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On the Efficiency of the Markets for Gold and Silver

Gold and silver are precious metals that have come to be viewed as possible investment assets. From the investor's point of view, it is important to know that gold and silver prices reflect the relevant information set at every point in time. A sequence of price changes for each metal is analyzed in this paper in order to establish if these markets are efficient in this sense. The distributions generating the price changes appear to exhibit characteristics of nonnormality and nonstationarity in the means and variances. There appears to be some positive dependence in the series, though it does not seem that investors can easily exploit this dependence. These results have important implications for the role of gold and silver as investment assets.

I. Introduction

Gold and silver have wide-ranging utility, from their use in art and jewelry to their use in the production of electronic equipment. The holding of gold and silver has always been considered a great treasure. Within the last decade a new use has developed for these metals: Individual investors and portfolio managers have recently begun to view them as possible investment assets. This paper examines the workings of the gold and silver markets from the investor's point of view. The prices of goods are important in their allocation among competing uses, and the same is true for gold and silver. Investors use the prices of the metals in making buy-and-sell decisions; however, the price changes for gold and silver permit these markets to be analyzed. Current theory maintains that market-determined changes in price-of-investment assets result from a changing set of information and that, given a competitive market system, the price changes tend to move the markets toward equilibrium. This is the essence of the efficient-markets theory. The purpose of this paper is to analyze the price changes for gold and silver to establish if the efficient-markets theory can be applied to the markets for these metals.

This paper is divided into four sections and a conclusion. The relationship between the gold and silver markets and market efficiency is ex-

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plained in Section II, and background on these markets is also provided. Evidence regarding the properties of the distributions generating the price and price change series is presented in Section III, while the tests which examine the dependencies between successive price changes are discussed in Section IV. The final section offers concluding comments concerning both the efficiency of the markets for these metals and the use of gold and silver as investment assets.

II. Trading in Gold and Silver

Gold and silver have each at different times been designated as an official medium of exchange. During these periods the prices of the metals have been determined by fiat. Since 1971, when the U.S. government suspended the convertibility of dollars into gold even for official transactions, gold has been traded freely in the commodity markets; silver has been traded as such for most of this century. This has added a new dimension to their function as stores of value since the prices of the metals are now determined in the marketplace. Figures 1 and 2 show that the markets have placed an increasing value on both metals over the last decade. There is considerable interest in holding gold and silver as investments in stores of value, and this makes gold

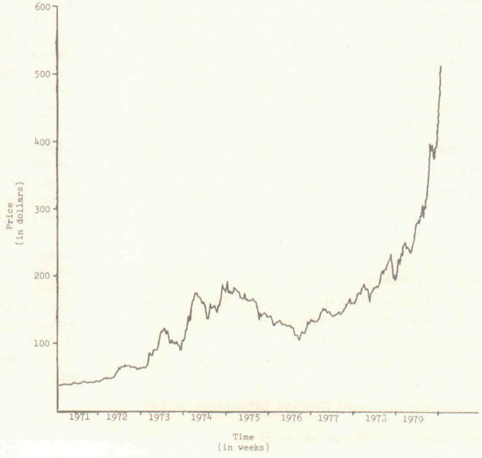

FIG. 1.—Weekly gold prices for the period 1971–79

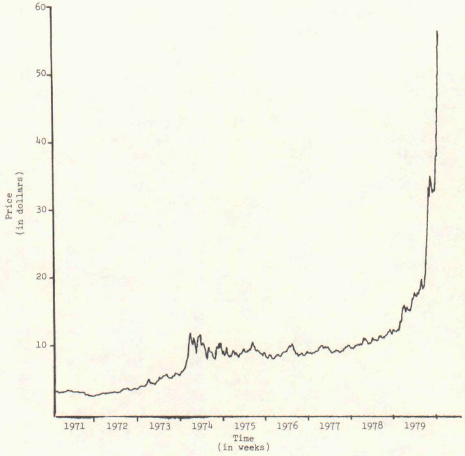

FIG. 2.—Weekly silver prices for the period 1971–79

and silver tangible substitutes for financial assets. Table 1 provides evidence indicating that the metals would have been outstanding investments over the decade. For almost any period between 1971 and 1979, the holding of gold and silver would have provided high annual rates of return, particularly for those holding periods that include 1979. The variability of the prices of the metals has been substantial, as figures 1 and 2 reveal, and the presence of some negative returns in table 1 indicate that these investments are not without risk. Trading in gold and silver is similar to trading financial assets, in that buy-and-sell decisions at a point in time are based on expectations concerning uncertain future outcomes.

Although there are similarities between trading in gold and silver and trading in financial assets, there is a difference, as financial assets represent claims on income streams while gold and silver are real assets that embody value. However, this distinction does not necessarily affect the behavior of the prices of the metals, since both the metals and the financial assets are "worth" only that for which they can be bought or sold. Prices are the key factors in allocating the supply of goods in competitive markets, and the prices of gold and silver can be said to allocate value in such a manner. If these metals are to be treated as investment assets, it is necessary to consider them in a

TABLE 1 **Average Annual Geometric Rates of Return from the Holding of Gold and Silver over Different Time Periods**

Beginning of Year	End of Year 1971	1972	1973	1974	1975	1976	1977	1978	1979
					A. Gold Returns				
1971	.189	.276	.392	.386	.264	.214	.212	.225	.291
1972		.362	.493	.451	.283	.218	.216	.230	.303
1973			.624	.500	.257	.183	.187	.208	.295
1974				.367	.073	.035	.077	.125	.240
1975					−.222	−.130	−.020	.064	.215
1976						−.040	.081	.159	.324
1977							.202	.258	.445
1978								.314	.556
1979									.818
					B. Silver Returns				
1971	−.173	.109	.229	.270	.191	.161	.153	.162	.315
1972		.391	.430	.417	.282	.228	.207	.209	.376
1973			.469	.430	.246	.188	.170	.179	.374
1974				.391	.135	.094	.096	.121	.358
1975					−.121	−.054	−.003	.053	.352
1976						.012	.056	.112	.470
1977							.101	.162	.623
1978								.222	.884
1979									1.540

risk-return framework. This requires an understanding of the process that generates the sequences of prices for the metals. One important aspect of this process to investors concerns how the markets work to establish gold and silver prices. Also of interest to investors is how new information is used by the markets to set new prices; this means that investors are also concerned with the sequences of price changes for the metals. If these markets are functioning properly in establishing prices and are efficient in processing information, then investors can evaluate the risks of holding the metals and make appropriate investment decisions.

