John D. C. Little and
Jeremy F. Shapiro
Massachusetts Institute of Technology

A Theory for Pricing Nonfeatured Products in Supermarkets

A two-stage theory of price setting postulates that customers once in the store purchase goods to maximize utility. This determines short-run price response. The store then maximizes short-run profit subject to a constraint on customer utility delivered. Utility level becomes a policy parameter that determines, in part, the long-run attractiveness of the store to the customers.

Introduction

If a management science team from another galaxy should enter a typical U.S. supermarket and measure sales response to price by varying shelf prices and observing sales, it would be surprised to find that the store seemed to be pricing each item much too low to maximize profit. Thus, for example, a typical store markup for a grocery product might be in the 15%–25% range. This would represent profit-maximizing behavior if the price elasticity were in the range 5–7. However, price elasticities obtained by in-store experiment usually give numbers less than two and sometimes much less. For example, the measurements of Henderson, Brown, and Hind (1961) imply elasticities for fruits as low as 0.2. Thus, the store acts as if the customer is considerably more sensitive to prices than customer actions in the store seem to indicate.

There is a reason for this. The store has a special apprehension. Even though a customer, once in the store, may buy the item at a higher price, will the customer come back? A pricing policy that maximizes profits taking into account the problem of store loyalty may call for much lower prices than implied by strictly in-store measurements.

Supermarket chains go to great effort to project an overall impression of low prices and good values for the money. They run newspaper ads

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featuring reduced prices and set up special displays in the stores to emphasize sale items. These are often real bargains, with stores frequently taking a net loss on featured items.

However, the great bulk of items are not featured and it is necessary to have a pricing policy for them. By and large, stores use a simple markup over cost. This does not take into account customer price response, and so, in an important sense, ignores customer preferences. However, since the store does not maximize profit on individual items, the question arises whether individual price measurements have any meaning or usefulness to the store.

The purpose of this paper is to contend that they do and to develop the beginnings of a theory of how to use such information.

Notation and Definitions

We suppose each customer of the store to have a utility function that depends on the quantities of various items bought and the total amount of money spent. Some fixed time period is presumed. The quantities bought also represent the sales of the store since the customers' spending is the store's revenue. Let

n = number of items carried by store;
s_{ij} = quantity of item i bought by customer j (units);
p_i = price of item i (dollar unit);
$\mathbf{s}_j = (s_{ij}, \ldots, s_{nj})$;
$\mathbf{p} = (p_1, \ldots, p_n)$;
$s_i = \sum_j s_{ij}$;
$\mathbf{s} = (s_1, \ldots, s_n)$;
$d_j = \mathbf{p} \cdot \mathbf{s}_j = \sum_i p_i s_{ij}$ = total dollars spent by customer j (dollars);
$v_j(\mathbf{s}_j)$ = utility received by customer j as a result of acquiring items $\mathbf{s}_j$ at store;
$w_j(d_j)$ = utility lost by customer j as a result of spending d_j dollars on store's items; and
$u_j(\mathbf{s}_j, d_j)$ = customer j's net utility after making purchases $\mathbf{s}_j$ in the amount d_j at the store.

We suppose that $u_j = v_j(\mathbf{s}_j) - w_j(d_j)$, and further that $v_j(\mathbf{s}_j)$ is concave and $w_j(d_j)$ convex so that $u_j(\mathbf{s}_j, d_j)$ is concave. For example the v_j and w_j functions might have the shapes shown in figure 1. Practically, the shape of v_j means that the more units of any item a customer buys, the less he or she values an additional unit. The shape of w_j says the more money spent in total, the more the customer prefers not to spend another dollar on these items.

A more common approach would be to let utility represent only the value received from the goods and let the customer maximize this value

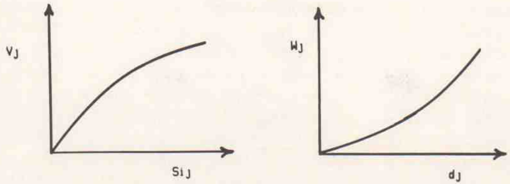

FIG. 1.—Components of customer j's utility function; v_j is the utility received by acquiring s_{ij} units of item i, w_j is the utility lost by spending d_j dollars in the store.

subject to a budget constraint. Such a formulation seems too rigid. The items bought in a supermarket are only a part of a customer's total spending. It seems likely that, as a person's purchases on supermarket items approach or exceed some habitual amount, a trade-off takes place between spending on these goods and other uses of money. Such a trade-off is explicitly modeled by the v_j and w_j functions.

