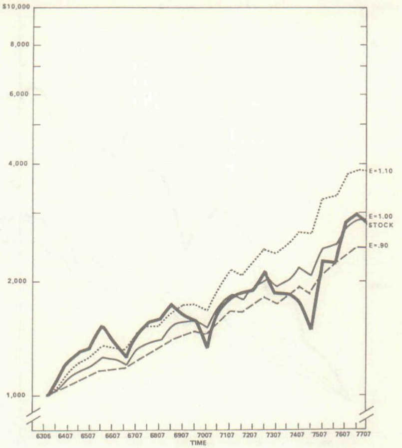

FIG. 11.—Growth of $1,000 invested in each of three uncovered put strategies and stock (DJ sample).

uncovered put strategy was about 1,300 basis points (13%) smaller than the largest loss on the underlying stock portfolio. However, for the same strategy, the largest gain was approximately 3,400 basis points (34%) smaller than the largest gain on the stock portfolio. This asymmetric reduction is also reflected by the significant reduction in positive skewness in switching from the stock portfolio to the uncovered put strategies. As with fully covered call-writing strategies, the lower return volatility of the uncovered put strategies and their somewhat higher returns when stocks either decline or appreciate moderately are earned at the expense of a substantially smaller return than on the stocks during periods of large stock appreciation.

As expected, the patterns of returns for the uncovered put-writing strategies are similar to those of the fully covered call-writing strategies. However, the return volatilities of the uncovered put strategies are systematically lower than the volatilities of the corresponding fully covered call strategies. There are essentially two reasons for the lower volatility. First, the initial dollar investment per 100 share option position is larger for the uncovered put-option strategy than for the corresponding fully covered call strategy, and therefore less leveraged. If, as assumed, the call-option premium received is used to reduce the investment required in the position, then the net investment required for a 100-share fully covered call-writing position in stock j, I'_j, can be written as

$$I'_j = 100S_{oj} - C_j, \tag{8}$$

where C_j is the call price on 100 shares of stock j and S_{oj} is the stock price per share. From formulas (6) and (8), we have that

$$I_j - I'_j = 100(E_j - S_{oj}) + C_j - P_j, \tag{9}$$

which will be positive for all maturities and exercise price/initial stock price ratios provided that the dividend rate on the stock is not unusually high.[23] Neglecting, for the moment, the early exercise possibility for the puts and assuming dividend payouts are known and fixed over the life of the options, the difference on the expiration date between the dollar value of a 100-share uncovered put-writing position and the dollar value of a corresponding fully covered call-writing position will be a fixed amount, independent of the stock price. Hence, in this case, the variance of the dollar return per 100-share position will be the same for the two strategies. But, I_j is generally larger than I'_j. Therefore, the variance of the return per dollar of investment for the uncovered put strategy will be smaller than the variance of the return per dollar for the corresponding fully covered call strategy. Second, including the early exercise provision will tend to reinforce this result, because any position that is exercised early will no longer be a source of variation for the portfolio over the balance of the investment period.[24] Therefore, our simulations empirically verify that uncovered put writing is a somewhat more conservative strategy than fully covered call writing, and both are more conservative than holding the underlying stocks.

As pointed out in our earlier study (Merton et al. 1978, p. 210), "the observed average returns for the strategies reflect the combined influence of two factors: (1) degree of sensitivity of the returns on the strategies to the returns on the underlying stocks and (2) the performance of the underlying stocks during the simulation period." As was true for our earlier study, the average returns reported here should be interpreted as reasonable estimates of the expected returns on the strategies conditional on the return experience of the underlying stocks. That is, in making comparisons of the average returns among

23. In Merton (1973*b*, p. 159, eq. [20]), it is shown for non-dividend-paying stocks that $(100(E_j - S_j) + C_j \geq P_j$. For dividend-paying stocks, the bounding inequality is $\Sigma_1^m d_i \exp(-rt_i) + 100(E_j - S_j) + C_j > P_j$. As noted in n. 10, there were some periods in which $C_j < P_j$ for $E_j = S_j$ because of large dividends.

24. This argument assumes that replacing a positive-variance asset with a riskless asset will reduce the variance of a portfolio. This need not be the case in the (unlikely) event that the position replaced has a negative correlation with at least some of the other positions in the portfolio. The early exercise in the put strategy will also "cut off" extreme losses when stocks decline.

the various strategies, the reader should bear in mind the (ex post) return experience of the underlying stocks. If stocks perform well, then a more bullish strategy will have higher average returns than a less bullish one. However, this need not be the case if stocks do not perform as well ex post as was expected ex ante.

During the simulation period, both the 136- and DJ stock portfolios outperformed the Standard & Poors 500 on an absolute and risk-adjusted basis.[25] Hence, it is not surprising to find that among the uncovered put-writing strategies, the more bullish, in-the-money strategy had a higher average return that the at-the-money strategy, which, in turn, had a higher average return than the less bullish, out-of-the-money strategy. Similarly, the more bullish, fully covered strategies had higher average returns than the less bullish ones. However, at first glance it is not clear why across the two stock samples the average returns on each uncovered put-writing strategy were virtually the same even though the 136-stock portfolio had an average return 300 basis points higher than the return on the DJ stock portfolio. The fully covered strategies did not exhibit this relative insensitivity to the difference in average returns between the two stock samples.

A significant systematic difference in the return patterns between the uncovered put-writing strategy and the comparable fully covered call strategy is the behavior at the extremes. Although the highest return figures are essentially the same, the largest loss figures are always much smaller for the uncovered put-writing strategy. The reason is that the optimal early exercise point in the put strategies creates, in effect, a "stop-loss" strategy. Hence, at the extremes, the losses on such a strategy will be much smaller than for a hold-until-expiration strategy. To see the effect of early exercise on the return patterns, we computed the returns on the uncovered put-writing strategies under the assumption that none of the puts is exercised prior to expiration. Table 6 displays these returns together with the returns from the fully covered call-writing strategies.

As expected, by eliminating the "stop-loss" feature induced by early exercise, the largest losses to the uncovered put-writing strategies increase substantially and are comparable in magnitude, although always smaller, to those on the fully covered strategies. Moreover, the standard deviation and skewness of the returns change such that overall return patterns on the uncovered put-writing strategies become, as expected, quite similar to those on the fully covered call strategies. Further, the previously observed insensitivity of the average returns on the put-writing strategies to the difference in average returns on the

25. During the simulation period, the Standard & Poors 500 stock portfolio had a semiannual average rate of return of 3.87% with a standard deviation of 13.05%. The measured "beta" and "alpha" of the 136- and DJ stock portfolios over this period were 1.17, 3.68% and 1.00, 0.71%, respectively.

TABLE 6 Summary Statistics for Semiannual Rate-of-Return Simulations, Uncovered Put-Writing Strategies with No Early Exercise, July 1963–June 1977

	Exercise Price = .9 Stock Price	Exercise Price = 1.0 Stock Price	Exercise Price = 1.1 Stock Price
136-stock sample:			
Average rate of return (%)	3.5	4.1	5.2
Standard deviation (%)	4.5	6.6	8.7
Highest return (%)	14.4	19.1	24.7
Lowest return (%)	−9.1	−13.2	−15.7
Average compound return (%)	3.3	3.8	4.7
Growth of $1,000 ($)	2,550	2,906	3,745
Coefficient of skewness	−.64	−.44	−.20
DJ stock sample:			
Average rate of return (%)	3.2	3.6	4.4
Standard deviation (%)	3.5	5.9	8.2
Highest return (%)	12.1	16.9	23.1
Lowest return (%)	−5.0	−8.1	−10.0
Average compound return (%)	3.1	3.4	4.0
Growth of $1,000 ($)	2,400	2,571	3,098
Coefficient of skewness	−.22	−.21	.05

put-writing strategies to the difference in average returns on the two underlying stock portfolios disappears. These rather significant changes in the patterns of returns underscore the importance of taking into account early exercise when examining put strategies.

In concluding our study of uncovered put-writing strategies, we analyze how their returns are affected by different levels of put premiums. This is particularly important because the simulations used premium levels generated by a model and not actual transaction prices. Moreover, because organized put trading has only just started, there are few data available to test the reliability of the model in predicting market prices. This sensitivity analysis, together with figures 2 and 3, provides the reader with the means to adjust our results to his own estimates of put premium levels and to the effects of commission costs.

The sensitivity analysis was applied to the at-the-money uncovered put strategy using six premium estimates different from the model estimates. The estimates used were in intervals of 10% changes in the premium level ranging from 30% below the model estimate to 30% above. Table 7 displays the summary return statistics of this sensitivity study.

Of course, other things the same, the higher the premium, the higher will be the returns on an uncovered put-writing strategy. However, it is the magnitude of the change which is important. For the 136-stock sample, a 10% increase in the premium received corresponds roughly to an additional 80 basis points (0.80%) in semiannual average return. For the DJ stock sample, the corresponding increase in average return

TABLE 7 Summary Return Statistics for Different Premium Levels, Uncovered Put-Writing Strategy with Exercise Price = Stock Price, July 1963–June 1977 (Option Premium Received Equal to X% of Model Estimate)

	X = 70	X = 80	X = 90	X = 100	X = 110	X = 120	X = 130
136-stock sample:							
Average rate of return (%)	1.7	2.5	3.3	4.1	4.9	5.7	6.5
Standard deviation (%)	5.2	5.3	5.4	5.6	5.7	5.9	6.1
Highest return (%)	14.1	15.7	17.2	18.8	20.4	22.2	23.9
Lowest return (%)	−8.4	−7.8	−7.1	−6.4	−5.7	−5.0	−4.3
Average compound return (%)	1.6	2.3	3.1	3.8	4.6	5.4	6.2
Growth of $1,000 ($)	1,556	1,917	2,368	2,930	3,634	4.516	5,626
DJ stock sample:							
Average rate of return (%)	2.1	2.7	3.4	4.0	4.7	5.4	6.1
Standard deviation (%)	4.5	4.6	4.7	4.8	5.0	5.1	5.2
Highest return (%)	13.1	14.4	15.6	16.9	18.2	19.6	21.0
Lowest return (%)	−5.7	−5.1	−4.6	−4.0	−3.4	−2.8	−2.3
Average compound return (%)	1.9	2.6	3.2	3.8	4.5	5.1	5.8
Growth of $1,000 ($)	1,723	2,052	2,448	2,925	3,499	4,190	5,027

is about 70 basis points (0.70%). These changes are significant. Moreover, because the standard deviation is virtually unaffected, the average compound return will change in about the same way the average return does. The observed sensitivity of the average returns to the level of premiums should also serve as a further warning against placing great significance on the levels of measured average return even over a 14-year period.

