

Stephen F. LeRoy

University of California, Santa Barbara

C. J. LaCivita

University of North Carolina at Greensboro

Risk Aversion and the Dispersion of Asset Prices*

I. Introduction

Several recent papers have investigated the implications for asset price dispersion of conventional security valuation models. LeRoy and Porter (1981) used the present-value equation to demonstrate that the coefficient of dispersion of stock prices must be lower than that of dividends. At the same time Shiller (1979), employing the expectations hypothesis of the term structure of interest rates, derived upper bounds for the dispersion of long-term interest rates and for the one-period rate of return relative to the dispersion of short-term interest rates. Shiller subsequently extended the analysis to stock prices (1981). All of these papers, as well as those of Singleton (1980) and Huang (1981), conducted empirical tests of the implied variance bounds. For the most part, they found that the implications were flagrantly violated: stock prices and

Several recent papers have concluded that asset price volatility is too great to be consistent with conventional security valuation models. We observe that the conventional models are based on risk neutrality or near risk neutrality and demonstrate that, in an exchange setting, risk aversion generally leads to greater dispersion of asset prices than does risk neutrality. It follows that volatility tests based on models assuming risk neutrality will generate downward-biased estimates of implied price variances to the extent that individuals are risk averse.

*We are grateful for helpful comments by members of the Workshop in Money and Banking of the University of Chicago, particularly Robert E. Lucas, Jr., Edward C. Prescott, and Frederic Mishkin; and by participants in the MIT Finance Workshop, especially Robert Merton, Fischer Black, and Robert Shiller. After circulating this paper in March 1980, we learned that Sanford J. Grossman and Robert J. Shiller were working along parallel lines—see Grossman and Shiller (1981). Also Ron Michener has independently derived similar results. This paper draws heavily on LaCivita (1981), where the reader may find a more complete treatment of the material summarized here.

(Journal of Business, 1981, vol. 54, no.4)
© 1981 by The University of Chicago
0021-9398/81/5404-0003\$01.50

long-term interest rates are far too volatile to be consistent with the hypothesis. LeRoy and Porter (1981) concluded that the actual coefficient of dispersion of stock prices was almost 10 times greater than its theoretical upper bound. Shiller (1979) showed that one-period holding yields were on the order of three times greater than their theoretical upper bound, and Shiller (1981) estimated that the standard deviation of stock prices was 6–12 times greater than its upper bound. Singleton (1980) rejected the hypothesis that the variance of actual long-term bond yields is less than its theoretical upper bound at the 5% level and rejected a similar restriction on holding period yields at the 1% level.

These results on asset price dispersion are troublesome because they conflict so sharply with the bulk of the evidence on market efficiency. Although the tests are subject to econometric and data problems, as we and the authors cited above have been reminded by proponents of efficient capital market models, it is hard to believe that these difficulties could account for such consistent and dramatic rejection of the implications of the model. We are led to reexamine the theory underlying the tests. Efficient markets tests, both those reported above and most or all others, are joint tests of three assumptions: (1) stationarity, (2) rational expectations, and (3) an "expected-value" model. Each element of the joint hypothesis may be challenged. The stationarity assumption requires that probability distributions be specified for all variables, that these probability distributions be time invariant, and that autocorrelations approach zero as the lag increases. The first requirement is merely a convention on modeling, but the others are not innocuous. There is some casual evidence that the risk premiums on stock (assume for the purpose of discussion that we know what this means) underwent an unforeseen but pronounced decline in the first 20 years of the postwar period. Similar informal considerations suggest that structural changes may have occurred in the bond market: Can it possibly be assumed that interest rates in the noninflationary 1950s and the inflationary 1970s are drawn from a common population?

The rational expectations assumption may also be questioned. Changes in stock prices are fairly strongly correlated with those of dividends, implying that a model in which stock prices are explained by dividends but in which the coefficients are not constrained by the assumption of rational expectations will account for a fairly high proportion of the variation in stock prices. An apparently plausible inference is that investors have information which is relevant for forecasting future dividends but are grossly overreacting to this information. Such overreaction would be explained by the hypothesis that investors have an extremely exaggerated estimate of the degree of persistence of dividend changes (as, indeed, Keynes [1936] believed). But even though such informal argumentation suggests that the assumptions of

stationarity and rational expectations are not inconsequential, economic modeling and hypothesis testing become much more difficult, both for philosophical and scientific reasons, when they are relaxed. Consequently, we prefer to maintain these assumptions and concentrate our critical attention on the third element of the joint hypothesis, the assumption of expectations models of asset prices.