The prices are set in a competitive situation, yet the sequences of price changes implied by figures 1 and 2 raise questions about the workings of these markets. The purpose of this paper is to evaluate these markets with respect to the investor's point of view by analyzing the price series generated over the 1971–79 period. This research proceeds by focusing on the question regarding market efficiency. According to Fama (1970), in an efficient market, all information available to the market is impounded in the current price, and new information, as it is produced, is rapidly and without bias reflected in the market price. The efficiency criterion provides a method of analysis.

The price changes for gold and silver are subjected to standard tests of market efficiency in order to evaluate the usefulness of the metals to investors. The concept of market efficiency is particularly apt for the gold and silver markets since neither metal bears any financial obligation; a nonzero return on investment can be realized from holding the metals only if price changes occur.

While it is of interest to analyze how information is used by the market in generating price changes, there is an implicit assumption made by this form of analysis concerning the workings of these economic markets. It is assumed here that an equilibrating market mechanism exists. No model of the price formation process for gold and silver is offered explicitly in this paper, but the results of the research will have implications regarding how these markets function. For example, finding the markets for metals to be efficient would imply that the current price is the best estimator of the next price. This would mean that the markets correctly use the available set of information in establishing prices; such a finding would provide pertinent knowledge to investors. Unfortunately, the results presented below are not consistent with what would be expected in perfectly efficient markets, yet the results shed light on the degree of efficiency of these markets and thus are of interest to current or potential investors. The structure of each market is briefly discussed in the next two subsections in order to provide an understanding of the price formation process. The important aspects of the market structures in relation to the topic of market efficiency are then considered in the third subsection.

The Gold Market

Official gold holdings by monetary institutions throughout the world remain substantial. According to Pick's Currency Yearbook (Pick 1978), official holdings at the end of 1976 were placed at nearly 1.25 billion ounces, and this represented almost two-thirds of the estimated total world gold stocks at that time. The world gold markets could not absorb the increase in supply without a severe price adjustment. However, even in the face of large sales of millions of ounces annually since 1976 by the International Monetary Fund (IMF) and the U.S. government, it seems that no permanent effect on gold prices has been produced. Central banks apparently see gold as having an important role in the scheme of things, since they are large purchasers at the IMF auctions. Some would have an expanded role for gold in the international monetary system, and this would eliminate the risk of sales of official gold stocks. However, Mundell (1973) states that when the price of gold is fixed and foreign exchange pressures are pointing to an increase in its price, gold is hampered in its ability to function as a reserve asset for central banks. This is because the marginal utility of gold increases relative to that for the currency in which it is defined.

This happened to the British pound in 1931 and to the U.S. dollar in 1971. Central banks are not indifferent to the composition of their international reserves, and Makin (1971) has shown that the changes in the composition of the international reserves of central banks behave as if the reserves were part of a portfolio that included gold. Central banks would be as willing to sell gold as to buy it, and they are likely to assess market conditions as they manage their reserves.

McDonald and Solnick (1977) report that the average monthly return to holding gold was nearly five times that for an investment in the Standard & Poor's 500 over the 1970–76 period, and the total risk, defined as the standard deviation of monthly returns, was also higher for gold. This is not surprising, but it is surprising that the correlation coefficient between the two return streams is $-.07$. Over the 1971–79 period this correlation coefficient is $-.11$. The diversification benefits are obvious, and standard portfolio theory offers an explanation for gold's high prices if indeed price is the adjusting mechanism. What is not explained by the theory is the sequences of both high prices and high returns observed in the last decade for an asset that yields no income. Part of the explanation is that gold serves as an effective hedge against inflation. McDonald and Solnick also report that the correlation between gold prices and inflation is positive from 1970 to 1976, and Jastram (1977) has shown that since 1970 gold actually increased in purchasing power. The same is true for the latter years of the decade, as table 1 suggests. It could be that speculation has caused the high prices and high returns. All that can be said is that over the last decade, gold has been a proven hedge against inflation, and the negative correlation of gold returns with those of the market means that the addition of gold holdings to a portfolio would have reduced the standard deviation of the portfolio's return.

The analogy between gold and commodities such as copper, porkbellies, and corn is easy to make. Gold metal is mined as are many other commodities; U.S. citizens since 1975 can buy or sell gold as if it were any commodity; and as active a futures market exists for gold as for many other commodities. Also, there is demand for gold as a raw material by the electronics industry, by dentistry, by governments for coinage, and by artists and jewelers. It is an exhaustible resource, and the costs of extraction dictate a floor price below which it is not economically feasible to mine gold. Demand for gold coins has grown steadily in the last decade, and in 1976 160 tons of gold were purchased internationally to mint almost 100 different coins. The major coins are at least 90% fine and, aside from coins that are valued as collector's items, maintain fairly consistent premiums for both buyers and sellers from the basic gold bar price. Investment in gold coins, as well as in 5-gram wafers, is a portable way of storing value and has allowed the small investor to have an impact on the market price of gold. The gold futures market allows investors to benefit from gold price movements without actually taking possession of any gold, and it allows the use of margin accounts. Many dealers will physically maintain investors' gold bullion for modest insurance and storage charges, and this allows the avoidance of assaying charges. The benefits of treating gold as a commodity for investors include the ability to buy and sell gold in amounts ranging from a few 1-ounce coins to 400-ounce gold bars without a large difference in the price per ounce.