In summary, investors who choose to follow the uncovered put-writing strategies will have significantly lower risk exposure than if they hold a portfolio of the underlying stocks. However, as with fully covered call strategies, this lower risk is accompanied by lower expected returns. Although the qualitative characteristics of the returns on the uncovered put-writing strategies are similar to their corresponding fully covered call-writing strategies, there are significant quantitative differences caused by the incidence of early exercise of the puts and the differences in leverage.

V. "Protective-Put" Buying Strategies

The "protective-put" buying strategies[26] all have the common property that the investor holds a specific number of shares of stock and buys put options on the same number of shares of the same stock. In the discussion of the functional characteristics of options in our earlier paper (Merton et al. 1978), we described the principal function of a put option as term insurance against a decline in the underlying stock where the length of the term is the time until the expiration date. So, in effect, investors who follow these strategies purchase "insurance" to limit their losses from declines in the prices of the underlying stocks. The maximum gross loss per share on a protective put-buying position is the difference between the initial stock price and the exercise price of the put less any dividends received on the stock. The maximum net loss per share is computed by adding to the gross loss the cost of the put per share. The magnitude of the maximum loss will depend on the relationship between the exercise price of the put and the market price of the stock at the time the put is purchased. As the exercise price/initial stock price ratio is increased, both the gross and net maximum losses are decreased. Indeed, for in-the-money puts, the maximum gross loss is negative (i.e., a gain). Of course, the maximum net loss is always positive.

All protective-put strategies are "bullish" on the underlying stock; although there is a floor" on losses, there is no "ceiling" on gains. Hence, the protective-put buyer, like any buyer of insurance, hopes

26. The term "protective put" to describe these strategies was coined by Robert C. Pozen, New York University. An alternative descriptive term would be "insured-equity" strategies.

that he will not have to collect on the "policy" and exercise his put. He hopes that the stock will rise because the larger the rise, the larger are his gains.

While all protective-put strategies are bullish, the degree of dependence on the stock is not the same for different exercise price/initial stock price ratios. For a fixed maturity, this dependence is monotonic in the exercise price/initial stock price ratio. As previously discussed, the maximum loss will be smallest on an in-the-money strategy and largest on an out-of-the-money strategy. However, given any probability distribution for the underlying stock, the probability of realizing this maximum loss is highest for the in-the-money strategy and lowest for the out-of-the-money strategy. The volatility of the returns will be lowest for the in-the-money strategy and highest for the out-of-the-money strategy. Overall, the in-the-money strategy returns will be the least sensitive to movements in the underlying stock's price and the out-of-the-money strategy will be the most sensitive.

The change in the rate-of-return distribution for an investor who switches from a straight stock position to a protective-put position is qualitatively the same for all such strategies. Clearly, the protective-put position is more conservative. In those periods when the underlying stock declines sharply, the protective-put position will have much smaller losses than the straight stock position. However, these smaller losses are earned at the cost of somewhat larger losses on the protective-put position when the stock decline is moderate and at the cost of smaller gains when the stock rises. These shifts in the return distribution lead to a smaller variance and narrower range of observed returns for the protective-put strategy. It is generally accepted that to earn a larger expected return on correctly priced securities, one must bear larger risks. Therefore, if the put and the stock are correctly priced, then the expected returns on any protective-put strategy will be lower than on the underlying stock.

The broad qualitative return characteristics of the protective-put strategies are similar to those of the call options–paper buying strategies which were simulated in our earlier paper (Merton et al. 1978). For example, both types of strategies have a "floor" on losses and no "ceiling" on gains. However, unlike the protective-put strategies examined here, the call options–paper strategies examined there allocated a fixed proportion (10%) of the portfolio to option purchases. Therefore, the "floor" or maximum net loss is always the same on these call strategies, namely, 10% of the portfolio's initial value less interest earned on the commercial paper part of the portfolio.

Moreover, the relative risk characteristics across different exercise price/initial stock price versions of the call strategies change monotonically in the opposite direction to the changes in the risk characteristics of the corresponding protective-put strategies. For example, the volatility and sensitivity to stock price movements of the call options–paper strategies increase with an increase in the exercise price/initial stock ratio, and the same is true for the observed range of returns. Although protective-put strategies are always less volatile than the underlying stocks, the call options–paper strategies can have return volatilities that are either larger or smaller than the volatility of the underlying stocks.

For sufficiently low exercise price/initial stock price ratios, the volatility of a particular protective-put strategy will be higher than the volatility of a corresponding call option–paper strategy with the same maturity and exercise price. And, for sufficiently high ratios, the reverse will be true. However, there appears to be no simple rule for determining the "watershed" exercise price/initial stock ratio where the two volatilities are just equal.

To help answer these and other quantitative questions, we simulated the returns to these strategies for the period beginning in July 1963 and ending in June 1977. As with the other simulations, the three strategies examined for each stock sample have the same 6-month maturity but different exercise price/initial stock price ratios. As before, the three ratios used are 0.9, 1.0, and 1.1. For each stock sample, three portfolios of protective-put positions were constructed to represent the three strategies. Within each portfolio, all options had the same exercise price/initial stock price ratio. The investment required for a 100-share protective-put position in stock j, I_j, is given by

$$I_j = 100S_{oj} + P_j, \tag{10}$$

where P_j is the put price on 100 shares of stock j and S_{oj} is the stock price per share at the time that the position is established. If there are n stocks in the sample and if the value of the portfolio is $\$W_o$, then to create a portfolio with an equal dollar investment in protective-put positions for each stock, the number of puts purchased on stock j, N_j, is given by

$$N_j = \frac{W_o}{nI_j}, j = 1, 2, \ldots, n, \tag{11}$$

and the number of shares of stock j purchased is 100 N_j.

If during the investment period the stock price on a particular position falls to a level where (according to our model) the put should be exercised, then the put is exercised by delivering the stock held in the portfolio, and the proceeds are reinvested in prime commercial paper for the balance of the investment period. All dividends received from the stocks are reinvested in commercial paper. Otherwise, the portfolios are not revised until the end of the 6-month investment period when the remaining options will expire.

At the end of the investment period, the commercial paper investments mature and the puts not already exercised either expire worthless or are exercised by delivering the underlying stock held in the portfolio. The net value of these proceeds added to the value of all stocks still held in the portfolio equals the end-of-period value of the portfolio, $\$W$. Each portfolio is then rebalanced to have an equal dollar investment in protective-put positions for each stock by using formulas (10) and (11), where "W" is substituted for "W_0." Each time a portfolio is rebalanced, the options in that portfolio will have the same exercise price/initial stock price ratio.

In table 8, the summary semiannual rate-of-return statistics are presented for the three strategies over the entire simulation period. To allow direct comparisons between the two types of strategies, we include the summary return statistics for the corresponding options–paper buying strategies. We also include the return statistics for each stock sample. In Appendix C, the average compound returns and growth of a \$1,000 investment for the at-the-money protective-put strategy are presented for both stock samples and every subperiod of the simulation period. Figure 12 displays the time paths of values for hypothetical initial investments of \$1,000 in each of the three protective-put strategies and the underlying stock portfolio for the 136-stock sample. Figure 13 provides the corresponding time paths for the DJ stock sample.

The overall return volatility of the protective-put strategies, measured either by standard deviation or range of observed returns, decreases as one moves in the direction of buying puts with higher exercise price/initial stock price ratios. As expected, this behavior is just the opposite of the corresponding options–paper portfolios. For the out-of-the-money protective-put strategy, the range of observed returns was about 5,200 basis points (−8.2 to 43.6%) for the 136-stock sample and about 4,800 basis points (−7.0 to 40.8%) for the DJ stock sample. For the in-the-money strategy, the range of observed returns dropped to about 3,300 basis points for the 136-stock sample and to 2,800 basis points for the DJ sample. The standard deviation of the out-of-the-money strategy was larger than the standard deviation of the in-the-money strategy by about 78%. The volatility of the at-the-money strategy was in between these two strategies' volatilities.

There are three changes that cause the volatility to increase as one moves in the direction of out-of-the-money strategies. First, the premium charged for a put, P_j, goes down. So, per 100-share position, the investment required, I_j, is lower, and therefore, the number of shares of stock held, $100N_j$, is larger. Hence, when stocks do well, the more out of the money is the strategy, the larger will be the return. Second, because the exercise price of the put will be smaller relative to the initial stock price, the magnitude of the loss on a position will be larger when stocks do poorly. Third, as one moves in the direction of out-

TABLE 8 Summary Statistics for Semiannual Rate-of-Return Simulations, Protective-Put-Buying Strategies, Call Options–Commercial Paper Buying Strategies, July 1963–June 1977

	Exercise Price = .9 Stock Price		Exercise Price = 1.0 Stock Price		Exercise Price = 1.1 Stock Price		
	Protective Put	Options-Paper	Protective Put	Options-Paper	Protective Put	Options-Paper	Stock Portfolio
136-stock sample:							
Average rate of return (%)	7.3	6.1	6.7	7.7	5.9	10.4	7.7
Standard deviation (%)	12.0	7.7	9.5	10.5	7.1	15.4	16.1
Highest return (%)	43.6	25.7	37.9	34.7	31.2	60.0	54.6
Lowest return (%)	−8.2	−4.7	−4.0	−5.2	−1.5	−6.3	−21.0
Average compound return (%)	6.5	5.7	6.2	7.0	5.5	9.0	6.3
Growth of $1,000 ($)	6,187	4,882	5,628	7,125	4,654	12,519	5,904
Coefficient of skewness	1.13	.83	1.47	.87	1.93	1.27	.75
DJ stock sample:							
Average rate of return (%)	4.7	4.5	4.5	5.4	4.0	7.3	4.6
Standard deviation (%)	10.4	7.4	7.9	10.4	5.6	14.9	13.7
Highest return (%)	40.8	27.2	35.1	34.4	28.1	42.6	49.1
Lowest return (%)	−7.0	−4.6	−1.8	−5.7	.3	−7.5	−16.4
Average compound return (%)	4.2	4.1	4.2	4.9	3.8	6.2	3.7
Growth of $1,000 ($)	3,218	3,178	3,209	3,895	2,884	5,646	2,829
Coefficient of skewness	1.62	1.18	2.25	1.05	3.02	.97	1.16

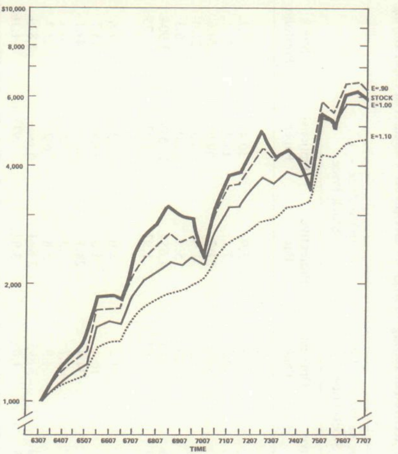

FIG. 12.—Growth of $1,000 invested in each of three protective-put strategies and stock (136-stock sample).

of-the-money strategies, the likelihood of an early exercise decreases. Because a position that is exercised early is not a source of variation to the portfolio over the balance of the investment period, fewer early exercises cause the volatility to increase.