It is known that in exchange models with unrestricted probability distributions of endowments the present-value equation for stock prices is valid only under risk neutrality (LeRoy 1971, 1973).¹ Similarly, LeRoy (1980*b*) showed that the expectations hypothesis of the term structure of interest rates is a derivable consequence of what was called "near risk neutrality." The question arises whether the LeRoy-Porter-Shiller restrictions on asset price dispersion can still be derived when risk neutrality is not zero or vanishingly small, or whether, on the contrary, there is some positive connection between risk aversion and asset price volatility. In this paper it is shown that the latter is the case. Our model is a special case of Lucas's (1978) recursive equilibrium model: instead of assuming a continuum of possible states, as Lucas did, we specify that only two states can occur. At each date the state of the world is then either good or bad, depending on the size of the endowment. As in Lucas, utilities are assumed to be additively separable and probabilities to follow a Markov process. Under these assumptions it is evident that for given probabilities, contingent endowments and discount factor, the price of stock may be written as a function of the state and a measure of the degree of risk aversion. This characterization makes it particularly easy to investigate the connection between risk aversion and stock price volatility.

We show that the price of stock in the good state is an increasing linear function of the risk-aversion parameter, therefore rising without bound for very high risk aversion. The price of stock in the bad state is a hyperbola, decreasing toward a lower asymptote as risk aversion increases. Subject to the proviso that the price of stock in the good state is higher than that in the bad state (this is not always true when risk aversion is low; see below) it follows that stock-price volatility unambiguously increases with risk aversion. Independently of the qualification it is proved that for any endowment and probabilities the coefficient of dispersion of stock prices is less (greater) than that of the endowment if the Arrow-Pratt measure of relative risk aversion is less (greater) than one (this is a generalization of LeRoy-Porter's theorem 1).

Why should the volatility of stock prices be higher under risk aversion than under risk neutrality? There are two ways to see the connec-

1. Under near risk neutrality, stocks in the aggregate will still obey the LeRoy-Porter restrictions, but those of individual stocks with very stable dividends may not; see LeRoy (1980*b*).

tion. First, suppose that a risk-averse individual faced the prices appropriate to a world of risk neutrality. He would attempt to smooth his consumption stream over time by buying stocks in the good state (saving) and selling them in the bad state (dissaving). But in a setting of exchange, individuals in the aggregate cannot smooth their consumption streams in this way, and securities prices must be such as to induce individuals instead to consume their endowment in either state. To induce a risk-averse individual to consume his endowment in the good state instead of saving part of it and buying stock, the price of stock must be higher in the good state and lower in the bad state than would be the case if he were risk neutral, since the prospect of a major capital loss if the good state is followed by the bad state would then offset the risk-averse individual's desire to smooth his consumption over time. Similar reasoning applies to the analysis of the price of stock in the bad state.

To see the result another way, observe that stock prices at any state and date are equal to the summed value of all the contingent claims to consumption in states of the world that can occur given the initial realization, measured in units of consumption at the assumed state and date (this specification will be made precise below). In a world composed of individuals who are very risk averse, the Arrow-Debreu prices of consumption in the good state will be low relative to those of consumption in the bad state. With the denominator of the ratio low, the price of stock will be high in the good state. Near the limit, almost the entire value of the stock derives from the fact that it provides a contingent claim on consumption in the bad state; these claims, measured in units of the almost worthless claims to consumption in the good state, therefore trade at a very high price. For the same reason, the value of stock in the bad state measured in units of consumption in the bad state decreases toward a lower asymptote as risk aversion rises, the claims to consumption in the good state being of decreasing relative importance in determining the value of the stock. In the limit, the claims to consumption in the good state become irrelevant to the determination of stock prices in the bad state, explaining the convergence toward an asymptote.