The Silver Market

Central banks have been large holders of silver and still maintain large supplies. According to Handy and Harman (1979), the U.S. government in 1978 held 139.5 million ounces of silver, and other governments held 80.0 million ounces. These holdings of silver are not nearly as substantial at the present time as those for gold, yet the behavior of the governments with respect to the buying of gold and silver is interesting and has implications for the buying and selling of official gold. Governments have not hesitated to trade in silver. Over the period 1873–79, the German government sold large amounts of silver at the same time as mines in the United States were producing large amounts of the metal. According to Leavons (1939), the effect on the price of silver was substantial, and the action of the German official sales hastened the end of the silver standard. The U.S. government accommodated U.S. silver producers at various times through various coinage acts in the late nineteenth and early twentieth centuries. After World War I, the U.S. government sold silver to India in order to provide that country with a much needed currency, but in the 1920s and 1930s the U.S. government again purchased large amounts of silver. The Silver Purchase Act of 1934 was aimed at increasing the amount of the official U.S. holdings in silver, and by the end of 1938 official holdings had increased to nearly 1.7 billion ounces. In the 1960s this policy was changed, and the U.S. government began selling this large supply.

The above exemplifies the willingness of governments and central banks to trade silver. Most of the impetus to buy or sell is tied to the rise or fall of the role of silver in the monetary systems of the world. This serves to underscore the point that the sale of the large official stocks of gold represents a serious downside risk to the price of gold, just as it has in the past for silver.

It is possible to treat silver, like gold, strictly as a commodity. The mechanics of trading gold described above apply to silver also. Silver is used in electronics, dentistry, photography, and in other ways as an industrial commodity. Silver coins are still struck by various countries, though the production of gold coins is currently on a larger scale. However, the demand for silver far exceeds the production of new silver by a ratio of over two to one. Much silver is recycled, but large amounts of silver for industrial use must come from private stocks. Although no shortage of silver currently exists, this is a potential problem for traders in silver.

Implications for Market Efficiency

Two previous research efforts concerning the workings of the gold market are relevant here. Lipschitz and Otani (1977) set forth a model of the gold market that fits their data well. Abken (1980) provides evidence that gold price changes are consistent not with "mob psychology" but with economic rationale. In the Lipschitz and Otani model, the price of gold is made the dependent variable. They assume that since gold stocks are large, relative to annual flows, gold prices adjust such that investors (or speculators or hoarders) absorb any excess supply existing after commodity demand has been satisfied. In addition, they assume a portfolio balance model which relates the demand for gold as a financial asset to rates on other financial assets, the rate of inflation, and an expected price for gold. In an in-sample simulation, their model tracked the price of gold closely. Abken relates the expected price change in gold to the marginal cost of holding gold. He acknowledges that the stocks substantially outweigh gold flows but maintains that both flow supply and flow demand are relatively insensitive to gold price changes. He contends that the stock-flow relationship makes gold price changes sensitive, and he attributes gold price changes to changes in the expected price of gold. The implication is that new information is rapidly impounded in the price of gold. Finding that gold price changes are not related to past changes, Abken concludes that the gold market responds quickly to new information. As further support of his model of gold price movements, he offers evidence which shows that gold price changes are related to changes in the price of gold futures contracts.

These results provide justification for assuming that an equilibrating market mechanism exists, at least for gold. Although no equivalent prior research has been performed regarding the silver market, this assumption is applied to the silver market. As noted, the gold and silver markets possess similar components of demand and supply. (The parameters of any model of the gold and silver price formation processes are not likely to be equal; there is no requirement that they must be, though.) Also, the sample correlation coefficient between gold and silver price changes is .83. Silver returns show low correlation with the return on the market (the coefficient is .03), and table 1 shows that metal appears to be a hedge against inflation. Therefore, the assumption seems to be a reasonable one.

The null hypothesis being tested is that the markets for gold and silver are efficient, and this provides a benchmark for the testing. For each metal, the implications are that the current price is the best estimator of the next price, that the price changes conform to some distribution, and that the price change series contain no dependence. Sections III and IV below provide evidence regarding these implications. Since the information set being used is the set of historical prices, the tests conducted below are called weak form tests. This research extends the work of both Lipschitz and Otani (1977) and Abken (1980). The silver market was not tested in either of their papers, nor did the authors examine the properties of the price change series for gold. Also, this paper offers refinements to the prior works. Lipschitz and Otani use an adaptive expectations model to generate expected gold prices. This does not conform with the tenets of market efficiency; their paper cannot be said to consider market efficiency at all. Both papers report the use of regression analysis but neither establishes that the data fulfill the assumptions of the regression model. The tests below provide both an in-depth analysis of the efficiency of the markets for these metals and insights into the use of gold and silver as investment assets. Thus the results of this research will complement the existing literature regarding security price changes.

III. Analysis of the Distributions of Gold and Silver Price Changes

The tests performed here require the use of a price series for both gold and silver. The analysis covers the period of the last decade over which both gold and silver prices were determined by the market. The gold price series is taken from the 469 observations on the Friday P.M. London fixing price per ounce for gold bullion, for the period beginning January 1971 and ending December 1979. The silver series is obtained from the Friday closing price per ounce for silver bullion quoted by Handy and Harman for the same period. The London Gold Market establishes the London gold price and is a dealer rather than an auction market. Although the gold price is "fixed" in the morning and afternoon, market conditions still determine the price of gold. London has historically been the leading international financial and gold-trading center, and the quoted price for gold used here remains the international standard. Handy and Harman are dealers in silver bullion, and their price for silver is a recognized standard. Market conditions also determine the silver price. While gold and silver are not traded solely as bullion, the bullion markets are orderly markets and the prices for the metals obtained from these markets reflect the underlying supply and demand forces. The prices for gold and silver traded in other markets and in other forms are closely related to the bullion prices used here.

Because most classical statistical techniques in the analysis of time series require that the variables being tested each possess a constant, finite variance, it is appropriate at the outset to determine if the data are likely to reflect this. The strong price movements over the decade indicated by figures 1 and 2 suggest that the process generating these series might not be characterized by constant variance, or homoscedasticity. Although there is no test that proves the presence or absence of a nonconstant variance, or heteroscedasticity, the Goldfeld-Quandt test is well suited to provide insights regarding this problem. Essentially this test requires that the data be ordered according to some criterion and then be partitioned into three subsamples, with the central observations comprising a holdout subsample. The test statistic is obtained from the ratio of the sum of the squared residual values for the two remaining subsamples. If this ratio deviates substantially from unity, there is an indication that the variability of the process generating the observations has not remained constant. The implication of figures 1 and 2 is that, for both gold and silver, the logical way to order the variables is sequentially. The results of this test will provide information about the variability of the prices and price changes for the metals over time. Because the figures indicate that 1979 is the year of most extreme price movement, the results of the tests could be sensitive to the choice of the sample period. In order to allow for this, the tests for stationarity are conducted both with and without 1979.

