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Observed Option Mispricing and the Nonsimultaneity of Stock and Option Quotations

Empirical research on options usually relies on published stock and option quotations. A well-known problem with these data is that the option price quotations are often from a different time of day than the stock price quotation. The nonsimultaneity of the option and stock quotations can cause inaccuracies in empirical work, especially in work to detect the mispricing of options. In this paper we develop a method for evaluating the probability that observed option mispricing is actually the result of nonsimultaneity in the option and the stock quotations and thus that the observed profit from the option strategies is unattainable in practice. We then present simulations of this method and use it to evaluate several empirical works on option market efficiency.

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If the stock price is used to determine the correct option price (using a theoretical pricing model such as the Black-Scholes [1973] pricing formula), the option may appear to be mispriced because the stock price that was used in the formula, the stock price quoted as the closing price, is not the stock price that existed in the market when the last option price was quoted. The problem of nonsimultaneity will also lead to apparent profitable trading strategies when in fact no such strategy exists. When the researcher takes an option position based on the published quotation, an apparent profit may be illusory, since trade may not have been possible at that price at the same time that the stock price equaled its published quote.

The problems with nonsimultaneity have been discussed in a number of empirical papers. Studies of option market efficiency by Trippi, by

*I am indebted to James McDonald, Roger Clarke, Steven Manaster, and an anonymous referee for helpful comments, and to Richard Guthrie for computational assistance.

(*Journal of Business*, 1981, vol. 54, no. 1)
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0021-9398/81/5401-0005$01.50

Chiras and Manaster, and by Galai have questioned whether observed profit opportunities are realizable in practice or are the artifacts of using nonsimultaneous quotations. While the possibility of errors from nonsimultaneity are well recognized, it remains difficult to determine the extent of any nonsimultaneity bias. In this paper we develop a method for evaluating the probability that an observed option mispricing is actually the result of nonsimultaneity in the option and the stock quotations, and that the observed profit from the option strategies is unattainable in practice.[1] We then present simulations of this method and use it to evaluate the empirical results discussed above.[2]

I. The Nonsimultaneity Problem: An Example

There are two nonsimultaneity problems to consider. The first and most critical problem is that nonsimultaneity between the stock price at closing and the stock price at the time of the last option quote will cause mispricing in the option when the stock price is entered into the option-pricing formula.

When two options are combined in a hedging strategy there is the second problem that at the time of the last price quote for the one option used in the hedge, the other option may have a price that differs enough from its price at the time of its last quote to account for the observed (but in this case unrealizable) profit.[3]

To illustrate these two problems, refer to the example presented in table 1. Here two options are considered on a stock. The two options have exercise prices of \$50 and \$60 and have 6 months to maturity. The volatility of the stock is assumed to be 0.4, and the riskless interest rate is 6%. Say the last trade on the \$50 option occurs at 1:00 P.M., and the closing quote on the \$50 option is equal to this price of \$6.16. This option is correctly priced according to the Black-Scholes formula, given that the stock price at this time is 49¾. Further, suppose that the last trade on the \$60 option occurs at 3:00, with a stock price at that time of 50¼. That option is also correctly priced according to the

1. While we use a theoretical methodology to analyze the problem of nonsimultaneity, the availability of transaction-by-transaction quotes provides the data necessary to consider the nonsimultaneity problem empirically. Galai uses intraday quotations to test his results for nonsimultaneity.

2. This study concentrates on the nonsimultaneity problem in option transactions. The nonsimultaneity problem has also been recognized in studies of stock markets, and the effect of nonsimultaneity in the measurement of stock betas is reported by Scholes and Williams (1977). The methodology developed in the present paper can be extended to consider broader questions of nonsimultaneity such as this one.

3. Some of the option positions will suffer from this second source of nonsimultaneity bias both when the position is initiated and when it is closed. If the bias occurs both times, the bias incurred at the initiation of the position may be accentuated (if the bias at the close is in the same direction as that at the initiation), or it may be reduced (if the bias at the close is in the opposite direction).