The analysis of bond interest rates is somewhat different. Let us first consider the interest rate on a discount bond for differing degrees of risk aversion, holding constant the maturity of the bond under consideration. Under risk neutrality the contingent interest rates are the same in both states, equaling the interest rate implicit in the constant at which utilities are discounted. The interest rate in the good state declines with risk aversion, becoming negative and eventually approaching -100% as risk aversion increases. The interest rate in the bad state increases toward an upper asymptote as risk aversion rises. The economic reasoning is the same here as for stocks: If risk-averse

individuals are faced with the interest rates appropriate to risk neutrality (i.e., if the interest rate is the same in either state), in the good state they will want to lend in order to be able to increase consumption if the bad state subsequently occurs. To induce them instead to consume their endowments, interest rates in the good state must be lower under risk aversion than under risk neutrality. Turning to the analysis of the term structure of interest rates for given risk aversion, it will be shown that the interest rate in the good state rises asymptotically with maturity toward the certainty interest rate, while the interest rate in the bad state decreases asymptotically toward the same limit. Since interest rates approach their limit geometrically, it follows that the relative volatility of interest rates on different maturities depends only on the maturities and not on the degree of risk aversion. Thus the existence of risk aversion does not impair Shiller's result that long-term interest rates should be less volatile than short rates. However, the applicability of this conclusion to Shiller's tests is not clear since our results apply to real interest rates, whereas Shiller analyzed nominal rates.

In the following section we set out the model. Since, as has already been suggested, it is useful to employ both the recursive and the state-preference approaches in deriving and interpreting our results, we provide an interpretation of the former in terms of the latter. In the third section the volatility of stock prices is examined, while in the fourth section our attention shifts to the term structure of interest rates. In the final section we offer concluding observations.

II. Recursive and State-Preference Representations of Equilibrium²

We will first develop the recursive equilibrium representation of stock prices. Suppose that all individuals have identical endowments and identical additively separable utilities of the form

$$EU(\{x_t\}) = E\left[\sum_t \beta^t V(x_t)\right], \quad (1)$$

where x_t is consumption at date t , and β is the discount rate. Assume that probabilities are Markov, so that the probability distribution of the state of the world at date $t + j$ given the realization up to date t depends only on the state at date t and not on the state at earlier dates. Under these conditions, the price of stock at any date will depend only on the state of the world at that date. This symmetry between the present and future distributions of stock prices, plus the requirement that at

2. The development of recursive equilibrium theory is summarized very briefly here. For more extended discussion the reader is referred to LeRoy (1971), (1973), (1980a); Lucas (1972), (1978); Mehra and Prescott (1979); and Prescott and Mehra (1980). Lucas's (1978) paper in particular provides an excellent discussion of a general version of the model adopted here.

equilibrium prices a representative individual will not change expected discounted utility by transferring a small amount of consumption across time via a sale and subsequent repurchase of stock (or vice versa), allows derivation of equilibrium prices. If one share of stock is sold at date t and the proceeds consumed, utility at date t goes up by pV' , where p is the stock price appropriate to the initial state and V' is the marginal utility of consumption evaluated at the endowment at that state. On the other hand, at date $t + 1$ the utility of consumption must drop by $(x' + \tilde{p})\tilde{V}'$, which, of course, depends on the state that occurs at date $t + 1$. But in equilibrium the discounted expected value of the utility loss at date $t + 1$ must equal the gain at date t , so we have

$$pV' = \beta E (\tilde{x} + \tilde{p})\tilde{V}',$$

where the expectation is taken over the distribution of the next-period state-of-the-world conditional on the current state. This is Lucas's (1978) equation 6 and is essentially identical to LeRoy's (1973) equation 8. In the two-state version, these equations take the agreeably simple form

$$\begin{aligned} p_g V'_g &= \beta [(p_g + \bar{x}_g)\pi(g|g)V'_g + (p_b + \bar{x}_b)\pi(b|g)V'_b], \\ p_b V'_b &= \beta [(p_g + \bar{x}_g)\pi(g|b)V'_g + (p_b + \bar{x}_b)\pi(b|b)V'_b], \end{aligned} \quad (2)$$