It is not clear what the size of the holdout sample should be. Because the concern here is with the attributes of the process generating the series for each metal, the number of observations withheld is allowed to vary. Table 2 displays the results of the Goldfeld-Quandt tests for each price and price change series, both in dollars and in logarithms, for the different holdout periods. The first holdout period includes the most central 52 observations; this number is increased until the three subsamples have approximately the same number of observations. (Slightly more than one-third of the observations are omitted in the 1971–78 subperiod.) The values presented in the table are the ratio of the squared residuals from the later subsample divided by the squared residuals from the earlier subsample. The residuals for each series are defined as the deviations from the means of the subsample. These statistics possess an F-distribution with equal degrees of freedom in the numerator and the denominator. Because the alternative hypothesis (that the variance is nonconstant) is being tested against the null hypothesis (that it is constant), this is a two-tailed test. The degrees of freedom vary between 130 and 208. The corresponding critical values for the F-distribution at the .05 level of significance which indicate increasing variation range between 1.42 and 1.33, and those which indicate decreasing variation range between .70 and .75. At the .01 level of significance the critical F-values are 1.61–1.50 and .61–.67 (see Pearson and Hartley 1972).

TABLE 2 Goldfeld-Quandt Statistics for the Price and Price Change Series

A. Prices in Dollars

1971–79 Period			1971–78 Period		
Holdout Period	Gold Prices	Silver Prices	Holdout Period	Gold Prices	Silver Prices
1975.1–1975.4	2.89**	8.15**	1974.3–1975.2	.52**	.18**
1974.4–1976.1	3.59**	9.53**	1974.2–1975.3	.76	.32**
1974.3–1976.2	4.13**	11.03**	1974.1–1975.4	1.40*	1.04
1974.2–1976.3	5.52**	18.94**	1973.4–1976.1	1.64**	1.41*
1974.1–1976.2	9.45**	64.57**	1973.3–1974.2	2.25**	2.39**

B. Price Changes

1971–79 Period			1971–78 Period		
Holdout Period	Gold Price Changes	Silver Price Changes	Holdout Period	Gold Price Changes	Silver Price Changes
1. In Dollars					
1975.1–1975.4	3.14**	7.32**	1974.3–1975.2	1.48*	.52**
1974.4–1976.1	4.13**	9.18**	1974.2–1975.3	1.52*	.80
1974.3–1976.2	4.81**	11.38**	1974.1–1975.4	2.22**	2.91**
1974.2–1976.3	5.41**	19.84**	1973.4–1976.1	2.97**	3.32**
1974.1–1976.2	7.94**	80.55**	1973.3–1974.2	4.45**	3.74**
2. In Logarithms					
1975.1–1975.4	.66*	.61*	1974.3–1975.2	.59**	.35**
1974.1–1976.1	.71*	.71*	1974.2–1975.3	.55**	.42**
1974.3–1976.2	.75	.82	1974.1–1975.4	.62*	.57**
1974.2–1976.2	.72	1.03	1973.4–1976.1	.74	.58**
1974.1–1976.4	.78	1.60*	1973.3–1974.2	.93	.59**

NOTE.—For holdout period, the number after the decimal point refers to year preceding decimal point, e.g., 1975.1 refers to first quarter of 1975.
*Significant at .05 level for two-tailed test.
**Significant at .01 level for two-tailed test.

The results presented in parts A and B1 of table 2 show substantial increases in variability in the prices and in the simple difference price changes (in dollars) for each metal over the decade. Almost all the entries in these parts are significant at the .01 level, and some of the statistics are quite large, particularly for the longer holdout periods. This is consistent with what is suggested by figures 1 and 2. The result—that the values of the statistics increase as do the number of observations withheld—indicates that the differences in variability are most extreme between the beginning and the end of the decade. Excluding 1979 from the sample demonstrates that it accounts for much of the change in variability for each series, particularly for silver. It is unusual that when 1979 is excluded the results indicate a decrease in the variability over the decade for three of the four series for the smaller holdout periods. This decrease is significant, particularly for silver prices. Figures 1 and 2 indicate that 1974 was also a year of large price movements, and as more and more of 1974 is withheld from the computations the variability of the latter period again exceeds that of the earlier one. The results presented in parts A and B1 of table 2 indicate clearly the extreme degree to which differences exist in the process generating both the price series and the simple difference price change series over the various parts of the decade.

When the gold and silver price changes are viewed in a relative sense, the results contrast sharply with those obtained from using the simple difference price changes. Part B2 of table 2 presents the results for the logarithmic price changes. These series measure price changes relative to the level of the prices for the metals, while the simple difference series measure the absolute price changes. Nearly all of the ratios of the sums of squares are less than unity. This indicates that relative variability has, in fact, decreased from the earlier to the latter part of the decade. This on the surface contradicts the results presented in part B1. However, the results in part B2 show that as the holdout period is increased, the ratio tends toward one. The prices have trended upward and the price changes have increased in size over the period, so the implication is that relative variability in the latter part of the period is approaching that of the earlier part. Part B1 demonstrates that the absolute variability of the price changes has in fact increased over the period, and this explains why the relative variability is also found to be increasing.

The attributes of the gold and silver price series reflect such significant changes in variability that these series are not suitable for use in most tests of classical statistics. This is the case even if the price series are transformed into logarithms. Although not reported, the results for this transformation also indicate substantial changes in variability over both test periods. A word of caution is imperative: Researchers who model these prices do so at their own risk. For example, the use of these series in regression analysis is not appropriate because the ordinary least-squares assumption pertaining to a constant finite variance is violated. Further research of these series may require the use of nonparametric statistics to avoid the problem of nonstationarity.

