John P. Gould
University of Chicago

The Economics of Markets: A Simple Model of the Market-making Process*

A simple model involving two traders, each of whom stands to gain by trading, is developed. The traders must expend resources to find each other so that trade can take place. The effects of changing the gains and costs from trading on the amount of "marketing" effort expended by each of the traders are examined. It is found that increasing the gains from trading need not increase the amount of search expenditures made by the traders and may in fact result in a reduction in the probability that trade occurs. This is because there are external effects that the traders do not take into account when making search decisions that are optimal from their individual viewpoints. These externalities suggest that both traders may gain from the emergence of a middleman who acts as a market maker and takes the externalities into account.

I. Introduction

Markets have played a central role in economic analysis at least since the publication of Adam Smith's *Wealth of Nations*. Various forms of market organization—pure competition, duopoly, oligopoly, and monopoly—have been considered in great detail in the literature, and virtually every course in microeconomic theory devotes a substantial amount of time to explaining the nature and implications of these different kinds of markets.

The traditional approach in economic analysis has been to assume a particular form of market organization and then to analyze output, price, and cost behavior in the context of the assumed market form. A closely related matter, though one which seems to have received less attention, is the question of where the market comes from in the first place. The process of making a market is, after all, itself an economic activity in the sense that it uses scarce resources. In the usual

* This paper was written for the Rochester Conference on Interfaces between Marketing and Economics, April 7–8, 1978. The author has received several useful comments from members of the Applied Price Theory workshop at the University of Chicago and from the Economic Theory seminar at the University of Toronto. Comments and suggestions from Thomas Borcherding, Dennis Carlton, Jack Carr, Arthur De Vany, George Haines, Albert Madansky, Naser Saïdi, and Lester Telser have been particularly helpful. All errors are, of course, the responsibility of the author.

(*Journal of Business*, 1980, vol. 53, no. 3, pt. 2)

0021-9398/80/5332-0019$01.50

competitive model, however, the Walrasian auctioneer is treated as a free good.[1]

This is not to say that the costs of using (and making) the market have been systematically ignored by economists. There is a substantial literature, especially in recent years, that deals with the economics of advertising, and much of this work clearly interprets advertising as a part of the market-making process.[2] In a particularly interesting contribution, Telser (1964) examined how advertising by firms may be compatible with competitive markets. Nelson (1974) has analyzed similar issues treating advertising as a form of search behavior undertaken by producers looking for potential customers who otherwise may be unaware of the advantages of the advertised product. Nerlove and Arrow (1962) and Gould (1970) treated advertising as investment in "goodwill" or consumer information just as investment in physical capital is treated as accumulation of a factor of production. In a very important contribution, Butters (1977) derives equilibrium price distributions that reflect different advertising policies on the part of sellers. The role of advertising as a market-making function stands out clearly in Butters's work.

At a more general level, the seminal works of Stigler (1961) and Alchian (1970) on the economics of information examine how economic agents will devote resources to the accumulation of information by searching and other means. These papers have motivated a great deal of subsequent research, particularly in the area of determining equilibrium price distributions, but numerous questions remain open.

Recent work by De Vany (1976), Carlton (1978), Gould (1978), and others have dealt with the role that inventories and queues play in establishing equilibrium in a market. In Gould (1978), I analyze how inventory practices of firms serve a clearinghouse or customer/seller matching function of the kind that is usually implicitly assumed to be performed by the Walrasian auctioneer. That paper indicates how sellers and buyers each perform two functions: the usual function of actually trading in the underlying good and the related function of establishing a market for the good. The market-making problem arises when there is no exogenous market maker, such as the Walrasian auctioneer, to match sellers and buyers. Sellers can increase the availability of the product (i.e., raise the probability that any given customer will find a seller that has the product on hand) by increasing inventory levels. In a competitive equilibrium this will mean that the price of the product will be greater to compensate sellers for the costs of carrying larger inventories. Other sellers may carry smaller inventories and set

1. The standard assumption that buyers and sellers have complete information is tantamount to assuming that information is a free good.

2. Among the papers and studies dealing with this topic are those by Nerlove and Arrow (1962), Telser (1964), Gould (1970), Schmalensee (1972), and Nelson (1974).

lower prices, but customers of these sellers are more likely to find that the product is out of stock on any given visit. This comparison indicates how product price and related seller and customer behavior may reflect two separate economic activities. The relatively high-price sellers are taking on a relatively larger part of the market-making activity, and part of the price they receive is compensation for this service. On the other hand, the customers of the relatively low-price sellers incur more search costs (or more disappointment), and in this sense the relatively low price they pay for the product is compensation for the fact that they are shouldering a larger share of the market-making activity.

As the above discussion suggests, the traditional assumption that the market is exogenous to the model and costless to use leaves unanswered a number of questions that are of considerable interest. For example:

Given that market making is an economic activity, who will undertake this activity and to what degree?