As expected, all three strategies had smaller volatilities than the underlying stock portfolio. The observed range of returns is reduced because the downside "insurance" of the put eliminates large losses and because the cost of purchasing the "insurance" reduces the upside rate of return. By switching from the stock portfolio to one of the protective-put strategies, an investor would have reduced the standard deviation of his returns by about 25% for the out-of-the-money strategy, by about 41% for the at-the-money strategy, and by about 57% for the in-the-money strategy.

As discussed in Section IV, the reader should take into account the ex post performance of the underlying stocks before drawing any conclusions from the average returns reported here about the levels of returns that one might reasonably expect to earn from these strategies in the future. Because both the 136- and DJ stock portfolios performed well during the simulation period, the more bullish strategies have

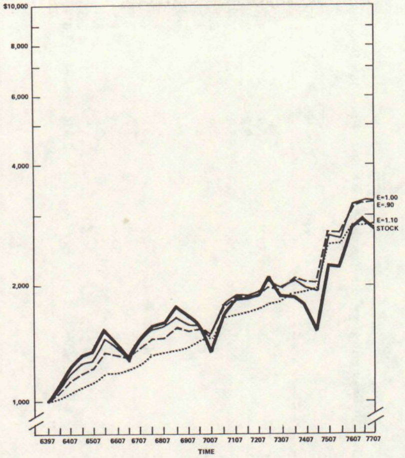

FIG. 13.—Growth of $1,000 invested in each of three protective-put strategies and stock (DJ sample).

higher average returns than the less bullish ones. Because of the superior stock performance of the 136-stock sample, the average returns on the strategies applied to this sample were significantly higher than the average returns on the corresponding strategies applied to the DJ stock sample.

Because the average returns are sensitive to premium levels, we complete our study of the protective-put strategies with a sensitivity analysis of the impact on the returns of different premium levels. Table 9 summarizes the return experience on an at-the-money protective-put strategy for different premiums ranging from 30% below the model estimates to 30% above. The results are similar to those reported in table 7 for the uncovered put-writing strategies, although, of course, the sign of the change is in the opposite direction. For the 136-stock sample, a 10% increase in the put premium reduces the average return by about 70 basis points, and for the DJ sample, the average return is reduced by about 60 basis points. Since these changes have little effect on the volatility of the returns, the average compound returns will change by approximately the same amount.

In summary, by insuring against large losses on the underlying

TABLE 9 **Summary Return Statistics for Different Premium Levels, Protective-Put-Buying Strategy with Exercise Price = Stock Price, July 1963–June 1977 (Option Premium Paid Equal X% of Model Estimate)**

	X = 70	X = 80	X = 90	X = 100	X = 110	X = 120	X = 130
136-stock sample:							
Average rate of return (%)	8.9	8.2	7.4	6.7	6.0	5.4	4.7
Standard deviation (%)	10.0	9.8	9.7	9.5	9.4	9.2	9.1
Highest return (%)	42.5	40.9	39.4	37.9	36.4	35.0	33.6
Lowest return (%)	−1.5	−2.4	−3.2	−4.0	−4.8	−5.6	−6.3
Average compound return (%)	8.1	7.5	6.8	6.2	5.5	4.9	4.2
Growth of $1,000 ($)	9,783	8,123	6,756	5,629	4,695	3,922	3,281
DJ stock sample:							
Average rate of return (%)	6.3	5.7	5.1	4.5	3.9	3.4	2.8
Standard deviation (%)	8.3	8.2	8.0	7.9	7.8	7.7	7.5
Highest return (%)	38.9	37.6	36.3	35.1	33.8	32.6	31.5
Lowest return (%)	−.4	−.9	−1.3	−1.8	−2.3	−2.8	−3.6
Average compound return (%)	5.9	5.3	4.7	4.2	3.6	3.1	2.5
Growth of $1,000 ($)	5,146	4,392	3,753	3,209	2,747	2,354	2,019

stocks, the investor who follows a protective-put strategy can significantly reduce the risk of his stock portfolio. However, the cost of this insurance is reflected by a reduction in expected return.

VI. Conclusion

As in our earlier paper on call-option strategies, our analysis here of the uncovered put-writing and protective-put buying strategies demonstrates that option strategies can be used by investors to produce patterns of returns which are not reproducible by any simple strategy of combining stocks with fixed-income securities.

If one views the put option as essentially term insurance against a loss in value of the underlying stock, then an investor who follows an uncovered put-writing strategy is supplying the service of providing insurance. This strategy is "bullish" on the stock. Like any writer of insurance, he hopes that he will not have to pay any claims, and therefore hopes that the stock does not decline. Moreover, he expects to be compensated for this service, and the form of the compensation is an expected return in excess of the return on riskless fixed-income securities.

An investor who follows the protective-put buying strategy purchases the insurance to protect the value of his stock portfolio. This strategy is also bullish on the stock. Like a buyer of traditional insurance, he hopes that he will not have to make a claim on the policy and that the underlying stocks rise in price. Moreover, he expects to pay for this insurance, and therefore he expects to earn a lower return than if he had not purchased the insurance.

As tables 7 and 9 amply demonstrate, if the price charged for the put insurance is "too low," then the protective-put investor can reduce his risk with less than the appropriate reduction in expected return. However, in this case, the uncovered put writer will not be adequately compensated for his service and would, therefore, be unwilling to continue to provide insurance at that price. Of course, just the opposite would be expected to occur if put premiums are "too high."

As with any study based on past experience, one should exercise great care in using our simulations as an indicator of future performance of the various strategies. One reason is that our results are clearly dependent upon the (ex post) trend and volatility of the actual paths for our sample stocks during the simulation period. Of course, the paths traversed by these stocks in the future could be quite different. If, for example, stocks have a steeper trend in the future, then the positive difference in average returns between the more bullish and less bullish strategies reported here will be larger. If the trend in stocks is less steep, then the differences will be smaller, and in some cases, could become negative. Similarly for the same trend, if, ex post, stock returns are less volatile than expected, then option-writing strategies will tend to outperform option-buying strategies. If stock returns are more volatile than expected, then the reverse will obtain.

A second reason to exercise such care is that our results depend significantly on the assumed option premiums, and actual premiums may be different. As our sensitivity analyses amply demonstrate, the principal effect of a change in premium levels is to change the average returns. We have not formally compared our assumed put premiums with actual market prices. However, in our earlier study on call strategies which used a similar model, we did compare our model call premiums with market prices and found that on average, the differences between the two was not significant. Hence, in our opinion, the premiums used in our simulations here are reasonable, and we would not expect to find large and persistent differences between our premium estimates and market prices.

Finally, as was stressed in our earlier paper, the reader should not infer from our findings that any one of these strategies dominates the others. If options and their underlying stocks are properly priced, there will be no single best strategy for all investors. This, of course, does not imply that an investor should be indifferent in his choice among strategies. Because the patterns of returns exhibited by the various strategies are not the same, investors with different tolerances for risk, tax brackets, and commission costs will find that their optimal choice among the strategies will also be different.

Appendix A

Black and Scholes (1973) derived the following partial differential equation for the price of a call option, $C(S,T)$,

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC - \frac{\partial C}{\partial T} = 0 \tag{A1}$$

subject to the boundary conditions

$$C(0,T) = 0,$$
$$C(S,0) = \max(0, S - E).$$

Equation (A1) can be solved to give a formula for the call price.

$$C(S,T) = S\phi(X_1) - Ee^{-rT}\phi(X_2), \tag{A2}$$

where

$$X_1 \equiv \left[\log(S/E) + \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sqrt{\sigma^2 T},$$
$$X_2 \equiv X_1 - \sigma\sqrt{T},$$

and $\phi(\ )$ is the cumulative normal density function. Formula (A2) assumes no cash dividends on the stock. In our earlier study and the present one, we adjust the formula for each dividends by computing the present value of the actual dividends paid during the life of the call option using the interest rate to discount,

$$\Sigma_1^m d_i \exp(-rt_i),$$

and subtract this present value from the current stock price. In effect, this assumes that the dividends paid during the life of the option were escrowed at the beginning of the period.

In Merton (1973*b*, p. 173), a corresponding partial differential equation is derived for the value of a put, $P(S,T)$, namely,

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P}{\partial S^2} + rS\frac{\partial P}{\partial S} - rP - \frac{\partial P}{\partial T} = 0 \tag{A3}$$

subject to the boundary conditions

$$P(S,0) = \max[0, E - S],$$
$$P[\bar{S}(T),T] = E - \bar{S}(T),$$
$$\frac{\partial P}{\partial S}[\bar{S}(T),T] = -1,$$

where $\bar{S}(T)$ is the stock price such that it is optimal to exercise the put when the put has time T to go until expiration. Except for the perpetual-put case, there is no known close-form solution to (A3), and it must be solved for by numerical methods. The solution includes the simultaneous determination of both $P(S,T)$ and $\bar{S}(T)$.