which will be the basis of our analysis of stock prices. Here p_g is the price of stock in the good state, $\pi(g|b)$ is the probability that the system goes to the good state given that it was in the bad state at the preceding date, $\bar{x}_g$ is the endowment in the good state ($> \bar{x}_b$), and similarly for the other variables. It will be convenient to express p_g and p_b as functions of the ratio of marginal utilities of consumption in either state, since this parameter provides a measure of the degree of risk aversion relative to existing social risk.³ If we define λ as V'_b/V'_g , equations (2) become

$$\begin{aligned} p_g &= \beta [(p_g + \bar{x}_g)\pi(g|g) + (p_b + \bar{x}_b)\pi(b|g)\lambda], \\ p_b &= \beta [(p_g + \bar{x}_g)\pi(g|b)/\lambda + (p_b + \bar{x}_b)\pi(b|b)]. \end{aligned}$$

The simplest derivation of expressions for interest rates analogous to those just presented for stock prices involves a construction using the underlying Arrow-Debreu prices. Consequently, it is appropriate to defer their derivation until after we have presented a state-preference

3. If

$$V(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ \ln(x) & \gamma = 1 \end{cases},$$

where γ is the Arrow-Pratt measure of relative risk aversion, then $\lambda = V'_b/V'_g = (\bar{x}_g/\bar{x}_b)^\gamma$.

interpretation of the recursive equilibrium model. In doing so, it is convenient (but not necessary) to assume that there exist initial and terminal dates, since then all realizations of the system which are possible have positive probability. Since initial conditions will be ignored, our treatment implicitly assumes that the state of the world at the first date is drawn from the limiting state distribution. Given this understanding, the assumption that there exist only a finite number of dates does not affect any variable of interest due to the assumptions of exchange, Markov probabilities, and additively separable utilities.

Some matters of notation need to be set out if we are to obtain a compact representation of equilibrium prices. Let $\tilde{i}_t$ be the state of the world at date t . We will distinguish between $\tilde{i}_t$, the random variable, and i_t , the value it takes on (so that $i_t = g$ or b in the two-state version). Also, we will denote by $\tilde{i}_t$ the ordered t -tuple $(\tilde{i}_1, \tilde{i}_2, \dots, \tilde{i}_t)$, and similarly for i_t . Thus $\tilde{i}_t$ is the random variable representing the entire evolution of the system up to date t , and $\{i_t\}$ is the set of realizations of $\tilde{i}_t$. If the first t elements of i_τ are denoted by i_t ($t \leq \tau$), then i_t will be referred to as a subhistory of i_τ . Finally, denote by I_t the set of all i_t . Using this notation, the prices of the Arrow-Debreu securities may now be defined and analyzed. Define $q_t(i_t)$ to be the price of one unit of consumption at date t in the event that $\tilde{i}_t = i_t$, and zero units otherwise. In the usual manner, the equilibrium values of the $q_t(i_t)$ will be the support of the endowment. We now derive expressions for the $q_t(i_t)$, temporarily assuming a general utility function and probability distribution (i.e., deferring the imposition of additively separable utilities and Markov probabilities). Under the expected utility hypothesis individuals may be viewed as choosing contingent consumptions $x_t(i_t)$ to maximize

$$\sum_{i_T \in I_T} \text{prob}(i_T) U[x_1(i_1), \dots, x_T(i_T)],$$

where the i_t are all the subhistories of each i_T . The expression $\text{prob}(i_T)$ denotes the probability that $\tilde{i}_T$ equals i_T . The constraint that the value of the consumption set equals that of the endowment may be written

$$\sum_{i_T \in I_T} q_t(i_t) [x_t(i_t) - \bar{x}(i_t)] = 0,$$

where, again, the i_t are the T subhistories of i_T . The contingent endowments $\bar{x}(i_t)$ are just x_g for the good state and $\bar{x}_b$ for the bad state. In competitive equilibrium the Arrow-Debreu prices are given by

$$q_t(i_t) = \sum \text{prob}(i_T) U_t[\bar{x}_1(i_1), \dots, \bar{x}_T(i_T)], \quad (3)$$

where the summation is over all the i_T for which i_t is a subhistory, and U_t denotes the partial derivative of U with respect to its t th argument.