Given that markets are a scarce resource, which markets will come into existence and in what situations will markets fail to emerge? Many economists have asserted that markets (particularly financial and state-preference markets) are incomplete, but there seems to be very little by way of explanation as to why this is the case. Telser and Higinbotham (1977) provide several interesting insights relevant to this question in the case of futures markets.

What determines the nature and characteristics of different markets? Why do some markets have fixed nonnegotiable prices (restaurant menus), while others have constantly varying prices (commodity markets, art auctions)?

Adam Smith noted that the division of labor is limited by the extent of the market, but what determines the "extent" of the market? Similarly, is there any meaningful economic content to the phrase "it is a thin market"?

In what cases and for what commodities will brokers and other middlemen be used to make the market?

What role do exclusive sales franchises and resale price maintenance policies play in making markets? In what ways are legislated price restrictions such as price ceilings and minimum-wage laws similar to resale price maintenance policies?

The above questions obviously involve a larger number of considerations than can be handled reasonably in a single paper. Indeed, the objectives of this paper are a good deal more modest. The main purpose is to present and discuss a very simple model of market making which, it is hoped, will shed some light on how these broader questions may be analyzed. The next section introduces the basic model. The remaining sections consider some of the above questions in terms of this simple model.

II. The Basic Model

It is common, when dealing with questions of advertising and other market-making activities, to separate economic agents into two classes: buyers and sellers. Typically the seller is assumed to undertake the active role in locating and persuading the buyer to purchase the product, while the buyer remains largely passive except with regard to the decision of whether to purchase the good and in what quantity. It seems to me that classifying economic agents into the categories of buyers and sellers is an artificial and potentially misleading way to begin.[3] In a fundamental sense there are neither buyers nor sellers, simply traders. Each trader is simultaneously a buyer and a seller. The fact that in advanced economies one trader gives up money in exchange for a good or service while the other gives up the good or service in exchange for money should not obscure the basic trading relationship that is involved. It is widely recognized in economics that trading occurs because both parties to the trade realize gains from the transaction. This is important because it means that both parties have an interest in making a market. Thus while we frequently think of the producer of the good or service as the trader that undertakes all of the marketing activity, it is more accurately the case that the purchaser of the good also contributes, perhaps significantly, to the trading process. The observation that product or service producers (or their agents) often seem to take most of the responsibility for making markets is to be treated as a datum to be explained not as an assumption of the model. Moreover, it is incorrect to treat this datum as universally true, since in many markets (such as labor markets) it appears that the purchaser does much of the searching and advertising.

The model examined here has two individuals, A and B, both of whom gain by trading. The gain to trader A is denoted g_A and trader B's gain is denoted g_B. Both g_A and g_B are positive.

To give content to the notion of market making, it is necessary to assume that traders A and B do not get together costlessly. There may be temporal, locational, and informational differences between the traders. The implicit assumption in the standard Walrasian analysis is that it is costless to get all interested traders together in the same place and at the same time. As a practical matter it is extremely difficult to arrange such a series of coincidences, and much of the work that has been done on the theory of search and information deals with how

[3] Kormendi (1979) has developed several useful results that relate to the economics of markets based on a pure trader model.

traders go about locating each other in time and space. No effort is made here to examine the details of this searching process. Instead, it is assumed simply that by spending resources to search, trader A has some probability of locating trader B and vice versa. Even at this level of generality the results turn out to be sensitive to technical assumptions about how the search behavior of each party affects the probability of being located by the other party. To illustrate, consider the following two scenarios.

Scenario 1

In this case, it is assumed that by spending resources in the amount s_A, trader A has probability $\theta(s_A)$ of locating trader B. Similarly, by spending s_B, trader B has probability of $\psi(s_B)$ of locating A. In a more complete analysis, it would be useful to develop the underlying determinants of the probabilities θ and ψ.[4] For purposes of this paper, I shall simply assume that the first derivatives of θ and ψ have the signs

$$\theta'(s_A) > 0, \ \psi'(s_B) > 0,$$

with second derivatives $\theta''(s_A) < 0$, $\psi''(s_B) < 0$. The global concavity of θ and ψ is assumed mainly for expositional convenience.

If both A and B search, the probability that they do *not* find each other is $(1 - \theta)(1 - \psi)$. Thus, the probability that they do find each other is

$$\pi(s_A, s_B) = 1 - (1 - \theta)(1 - \psi) = \theta(s_A) + \psi(s_B) - \theta(s_A)\psi(s_B).$$

To simplify notation, the partial derivatives of π will be designated only by "A" and "B" subscripts. Thus

$$\pi_A \equiv \frac{\partial \pi}{\partial s_A}$$

and

$$\pi_{AB} \equiv \frac{\partial^2 \pi}{\partial s_A \, \partial s_B}.$$

Using the assumptions of this scenario, we see that

$$\pi_A = \theta'(s_A)[1 - \psi(s_B)] > 0,$$
$$\pi_{AA} = \theta''(s_A)[1 - \psi(s_B)] < 0,$$
$$\pi_{AB} = -\theta'(s_A)\psi'(s_B) < 0.$$

The signs of π_B, π_{BB} are positive and negative, respectively. The negative sign of π_{AB} is of some interest. It arises because the marginal

[4] An example of how economic assumptions lead to endogenously determined probability distributions may be found in Gould (1978).