TABLE 1 **Nonsimultaneity Problems**

	Time of Trade			Listed Closing Price ($)
	1:00	3:00	4:00	
Stock price ($)	49 3/4	50 1/4	50	50
50 option price ($)	6.16	No trade	No trade	6.16
60 option price ($)	2.74	2.91	No trade	2.91

Black-Scholes formula. The last stock trade occurs at 4:00. At that time the stock is priced at $50; it has dropped a quarter of a point from its price at 3:00 and has increased a quarter of a point from its 1:00 price. If we use the closing price of the stock to determine the prices of the options, we will find that the $60 option appears slightly overpriced, while the $50 option is underpriced. This is the first nonsimultaneity problem.

The appropriate strategy to profit from the apparent mispricing will involve selling the overpriced $60 option and buying the underpriced $50 option. The prices used for the hedge will be $2.91 and $6.16, respectively. This will bring a second inaccuracy into the strategy, since if a hedge were put on at the time that the $50 option were selling at $6.16, the $60 option would have been at $2.74.

The options appear mispriced because of nonsimultaneity between the stock price and the option prices, and the strategy will yield an observed profit because of the nonsimultaneity between the two options. But in practice, if simultaneous prices were used, the options would not be found to be mispriced, and the hedge would not yield a profit.

II. Methodology

The mispricing of the option will be due to nonsimultaneity if the stock price at the time of the last option quote differs significantly from the last stock price quote. Let the time of the last stock trade be S, and the quoted stock price of that period be $P(S)$. This will be the price that is posted as the closing stock price. The stock price that existed at the time of the last option trade is not known. If we assume that the option was correctly priced at the time of the closing option quote, then the stock price that existed at that time can be determined implicitly from the Black-Scholes option-pricing formula. Let us denote the implicit stock price by P^*.

The probability that the option mispricing is due to nonsimultaneity is the probability that the stock price was P^* at the time of the last option quotation given that the stock price was $P(S)$ at closing. If we denote the time of the last option trade as T, and the stock price at the time of the last option trade as $P(T)$, then the probability that the option mispricing is due to nonsimultaneity is the probability that the stock price changed enough to account for the discrepancy between the final stock quote and the implicit stock price P^* between the time of the option trade and the final stock trade. That is,

$$\text{prob (mispricing is due to nonsimultaneity)} = \text{prob}\left[\,|P(T) - P(S)| \geq b\right], \quad (1)$$

where b is chosen to represent a suitable stock price movement to account for the mispricing in the option formula.[4]

The question of nonsimultaneity thus rests on evaluating the probability that the closing stock price could significantly differ from the stock price that existed at the time of the closing option quote. The times of the final quotes, T and S, are unknown, and the stock price in period T is also unknown. However, we know that the variation in the stock price between period T and period S will depend on two factors.

First, it will depend on the passage time between the final option quote and the final stock quote. While the length of the passage time is not known, we can get an estimate of its upper bound through the volume of trade in both the stock and the option. (Block trades will make such a proxy exaggerate the actual number of trades. However, as we will see, an overestimation of trading volume will cause an underestimation of the extent of any nonsimultaneity bias.)

Second, for any passage time between quotes, the change in the stock price will depend on the volatility of the stock. The more volatile the stock, the more likely a large change in the stock price over any number of time periods.

We will consider these two factors in turn in order to obtain a usable expression for equation (1).

Probability of a Given Time Passage between the Closing Option and Stock Quotes

The time of any given option trade is a random variable. We assume that the time of each of N trades is uniformly distributed. Thus the probability that the last trade is at a fraction t of the trading day is Nt^{N-1}, $0 \leq t \leq 1$.