Like the call equation (A1), equation (A3) assumes no cash dividends. In taking into account dividends, (A3) becomes a system of $m + 1$ recursive partial differential equations, where m is the number of dividend payments during the life of the option. This system can be written as follows:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P_i}{\partial S^2} + rs\frac{\partial P_i}{\partial S} - rP_i - \frac{\partial P_i}{\partial \tau} = 0, i = 1, 2, \ldots, m \tag{A4}$$

subject to the boundary conditions:

$$P_i[\bar{S}(\tau),\tau] = E - \bar{S}(\tau),$$
$$\frac{\partial P_i}{\partial S}[\bar{S}(\tau),\tau] = -1,$$
$$P_i(S,T - t_i) = P_{i+1}(S - d_i, T - t_i),$$
$$\frac{\partial P_i}{\partial S}(S,T - t_i) = \frac{\partial P_{i+1}}{\partial S}(S - d_i, T - t_i),$$

where $P_i(S,\tau)$ is the value of the put option with time τ left to go before expiration, and $T - t_{i-1} > \tau \geq T - t_i$. If, by convention, $t_0 = 0$ and $t_{m+1} = T$, then $P_{m+1}(S,\tau)$ will satisfy equation (A3).

In performing the simulations, we solve (A4) numerically for each stock in each period using a method adapted from Parkinson (1977).

Appendix B

TABLE B1 Uncovered Put-Writing Strategy: Exercise Price/Stock Price = 1.0, Growth of $1,000 for 136-Stock Sample ($)

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	1,042	1,099	1,160	1,210	1,295	1,312	1,293	1,424	1,504	1,529	1,662	1,681	1,720	1,610	1,833	2,010	2,008	2,135	2,243	2,137	2,220	2,301	2,223	2,641	2,628	2,899	2,987	2,930
6401		1,055	1,113	1,161	1,243	1,259	1,241	1,367	1,443	1,468	1,595	1,613	1,651	1,545	1,760	1,930	1,928	2,050	2,153	2,051	2,131	2,208	2,134	2,535	2,523	2,783	2,867	2,813
6407			1,055	1,101	1,178	1,194	1,177	1,296	1,368	1,391	1,512	1,529	1,565	1,465	1,668	1,829	1,827	1,943	2,041	1,944	2,020	2,093	2,023	2,403	2,392	2,638	2,718	2,666
6501				1,043	1,116	1,131	1,115	1,228	1,296	1,318	1,433	1,449	1,483	1,388	1,581	1,733	1,731	1,841	1,933	1,842	1,914	1,983	1,917	2,277	2,266	2,500	2,575	2,526
6507					1,070	1,084	1,069	1,177	1,243	1,264	1,374	1,389	1,422	1,330	1,515	1,662	1,660	1,765	1,854	1,766	1,835	1,902	1,838	2,183	2,173	2,396	2,469	2,422
6601						1,013	999	1,100	1,162	1,181	1,284	1,298	1,329	1,243	1,416	1,553	1,551	1,650	1,732	1,651	1,715	1,777	1,717	2,040	2,030	2,240	2,308	2,263
6607							986	1,086	1,146	1,166	1,267	1,281	1,311	1,227	1,398	1,533	1,531	1,628	1,710	1,629	1,693	1,754	1,695	2,014	2,004	2,211	2,277	2,234
6701								1,101	1,163	1,182	1,285	1,300	1,330	1,245	1,418	1,555	1,553	1,651	1,734	1,653	1,717	1,779	1,719	2,043	2,033	2,242	2,310	2,266
6707									1,056	1,074	1,167	1,180	1,208	1,130	1,288	1,412	1,410	1,500	1,575	1,501	1,559	1,616	1,561	1,855	1,846	2,036	2,098	2,058
6801										1,017	1,105	1,118	1,144	1,070	1,219	1,337	1,336	1,420	1,491	1,421	1,477	1,530	1,479	1,757	1,748	1,928	1,987	1,949
6807											1,087	1,099	1,125	1,053	1,199	1,315	1,313	1,397	1,467	1,398	1,452	1,505	1,454	1,727	1,719	1,896	1,954	1,916
6901												1,011	1,035	968	1,103	1,210	1,208	1,285	1,349	1,286	1,336	1,384	1,338	1,589	1,582	1,745	1,797	1,763
6907													1,023	958	1,091	1,196	1,195	1,271	1,334	1,271	1,321	1,369	1,323	1,572	1,564	1,725	1,777	1,743
7001														936	1,066	1,169	1,168	1,242	1,304	1,242	1,291	1,338	1,293	1,536	1,528	1,686	1,737	1,703
7007															1,139	1,249	1,248	1,327	1,393	1,328	1,379	1,429	1,381	1,641	1,633	1,801	1,856	1,820
7101																1,097	1,095	1,165	1,223	1,166	1,211	1,255	1,213	1,441	1,434	1,581	1,629	1,598
7107																	999	1,062	1,116	1,063	1,104	1,144	1,106	1,314	1,307	1,442	1,486	1,457
7201																		1,063	1,117	1,064	1,106	1,146	1,107	1,315	1,309	1,444	1,488	1,459
7207																			1,050	1,001	1,040	1,077	1,041	1,237	1,231	1,358	1,399	1,372
7301																				953	990	1,026	991	1,178	1,172	1,293	1,332	1,307
7307																					1,039	1,077	1,040	1,236	1,230	1,357	1,398	1,371
7401																						1,036	1,001	1,190	1,184	1,306	1,345	1,320
7407																							966	1,148	1,143	1,260	1,298	1,274
7501																								1,188	1,182	1,304	1,344	1,318
7507																									995	1,098	1,131	1,109
7601																										1,103	1,136	1,115
7607																											1,030	1,011
7701																												981

TABLE B2 Uncovered Put-Writing Strategy: Exercise Price/Stock Price = 1.0, Semiannual Percentage Rate of Return for 136-Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	4.2	4.8	5.1	4.9	5.3	4.6	3.7	4.5	4.6	4.3	4.7	4.4	4.3	3.5	4.1	4.5	4.2	4.3	4.3	3.9	3.9	3.9	3.5	4.1	3.9	4.2	4.1	3.9
6401		5.5	5.5	5.1	5.6	4.7	3.7	4.6	4.7	4.4	4.8	4.4	4.3	3.4	4.1	4.5	4.2	4.3	4.4	3.9	3.9	3.8	3.5	4.1	3.9	4.2	4.1	3.9
6407			5.5	4.9	5.6	4.5	3.3	4.4	4.6	4.2	4.7	4.3	4.2	3.2	4.0	4.4	4.1	4.2	4.3	3.8	3.8	3.8	3.4	4.1	3.9	4.1	4.1	3.8
6501				4.3	5.6	4.2	2.8	4.2	4.4	4.0	4.6	4.2	4.0	3.0	3.9	4.3	4.0	4.2	4.2	3.7	3.7	3.7	3.3	4.0	3.8	4.1	4.0	3.8
6507					7.0	4.1	2.2	4.2	4.4	4.0	4.6	4.2	4.0	2.9	3.9	4.3	4.0	4.1	4.2	3.6	3.6	3.6	3.3	4.0	3.8	4.1	4.0	3.8
6601						1.3	−0.1	3.2	3.8	3.4	4.3	3.8	3.6	2.4	3.5	4.1	3.7	3.9	4.0	3.4	3.4	3.4	3.0	3.8	3.6	3.9	3.9	3.6
6607							−1.4	4.2	4.7	3.9	4.8	4.2	3.9	2.6	3.8	4.4	3.9	4.1	4.2	3.5	3.6	3.6	3.2	4.0	3.7	4.0	4.0	3.7
6701								10.1	7.8	5.7	6.5	5.4	4.9	3.2	4.5	5.0	4.5	4.7	4.7	3.9	3.9	3.9	3.4	4.3	4.0	4.3	4.3	4.0
6707									5.6	3.6	5.3	4.2	3.9	2.1	3.7	4.4	3.9	4.1	4.2	3.4	3.5	3.5	3.0	3.9	3.7	4.0	4.0	3.7
6801										1.7	5.1	3.8	3.4	1.4	3.4	4.2	3.7	4.0	4.1	3.2	3.3	3.3	2.8	3.8	3.6	3.9	3.9	3.6
6807											8.7	4.8	4.0	1.3	3.7	4.7	4.0	4.3	4.3	3.4	3.4	3.5	2.9	4.0	3.7	4.1	4.0	3.7
6901												1.1	1.7	−1.1	2.5	3.9	3.2	3.6	3.8	2.8	2.9	3.0	2.5	3.6	3.3	3.8	3.7	3.4
6907													2.3	−2.1	2.9	4.6	3.6	4.1	4.2	3.0	3.1	3.2	2.6	3.8	3.5	4.0	3.9	3.5
7001														−6.4	3.2	5.3	3.9	4.4	4.5	3.1	3.2	3.3	2.6	4.0	3.6	4.1	4.0	3.6
7007															13.9	11.8	7.7	7.3	6.9	4.8	4.7	4.6	3.7	5.1	4.6	5.0	4.9	4.4
7101																9.7	4.7	5.2	5.2	3.1	3.2	3.3	2.4	4.1	3.7	4.3	4.2	3.7
7107																	−0.1	3.1	3.7	1.5	2.0	2.3	1.4	3.5	3.0	3.7	3.7	3.2
7201																		6.3	5.7	2.1	2.5	2.8	1.7	4.0	3.4	4.2	4.1	3.5
7207																			5.0	0.0	1.3	1.9	0.8	3.6	3.0	3.9	3.8	3.2
7301																				−4.7	−0.5	0.9	−0.2	3.3	2.7	3.7	3.6	3.0
7307																					3.9	3.8	1.3	5.4	4.2	5.2	4.9	4.0
7401																						3.6	0.1	6.0	4.3	5.5	5.1	4.0
7407																							−3.4	7.1	4.5	6.0	5.4	4.1
7501																								18.8	8.7	9.3	7.7	5.7
7507																									−0.5	4.8	4.2	2.6
7601																										10.3	6.6	3.7
7607																											3.0	0.5
7701																												−1.9

TABLE B3 Uncovered Put-Writing Strategy: Exercise Price/Stock Price = 1.0, Growth of $1,000 for DJ Stock Sample ($)