Also, the simple difference price changes provide strong evidence supporting nonstationarity. It is suggested that this conclusion is less valid in the case of the logarithmic transformations of the price changes. This is because the entries in part B2 of table 2 indicate much lower levels of significance than do those for the other series. The use of these series for gold and silver presents a reasonably stationary picture of the variance of each series. Figures 1 and 2 signal the large price movements in 1979, and this is why the 1971–78 period was

TABLE 3 Means of the Price Change Series for Periods of Increasing Length Beginning in 1971

	Gold Price Changes		Silver Price Changes	
Ending Year	In Dollars	In Logarithms	In Dollars	In Logarithms
1971	.119	.003	−.005	−.003
1972	.261	.005	.004	.002
1973	.478	.007	.010	.004
1974	.748	.008	.014	.005
1975	.400	.005	.010	.004
1976	.304	.004	.009	.003
1977	.335	.004	.009	.003
1978	.421	.004	.010	.003
1979	1.012	.006	.056	.006

examined separately. It seems that the results of the analysis are at least somewhat sensitive to the choice of the sample period for all of the series. The effects of the contribution of 1979 to the variance of the series are not considered sufficient to destroy the usefulness of the series over the whole period. Part B2 shows that the results for relative price changes are not as sensitive to the choice of the sample period as are the other series. Also, for each series in part B2 the range of statistics is narrower, indicating that the results for the logarithmic price changes are not as sensitive to the choice of the holdout period. For the other series, table 2 shows that the significance of the results can be influenced by adjusting the width of the holdout period. For these reasons the remainder of the paper examines the logarithmic price change series for each metal over the 1971–79 period. We believe that this period presents a cohesive and interesting data set that can be used without doing violence to the assumption of a stationary variance.

The issue of nonstationarity also is pertinent with regard to the means of the price change series. Table 3 shows for each price change series that as more of the sample period is included in computing the means, these mean values tend to increase. For series that are stationary, it would be expected that these means converge to some value as the sample size is increased. Table 3 shows this is not the case for any of the price change series. This table also shows that for the whole period the mean of the price changes for either metal is nonzero, being $1.012 for gold and $.056 for silver. Dividing these values by the standard error of the means produces standardized values that are 3.9 for gold and 3.3 for silver. The large size of these statistics supports the contention of nonzero means for the price change sequences. The existence of a nonzero mean for the first differences often indicates the presence of a deterministic trend in the data which suggests that the sequences do not simply drift upward. This is hard to demonstrate because of the apparent nonstationarity in the variances of the sequences. However, the price changes do not vibrate about zero, and the price changes for either metal cannot be said to follow a strict random walk. The entries in table 3 for the price changes in logarithms can be considered as average weekly geometric rates of return. These price changes, as noted above, are to be interpreted in a relative sense. The average values for these series over the whole period are .006 for both gold and silver. The associated standardized values for these series are 3.12 for gold and 2.40 for silver. For assets that guarantee no rate of return, it is surprising that the average logarithmic price change denotes a significant positive rate of return. The implications of this with regard to the question of stationarity are the same as for the simple difference price changes: There seems to be a deterministic element in the means of these series. Table 3 also shows that 1974 and 1979 are years of unusual price movement for both types of price changes for each metal.

Knowledge of the shape of the distributions of the price changes of the metals is relevant to the process of forming expectations about future outcomes. Fama (1965) points out that the efficient market models require that the price changes be generated from some distribution. The weekly price changes aggregate the effects of the daily and intradaily price changes, and by employing the central limit theorem, this leads naturally to the assumption that the weekly gold and silver price changes are normally distributed. If this were true, the distributions would be defined completely by their expected values and variances. Granger and Morgenstern (1970) show that the normal distribution is necessary to establish a strict random walk. They state further that this type of process is consistent with markets that function in a perfectly competitive manner. Finding the price changes to be normally distributed would impose strong conditions on these series and would allow sharp conclusions to be drawn concerning the statistical nature of the underlying processes generating gold and silver price changes.

Frequency distributions for gold and silver price changes are presented in figures 3 and 4, respectively. Figures 5 and 6 display the frequency distributions for the logarithmic price changes. Contained in each of the figures is a normal frequency distribution having the same expected value and variance as that suggested by the price change data. As can be seen by inspection, each price change series seems to exhibit a nonnormal frequency distribution. Each of the empirical distributions has more observations than does the associated normal frequency distribution that falls near the (nonzero) mean and that lie in the tails. Indeed, when the Kolmogorov-Smirnov (K-S) goodness-of-fit test is performed on these frequency distributions, the results indicate departures from normality. Table 4 displays the K-S statistics. For the