Although virtually all items in the supermarket are sold in discrete units, we take all quantities to be continuous to avoid useless complexities.

Customer's Problem Once in the Store

The customers are assumed to make purchases to maximize their individual utilities given the prevailing prices in the store. In other words, for fixed $\mathbf{p} = (p_1, \ldots, p_n)$, customer j solves:

$$\begin{aligned}&\text{C1: Find } \mathbf{s}_j \text{ to}\\ &\max u_j = v_j(\mathbf{s}_j) - w_j(\mathbf{p} \cdot \mathbf{s}_j)\\ &\text{subject to: } \mathbf{s}_j \geq 0.\end{aligned}$$

The concavity of u_j guarantees a global maximum for some set of purchased quantities $\mathbf{s}_j$. At the maximum we shall have

$$\frac{\partial u_j}{\partial s_{ij}} \equiv \frac{\partial v_j}{\partial s_{ij}} - w_j'(d_j)p_i \begin{cases} = 0 & \text{for } i \text{ such that } s_{ij} > 0 \\ \leq 0 & \text{for } i \text{ such that } s_{ij} = 0 \end{cases} \tag{1}$$

where $w_j' = \partial w_j / \partial d_j$ and $d_j = \mathbf{p} \cdot \mathbf{s}_j$. The reason for the top equality is that anything else would imply u_j could be increased by changing s_{ij}, contrary to $\mathbf{s}_j$ producing the maximum. Similarly, the bottom $\leq$ must hold because $>$ would imply s_{ij} should be increased from zero, again contrary to $\mathbf{s}_j$ providing the maximum. Alternatively, the results follow immediately from the Kuhn-Tucker conditions for C1.

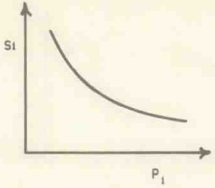

FIG. 2.—The store sees a price-response function that results from the customers' making purchases to maximize their utilities.

Repeated solutions of C1 for various prices **p** develop the customer's price response functions:

$s_{ij}(\mathbf{p})$ = sales of item i to customer j when items are priced $\mathbf{p} = (p_1, \ldots, p_n)$.

These, in turn, imply the customer's total spending $d_j(\mathbf{p}) = \sum_i p_i s_{ij}(\mathbf{p})$ as well as store sales by item, $s_i(\mathbf{p}) = \sum_j s_{ij}(\mathbf{p})$, at each set of prices **p**.

The functions $s_i(\mathbf{p})$ are those we believe a store might feasibly measure by in-store experiment. The general shape of $s_i(\mathbf{p})$ would presumably look like the sketch in figure 2.

Store's Problem Given Customers Trade at Store

The store's problem is, given the way customers respond to price, to set those prices. We investigate several methods for doing this. In this section we again assume that the customers regularly trade at the store.

Maximizing Customers' Utility Subject to Profit Constraint

Suppose the store somehow knows the customers' utility functions and chooses to maximize their sum subject to making a profit of at least some fixed amount. Then the store solves:

S1: Find **p** to

$$\max u = \sum_j \{v_j[s_j(\mathbf{p})] - w_j[d_j(\mathbf{p})]\} \tag{S1.1}$$

$$\text{subject to } \Sigma(p_i - c_i)\, s_i\,(\mathbf{p}) \geq \pi_0, \tag{S1.2}$$

$$\mathbf{p} \geq 0, \tag{S1.3}$$

where

c_i = incremental cost to the store of item i (dollar/item);

$\mathbf{c} = (c_1, \ldots, c_n)$;

π_0 = minimum profit for store before fixed costs (dollars); and

$d_j(\mathbf{p}) = \mathbf{p} \cdot \mathbf{s}_j(\mathbf{p})$.

Since, under our concavity assumptions, customer utility can always be increased by decreasing prices, maximum utility will be reached when the profit constraint is satisfied by equality and so, without loss of optimality, we can replace the inequality with equality in (S1.2). Furthermore, a solution with any prices zero would mean the store should not stock the product. Therefore, we suppose that in the solution of any real problem, $\mathbf{p} > 0$.