Assume that the number of option trades above one is Poisson distributed, so[5]

$$\text{prob}\ (N = n) = \lambda^n / (e^\lambda - 1)n!,$$

where λ is the parameter of trades per trading day. The probability that all trades in the option have occurred at time t is then prob (last option

4. A reasonable value to use for b might be $|P^* - P(S)|$.

5. The distribution is derived under the assumption that at least one trade occurs during the day. The option quotations will indicate if no trades occur during the day.

trade is at time t) $= e^{\lambda t}/(e^{\lambda} - 1)$. This expression is the cumulative distribution function for the probability that the last trade of the day occurs at time t. Differentiating, we obtain the density function $\lambda e^{\lambda t}/(e^{\lambda} - 1)$.

Similarly, if stock trades follow a Poisson distribution with δ as the parameter for stock trades over the day, then the density function for the probability that the last stock trade occurs when a fraction s of the trading day has passed is $\delta e^{\delta s}/(e^{\delta} - 1)$.

Assuming that option trades and stock trades are statistically independent, we can use these expressions to determine the joint density function for the probability that the last option occurred at time t and the last stock trade occurred at time s, $f(t,s)$,

$$f(t,s) = \gamma\delta e^{-\lambda(1-t)-\delta(1-s)}. \tag{2}$$

We are interested in the length of time between the final trades, not the particular time periods when they occur. Denote the random variable of the length of the passage time between final trades by γ, that is, define

$$\gamma \equiv | t - s |. \tag{3}$$

To derive the density function for γ, we first define the variable $z \equiv t - s$. The joint density of (z,s) will be

$$h(z,s) = f(s,z + s) = \frac{\lambda\delta e^{\lambda z + s(\lambda+z)}}{(e^{\lambda} - 1)(e^{\delta} - 1)}.$$

Noting the Jacobian of the transformation,

$$\frac{\partial(s,t)}{\partial(z,s)} = \left\| \begin{matrix} 0 & 1 \\ 1 & 1 \end{matrix} \right\| = 1,$$

we can derive the density of z for the two cases, $0 \leqslant z \leqslant 1$ and $-1 \leqslant z \leqslant 0$, to obtain:

For $0 \leqslant z \leqslant 1$:

$$l(z) = \int_0^{1-z} h(z,s)ds = \frac{\lambda\delta e^{\lambda z}}{(e^{\lambda} - 1)(e^{\delta} - 1)} \int_0^{1-z} e^{s(\lambda+\delta)}\, ds \tag{3A}$$

$$= \frac{\lambda\delta e^{\lambda z}\, [e^{(1-z)(\lambda+\delta)} - 1]}{(\lambda + \delta)(e^{\lambda} - 1)(e^{\delta} - 1)};$$

For $-1 \leqslant z \leqslant 0$:

$$l(z) = \int_{-z}^{1} h(z,s)ds = \frac{\lambda\delta e^{\lambda z}}{(e^{\lambda} - 1)(e^{\delta} - 1)} \int_{-z}^{1} e^{s(\lambda+\delta)}\, ds \tag{3B}$$

$$= \frac{\lambda\delta e^{\lambda z}\, [e^{\lambda+\delta} - e^{-z(\lambda+\delta)}]}{(\lambda + \delta)(e^{\lambda} - 1)(e^{\delta} - 1)}.$$

Defining $\gamma \equiv |z|$ we arrive at the density function for γ:[6]

$$g(\gamma) = \frac{\lambda\delta}{(\lambda + \delta)(e^{\lambda} - 1)(e^{\delta} - 1)} \left(e^{\lambda+\delta-\gamma\delta} - e^{\lambda\gamma} + e^{\lambda+\delta-\lambda\gamma} - e^{\delta\gamma}\right). \quad (4)$$

The Probability That |P(T) − P(S)| *Exceeds* b *for a Given Passage of Time*

We assume that the stock price movement is lognormally distributed over the time intervals, with a mean return per period of r and a per period variance of return equal to σ^2.[7]

Let us define the absolute difference between the stock price at the time of the last option quotation and at the time of the last stock quotation by ΔP, that is, $\Delta P \equiv |P(T) - P(S)|$.