In the argument of U_t , the $\underline{i}_\tau$ are the subhistories of $\underline{i}_t$ for $\tau \leq t$, and $\underline{i}_t$ is a subhistory of $\underline{i}_\tau$ for $\tau \geq t$. Here the Lagrange multiplier has been set equal to unity as a numeraire choice.

The contingent claims prices determined by (3) may now be used to define the prices and yields on real-world securities. Let us begin with stocks. The aggregate value of stock at a given date and state may be identified as the total value of future endowments measured in units of the endowment at the assumed date and state (the Miller-Modigliani theorem guarantees that nothing of substance is lost if we ignore bonds). That is, we have that $p_t(\underline{i}_t)$, the aggregate value of stock at date t if $\tilde{\underline{i}}_t = \underline{i}_t$, is given by

$$p_t(\underline{i}_t) = \frac{\sum_{\tau} q_{\tau}(\underline{i}_{\tau}) \bar{x}(\underline{i}_{\tau})}{q_t(\underline{i}_t)}, \quad (4)$$

where the summation is over all $\underline{i}_{\tau}$ ($\tau > t$), such that $\underline{i}_t$ is a subhistory of $\underline{i}_{\tau}$ (the strong inequality reflects an arbitrary convention that stocks are valued exdividends).

The assumption of additively separable utilities may now be used to simplify (3) and (4). First, from (1) we have that $U_t(\dots) = \beta^t V'_t$ where, of course, V'_t is evaluated at the endowment. Since V'_t depends only on $\underline{i}_t$, we may sum over the $\underline{i}_{\tau}$ for $\tau > t$ in (3). We obtain

$$q_t(\underline{i}_t) = \text{prob}(\underline{i}_t) \beta^t V'_t, \quad (5)$$

where $\text{prob}(\underline{i}_t)$ denotes the sum of the probabilities of all the $\underline{i}_{\tau}$ for which $\underline{i}_t$ is a subhistory. Using (5), (4) becomes

$$p_t(\underline{i}_t) = \frac{\sum_{\tau} \text{prob}(\underline{i}_{\tau}) \beta^{\tau} V'_{\tau} \bar{x}(\underline{i}_{\tau})}{\text{prob}(\underline{i}_t) \beta^t V'_t},$$

or, equivalently,

$$p_t(\underline{i}_t) = \sum_{\tau} \text{prob}(\underline{i}_{\tau} | \underline{i}_t) \beta^{\tau-t} \bar{x}(\underline{i}_{\tau}) V'_{\tau} / V'_t,$$

where, again, the summation is over all the $\underline{i}_{\tau}$ for which $\underline{i}_t$ is a subhistory.

The specification of Markov probabilities allows further simplification. We have that $\text{prob}(\underline{i}_{\tau} | \underline{i}_t)$ depends only on $\underline{i}_t$, not on $\underline{i}_{t-1}$. This fact allows us to write the $p_t(\underline{i}_t)$ as $p_t(\underline{i}_t)$, so we arrive at

$$p_t(\underline{i}_t) = \sum \text{prob}(\underline{i}_{\tau} | \underline{i}_t) \beta^{\tau-t} \bar{x}(\underline{i}_{\tau}) V'_{\tau} / V'_t. \quad (6)$$

Since $\text{prob}(\underline{i}_{\tau} | \underline{i}_t) = \text{prob}(\underline{i}_{\tau} | \underline{i}_t)$ for all values of $\underline{i}_{t-1}$, this probability can be interpreted indifferently as a probability conditional on $\underline{i}_{t-1}$ or as a marginal probability (with the $\underline{i}_{t-1}$ integrated out). By stationarity, the prices $p_t(\underline{i}_t)$ do not depend on the date; we acknowledge this fact by deleting the time subscript.

Now we are able to derive the recursive equilibrium equations (2).