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	052	1,112	1,158	1,195	1,265	1,250	1,216	1,328	1,383	1,416	1,525	1,564	1,586	1,523	1,701	1,817	1,802	1,896	2,028	1,948	2,006	2,127	2,070	2,420	2,493	2,753	2,893	2,925
6401		1,057	1,101	1,136	1,202	1,189	1,156	1,263	1,315	1,347	1,450	1,487	1,508	1,448	1,617	1,728	1,713	1,802	1,928	1,853	1,907	2,022	1,968	2,301	2,371	2,617	2,751	2,781
6407			1,041	1,075	1,137	1,124	1,093	1,194	1,244	1,274	1,371	1,406	1,426	1,369	1,529	1,634	1,620	1,705	1,823	1,752	1,803	1,912	1,862	2,176	2,242	2,475	2,601	2,630
6501				1,032	1,092	1,080	1,050	1,147	1,195	1,223	1,317	1,351	1,370	1,315	1,469	1,569	1,556	1,637	1,751	1,683	1,732	1,837	1,788	2,090	2,154	2,378	2,499	2,526
6507					1,058	1,046	1,017	1,111	1,157	1,185	1,276	1,308	1,327	1,274	1,423	1,520	1,507	1,586	1,697	1,630	1,678	1,779	1,732	2,025	2,086	2,303	2,420	2,447
6601						989	961	1,050	1,094	1,120	1,206	1,236	1,254	1,204	1,345	1,437	1,425	1,499	1,603	1,541	1,586	1,682	1,637	1,914	1,972	2,177	2,288	2,313
6607							972	1,063	1,106	1,133	1,220	1,251	1,269	1,218	1,360	1,454	1,441	1,516	1,622	1,559	1,604	1,701	1,656	1,936	1,995	2,202	2,314	2,340
6701								1,093	1,138	1,165	1,255	1,286	1,305	1,253	1,399	1,495	1,482	1,560	1,668	1,603	1,650	1,750	1,703	1,991	2,051	2,265	2,380	2,406
6707									1,041	1,066	1,148	1,177	1,194	1,146	1,280	1,368	1,356	1,427	1,527	1,467	1,510	1,601	1,559	1,822	1,877	2,072	2,178	2,202
6801										1,024	1,103	1,131	1,147	1,101	1,230	1,314	1,303	1,371	1,466	1,409	1,450	1,538	1,497	1,750	1,803	1,990	2,092	2,115
6807											1,077	1,104	1,120	1,075	1,201	1,283	1,272	1,338	1,432	1,376	1,416	1,501	1,462	1,709	1,760	1,943	2,043	2,065
6901												1,025	1,040	998	1,115	1,191	1,181	1,243	1,329	1,277	1,315	1,394	1,357	1,587	1,635	1,805	1,897	1,918
6907													1,014	974	1,088	1,162	1,152	1,212	1,297	1,246	1,283	1,360	1,324	1,548	1,595	1,760	1,850	1,870
7001														960	1,072	1,146	1,136	1,195	1,279	1,229	1,265	1,341	1,305	1,526	1,572	1,736	1,824	1,844
7007															1,117	1,193	1,183	1,245	1,332	1,279	1,317	1,397	1,360	1,589	1,637	1,808	1,900	1,921
7101																1,068	1,059	1,115	1,192	1,146	1,179	1,250	1,217	1,423	1,466	1,619	1,701	1,720
7107																	992	1,043	1,116	1,072	1,104	1,170	1,139	1,332	1,372	1,515	1,592	1,610
7201																		1,052	1,126	1,081	1,113	1,180	1,149	1,343	1,384	1,528	1,606	1,623
7207																			1,070	1,028	1,058	1,122	1,092	1,277	1,315	1,452	1,526	1,543
7301																				961	989	1,049	1,021	1,194	1,230	1,358	1,427	1,442
7307																					1,029	1,091	1,063	1,242	1,280	1,413	1,485	1,501
7401																						1,060	1,032	1,207	1,243	1,372	1,442	1,458
7407																							974	1,138	1,173	1,294	1,360	1,375
7501																								1,169	1,204	1,330	1,397	1,413
7507																									1,030	1,137	1,195	1,208
7601																										1,104	1,160	1,173
7607																											1,051	1,062
7701																												1,011

TABLE B4 Uncovered Put-Writing Strategy: Exercise Price/Stock Price = 1.0, Semiannual Percentage Rate of Return for DJ Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	5.2	5.5	5.0	4.6	4.8	3.8	2.8	3.6	3.7	3.5	3.9	3.8	3.6	3.0	3.6	3.8	3.5	3.6	3.8	3.4	3.4	3.5	3.2	3.8	3.7	4.0	4.0	3.9
6401		5.7	4.9	4.4	4.7	3.5	2.4	3.4	3.5	3.4	3.8	3.7	3.5	2.9	3.5	3.7	3.4	3.5	3.7	3.3	3.3	3.4	3.1	3.7	3.7	3.9	4.0	3.9
6407			4.1	3.7	4.4	3.0	1.8	3.0	3.2	3.1	3.6	3.5	3.3	2.7	3.3	3.6	3.3	3.4	3.6	3.2	3.2	3.3	3.0	3.6	3.6	3.8	3.9	3.8
6501				3.2	4.5	2.6	1.2	2.8	3.0	2.9	3.5	3.4	3.2	2.5	3.3	3.5	3.2	3.3	3.6	3.1	3.1	3.3	2.9	3.6	3.5	3.8	3.9	3.8
6507					5.8	2.3	0.6	2.7	3.0	2.9	3.5	3.4	3.2	2.5	3.3	3.6	3.2	3.3	3.6	3.1	3.1	3.3	2.9	3.6	3.6	3.9	3.9	3.8
6601						-1.1	-2.0	1.7	2.3	2.3	3.2	3.1	2.9	2.1	3.0	3.3	3.0	3.2	3.4	2.9	2.9	3.1	2.8	3.5	3.5	3.8	3.8	3.7
6607							-2.8	3.1	3.4	3.2	4.1	3.8	3.5	2.5	3.5	3.8	3.4	3.5	3.8	3.2	3.2	3.4	3.0	3.7	3.7	4.0	4.1	3.9
6701								9.3	6.7	5.2	5.8	5.2	4.5	3.3	4.3	4.6	4.0	4.1	4.4	3.7	3.6	3.8	3.4	4.1	4.1	4.4	4.4	4.3
6707									4.1	3.3	4.7	4.2	3.6	2.3	3.6	4.0	3.4	3.6	3.9	3.2	3.2	3.4	3.0	3.8	3.8	4.1	4.2	4.0
6801										2.4	5.0	4.2	3.5	1.9	3.5	4.0	3.4	3.6	3.9	3.2	3.1	3.4	2.9	3.8	3.8	4.1	4.2	4.0
6807											7.7	5.1	3.8	1.8	3.7	4.2	3.5	3.7	4.1	3.2	3.2	3.4	3.0	3.9	3.8	4.2	4.3	4.1
6901												2.5	2.0	-0.1	2.8	3.6	2.8	3.2	3.6	2.8	2.8	3.1	2.6	3.6	3.6	4.0	4.1	3.9
6907													1.4	-1.3	2.8	3.8	2.9	3.3	3.8	2.8	2.8	3.1	2.6	3.7	3.7	4.1	4.2	4.0
7001														-4.0	3.6	4.6	3.2	3.6	4.2	3.0	3.0	3.3	2.7	3.9	3.8	4.3	4.4	4.2
7007															11.7	9.2	5.8	5.6	5.9	4.2	4.0	4.3	3.5	4.7	4.6	5.1	5.1	4.8
7101																6.8	2.9	3.7	4.5	2.8	2.8	3.2	2.5	4.0	3.9	4.5	4.5	4.3
7107																	-0.8	2.1	3.7	1.8	2.0	2.7	1.9	3.6	3.6	4.2	4.3	4.0
7201																		5.2	6.1	2.6	2.7	3.4	2.3	4.3	4.1	4.8	4.9	4.5
7207																			7.0	1.4	1.9	2.9	1.8	4.2	4.0	4.8	4.8	4.4
7301																				-3.9	-0.5	1.6	0.5	3.6	3.5	4.5	4.5	4.2
7307																					2.9	4.5	2.0	5.6	5.1	5.9	5.8	5.2
7401																						6.0	1.6	6.5	5.6	6.5	6.3	5.5
7407																							-2.6	6.7	5.4	6.7	6.3	5.5
7501																								16.9	9.7	10.0	8.7	7.2
7507																									3.0	6.6	6.1	4.8
7601																										10.4	7.7	5.5
7607																											5.1	3.1
7701																												1.1