With the stock price movement as described above, the probability that ΔP is greater than b is given by evaluating the two-tailed standard normal distribution,

$$\text{prob}(\Delta P > b\,|\,\gamma) = 2\,\Phi\,\{(-\,b\, -\, r\gamma)/[(\gamma)^{1/2}\sigma]\}, \quad (5)$$

where $\Phi\,(\cdot)$ is the cumulative standard normal distribution function.[8]

The Probability That Option Mispricing Is due to Nonsimultaneity

We can combine the results of the preceding subsections to derive the expression for the probability that ΔP, the difference between the stock price at the time of the last option trade and at the time of the last stock trade, is greater than the benchmark b and therefore that the mispricing of the option is due to nonsimultaneity in the security quotes.

The probability is obtained by summing the conditional probability in equation (5) over the probability of γ taking on each possible value, so that

$$\text{prob}(\Delta P > b) = \int \text{prob}(\Delta P > b\,|\,\gamma)\; g(\gamma)\; d\gamma. \quad (6)$$

Substituting into equation (6) from equations (4) and (5), we obtain an expression that can be used for empirical tests of the probability that option mispricing is due to nonsimultaneity. The value of σ is comput-

6. I am indebted to James McDonald for this derivation.

7. The value σ will be the same as that used in determining the option price with the Black-Scholes formula, adjusted for the difference in the time period and expressed in dollar rather than in return terms.

8. The Poisson distribution can be used as an alternative to the normal distribution in this model. However, even for a moderate number of time periods, the value of the probability of mispricing being due to nonsimultaneity will not differ significantly when the one function is used in place of the other. Also, the use of the normal distribution is more conventional within the option literature and facilitates the use of published stock volatilities for option pricing.

able from the stock and option price data.[9] The parameter b is our choice. While the values for γ and δ are not directly observable, the volume of trade can be used to obtain an upper bound to the number of trades. (Operationally, we will later assume that stock trades occur in lots of 100 shares and that option trades occur in lots of one option contract.)

III. Simulations

To illustrate this model, we have computed the probability of nonsimultaneity as presented in equation (6) for various values of the parameters. Since the integral cannot be evaluated analytically, we use a numerical solution in this simulation.

Table 2 presents the probability of mispricing due to nonsimultaneity for various values of b and for several values of σ. For example, with b set equal to 0.25, this table assesses the probability that the stock price at the time of the closing option quote and at the time of the closing stock quote differ by more than a quarter of a point. The number of stock trades is set equal at 1,000 per day in table 2. As would be expected, the probability of nonsimultaneity is a decreasing function of the number of option trades and is an increasing function of σ.

The standard deviation is measured in dollar terms. For a stock price of $100, a standard deviation of $1.00 per day corresponds to an annual standard deviation on returns of approximately 20%; a standard devia-

TABLE 2 **The Probability of Mispricing due to Nonsimultaneity for Various Values of λ, σ, and b**

	λ				
σ	1	5	10	20	50
b = .25:					
1	.647	.472	.289	.150	.030
2	.850	.611	.436	.297	.108
3	.862	.711	.524	.317	.124
b = .50:					
1	.455	.189	.090	.023	.003
2	.695	.415	.262	.144	.038
3	.765	.524	.400	.241	.075
b = 1.00:					
1	.161	.019	.004	.001	0
2	.436	.167	.078	.020	.003
3	.566	.312	.158	.088	.012

NOTE.—The parameter for the number of stock trades, δ, is set at 1,000. The value of σ is the daily standard deviation in dollar terms.

9. The estimate of σ may be improved by using the high and low quotes of the particular day in question.

tion of \$2.00 per day corresponds to an annual standard deviation on returns of approximately 40%; and a standard deviation of \$3.00 per day corresponds to an annual standard deviation on returns of just under 60%. Assuming a stock price of \$100, these standard deviations cover a representative range for stock volatilities.