Maximizing Profit Subject to Utility Constraint

In this case the store would solve:

$$\text{S2: Find } \mathbf{p} \text{ to} \tag{S2.1}$$
$$\max \pi = \sum_i (p_i - c_i)\, s_i\,(\mathbf{p})$$
$$\text{subject to } \sum_j \{v_j[\mathbf{s}_j(\mathbf{p})] - w_j[d_j(\mathbf{p})]\} \geqslant u_0, \tag{S2.2}$$
$$\mathbf{p} \geqslant 0, \tag{S2.3}$$

where u_0 = smallest permitted customer utility.

We shall assume that $\Sigma_i(p_i - c_i)\, s_i\,(\mathbf{p})$ is concave in $\mathbf{p}$. This implies conditions on the customer utility functions giving rise to the $s_i(\mathbf{p})$. It is mathematically conceivable that we could pick a value of u_0 so low that the store's profit is maximized with customers' utility greater than u_0. There would be no point in the store setting up S2 with such a low u_0 and we presume this does not happen. Then there is a true trade-off between profit and customer utility and (S2.2) can be replaced by an equality without loss of optimality. Finally, we can again assume that real situations give rise to $\mathbf{p} > 0$.

Equivalence of S1 and S2

The store's price-setting problems S1 and S2 are equivalent in a certain important sense. Suppose u_0 is the customer utility achieved in S1 by maximizing utility subject to the store profit constraint $\pi = \pi_0$. If the same value of u_0 is made the utility constraint in maximizing store profit in S2, then maximal profit in S2 is π_0.

The situation is clarified by considering S3 where the store sets prices to maximize a weighted combination of store profit and customer utility.

$$\text{S3: Find } \mathbf{p} \text{ to}$$
$$\max \lambda\pi(\mathbf{p}) + (1 - \lambda)u(\mathbf{p}) \tag{S3.1}$$
$$\text{subject to } \mathbf{p} \geqslant 0 \tag{S3.2}$$

where λ is a constant.

As λ moves from 0 to 1 we develop sets of prices that go from pure utility maximizing to pure profit maximizing. If, for some λ, S3 produces an optimal solution $\pi = \pi_0$ and $\mathrm{u} = \mathrm{u}_0$ with prices $\mathbf{p} = \mathbf{p}_0$, these prices are optimal in both S1 and S2. This follows because $\mathbf{p}_0$ is feasible in both problems and further, if a better solution existed to S1 or S2, it would also be better in S3, contrary to the optimality we started with. (For a more formal development underlying the equivalence of S1 and S2, see Varian [1978].)

The main points for our purposes are two: first, we have explicitly developed the trade-off between customer utility and store profit. Second, and more important, *there is a set of efficient prices that permit each of these objectives to be as high as possible given a constraint on the other*. However, the development so far supposes that the customers trade at the store and then sales response to price $\mathbf{s}(\mathbf{p})$ is measured there. As discussed previously, a major issue for the store is whether an individual customer comes back to the store after shopping there or ever shops there in the first place.

Customers' Propensity to Shop at Store

We now hypothesize that although there are many ways to bring customers to the store and keep them coming (e.g., special features, product variety, cleanliness, pleasant surroundings, and friendly service) one important way is to provide low prices throughout the store, or, better yet, provide high value of customer utility. Thus the store might choose to maximize customer utility subject to a profit constraint or, as we have seen to be equivalent, to maximize profit subject to a utility constraint. In the last form, the value of utility, if it could be made operational, would become a parameter which the store could adjust. Therefore, we proceed to find the prices a store should charge according to the following assumptions: (1) the customer is buying to maximize utility (i.e., behaving according to $s_i = s_i[\mathbf{p}]$ from C1); (2) the store is pricing to maximize profit, subject to a customer utility constraint as in S2.