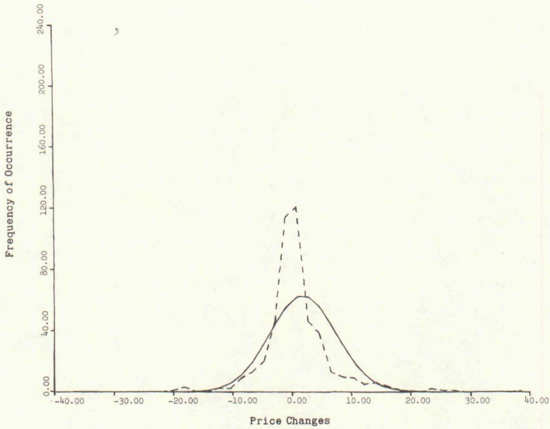

FIG. 3.—Frequency distribution for gold price changes

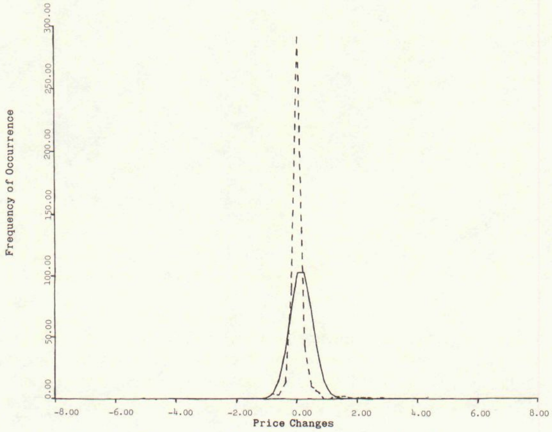

FIG. 4.—Frequency distribution for silver price changes

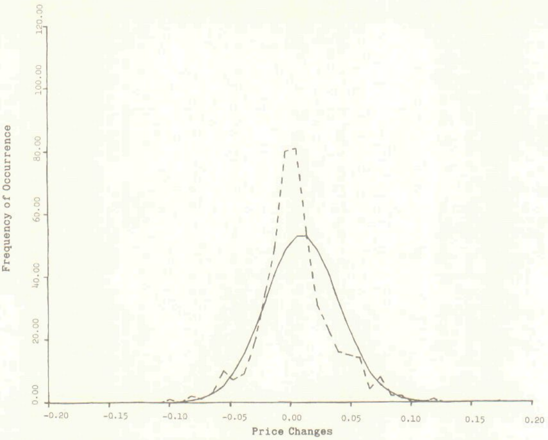

FIG. 5.—Frequency distribution for gold price changes in logarithms

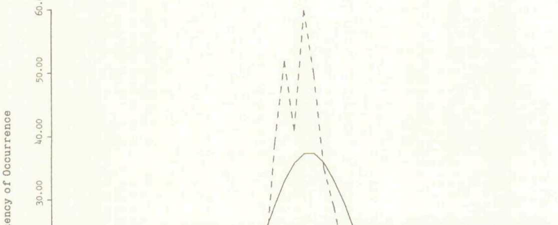

FIG. 6.—Frequency distribution for silver price changes in logarithms

TABLE 4 Kolmogorov-Smirnov Statistics

	Gold Price Changes	Silver Price Changes
Simple differences	.207*	.354*
Logarithmic differences	.137*	.112*

*Significant at the .01 level.

sample size of 468 the critical value at the .01 level is approximately .07. The statistics contained in table 4 are each quite significant though the logarithmic transformations are less extreme. The results for the logarithmic price changes are more reliable, yet the statistics for the price changes in dollars are so large that they cannot be overlooked.

If the process generating the means and variances is actually nonstationary, the interpretation of the results of the K-S test is limited. Thus it would not be possible to reject the hypothesis that the price change distributions are not nonnormal. Other researchers have found "peakedness" and "fattails" in the distribution of stock price changes that are similar to those displayed in figures 3 through 6 (see Fama 1965). Although the changing variances here might be construed to indicate "infinite variances," the nonzero mean of the price changes do not permit this distinction to be made since it does not appear that the means simply drift. An explanation of the nonzero means is that the price changes represent nonsymmetric drawings from two normal distributions. However, this would not necessarily explain the high degree of heteroscedasticity displayed by the series. A possibility that cannot be ruled out entirely is that the price change series are generated from a normal distribution that is confounded with a trend element. While it is discouraging that the analysis has not led to a suitable explanation of the behavior of the gold and silver price changes, the price change distributions have been found to possess interesting properties and are usable in logarithmic form. Further research is called for and is currently under way to resolve some of the issues raised here.

IV. Testing for Dependence in the Price Change Series

In this section another aspect of the process generating the price change series for gold and silver is analyzed. It is necessary to evaluate the association among the elements of the series in order to establish the degree of efficiency of these markets. The results of three tests for dependence are presented in this section. The first is a parametric test, the next is nonparametric in nature, and the third analyzes the economic value that existing dependencies might possess.

Tests for Autocorrelation

It is not possible in an efficient market to use the current information set to obtain expectations of future outcomes that are superior to the market-based expectations. This is because the market functions in a manner that makes successive price changes independent. Finding dependencies in the gold and silver price change sequences would imply that not all information is reflected by the prices. One method of testing for dependencies involves the calculation of autocorrelation coefficients. These coefficients measure the association between the price changes for either gold or silver and lagged values of the price changes. The definition of the coefficient for an m-week lag is:

$$r_m = \frac{v_m}{v_0}, \quad (1)$$

where

$$v_m = \frac{1}{468} \sum_{t=1}^{468-m} (e_t - u)(e_{t+m} - u) \quad \text{for } m = 1,2 \ldots M; \quad (2)$$

e = the price change for gold or silver and u = the mean price change for gold or silver. The v_0 represents a zero-week lag and is the variance of the price changes. The price changes used here are the differences in the logarithms of the prices of the metals as discussed above.

The results of these tests are reported in table 5. The evidence suggests that a positive relationship exists between successive price changes for both gold and silver because the bulk of the coefficients are greater than zero. For gold the positive relationship is more obvious. It appears to be stronger for the smaller lags. The 1-week lag for silver produces the largest observed coefficient, and though the remaining coefficients are generally not large, it appears that the silver price change sequence also contains positive dependencies. Coefficients

TABLE 5 Autocorrelation Coefficients for Gold and Silver Price Changes in Logarithms

Lag	Gold Price Change	Silver Price Change
1	.11	.16
2	.07	.01
3	.11	.03
4	.01	.01
5	.10	−.06
6	.01	.05
7	.02	−.06
8	.01	−.07
9	−.06	.04
10	−.02	.08
11	.09	.11
12	.03	.05

NOTE.—SE for both sequences is .05.

were calculated using longer lags than those reported, and they tended to be small in size and to fluctuate in sign. Also, the tests conducted using the simple difference price changes produce results that are consistent with those reported here. In fact, the autocorrelation coefficients based on the simple differences indicate even more strongly that positive dependencies exist in the price change sequences for the metals.

The coefficients in table 5 must be evaluated carefully. The finding in the subsection above raises questions about the reliability of the standardized values. This is why they were not reported here, even though few of the coefficients appear to be significant. Because the logarithmic price change series were analyzed, the results are as reliable as possible. In addition, for series that are autocorrelated, standardized values would be biased upward. Even so, the coefficients do indicate that a positive association exists between successive price changes for the metals. This implies that all information may not be impounded in the market prices since successive price changes for the metals do not appear to be totally unrelated. The results of this subsection indicate some departures from market efficiency for both gold and silver.