The most immediate result from table 2 is that the problem of nonsimultaneity is significant when there are less than 20 option trades during the day. Even for the lowest value of σ, for $\lambda = 20$ there is a 15% chance that the stock price at the time of the last option quote differed by at least a quarter of a point from the stock price at the time of the last stock quote. For a highly volatile stock with $\sigma = 3$, that probability increases to 30%. For in-the-money options, a quarter-point variation in the stock price will cause nearly a quarter of a point mispricing in the option since the option price and the stock price will move nearly one to one. However, for at-the-money or out-of-the-money options, a quarter-point difference in the stock price may not lead to significant mispricing.

Table 2 also presents the probability that ΔP will exceed 0.5 and that ΔP will exceed one. The probabilities decrease with an increase in b. For example, 50 option trades are enough to make the nonsimultaneity problem insignificant when b is set equal to one.

For at-the-money or near-the-money options, a variation of one-half a point in the stock price will usually bring a significant change in the Black-Scholes value of the option. And the probability that the difference between the stock price at the time of the last option trade and at the time of the stock close will differ by more than half a point exceeds 5% in all but three instances presented in the table. For highly volatile stocks, with $\sigma = 3$ and $\gamma = 20$, there is a 25% chance that ΔP will exceed \$0.50. Even for a moderately volatile stock with $\sigma = 2$ and with $\lambda = 50$, there is nearly a 4% chance that ΔP will exceed \$0.50. If we use the \$0.50 benchmark for b, at least 50 option trades will be needed to make the nonsimultaneity problem insignificant for stocks of moderate or high volatility.

A significant number of options have the low trading volumes used in table 2. Sampling from the quotations over 10 trading days, over two-thirds of the options traded had under 50 contracts traded, one-half had under 20 contracts traded, and nearly one-third of the options had less than 10 contracts traded. With this low volume being commonplace, the results of the simulation suggest there will be cases of apparent mispricing caused by nonsimultaneity in any given day.

In table 2 we have set the number of stock trades equal to 1,000. If each trade involves 100 shares of stock, this represents a volume of 100,000 shares per day. To see how variations in the volume of stock trades will affect the probability of mispricing due to nonsimultaneity, we turn to table 3. In this table, we have set b equal to 0.5 and have set

TABLE 3 **The Probability of Nonsimultaneity for Various Values of δ***

δ	λ: 1	5	10	20	50
50	.660	.404	.263	.150	.050
100	.684	.412	.260	.146	.042
1,000	.695	.415	.262	.144	.038

NOTE.—The volatility, σ, is set as 2, and b is set at 0.5.

σ at \$2.00 per day. These represent the intermediate values from table 2. Table 3 indicates that the probabilities are not particularly sensitive to the number of stock trades. Since the underlying stocks for listed options are selected partly for their high volume, it is likely that 50 stock trades represents a lower bound for most days. But even with such a low number of trades, the values for the probabilities do not differ substantially from the values when there are 100 or 1,000 trades. This is especially true as the number of option trades increases to a modest size. The small difference in the probabilities for δ equal to 100 and 1,000 indicates that, as might be expected, large-number properties take hold by the time the trading reaches 100 trades per day.[10]

IV. An Extension: The Case of Correlation between Option and Stock Trades

In the model of Section II, the trades of options and stocks were assumed to be independently distributed. An alternative assumption is that option and stock trades are correlated. This seems to be a plausible case because often option strategies involve taking a hedge position in the underlying stock.

Any correlation between option and stock trades will decrease the likelihood of nonsimultaneity in trades, since at least some of the option trades will be entered with a simultaneous transaction in the underlying stock.

In order to evaluate the effect of the independence assumption on the results of the previous section, we will modify the model to allow correlations between stock and option trades. We will consider the

10. The model used to develop these values employs several restrictive assumptions, most notably the assumptions that trading follows a Poisson distribution. Also, trading in the option and the stock will probably not be independent. Since option-trading strategies sometimes involve taking a simultaneous position in the option and the stock, there will be some positive correlation between the trading of the two securities. If significant, this will tend to reduce the nonsimultaneity. If in addition to this correlation, the trading of the securities follows a distribution that is highly concentrated at certain periods of the day, the probability of nonsimultaneity will be further reduced.