Starting from the Lagrangian for S2, $L(\mathbf{p},u) \equiv \Sigma_i\,(p_i - c_i)\Sigma_j s_{ij}(\mathbf{p}) + \mu\Sigma_j\{v_j[\mathbf{s}_j(\mathbf{p})] - w_j[d_j(\mathbf{p})]\}$, necessary and sufficient conditions for a maximum are $\partial L/\partial p_k = 0$, or

$$\sum_j s_{kj} + \sum_i (p_i - c_i)\sum_j \frac{\partial s_{ij}}{\partial p_k} + \mu\sum_j\left[\sum_i\left(\frac{\partial v_j}{\partial s_{ij}} - w_j' p_i\right)\frac{\partial s_{ij}}{\partial p_k} - w_j' s_{kj}\right] = 0, \qquad k = 1, \ldots, n, \tag{2}$$

where μ is chosen to satisfy constraint S2.2 as an equality. Because the customers are maximizing their utilities, we have from C1 and (1):

$$\left(\frac{\partial v_j}{\partial s_{ij}} - w_j' p_i\right)\frac{\partial s_{ij}}{\partial p_k} = 0.$$

This follows from (1) since either the quantity in parentheses is zero or $\partial s_{ij}/\partial p_k$ is zero (because $s_{ij} = 0$).

Now (2) simplifies to

$$s_k + \sum_i \left(p_i - c_i\right)\frac{\partial s_i}{\partial p_k} - \mu \sum_j w_j' s_{kj} = 0.$$

Defining elasticity and cross-elasticities,

$$\eta_k(\mathbf{p}) = -\,(p_k/s_k)\frac{\partial s_k}{\partial p_k};$$

$$\eta_{ik}(\mathbf{p}) = (p_k/s_i)\frac{\partial s_i}{\partial p_k};$$

and letting

$$z_k(\mathbf{p}) = \mu\ \{\sum_j w_j' s_{kj}\}/s_k; \tag{3}$$

(2) becomes

$$p_k = \frac{\eta_k + \sum_{i \neq k} [(p_i - c_i)/c_k][s_i/s_k]\eta_{ik}}{\eta_k - 1 + z_k}\, c_k. \tag{4}$$

This is the store's formula for setting prices. It is not really solved for p_k because p_k appears on the right-hand side in $\eta_k(\mathbf{p})$, $s_i(\mathbf{p})$, $\eta_{ik}(\mathbf{p})$, and $z_k(\mathbf{p})$. However, if we suppose that these quantities vary slowly with $\mathbf{p}$ or, more relevant, that we will measure η_k, η_{ik}, and s_i at the current $\mathbf{p}$ and make changes from current $\mathbf{p}$ rather gradually, then (4) is an appropriate calculation. If a p_k as calculated does not differ too much from current practice, then the calculated value can be expected to be very close. If the calculated p_k is quite different, a step in the direction of the new value is indicated, at which point new measurements would be taken.

Notice in (4) that, if the cross-elasticities, η_{ik}, were all zero and z_k were missing, the formula would reduce to $p_k = c_k\eta_k/(\eta_k - 1)$, which is simply the formula for a profit-maximizing price in the absence of constraints and interactions.

All quantities in (4) are presumed measurable or known except z_k. Essentially, z_k expresses the effect of the customer utility constraint. An important special case occurs when customers are homogeneous. Then w_j' is the same for each customer, say w' and (3) simplifies to $z_k = \mu w'$. This makes z_k independent of k. Under these circumstances, given the in-store measurements, *one constant sets all prices*. This constant expresses the store's desire to lower its price to keep the customers' satisfaction high and maintain store loyalty. In practice the constant might be initially set to keep average prices in the store near their current level. As time went on, the constant could be adjusted depending on how the store wishes to move. In either case, by using the formula (4), the store's prices have the property of efficiency, that is, for a given profit, no greater customer utility can be delivered.

Customers are certainly not completely homogeneous, although neighborhoods tend to contain people who are economically similar. However, even with some nonhomogeneity, z_k may not vary much with the item k. The pivotal question concerns w_j', the willingness of a customer to spend another dollar in the store (regardless of item): How does w_j' vary across the customer population? One might expect it to vary with ease of shopping at other stores. If w_j' is relatively constant or even if it varies, but the mix of items bought is similar across groups with homogeneous w', then z_k will tend to be independent of k, and again the prices are set by the measurements plus a single policy parameter.

Measurement Issues

Is it practical to measure price response as required by (4)? Today, no. Although many people have made in-store price response measurements, the task requires substantial time and expense, particularly to collect accurate sales data. The usual method is to take inventory at the start, tally all quantities restocked on the shelves, take inventory at the end, and then compute the net amount of product moving through the store. All in all, this is a cumbersome process.

By contrast, the introduction of equipment that records each sale at the point of purchase drastically simplifies the situation. Sales in great detail can be collected cheaply in machine-readable form. The quantity of data is, in fact, a little overwhelming and much needs to be learned about handling it effectively for analytic purposes.