Runs Tests

The gold and silver markets generate positive, negative, and zero price changes. A run is defined as a sequence of price changes of the same type preceded and followed by price changes of the other types. Dependencies in the price change series would be likely to produce runs. Positive dependencies would produce fewer runs in total and runs of longer length than expected, while negative dependencies would produce many more total runs that are not of long length. Since little is known about the structure of the price change distributions, the runs test is appropriate to use in order to investigate the nature of the dependencies in the price change series. The runs are analyzed in two ways: The first analyzes the total number of runs, and the second analyzes the runs by length. The method followed is that used by Fama (1965). The results of the analysis are consistent with the results of the subsection above. Table 6 shows that the sequences of price changes for both gold and silver contain fewer runs than expected for sequences

TABLE 6 Analysis of Total Number of Runs

	Gold Price Changes	Silver Price Changes
Actual number of runs	220.0	240.0
Expected number of runs	251.2	255.2
Standardized difference between actual and expected values	−3.1	−.9

TABLE 7 **Analysis of Runs by Length**

Length of Run	Positive Actual	Positive Expected	Negative Actual	Negative Expected	Zero Actual	Zero Expected
			A. Gold Prices			
1	50	46.9	51	58.0	17	18.1
2	20	25.7	22	23.2	0	.9
3	8	14.1	9	9.4	2	.0
4	11	7.8	11	3.8	0	.0
5	6	4.3	3	1.6	0	.0
6	3	2.4	0	.6	0	.0
7	1	1.3	1	.2	0	.0
8	4	.7	0	.1	0	.0
12	1	.0	0	.0	0	.0
Total runs	104	...	97	...	19	...
			B. Silver Prices			
1	50	53.4	57	63.9	16	20.8
2	26	28.4	31	21.8	0	1.1
3	22	15.0	14	9.1	0	.1
4	4	7.9	5	3.8	0	.0
5	5	4.2	3	1.6	0	.0
6	5	2.2	0	.7	0	.0
7	0	1.2	0	.0	0	.0
8	1	.6	0	.0	0	.0
10	1	.2	0	.0	0	.0
Total runs	114	...	110	...	16	...

of random numbers. For gold price changes the number appears to be significantly fewer. The positive autocorrelation found in the subsection above implies that price changes which are either positive or negative are followed by price changes of the same sign. The analysis of total runs also suggests that positive dependencies exist in both sequences because there are fewer total runs than expected.

All this implies that runs of longer length are to be expected. Table 7 provides evidence that supports this. The expected values in this table are computed from sample probabilities of positive, negative, and zero price changes obtained from the price change series. This is why the expected values differ both for the two metals and for the type of run (see Fama 1965). Although the bulk of the runs of all types are 3 weeks or less in length, there are apparent dependencies in the price changes of the metals that produce the longer runs for both gold and silver price changes. In the case of gold prices, successive increases have equaled or exceeded 8 weeks five times, and the same has occurred twice for silver price increases. Longer negative runs have occurred more often than expected, as table 7 also shows, but not to the same extent as have positive runs. However, this also indicates that a positive association exists between successive price changes for gold and silver. The analysis of runs provides results that are consistent with the results of the autocorrelation tests. Finding positive dependencies means that, again, complete support for market efficiency is lacking.

Testing Trading Strategies

The results of the last two subsections provide statistical evidence that some dependence exists in the price change series for gold and silver, but the tests in this section will examine the economic value of any existing dependence. The method here is to use mechanical trading rules in an attempt to increase returns beyond the market-determined returns. The trading strategy employs the filter rule of Alexander (1964) and Fama and Blume (1973), which works as follows: If the price of the metal being traded increases (decreases) by $X\%$, buy (sell) the metal and hold it (Treasury bills) until the price falls (rises) by $X\%$ from a subsequent high (low), and at that point reverse the position. The current position, whatever it is, is terminated at the end of 1979, and the proceeds are included as part of the return on the filter. The logic underlying the test is that if the market price does not adjust to new information immediately, the adjustment process will produce a discernible pattern in the prices of the metals. The filter rule seeks to measure any positive excess returns that might exist due to lack of market efficiency. There are no comparable tests other than the testing of other trading rules. However, it is believed that the tests conducted here suitably evaluate the economic significance of the dependencies found in the price change sequences.

Table 8 presents the results of using this trading strategy. Filters of various sizes are compared with the passive buy-and-hold strategy. The buy-and-hold position is taken at the same time the first position of the filter is taken. This provides comparable entries for each filter for the two strategies (see Fama and Blume 1973). Also the returns on the two strategies are presented in average annual geometric rates of return, both before and after transaction costs. This provides further justification for analyzing the logarithmic price changes. The transaction costs are broken down as follows: (1) The commission charged is 1% on buying and selling the metals and the commission on dealing in Treasury bills is considered to be negligible; and (2) storage and insurance charges are assumed to be half of 1% annually, and for periods of less than a year the charge is prorated. Delivery and assaying charges and sales taxes are ignored. Although these costs cannot be applied to all investors or to all forms of gold and silver trading, they do provide a standard of comparison with respect to trading in gold and silver bullion (see *Business Week* 1977). Professional arbitrageurs may approach such low transactions costs.

Table 8 presents results for the whole period from 1971 to 1979 and for the period from 1975 to 1979 when U.S. citizens could legally trade in all forms of gold. Generally, the filters for gold and silver perform poorly when compared with the buy-and-hold strategy. This is true whether the returns are before or after transaction costs or whether the whole period or the subperiod is used. The same also holds true for other filters for which the results are not presented. Of the filters presented, only the 10% filter shows some success for both gold and silver, particularly in the 1975–79 subperiod. For gold, it outperforms the buy-and-hold strategy after transactions costs. The 5% and 15% filters have some success before transaction costs for gold, but the transaction costs wipe out whatever gains there are. This holds for the filters that are smaller than 5% for both metals. The larger filters do not have the problem with transaction costs to the same extent, but these filters fail to perform at least as well as the buy-and-hold strategy on a gross basis unless, as is the case for the largest filters for silver, the two strategies make the same transactions. There appears to be no inefficiency that can be easily exploited.

A closer look at some "profitable" filters also supports this. As noted above, the 10% filter performs rather well over the 1975–79 subperiod, and the same is true for the unreported 8.5% filter. In order to see how well these filters work, the subperiods over which they are tested are allowed to vary in length. As table 9 shows, both filters do well before and after transaction costs for the 1971–75, 1971–76, and 1975–79 subperiods. However, these two filters do not outperform the buy-and-hold strategy for all the subperiods, so none of the trading rules can be said to dominate the passive buy-and-hold strategy for either gold or silver.