Let us begin by breaking out the values of the endowment at $t + 1$ from those at $t + 2, t + 3$, etc., in (6). Doing this and multiplying both sides of (6) by V'_t , we obtain

$$p(i_t)V'_t = \beta \sum_{i_{t+1}} \text{prob}(i_{t+1} | i_t) [\bar{x}(i_{t+1})V'_{t+1}] + \sum_{i_\tau} \beta^{\tau-t-1} \text{prob}(i_\tau | i_{t+1}) \bar{x}(i_\tau)V'_\tau, \quad (7)$$

where the second summation is over i_τ such that $\tau > t + 1$. The term

$$\sum_{i_\tau} \beta^{\tau-t-1} \text{prob}(i_\tau | i_{t+1}) \bar{x}(i_\tau)V'_\tau$$

is recognized as just $p(i_{t+1})V'_{t+1}$. Reverting to the two-state notation, (7) therefore becomes

$$p_g V'_g = \beta[(p_g + \bar{x}_g)\pi(g | g)V'_g + (p_b + \bar{x}_b)\pi(b | g)V'_b],$$

$$p_b V'_b = \beta[(p_g + \bar{x}_g)\pi(g | b)V'_g + (p_b + \bar{x}_b)\pi(b | b)V'_b],$$

coinciding with (2).

The equations (3) for the Arrow-Debreu prices may now be used to derive expressions for the contingent interest rates on pure discount bonds. The k -period interest rate at date t is the rate at which consumption at date t can be exchanged for consumption at date $t + k$ independently of the realization of the state of the world between dates t and $t + k$. It depends, of course, on the realization of the state of the world at date t . The contingent interest rates may be written

$$[1 + r_k(i_t)]^k = \frac{q_t(i_t)}{\sum q_{t+k}(i_{t+k})}, \quad (8)$$

where $r_k(i_t)$ is the k -period interest rate at date t and the summation is over all i_{t+k} for which i_t is a subhistory, for k fixed. This expression is analogous to those of LeRoy (1980b) and Woodward (1980), to which the reader is referred for further discussion. Passing immediately to the two-state version and imposing the restrictions of additively separable utilities and Markov probabilities, we may verify that expressions (8) reduce to

$$[1 + r_g^k]^k = \beta^{-k}[\pi_k(g | g) + \pi_k(b | g)\lambda]^{-1},$$

$$[1 + r_b^k]^k = \beta^{-k}[\pi_k(g | b)/\lambda + \pi_k(b | b)]^{-1}, \quad (9)$$

using (5). Here $\pi_k(b | g)$ is the probability that the state of the world will be bad at date $t + k$ given that it is good at date t .

III. Risk Aversion and Stock Prices

Equipped with equations (2), it is easy to demonstrate the relation between equilibrium stock prices and risk aversion. As has become de rigueur in recursive equilibrium analyses of stock prices (LeRoy 1971, 1973; Lucas 1978, Prescott and Mehra 1980), we begin by noting that under risk neutrality ($\lambda = 1$) stock prices plus cumulated dividends follow a martingale with drift equal to $(1 - \beta)/\beta$.

Under risk aversion, equations (2) may be solved for p_g and p_b as functions of λ :

$$p_g = \frac{\beta \bar{x}_g \{ \pi(g|g)(1 - \beta) + \beta[1 - \pi(b|b)] \} + \beta \bar{x}_b [1 - \pi(g|g)] \lambda}{(1 - \beta) \{ 1 + \beta - \beta[\pi(g|g) + \pi(b|b)] \}},$$

$$p_b = \frac{\beta \bar{x}_g \pi(g|b)/\lambda + \beta \bar{x}_b [\beta \pi(b|g) + \pi(b|b)(1 - \beta)]}{(1 - \beta) \{ 1 + \beta - \beta[\pi(g|g) + \pi(b|b)] \}}. \quad (10)$$

These equations show that, as indicated in the introduction, p_g rises linearly to infinity with risk aversion, and p_b decreases toward an asymptote. An interesting intermediate case occurs for $\lambda = \bar{x}_g/\bar{x}_b$, implying that the Arrow-Pratt measure of relative risk aversion is unity (see n. 3 above). In this case, stock prices are exactly proportional to the endowment for any specification of transition probabilities. To verify this, substitute $\lambda = \bar{x}_g/\bar{x}_b$ in (10) and simplify to arrive at $p_g = (\beta \bar{x}_g)/(1 - \beta)$ and $p_b = (\beta \bar{x}_b)/(1 - \beta)$. The interpretation is straightforward. For the assumed value of λ , the marginal utility of consumption declines in inverse proportion to the endowment. Since $V'_\tau \bar{x}(i_\tau)$ is the same for either state, the value relative to a common numeraire of the consumption to which stock is title is the same in both states. To calculate the value of stock in either state measured in units of consumption of that state, however, one must then divide by the Arrow-Debreu price of consumption in that state. But these prices are in inverse proportion to the endowments, so the result is proved.