TABLE 4 **The Probability of Mispricing due to Nonsimultaneity for Various Values of λ, σ, and *b* (All Option Trades Are Assumed to Coincide with Stock Trades)**

	λ				
σ	1	5	10	20	50
b = .25:					
1	.457	.320	.191	.101	.025
2	.524	.411	.268	.155	.078
3	.631	.502	.295	.192	.100
b = .50:					
1	.311	.159	.060	.015	.002
2	.378	.202	.188	.125	.022
3	.425	.296	.270	.166	.038
b = 1.00:					
1	.120	.005	.002	.001	0
2	.185	.070	.033	.010	.001
3	.276	.115	.056	.039	.008

extreme case where every option trade coincides with a simultaneous trade in the stock.

To derive the density function, we partition the stock trades into two groups, those that coincide with a trade in option and those that do not. We first redefine the parameter for stock trades to include only those trades that are not accompanied by an option trade, so that $\delta' = \delta - \lambda$. The derivation of the joint density function between the last option and the last stock trade then proceeds in a manner similar to that discussed in Section II, but with the limits for γ redefined: if $s > t$, so that a stock trade occurs after the last joint option-stock trade, then the density function is evaluated as before, while for $t > s$, the stock and option trade will be simultaneous, so that the difference in the time of trade will be zero.

Using the redefined parameter values, the joint density for the time between the last option trade and the last stock trade will be:[11]

$$g(\gamma') = \frac{\lambda\delta' e^{\lambda\delta'}[e^{\lambda+\delta'} - e^{-\gamma'(\lambda+\delta)}]}{(e^{\lambda} - 1)(e^{\delta'} - 1)(\lambda + \delta')},$$

where $\delta' = \delta - \lambda$ and $\gamma' = (s - t)$, $s > t$. The results of the simulations with this case are presented in table 4. As would be expected, the probability of mispricing being due to nonsimultaneity is reduced, since there is an increased chance that the last option trade and the last stock trade will coincide. However, the reduction is small, and the probability of nonsimultaneity still appears to have a significant value. The

11. The derivation of the density $g(\gamma')$ follows that for $g(\gamma)$, but with the integral evaluated only for the range $t - s < 0$, and setting $g(\gamma') = 0$ for the range $t - s > 0$.

reason for the reduction being slight is the relatively small volume of option trades as compared to stock trades. There is still a high probability that the final trade will be a stock trade, since many times more stock trades occur. So while the assumption of correlation between stock and option trades removes the possibility of nonsimultaneity being caused by an option trade following the final stock trade, the largest cause of the nonsimultaneity arises from the final stock trade following the final option trade.

V. Nonsimultaneity in Studies of Option Market Efficiency

The problems with nonsimultaneity have been discussed in a number of empirical papers dealing with the efficiency of option markets. Trippi (1977) used the Black-Scholes formula to spot mispriced options over a 6-month period, and bought underpriced options and wrote overpriced options. He closed all transactions after 1 week. Over the 6-month period, he found that his returns averaged 11% per week. Since these option positions were not hedged, they involved substantial risk, and the risk-adjusted return was not considered in his study. But even after risk adjustment such a high weekly return would seem extraordinary. Trippi noted in his paper that actual trade may not have been possible at the quoted option and stock prices. He checked for the possibility that the profits resulted from nonsimultaneity by comparing the closing prices for the stock and the option with the opening price the following day.

Galai (1977) used the closing quoted prices to determine if options fulfilled the boundary conditions of Merton (1973). Concerned with the nonsimultaneity problem, he noted that "closing prices do not always reflect the situation at closing time, but rather may only reflect the price of the last transaction which might have taken place a few hours before the closing round" (p. 194). To help evaluate the problems that this might cause, Galai used transaction-by-transaction data on the stock, and data that gave hour-by-hour quotes on the option. While such data could not guarantee simultaneity of the quotes, it is a finer sample than the closing data.