The tests of this subsection are aimed at finding economic significance in the dependencies found above. Even on a gross basis, the filters find little advantage that can be used to the benefit of investors. Also, the very modest transaction costs have enough of an impact on the filters to make their returns inferior to the buy-and-hold strategy. Larger transactions costs that would apply to the average investor would reduce the return on the filters even more. As a further test, the filters were run using moving averages of past prices. If the current price differed from the moving average by $X\%$, a buy-or-sell signal was generated as above. This was done in order to see if the effect of the trends in the prices could be captured. Moving averages of 26, 52, and 104 weeks showed no improvement in the returns for the filters. In fact, the use of moving averages often leads to the buy-and-hold strategy for the trading rules. While the results of these tests do not overwhelmingly support market efficiency, they do indicate that there are no inefficiencies that exist that can be easily exploited by investors.

TABLE 8 Average Annual Geometric Rates of Return on the Trading Strategies

	Gold Returns				Silver Returns			
	1971–79 Period		1975–79 Period		1971–79 Period		1975–79 Period	
	Annual Rates of Return	Net of Transaction Costs	Annual Rates of Return	Net of Transaction Costs	Annual Rates of Return	Net of Transaction Costs	Annual Rates of Return	Net of Transaction Costs
Filter = .005:								
F	.280	.109	.240	.062	.290	.072	.337	.123
BH	.290	.287	.215	.211	.312	.309	.382	.379
Filter = .01:								
F	.270	.118	.231	.078	.279	.096	.317	.131
BH	.290	.288	.215	.211	.312	.310	.383	.379
Filter = .02:								
F	.284	.183	.233	.132	.272	.141	.312	.183
BH	.290	.288	.218	.214	.312	.309	.383	.379
Filter = .05:								
F	.315	.264	.282	.235	.296	.231	.299	.239
BH	.295	.293	.237	.233	.323	.321	.390	.386
Filter = .10:								
F	.330	.314	.308	.294	.270	.236	.408	.390
BH	.292	.290	.263	.258	.352	.360	.396	.394
Filter = .15:								
F	.305	.291	.284	.270	.285	.265	.367	.353
BH	.294	.292	.279	.274	.362	.360	.389	.385
Filter = .20:								
F	.281	.269	.285	.276	.256	.240	.923	.912
BH	.303	.300	.289	.284	.370	.368	.923	.912
Filter = .30:								
F	.282	.275	.325	.320	.282	.270	1.279	1.263
BH	.299	.296	.313	.308	.373	.370	1.279	1.263
Filter = .50:								
F	.270	.264	.359	.353	.325	.319	1.562	1.50
BH	.288	.285	.442	.436	.351	.350	1.562	1.540

NOTE.—*F* denotes the return on the filter and *BH* denotes the return on the buy-and-hold strategy.

TABLE 9 Results for Two Filters over Different Time Periods for Gold Trading

	Filter = .085		Filter = .10	
	Annual Rates of Return	Net of Transaction Costs	Annual Rates of Return	Net of Transaction Costs
1971–73:				
F	.420	.400	.433	.408
BH	.410	.402	.433	.425
1971–74:				
F	.405	.378	.419	.396
BH	.400	.392	.414	.409
1971–75:				
F	.331	.309	.338	.320
BH	.264	.260	.272	.267
1971–76:				
F	.308	.287	.308	.290
BH	.210	.207	.215	.211
1974–79:				
F	.291	.267	.272	.256
BH	.217	.214	.217	.214
1975–79:				
F	.324	.307	.308	.294
BH	.248	.244	.262	.258
1976–79:				
F	.405	.382	.372	.355
BH	.401	.395	.408	.402
1977–79:				
F	.416	.387	.380	.359
BH	.447	.440	.444	.437

NOTE.—*F* denotes the return on the filter and *BH* denotes the return on the buy-and-hold strategy.

V. Concluding Comments

The major results of the analysis are summarized as follows. (1) There is evidence of considerable heteroscedasticity in the variances of the price changes for the metals except for the logarithmic price change series; (2) the means of the price change series for gold and silver are nonzero and not stationary, and they do not appear to simply drift; (3) positive dependence in each of the price change series is indicated by the data; (4) investors cannot, before or after transactions costs, easily use the positive dependence to generate superior returns. The remainder of this section provides a discussion of these four points.

Table 1 shows that holding gold and silver over the last decade generally provided high levels of returns, and the mechanical trading rules are pitted against prices that have experienced strong upward trends over the same period. The buy-and-hold strategy consequently does well because of this, and the filters have a tough job overcoming these trends. The intent of using the moving average trading rules was to incorporate information about the trends in the investment strategies; the (unreported) results underscore the contention that the market-determined return cannot be bettered. This is because many times the moving average trading rules produced the same decisions as the buy-and-hold strategy. The upward trend in prices apparently accounts for the success that trading in the metals provided in the last decade.

The large movements in prices that occurred during 1979 brought about, as table 3 shows, increases in the means of each of the price change series. Interestingly, though, excluding 1979 from the computations does not alter significantly the nature of the results of the investment strategies. The buy-and-hold strategy still provided performance that could not be improved upon by the mechanical trading rules. Also, the results of the tests for autocorrelation and the runs tests did not change to any great extent when 1979 was excluded. This supports the third point listed above, in that positive dependence in the series was encountered by this analysis, but again it seems that it was not possible to use this dependence to generate excess returns. Thus, including or excluding the year of large price movements, the results of the analysis support the fourth contention above.

Points 1 and 2 are concerned with the form of the distributions generating the price change series. These generating functions often appear to exhibit nonstationary aspects whether 1979 is excluded or included. The analysis of the price change series indicates that neither the gold market nor the silver market conforms to traditional models that are consistent with market efficiency. The attributes of the distributions with regard to nonzero means and nonstationarity are documented in Section II, but further research is required before it can be said that the nature of generating functions is understood.

Since it is not clear that the current information set is impounded in the gold and silver prices, these prices cannot be assumed to be equilibrium prices. Trading gold and silver could be considered more of a speculative activity than an investment activity. However, due to the nonstationarity observed in the data, this is a more valid description for some periods than for others. It is not clear how the prices for the metals are to be used in buy-and-sell decisions, not does it seem that investors can easily put either metal in a traditional investment framework. Gold and silver have provided excellent returns over the last decade, but this does not guarantee similar performance in the future. There appears to be considerable risk in trading the metals, not the least of which is the possibility of a change in the process generating the price changes for the gold and silver.

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