Appendix C

TABLE C1 Protective-Put Buying Strategy: Exercise Price/Stock Price = 1.0, Growth of $1,000 for 136-Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	1.092	1.169	1.216	1.261	1.565	1.608	1.597	1.897	2.080	2.146	2.281	2.244	2.326	2.256	2.750	3.113	3.184	3.444	3.712	3.674	3.894	3.799	3.818	5.265	5.054	5.696	5.727	5,628
6401		1.071	1.114	1.155	1.434	1.473	1.463	1.737	1.905	1.966	2.090	2.055	2.130	2.067	2.519	2.852	2.917	3.155	3.400	3.365	3.567	3.479	3,497	4.822	4.629	5.217	5.246	5.155
6407			1.040	1.079	1.339	1.376	1.366	1.622	1.779	1.836	1.951	1.919	1.989	1.930	2.352	2.663	2.724	2.946	3.175	3.143	3.331	3.249	3.266	4.503	4.323	4.872	4.899	4.814
6501				1.037	1.287	1.323	1.314	1.560	1.710	1.765	1.876	1.845	1.912	1.855	2.261	2.560	2.618	2.832	3,052	3,021	3.202	3.123	3.139	4.329	4.156	4.684	4.709	4.628
6507					1.241	1,275	.266	1.504	1.649	1.702	1.808	1.779	1.844	1.789	2.180	2.468	2.524	2.730	2.943	2.912	3.087	3.011	3.026	4.173	4,006	4.516	4.540	4.462
6601						1,028	.021	1.212	1.329	1.371	1.458	1.434	1.486	1.442	1.757	1.989	2.035	2.201	2.372	2.347	2.488	2.427	2.439	3.364	3.229	3,639	3,659	3,596
6607							993	1.179	1.293	1.334	1.418	1.395	1.446	1.403	1.710	1.936	1.980	2.141	2.308	2.284	2.421	2.362	2.373	3.273	3.142	3.541	3.561	3.499
6701								1.187	1.302	1.344	1.428	1.405	1.456	1.412	1.721	1.949	1.993	2.156	2.324	2.300	2.438	2.378	2.390	3.296	3.164	3.566	3.585	3.523
6707									1.096	1.132	1.203	1.183	1.226	1.190	1.450	1.642	1.679	1.816	1.957	1.937	2.053	2.003	2.013	2.776	2.665	3.003	3.020	2.967
6801										1.032	1.097	1.079	1.118	1.085	1.322	1.497	1.531	1.656	1.785	1.767	1.872	1.827	1.836	2.532	2.430	2.739	2.754	2.706
6807											1.063	1.045	1.083	1.051	1.281	1.450	1.483	1.605	1.729	1.712	1.814	1.770	1.779	2.453	2.355	2.654	2.668	2.622
6901												984	1.019	989	1.205	1.365	1.396	1.510	1.627	1.611	1.707	1.665	1.674	2.308	2.215	2.497	2.511	2.467
6907													1.036	1.006	1.225	1.387	1.419	1.535	1.654	1.637	1.735	1.693	1.701	2.346	2.252	2.538	2.552	2.508
7001														970	1.182	1.339	1.369	1.481	1.596	1.580	1.674	1.633	1.642	2.264	2.173	2.449	2.463	2.420
7007															1.219	1.380	1.411	1.526	1.645	1.628	1.726	1.684	1.692	2.333	2.240	2.524	2.538	2.494
7101																1.132	1.158	1.253	1.350	1.336	1.416	1.381	1.388	1.915	1.838	2.071	2.083	2.047
7107																	1.023	1.106	1.192	1.180	1.251	1.220	1.226	1.691	1.623	1.830	1.840	1.808
7201																		1.082	1.166	1.154	1.223	1.193	1.199	1.653	1.587	1.789	1.799	1.767
7207																			1.078	1.067	1.131	1.103	1.108	1.528	1.467	1.654	1.663	1.634
7301																				990	1.049	1.023	1.029	1.418	1.362	1.535	1.543	1.516
7307																					1.060	1.034	1.039	1.433	1.376	1.550	1.559	1.532
7401																						976	980	1.352	1.298	1.463	1.471	1.445
7407																							1.005	1.386	1.330	1.500	1.508	1.482
7501																								1.379	1.324	1.492	1.500	1.474
7507																									960	1.082	1.088	1.069
7601																										1.127	1.133	1.114
7607																											1.005	988
7701																												983

TABLE C2 Protective-Put Buying Strategy: Exercise Price/Stock Price = 1.0, Semiannual Percentage Rate of Return for 136-Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	9.2	8.1	6.7	6.0	9.4	8.2	6.9	8.3	8.5	7.9	7.8	7.0	6.7	6.0	7.0	7.4	7.1	7.1	7.1	6.7	6.7	6.3	6.0	7.2	6.7	6.9	6.7	6.4
6401		7.1	5.5	4.9	9.4	8.1	6.5	8.2	8.4	7.8	7.6	6.8	6.5	5.7	6.8	7.2	6.9	7.0	7.0	6.6	6.6	6.1	5.9	7.1	6.6	6.8	6.6	6.3
6407			4.0	3.9	10.2	8.3	6.4	8.4	8.6	7.9	7.7	6.7	6.5	5.6	6.8	7.2	6.9	7.0	7.0	6.6	6.5	6.1	5.8	7.1	6.6	6.8	6.6	6.2
6501				3.7	10.2	9.8	7.1	9.3	9.4	8.5	8.2	7.0	6.7	5.8	7.0	7.5	7.1	7.2	7.2	6.7	6.7	6.2	5.9	7.2	6.7	6.9	6.7	6.3
6507					24.1	12.9	8.2	10.7	10.5	9.3	8.8	7.5	7.0	6.0	7.3	7.8	7.4	7.4	7.5	6.9	6.9	6.3	6.0	7.4	6.8	7.1	6.8	6.4
6601						2.8	1.0	6.6	7.4	6.5	6.5	5.3	5.1	4.1	5.8	6.5	6.1	6.3	6.4	5.9	5.9	5.4	5.1	6.6	6.0	6.3	6.1	5.7
6607							−0.7	8.6	8.9	7.5	7.2	5.7	5.4	4.3	6.1	6.8	6.4	6.6	6.6	6.1	6.1	5.5	5.2	6.8	6.2	6.5	6.2	5.9
6701								18.7	14.1	10.3	9.3	7.0	6.5	5.1	7.0	7.7	7.1	7.2	7.3	6.6	6.6	5.9	5.6	7.3	6.6	6.9	6.6	6.2
6707									9.6	6.4	6.3	4.3	4.2	2.9	5.4	6.4	5.9	6.1	6.3	5.7	5.7	5.1	4.8	6.6	5.9	6.3	6.0	5.6
6801										3.2	4.7	2.6	2.8	1.6	4.8	5.9	5.5	5.8	6.0	5.3	5.4	4.7	4.4	6.4	5.7	6.1	5.8	5.4
6807											6.3	2.2	2.7	1.3	5.1	6.4	5.8	6.1	6.3	5.5	5.6	4.9	4.5	6.6	5.9	6.3	5.9	5.5
6901												−1.6	1.0	−0.4	4.8	6.4	5.7	6.1	6.3	5.4	5.5	4.7	4.4	6.6	5.8	6.3	5.9	5.5
6907													3.6	0.3	7.0	8.5	7.3	7.4	7.5	6.4	6.3	5.4	4.9	7.4	6.4	6.9	6.4	5.9
7001														−3.0	8.7	10.2	8.2	8.2	8.1	6.8	6.7	5.6	5.1	7.7	6.7	7.1	6.6	6.1
7007															21.9	17.5	12.2	11.2	10.5	8.5	8.1	6.7	6.0	8.8	7.6	8.0	7.4	6.7
7101																13.2	7.6	7.8	7.8	6.0	6.0	4.7	4.2	7.5	6.3	6.8	6.3	5.7
7107																	2.3	5.2	6.0	4.2	4.6	3.4	3.0	6.8	5.5	6.2	5.7	5.1
7201																		8.2	8.0	4.9	5.2	3.6	3.1	7.4	5.9	6.7	6.0	5.3
7207																			7.8	3.3	4.2	2.5	2.1	7.3	5.6	6.5	5.8	5.0
7301																				−1.0	2.4	0.8	0.7	7.2	5.3	6.3	5.6	4.7
7307																					6.0	1.7	1.3	9.4	6.6	7.6	6.5	5.5
7401																						−2.4	−1.0	10.6	6.7	7.9	6.6	5.4
7407																							0.5	17.7	10.0	10.7	8.6	6.8
7501																								37.9	15.1	14.3	10.7	8.1
7507																									−4.0	4.0	2.8	1.7
7601																										12.7	6.5	3.7
7607																											0.5	−0.6
7701																												−1.7

TABLE C3 Protective-Put Buying Strategy: Exercise Price/Stock Price = 1.0, Growth of $1,000 for DJ Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	1.050	1.130	1.188	1.205	1.327	1.309	1.285	1.385	1.448	1.462	1.532	1.507	1.524	1.497	1.757	1.822	1.829	1.857	1.986	1.957	2.054	2.036	2.033	2.746	2.709	3.180	3.206	3.209
6401		1.076	1.132	1.148	1.263	1.246	1.223	1.318	1.379	1.392	1.459	1.435	1.451	1.426	1.673	1.735	1.742	1.769	1.891	1.864	1.955	1.938	1.936	2.615	2.579	3.028	3.053	3.055
6407			1.052	1.067	1.175	1.158	1.137	1.226	1.282	1.294	1.357	1.334	1.349	1.326	1.556	1.613	1.619	1.644	1.758	1.733	1.818	1.802	1.800	2.431	2.398	2.815	2.838	2.841
6501				1.014	1.116	1.101	1.081	1.165	1.218	1.230	1.289	1.268	1.283	1.260	1.479	1.533	1.539	1.563	1.671	1.647	1.728	1.713	1.711	2.311	2.279	2.676	2.698	2.700
6507					1.101	1.086	1.066	1.149	1.201	1.213	1.271	1.251	1.265	1.242	1.458	1.511	1.517	1.541	1.648	1.624	1.704	1.689	1.687	2.278	2.247	2.638	2.660	2.662
6601						986	968	1.044	1.091	1.102	1.155	1.136	1.149	1.129	1.324	1.373	1.379	1.400	1.497	1.475	1.548	1.534	1.532	2.070	2.041	2.397	2.416	2.418
6607							982	1.058	1.106	1.117	1.171	1.152	1.165	1.144	1.343	1.392	1.398	1.419	1.518	1.496	1.569	1.556	1.554	2.099	2.070	2.430	2.450	2.452
6701								1.078	1.127	1.138	1.193	1.173	1.187	1.166	1.368	1.418	1.424	1.446	1.546	1.524	1.599	1.584	1.583	2.138	2.108	2.475	2.495	2.498
6707									1.046	1.056	1.107	1.089	1.101	1.082	1.269	1.316	1.321	1.341	1.434	1.414	1.483	1.470	1.468	1.983	1.956	2.297	2.315	2.317
6801										1.010	1.058	1.041	1.053	1.034	1.214	1.258	1.263	1.283	1.372	1.352	1.418	1.406	1.404	1.897	1.871	2.197	2.214	2.216
6807											1.048	1.031	1.043	1.025	1.202	1.246	1.251	1.271	1.359	1.339	1.405	1.393	1.391	1.879	1.853	2.176	2.193	2.195
6901												984	995	977	1.147	1.189	1.194	1.212	1.296	1.277	1.340	1.328	1.327	1.792	1.768	2.075	2.092	2.094
6907													1.011	993	1.166	1.209	1.213	1.232	1.317	1.298	1.362	1.350	1.349	1.822	1.797	2.110	2.127	2.129
7001														982	1.153	1.195	1.200	1.219	1.303	1.284	1.347	1.335	1.334	1.802	1.777	2.086	2.103	2.105
7007															1.173	1.217	1.221	1.240	1.326	1.307	1.371	1.359	1.358	1.834	1.809	2.124	2.141	2.143
7101																1.037	1.041	1.057	1.130	1.114	1.169	1.158	1.157	1.563	1.541	1.810	1.824	1.826
7107																	1.004	1.020	1.090	1.074	1.127	1.117	1.116	1.507	1.487	1.746	1.760	1.761
7201																		1.016	1.086	1.070	1.123	1.113	1.112	1.501	1.481	1.739	1.753	1.754
7207																			1.069	1.054	1.106	1.096	1.095	1.479	1.458	1.712	1.726	1.728
7301																				986	1.034	1.025	1.024	1.383	1.364	1.601	1.614	1.616
7307																					1.049	1.040	1.039	1.403	1.384	1.625	1.638	1.639
7401																						991	990	1.337	1.319	1.549	1.561	1.562
7407																							999	1.349	1.331	1.562	1.575	1.576
7501																								1.351	1.332	1.564	1.577	1.578
7507																									986	1.158	1.167	1.168
7601																										1.174	1.184	1.185
7607																											1.008	1.009
7701																												1.001