Chiras and Manaster (1978) used estimates of the implicit volatility of the stock to obtain estimates of the correct option price, and used option hedges when the published option price varied more than 10% from the formula price. During 22 holding periods, they formed 118 hedges and found that 93 of them were profitable. The average gain on the hedges was $9.96 per month, far above the fair market return. These results, if actually attainable, suggest substantial inefficiencies in the options market.

However, Chiras and Manaster are careful to point out that the observed gains from their hedging strategy may not equal the actual gain attainable from the strategy in practice. The reason for this is the problem with nonsimultaneity. As they state: "An option trader may not be able to capture all of the observed gain. The closing price of an option on any observation date may not accurately reflect the price at which a transaction could have occurred. . . . The observed closing . . . may not be the price at which the hedge position can be established. One would expect the observed price to be biased in favor of the trading rule and therefore expect the attainable gain to be less than the observed gain" (p. 229).

Nonsimultaneity in the Chiras-Manaster Study

Using the methodology described in Section I, we will consider the probability of nonsimultaneity in the Chiras-Manaster study.[12]

Nonsimultaneity and observed option mispricing. To determine the probability that the observed mispricing of the options is due to nonsimultaneity, we first selected the bounds on b to account for the difference between the observed and the implied market value of the options. For example, say the quoted price for the option is \$6.16, but the theoretical option value using the quoted stock price of \$50 in the option valuation formula was \$6.31. If a stock price of $49\frac{3}{4}$ would have given a theoretical value equal to the quoted option price of \$6.16, then we would set b equal to \$0.25.[13]

The volatility σ for the formula was computed using the future volatility from the Chiras-Manaster study. This volatility was put in daily terms, and then using the existing stock price was converted into dollar terms.

The parameter of the number of stock transactions, δ, was set equal to the stock volume divided by 100. This assumes that the average stock transaction is 100 shares. As was pointed out in the previous section, the results will not be particularly sensitive to the stock volume for actively traded stocks.

The parameter of the number of option transactions, λ, was set equal to the number of option trades. Since an option trade must involve at least one option contract, this will give an upper bound to the number of option trades. But since option trades will sometimes involve more than one contract, this will tend to exaggerate the number of option trades and give a downward bias to the probabilities. Thus, in equating the number of option trades with this upper bound, we bias out tests *against* finding the nonsimultaneity problem significant.

12. I am indebted to Steven Manaster for providing the data for the tests in this section.

13. When the deviations are expressed in terms of the deviation of the option price from the implied market value, b will be a nonlinear function, and the bands on b for a given amount of option mispricing will be nonsymmetrical. However, for the values of mispricing that we deal with, the degree of the nonsymmetry is insignificant.

TABLE 5 **Probability That Option Mispricing Is due to Nonsimultaneity in Stock and Option Quotations**

Number of Positions Having Mispricing due to Nonsimultaneity	Probability			
	5%	10%	20%	40%
For one option	47	32	17	4
For both options	30	14	5	1

The probability that the difference between the stock price at the time of the last option trade and at the time of the closing stock quote differed by enough to account for the apparent option mispricing—that is, prob ($\Delta P > b$)—was calculated for the 64 option hedges that had an observed return of greater than 10%. The results of these computations are presented in table 5.

The first row in this table shows the number of positions that had one option with a probability of mispricing due to nonsimultaneity that was greater than or equal to 5%, 10%, 20%, and 40%. Forty-seven of the 64 positions had at least one option with a 5% probability of mispricing due to nonsimultaneity. Half of the positions had prob ($\Delta P > b$) greater than or equal to 10%.