If, for unit relative risk aversion, stock prices are proportional to the endowments, the coefficient of dispersion of stock prices is exactly equal to that of the endowments. Further, since p_g (p_b) increases (decreases) with risk aversion, it is clear that, if the Arrow-Pratt relative risk-aversion parameter exceeds unity, the coefficient of dispersion of prices exceeds that of the endowment. Finally, the fact that under risk neutrality the coefficient of dispersion of prices is less than that of the endowment (LeRoy and Porter 1981, theorem 1) together with the monotonicity of p_g and p_b in λ , guarantees that the coefficient of dispersion of prices is less than that of the endowment for any value of the relative risk-aversion parameter less than unity.

It is worth noting that if the relative risk aversion parameter is less

than unity, there is no assurance that p_g is greater than p_b . Let us see why the reverse might be the case. Under risk neutrality, the difference between p_g and p_b is given by

$$p_g - p_b = \frac{\beta(\bar{x}_g - \bar{x}_b)[\pi(g|g) + \pi(b|b) - 1]}{\{1 + \beta - \beta[\pi(g|g) + \pi(b|b)]\}}, \quad (11)$$

as may be seen by setting $\lambda = 1$ in (10), subtracting and simplifying. But (11) guarantees that $p_g > p_b$ if and only if $\pi(g|g) + \pi(b|b) > 1$. In informal terms, this assumption requires that the state realization be positively autocorrelated over time; in the reverse case the price of stock in the bad state may exceed that of the good state. Two simple examples will make the connection clearer. If $\pi(g|g) + \pi(b|b) = 1$, the conditional state probabilities are equal to the unconditional probabilities ($\pi[g|g] = \pi[g]$ and $\pi[b|b] = \pi[b]$), and therefore are independent of the initial state. Consequently, the initial state is irrelevant for the pricing of stock.

Suppose, on the other hand, that $\pi(g|g) = \pi(b|b) = 0$, so that the system changes state at each date with certainty. Then, from (11),

$$p_g - p_b = \frac{\beta(\bar{x}_b - \bar{x}_g)}{1 + \beta},$$

so that $p_b > p_g$. Intuitively, stock in the bad state is worth more than stock in the good state because in the former case the state is certain to be good at the date when the first dividend is paid (i.e., at the next date), whereas in the latter case the first dividend is certain to occur in the bad state.

IV. Risk Aversion and the Term Structure of Interest Rates

We have already seen that the k -period interest rates on discount bonds are given by

$$\begin{aligned} (1 + r_g^k)^k &= \beta^{-k}[\pi_k(g|g) + \pi_k(b|g)\lambda]^{-1}, \\ (1 + r_b^k)^k &= \beta^{-k}[\pi_k(g|b)/\lambda + \pi_k(b|b)]^{-1} \end{aligned} \quad (12)$$

(eq. [9]). From (12) it is immediate that under risk neutrality we have

$$(1 + r_g^k)^k = (1 + r_b^k)^k = \beta^{-k},$$

so that interest rates are equal in the two states. The result that, in an exchange economy, interest rates are nonrandom under risk neutrality has been derived and commented on elsewhere (LeRoy 1980b). Further, r_g^k is a decreasing function of λ and r_b^k an increasing function of λ , so that greater risk aversion is unambiguously associated with greater volatility in interest rates. However, (12) also shows that long rates are less volatile than short rates for any degree of risk aversion. Con-

sequently, the presence of risk aversion does not distort the volatility implications of the expectations hypothesis of the term structure of interest rates in the same drastic manner as it biases those of the martingale model of stock prices.