TABLE C4 Protective-Put Buying Strategy: Exercise Price/Stock Price = 1.0, Semiannual Percentage Rate of Return for DJ Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	5.0	6.3	5.9	4.8	5.8	4.6	3.6	4.2	4.2	3.9	4.0	3.5	3.3	2.9	3.8	3.8	3.6	3.5	3.7	3.4	3.5	3.3	3.1	4.3	4.1	4.6	4.4	4.3
6401		7.6	6.4	4.7	6.0	4.5	3.4	4.0	4.1	3.7	3.9	3.3	3.2	2.8	3.7	3.7	3.5	3.4	3.6	3.3	3.4	3.2	3.0	4.3	4.0	4.5	4.4	4.2
6407			5.2	3.3	5.5	3.7	2.6	3.5	3.6	3.3	3.4	2.9	2.8	2.4	3.5	3.5	3.3	3.2	3.4	3.1	3.2	3.0	2.8	4.1	3.9	4.4	4.3	4.1
6501				1.4	5.7	3.3	2.0	3.1	3.3	3.0	3.2	2.7	2.5	2.1	3.3	3.3	3.1	3.0	3.3	3.0	3.1	2.9	2.7	4.1	3.8	4.4	4.2	4.1
6507					10.1	4.2	2.1	3.5	3.7	3.3	3.5	2.8	2.6	2.2	3.5	3.5	3.3	3.1	3.4	3.1	3.2	3.0	2.8	4.2	3.9	4.5	4.3	4.2
6601						−1.4	−1.6	1.4	2.2	2.0	2.4	1.8	1.7	1.4	2.8	2.9	2.7	2.6	2.9	2.6	2.8	2.5	2.4	3.9	3.6	4.3	4.1	3.9
6607							−1.8	2.9	3.4	2.8	3.2	2.4	2.2	1.7	3.3	3.4	3.1	3.0	3.3	2.9	3.1	2.8	2.6	4.2	3.9	4.5	4.4	4.2
6701								7.8	6.2	4.4	4.5	3.2	2.9	2.2	4.0	4.0	3.6	3.4	3.7	3.3	3.4	3.1	2.9	4.6	4.2	4.9	4.7	4.5
6707									4.6	2.7	3.4	2.1	1.9	1.3	3.5	3.5	3.1	3.0	3.3	2.9	3.1	2.8	2.6	4.4	4.0	4.7	4.5	4.3
6801										1.0	2.9	1.4	1.3	0.7	3.3	3.3	3.0	2.8	3.2	2.8	3.0	2.7	2.5	4.4	4.0	4.7	4.5	4.3
6807											4.8	1.6	1.4	0.6	3.8	3.7	3.3	3.0	3.5	3.0	3.1	2.8	2.6	4.6	4.2	5.0	4.7	4.5
6901												−1.6	−0.3	−0.8	3.5	3.5	3.0	2.8	3.3	2.8	3.0	2.6	2.4	4.6	4.2	5.0	4.7	4.4
6907													1.1	−0.3	5.2	4.9	3.9	3.5	4.0	3.3	3.5	3.0	2.8	5.1	4.6	5.5	5.2	4.8
7001														−1.8	7.4	6.1	4.7	4.0	4.5	3.6	3.8	3.3	2.9	5.5	4.9	5.8	5.5	5.1
7007															17.3	10.3	6.9	5.5	5.8	4.6	4.6	3.9	3.5	6.3	5.5	6.5	6.0	5.6
7101																3.7	2.0	1.9	3.1	2.2	2.6	2.1	1.8	5.1	4.4	5.5	5.1	4.7
7107																	0.4	1.0	2.9	1.8	2.4	1.9	1.6	5.3	4.5	5.7	5.3	4.8
7201																		1.6	4.2	2.3	2.9	2.2	1.8	6.0	5.0	6.3	5.8	5.2
7207																			6.9	2.7	3.4	2.3	1.8	6.7	5.5	7.0	6.3	5.6
7301																				−1.4	1.7	0.8	0.6	6.7	5.3	7.0	6.2	5.5
7307																					4.9	2.0	1.3	8.8	6.7	8.4	7.3	6.4
7401																						−0.9	−0.5	10.2	7.2	9.1	7.7	6.6
7407																							−0.1	16.2	10.0	11.8	9.5	7.9
7501																								35.1	15.4	16.1	12.1	9.6
7507																									−1.4	7.6	5.3	4.0
7601																										17.4	8.8	5.8
7607																											0.8	0.4
7701																												0.1

Appendix D

TABLE D1 Underlying Stock Portfolio: Semiannual Percentage Rate of Return for DJ Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	8.9	10.3	9.2	7.3	8.7	5.6	3.6	4.8	5.1	4.8	5.1	4.2	3.4	2.1	3.5	3.8	3.6	3.6	4.0	3.1	3.0	2.7	1.8	3.4	3.3	4.0	4.1	3.8
6401		11.7	9.4	6.7	8.7	5.0	2.7	4.2	4.7	4.3	4.7	3.8	3.0	1.6	3.2	3.5	3.3	3.3	3.7	2.8	2.7	2.4	1.5	3.2	3.0	3.9	3.9	3.6
6407			7.1	4.3	7.7	3.3	1.0	3.1	3.7	3.4	4.0	3.1	2.2	0.8	2.5	2.9	2.7	2.8	3.3	2.3	2.2	2.0	1.0	2.8	2.7	3.5	3.6	3.3
6501				1.6	7.9	2.1	−0.4	2.3	3.1	2.9	3.6	2.6	1.7	0.2	2.2	2.6	2.4	2.5	3.0	2.0	2.0	1.7	0.7	2.6	2.5	3.4	3.4	3.1
6507					14.7	2.4	−1.1	2.4	3.4	3.1	3.9	2.7	1.8	0.1	2.2	2.7	2.5	2.6	3.1	2.1	2.0	1.7	0.7	2.7	2.5	3.5	3.5	3.2
6601						−8.6	−8.2	−1.4	0.8	i.0	2.2	1.1	0.2	−1.4	1.0	1.6	1.5	1.7	2.4	1.3	1.2	1.0	−0.1	2.1	1.9	3.0	3.0	2.7
6607							−7.7	2.5	4.1	3.5	4.5	2.9	1.6	−0.5	2.2	2.7	2.5	2.6	3.3	2.0	1.9	1.6	0.5	2.7	2.5	3.6	3.6	3.3
6701								13.8	10.7	7.6	7.8	5.1	3.2	0.6	3.5	4.0	3.6	3.6	4.2	2.8	2.7	2.3	1.0	3.3	3.1	4.2	4.2	3.8
6707									7.6	4.6	5.9	3.1	1.2	−1.4	2.1	2.8	2.5	2.6	3.4	1.9	1.8	1.5	0.2	2.7	2.5	3.7	3.8	3.4
6801										1.7	5.1	1.6	−0.3	−3.1	1.2	2.1	1.9	2.1	3.0	1.4	1.4	1.1	−0.3	2.4	2.2	3.5	3.5	3.2
6807											8.6	1.5	−1.0	−4.3	1.1	2.2	1.9	2.1	3.1	1.4	1.4	1.0	−0.5	2.5	2.2	3.6	3.6	3.2
6901												−5.1	−5.4	−8.2	−0.7	1.0	0.8	1.2	2.5	0.6	0.7	0.3	−1.2	2.0	1.8	3.3	3.3	2.9
6907													−5.8	−9.8	0.8	2.5	2.0	2.3	3.6	1.4	1.3	0.9	−0.8	2.6	2.4	3.9	3.9	3.5
7001														−13.6	4.3	5.4	4.1	4.0	5.2	2.4	2.2	1.7	−0.3	3.4	3.1	4.7	4.7	4.1
7007															26.0	16.5	10.8	9.0	9.5	5.4	4.7	3.8	1.3	5.3	4.7	6.4	6.2	5.5
7101																7.8	3.9	3.8	5.7	1.7	1.6	0.9	−1.4	3.2	2.8	4.7	4.7	4.1
7107																	0.1	1.9	5.0	0.2	0.4	−0.2	−2.7	2.7	2.3	4.4	4.5	3.8
7201																		3.7	7.5	0.3	0.4	−0.2	−3.1	3.0	2.6	4.9	4.9	4.1
7207																			11.5	−1.4	−0.7	−1.2	−4.4	2.9	2.4	5.1	5.0	4.1
7301																				−12.9	−6.2	−5.1	−8.1	1.3	1.0	4.2	4.2	3.3
7307																					0.9	−1.0	−6.4	5.2	4.0	7.3	6.9	5.6
7401																						−2.8	−9.9	6.6	4.8	8.7	8.0	6.3
7407																							−16.4	11.7	7.4	11.8	10.3	7.9
7501																								49.1	21.7	23.1	18.2	13.5
7507																									−0.6	11.9	9.4	6.0
7601																										26.0	14.8	8.3
7607																											4.6	0.4
7701																												−3.5

TABLE D2 Underlying Stock Portfolio: Semiannual Percentage Rate of Return for 136-Stock Sample