Since each hedge position involved the pricing of two options, we also calculated the probability that both options were mispriced in each of the 64 hedges. Half of the hedges had at least a 5% chance that both of the options were mispriced due to nonsimultaneity, and 14 had at least a 10% chance. At the 5% confidence level, then, we must reject over three-fourths of the option hedges as being the result of nonsimultaneity in the security quotations.

Nonsimultaneity and observed profits from the hedge position. Once the options are found to be mispriced, the profitability of the hedge depends on the relationship between the two options. As is shown in table 1, if the option quotes for the two options are from different periods of the day, a simultaneous position in the two options may not have been possible at the quoted option prices. In order to assess the probability that the observed values of the hedge could have been realized, we apply the methodology of Section I to this simultaneity problem.

We ask what is the probability that, given the closing quotes on the two options, one option had a price that would have eliminated the profit opportunity at the time of the closing quote of the other. That is, what is the probability that the two options would have been correctly priced contemporaneously, given their prices at the time of their last trade.

This question is similar to that posed with the stock and the option, except that now another option is being used in place of the stock.

TABLE 6 **Probability That Relative Option Mispricing Is due to Nonsimultaneity in the Two Option Quotations**

Number of Positions Having Mispricing due to Nonsimultaneity	Probability 5%	10%	20%	40%
For opening position	30	20	19	7
For closing position	32	29	17	9
Total positions	50	43	28	12

The values for the number of trades in the two options are set equal to the volume of option contracts. Again, this will give an upper bound to the number of trades and will cause a downward bias in the probabilities.

The value of b is set equal to the difference between the two option prices and the theoretical value that is necessary to eliminate the profit opportunity.

Since the option volatility is unavailable, we use the return volatility of the stock from the Chiras and Manaster study. The volatility is translated into daily terms and then adjusted from percentage terms to the relevant dollar terms for the option that will be used in place of the stock. Since the option will be more volatile in percentage terms than the underlying stock, this will underestimate the true volatility for the option and will bring another downward bias to the probability calculations.[14]

An observed profit can occur from a relative mispricing at the time of the opening of the position or at the time that the position is closed 1 month later. Table 6 computes the probability that the nonsimultaneity caused the observed profit by determining the probability of mispricing due to nonsimultaneity both when the position was initiated and when the position was closed. A nonsimultaneity problem in either of the two times can cause the observed profit.

The profit is attributable to nonsimultaneity at the 10% level for over half of the positions. Over a third of the positions were significant at the 20% level.

Since some of the option positions had a nonsimultaneity problem at both the opening and the closing of the position, the total number of hedges that had a nonsimultaneity problem must be tabulated separately. The third row in table 6 does this. This row shows the total number of hedge positions that had a probability of mispricing due to nonsimultaneity at the initiation or close of the position or at both times. At the 5% level, we cannot reject the hypothesis that the profits

14. The option used in place of the stock is the less volatile of the two. This will be the one with the lower exercise price or the longer time to expiration.

of 50 of the 64 positions tested were due to nonsimultaneity problems between the two options. At the 10% level, we cannot reject the hypothesis that 43 of the positions, over 70% of those tested, have observed profits that are due to nonsimultaneity. The results presented in this table give strong support to the concerns of Chiras and Manaster that the observed profits were due to the noncontemporaneous data, and are not achievable in practice.

VI. Conclusion

We have presented a method for evaluating the probability of nonsimultaneity in empirical work using published quotations. This method has been used to evaluate the probability that the mispricing and profit opportunities from option strategies are due to nonsimultaneity and are thus not achievable in practice.

The question of nonsimultaneity is broader than that which we have treated. The methods suggested here may also be of value in time-series analysis when daily quotations are used. For example, the extent of the error induced from the nonsimultaneity of the data due to day-to-day variation in the time of the last closing quotation can be assessed.

In the area of empirical work on options, the methods described in this paper may be of value in determining option mispricing. For example, the degree of apparent mispricing may be weighted according to the probability that the mispricing is due to nonsimultaneity, and then the decision to act on the observed mispricing may be adjusted accordingly.

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