V. Conclusions

Our principal conclusions are that the validity of the theoretical results on stock prices of LeRoy and Porter is limited to the case of low risk aversion and that the inequalities are reversed for high risk aversion. Since there is no reason to presume that the degree of risk aversion is small enough relative to existing social risk to justify LeRoy-Porter's model, their empirical results do not constitute conclusive evidence against market efficiency or, in particular, stationarity or rational expectations. Indeed, the LeRoy-Porter results may instead be interpreted as evidence that the degree of risk aversion is in fact such that the present-value martingale model is an exceedingly bad approximation, at least in its implications for asset price volatility. This, of course, is not to deny that martingale models may provide useful approximations for many purposes; all we are asserting here is that they are inappropriate vehicles for investigating asset price dispersion. Nor have we denied that asset prices may in fact be too volatile to be consistent with market efficiency. We have, however, shown that LeRoy and Porter's results are without clear interpretation in the absence of a correction for increased volatility expected due to risk aversion.

There remains the empirical question of whether the existing degree of risk aversion is sufficient to account for the observed dispersion in asset prices. To shed light on this question one might compute the volatility of asset prices implied by existing quantitative measures of risk aversion and compare this figure with the corresponding measured volatility. Or, following Grossman and Shiller (1981), one could compute a measure of risk aversion directly from the observed volatility of asset prices and compare this estimate with measures derived by other means. However, our view is that such exercises are subject to a fundamental difficulty. All measures of risk aversion are computed from asset prices; consequently, it seems to us that if consistent methods are used the conclusion can only be that the estimated degree of risk aversion will inevitably be sufficient to account for the estimated volatility of asset prices. If the reverse conclusion is obtained, our presumption would be that some misspecification or econometric problem had biased the comparison, not that asset prices are too volatile to be consistent with measured risk aversion.

References

- Grossman, S. J., and Shiller, R. J. 1981. The determinants of the variability of stock market prices. *American Economic Review* 71, no. 2 (May): 222-27.
- Huang, R. D. 1981. Efficiency and the volatility of floating exchange rates. *Journal of Finance* 36, no. 1 (March): 31-41.
- Keynes, J. M. 1936. *General Theory of Employment Interest and Money*. New York: Harcourt, Brace & World.
- LaCivita, C. J. 1981. The dispersion of asset prices in a general equilibrium model with risk-aversion. Ph.D. dissertation, University of California, Santa Barbara.
- LeRoy, S. F. 1971. The determination of stock prices. Ph.D. dissertation, University of Pennsylvania.
- LeRoy, S. F. 1973. Risk-aversion and the martingale property of stock prices. *International Economic Review* 14, no. 2 (June): 436-46.
- LeRoy, S. F. 1980a. Expectations models of asset prices: a survey of theory. Photocopied. Center for Mathematical Studies in Business and Economics. Chicago: University of Chicago.
- LeRoy, S. F. 1980b. Securities prices under risk-neutrality and near risk-neutrality. Photocopied. Report 7956, Center for Mathematical Studies in Business and Economics. Chicago: University of Chicago.
- LeRoy, S. F., and Porter, R. D. 1981. The present value relation: tests based on implied variance bounds. *Econometrica* 49, no. 3 (May): 555-74.
- Lucas, R. E., Jr. 1972. Expectations and the neutrality of money. *Journal of Economic Theory* 4, no. 2 (April): 103-24.
- Lucas, R. E., Jr. 1978. Asset prices in an exchange economy. *Econometrica* 46, no. 6 (November): 1429-45.
- Mehra, R., and Prescott, E. C. 1979. Multiperiod asset pricing in a recursive general equilibrium setting. Photocopied. Chicago: University of Chicago.
- Prescott, E. C., and Mehra, R. 1980. Recursive competitive equilibrium: the case of homogeneous households. *Econometrica* 48, no. 6 (September): 1365-79.
- Shiller, R. J. 1979. The volatility of long-term interest rates and expectations models of the term structure. *Journal of Political Economy* 87, no. 6 (December): 1190-1219.
- Shiller, R. J. 1981. Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review* 71, no. 3 (June): 421-36.
- Singleton, K. J. 1980. Expectations models of the term structure and implied variance bounds. *Journal of Political Economy* 88, no. 6 (December): 1159-76.
- Woodward, S. E. 1980. The liquidity premium and the solidity premium. Working Paper no. 183, University of California, Los Angeles.

Copyright of Journal of Business is the property of University of Chicago Press and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.