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	13.0	12.0	10.4	9.2	13.2	10.9	9.2	11.2	11.6	11.1	11.2	9.6	8.8	6.3	7.9	8.6	8.3	8.5	8.7	7.5	7.3	6.6	5.6	7.3	6.7	7.2	7.0	6.5
6401		11.0	9.2	7.9	13.2	10.5	8.6	11.0	11.4	10.9	11.0	9.3	8.4	5.8	7.5	8.4	8.0	8.3	8.5	7.2	7.1	6.3	5.3	7.0	6.4	7.0	6.8	6.3
6407			7.4	6.4	13.9	10.4	8.1	11.0	11.5	10.9	11.0	9.1	8.2	5.4	7.3	8.2	7.9	8.1	8.3	7.0	6.9	6.0	5.0	6.9	6.2	6.8	6.6	6.1
6501				5.4	17.3	11.4	8.3	11.7	12.2	11.4	11.4	9.3	8.3	5.2	7.3	8.2	7.9	8.2	8.4	7.0	6.8	6.0	4.9	6.8	6.2	6.8	6.6	6.1
6507					30.7	14.6	9.3	13.4	13.6	12.4	12.3	9.8	8.6	5.2	7.5	8.5	8.1	8.4	8.6	7.1	6.9	6.0	4.9	6.9	6.2	6.9	6.7	6.1
6601						0.5	−0.0	8.1	9.7	9.1	9.5	7.1	6.1	2.7	5.4	6.6	6.4	6.8	7.1	5.6	5.6	4.7	3.6	5.8	5.1	5.8	5.7	5.2
6607							−0.5	12.2	13.0	11.4	11.4	8.2	6.9	3.0	5.9	7.3	6.9	7.4	7.7	6.0	5.9	5.0	3.8	6.1	5.4	6.1	5.9	5.4
6701								26.5	20.3	15.6	14.6	10.1	8.2	3.5	6.8	8.2	7.7	8.1	8.4	6.5	6.4	5.3	4.0	6.5	5.7	6.5	6.3	5.7
6707									14.5	10.5	10.9	6.3	4.9	0.1	4.2	6.1	5.8	6.4	6.9	5.0	5.0	4.0	2.7	5.4	4.6	5.5	5.3	4.7
6801										6.7	9.2	3.7	2.6	−2.6	2.6	4.9	4.8	5.6	6.1	4.2	4.2	3.2	1.9	4.8	4.0	4.9	4.8	4.2
6807											11.7	2.3	1.3	−4.8	1.8	4.7	4.5	5.4	6.1	4.0	4.0	2.9	1.5	4.6	3.8	4.8	4.7	4.1
6901												−6.4	−3.5	−9.7	−0.6	3.3	3.3	4.6	5.4	3.1	3.3	2.1	0.7	4.1	3.3	4.4	4.3	3.7
6907													−0.6	−11.4	1.4	5.9	5.4	6.5	7.2	4.4	4.4	3.0	1.4	5.0	4.1	5.2	5.0	4.3
7001														−21.0	2.5	8.1	7.0	8.0	8.5	5.1	5.1	3.5	1.6	5.6	4.5	5.7	5.5	4.7
7007															33.0	26.5	18.3	16.7	15.7	10.2	9.4	7.0	4.5	8.7	7.2	8.3	7.8	6.8
7101																20.3	11.6	11.8	11.7	6.2	5.9	3.7	1.4	6.3	4.9	6.3	6.0	5.0
7107																	3.5	7.7	9.0	2.9	3.2	1.2	−1.1	4.6	3.3	4.9	4.7	3.8
7201																		12.1	11.8	2.7	3.2	0.7	−1.8	4.8	3.3	5.1	4.9	3.8
7207																			11.4	−1.7	0.4	−1.9	−4.4	3.6	2.1	4.3	4.1	3.0
7301																				−13.3	−4.8	−6.0	−8.0	2.1	0.6	3.3	3.2	2.2
7307																					4.6	−2.1	−6.1	6.4	3.6	6.3	5.8	4.3
7401																						−8.5	−11.1	6.9	3.4	6.7	6.0	4.2
7407																							−13.6	15.6	7.6	10.8	9.1	6.5
7501																								54.6	20.1	20.4	15.7	11.1
7507																									−6.7	6.3	5.0	2.2
7601																										21.0	11.4	5.4
7607																											2.7	−1.6
7701																												−5.8

TABLE D3 Underlying Stock Portfolio: Semiannual Percentage Rate of Return for Commercial Paper

	6312	6406	6412	6506	6512	6606	6612	6706	6712	6806	6812	6906	6912	7006	7012	7106	7112	7206	7212	7306	7312	7406	7412	7506	7512	7606	7612	7706
6307	2.0	2.0	2.1	2.1	2.2	2.3	2.4	2.4	2.5	2.5	2.6	2.7	2.9	2.9	3.0	2.9	2.9	2.9	2.9	2.9	3.0	3.1	3.3	3.3	3.3	3.3	3.2	3.2
6401		2.1	2.1	2.1	2.2	2.3	2.5	2.5	2.5	2.6	2.7	2.8	2.9	3.0	3.0	3.0	3.0	2.9	2.9	3.0	3.1	3.2	3.3	3.3	3.3	3.3	3.3	3.3
6407			2.1	2.2	2.2	2.4	2.5	2.6	2.6	2.7	2.7	2.8	3.0	3.1	3.1	3.1	3.1	3.0	3.0	3.0	3.1	3.2	3.4	3.4	3.4	3.4	3.3	3.3
6501				2.3	2.3	2.5	2.6	2.7	2.7	2.7	2.8	2.9	3.1	3.2	3.2	3.2	3.1	3.1	3.0	3.1	3.2	3.3	3.4	3.4	3.4	3.4	3.4	3.4
6507					2.3	2.5	2.8	2.7	2.7	2.8	2.9	3.0	3.2	3.3	3.3	3.2	3.2	3.1	3.1	3.1	3.2	3.4	3.5	3.5	3.5	3.5	3.4	3.4
6601						2.8	3.0	2.9	2.8	2.9	3.0	3.1	3.3	3.4	3.4	3.3	3.3	3.2	3.1	3.2	3.3	3.4	3.6	3.6	3.6	3.5	3.5	3.5
6607							3.2	2.9	2.9	2.9	3.0	3.1	3.4	3.5	3.5	3.4	3.3	3.2	3.2	3.2	3.3	3.5	3.6	3.6	3.6	3.6	3.5	3.5
6701								2.7	2.7	2.9	2.9	3.1	3.4	3.5	3.5	3.4	3.3	3.2	3.2	3.2	3.3	3.5	3.6	3.6	3.6	3.6	3.5	3.5
6707									2.7	2.9	3.0	3.2	3.5	3.6	3.6	3.5	3.4	3.3	3.2	3.2	3.4	3.5	3.7	3.7	3.7	3.6	3.6	3.5
6801										3.1	3.2	3.4	3.7	3.8	3.8	3.6	3.5	3.3	3.2	3.3	3.5	3.6	3.8	3.8	3.7	3.7	3.6	3.6
6807											3.2	3.6	4.0	4.0	3.9	3.6	3.5	3.4	3.3	3.3	3.5	3.6	3.8	3.8	3.8	3.7	3.7	3.6
6901												3.9	4.3	4.2	4.1	3.7	3.6	3.4	3.3	3.3	3.5	3.7	3.9	3.9	3.8	3.8	3.7	3.6
6907													4.8	4.4	4.1	3.7	3.5	3.3	3.2	3.2	3.5	3.6	3.9	3.8	3.8	3.7	3.7	3.6
7001														4.1	3.8	3.3	3.2	3.0	2.9	3.0	3.3	3.5	3.8	3.8	3.7	3.7	3.6	3.5
7007															3.5	3.0	2.9	2.7	2.7	2.8	3.2	3.5	3.7	3.7	3.7	3.6	3.6	3.5
7101																2.5	2.6	2.5	2.5	2.7	3.1	3.4	3.8	3.8	3.7	3.6	3.6	3.5
7107																	2.8	2.5	2.5	2.8	3.3	3.6	4.0	3.9	3.9	3.8	3.7	3.6
7201																		2.1	2.3	2.8	3.4	3.8	4.2	4.1	4.0	3.9	3.8	3.7
7207																			2.5	3.1	3.8	4.2	4.6	4.4	4.3	4.1	4.0	3.8
7301																				3.6	4.5	4.8	5.1	4.8	4.6	4.3	4.1	4.0
7307																					5.4	5.3	5.6	5.1	4.8	4.4	4.2	4.0
7401																						5.3	5.7	5.0	4.6	4.3	4.0	3.8
7407																							6.2	4.9	4.4	4.0	3.8	3.6
7501																								3.6	3.5	3.3	3.2	3.1
7507																									3.4	3.1	3.0	2.9
7601																										2.8	2.8	2.8
7607																											2.8	2.8
7701																												2.7

References

Black, F., and Scholes, M. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81, no. 3 (May–June): 637–59.

Brennan, M., and Schwartz, E. 1977. The valuation of American put options. *Journal of Finance* 32 (March): 449–62.

Gould, J. P., and Galai, D. 1974. Transactions costs and the relationship between put and call prices. *Journal of Financial Economics* 1 (July): 105–30.

Latane, H. A., and Rendleman, R. J., Jr. 1976. Standard deviations of stock price ratios implied in option prices. *Journal of Finance* 31 (May): 369–81.

Merton, R. C. 1973*a*. The relationship between put and call option prices: comment. *Journal of Finance* 28 (March): 183–94.

Merton, R. C. 1973*b*. Theory of rational option pricing. *Bell Journal of Economics and Management Science* 4 (Spring): 141–83.

Merton, R. C.; Scholes, M.; and Gladstein, M. 1978. The returns and risk of alternative call option portfolio investment strategies. *Journal of Business* 51, no. 2 (April): 183–242.

Parkinson, M. 1977. Option pricing: the American put. *Journal of Business* 50, no. 1 (January): 21–36.

Schwartz, E. S. 1977. The valuation of warrants: implementing a new approach. *Journal of Financial Economics* 4 (January): 79–94.

Smith, C. W. 1976. Option pricing: a review. *Journal of Financial Economics* 3 (January–March): 3–51.

Stoll, H. R. 1969. The relationship between put and call option prices. *Journal of Finance* 24 (December): 802–24.