In supermarkets, optical scanning equipment that reads the Universal Product Codes (UPC) offers the prospect of automatic sales recording. As of this writing (April 1979), however, only 700 stores out of many thousands in the United States are equipped with scanners and so we are not talking about an immediate revolution. However, pilot price measurements can be made without scanners even though such measurements would be too expensive for ongoing operations. Many stores have electronic check-out equipment on which it is possible, with moderate special effort, to record sales by item. As part of a study to investigate the measurement potential of UPC, one of the authors (Little) ran a 16-week, eight-store experiment in a Boston supermarket chain in which daily sales and price data were collected for a frozen-food category consisting of 16 separate items.

It is not our purpose here to do a full-scale evaluation of the potential of UPC data for marketing measurements, but we wish to present preliminary indications that price measurements are feasible. Figure 3 shows daily market share for two of the national brands in the category in the eight test stores during the period July 12–November 6, 1976.

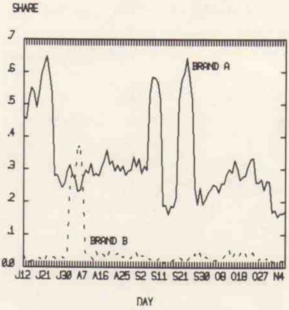

FIG. 3.—Daily market share of two national brands in eight stores of a Boston supermarket chain. Spikes are weekly specials.

Notice how weekly specials jump out of the data as bumps in share. The power of the data to support a variety of analyses is clear.

Insofar as price measurement goes, the experimental design examined the effect of changing prices of national brands relative to those of private labels in the category under study. Normally the chain maintains about 16% spread between these two sets of products, that is, the prices of national brands to private labels have a ratio 1.16. The experiment examined three different relative prices: normal, high, and low. For a given treatment prices were held constant for 4 weeks. Figure 4 shows what happened in two groups of stores, A and B. A stores received treatments normal, high, normal, and low in that order; B stores received high and low in reverse order. The plot shows national brand sales relative to private label (i.e., their ratio) for each group of stores over the experimental period. The values have been scaled to make the normal treatment periods average to 1.0. The price effects are evident—in the second period, the B stores with their higher priced national brands show lower national brand sales as compared to the A stores with their lower priced national brands (solid line above dashed line); in the third period, the stores come back together as both use normal prices; and in the fourth period, the treatments are reversed and so are the sales results (dashed line above solid line).

Several remarks can be made. First, the price effects were detectable. Second, response was quick, for example, it showed up in the first week. Third, the effect also appears to be reversible: It disappeared in the normal third period and was reversed in the fourth period. Detectability, speed of response, and reversibility are obviously all key issues

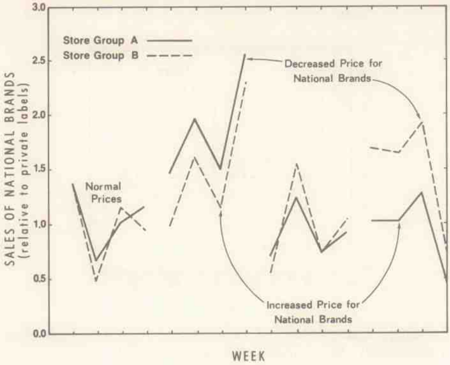

FIG. 4.—Measurement of a price effect on sales of selected items in a product category (data from a Boston supermarket chain).

in designing on-going systems. We certainly do not claim to have resolved all the operational problems; however, these initial results are encouraging.

Incidentally, the price response measured in the described experiment was low, confirming the observations made earlier: The price elasticities measured would imply that the store was pricing products too low for maximum profit.

Discussion

The theory developed here focuses on a central idea: a two-step process for relating in-store response measurements to price setting. First, the customers, once in the store, are considered to maximize their utility. This determines in-store price response. Then the store maximizes its profit subject to a constraint on customer utility determined so that the customer is motivated to trade at the store. The constraint puts a policy parameter in the store's pricing formula and, in effect, lowers the prices.

A number of theoretical issues deserve further investigation. One of these is to seek ways of setting the policy parameter (or eliminating it from the model) by explicitly modeling interstore competition. Such models have been built for other types of retail stores (see, e.g., Hlavac and Little [1969]). However, the main result is to identify a theory which has the promise of making in-store empirical measurements realistically useful in setting prices.

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