"product" (in terms of increased π) of more search by either party is reduced when search by the other party raises π. Thus, intuitively, if B is searching enough to get $\pi = .99$, additional search by A cannot have much effect on π (an increase of at most .01). If B searches very little, the effect of an increase in s_A could be very large. The negative sign on π_{AB} reflects the cross-product term $\theta(s_A)\psi(s_B)$, which has a negative sign in the definition of π. In this scenario, search by B acts as a substitute for search by A and vice versa.

Scenario 2

In the first scenario $\theta(s_A)$ and $\psi(s_B)$ depend, respectively, only on s_A and s_B. It is possible, however, that search efforts by B enhance A's search efforts and vice versa. If this is the case, the probability that A finds B is $\theta(s_A, s_B)$ with

$$\frac{\partial\theta}{\partial s_A} > 0, \; \frac{\partial^2\theta}{\partial s_A^2} < 0, \text{ and } \frac{\partial\theta}{\partial s_A\, \partial_B} > 0.$$

Similarly, ψ and ψ_B may be positively affected by the search efforts of A. If the cross-partials θ_{AB} and ψ_{AB} are positive and large, they may swamp the substitution phenomena of scenario 1, thereby making π_{AB} positive. In this case, search by B complements or enhances search by A.

In Sections III–VI the assumptions of scenario 1 (i.e., search as a substitute activity) are adapted. In Section VII the results of these sections are reconsidered to see how they are affected by the scenario 2 assumptions where search activities are complements. Summarizing the notation introduced to this point we have:

$$\begin{aligned}
g_A &= \text{gains from trade enjoyed by } A,\\
g_B &= \text{gains from trade enjoyed by } B,\\
s_A &= \text{search expenditures of } A,\\
s_B &= \text{search expenditures of } B,\\
\theta(s_A) &= \text{probability that } A \text{ finds } B,\\
\psi(s_B) &= \text{probability that } B \text{ finds } A,\\
\pi(s_A, s_B) &\equiv \theta(s_A) + \psi(s_B) - \theta(s_A)\psi(s_B) = \text{probability that } A \text{ and}\\
&\quad\; B \text{ do trade.}
\end{aligned}$$

For most of the following analysis it is not necessary to worry about $\theta(s_A)$ and $\psi(s_B)$ explicitly. Indeed, all we need to know are the signs of the partial derivatives and cross-partial derivatives of $\pi(s_A, s_B)$. In Sections III–VI we assume that π_A and π_B are positive, π_{AA} and π_{BB} are negative, and π_{AB} is negative. Obviously this is a more general case than the one described in scenario 1.

Assuming that A and B are expected value maximizers, A will choose s_A so as to maximize

$$\pi(s_A, s_B)g_A - s_A, \tag{1}$$

and B will choose s_B to maximize

$$\pi(s_A, s_B)g_B - s_B. \tag{2}$$

The resulting equilibrium is discussed in the following section.

III. The Market-making Decision

Given the model of the last section, it is reasonable to think of s_A and s_B as the resource costs incurred by A and B, respectively, in their efforts to make a market. These resource expenditures determine the probability π which may be viewed as an indicator of how successful A and B are in forming a market. At $\pi = 1$ there is a "perfect" market in the sense that trade will certainly occur. When $\pi = 0$, there is no market and hence no trading.

The determination of π will depend on how A and B react to each other's search efforts. Because there are only two traders in this simple model, this interaction involves the kinds of ambiguities associated with the familiar analysis of duopoly and other two-person games. I make no attempt to get into a detailed analysis of all these issues here, but simply examine the nature and existence of a Cournot-Nash equilibrium.

Following the Cournot-Nash assumptions, A will treat s_B as given in choosing s_A. Thus A will pick s_A to maximize (1) with s_B assumed constant. The first-order condition is

$$\pi_A g_A = 1. \tag{3}$$

The maximizing value of s_A (given s_B) need not always satisfy (3). If $\pi_A(0, s_A)$ is finite, there may be sufficiently large values of s_B or sufficiently small values of g_A such that A will decide not to search at all. This corner solution arises from the constraint that $s_A \geq 0$. Because we are now assuming $\pi_{AB} < 0$ and $\pi_{AA} < 0$, the amount of search A undertakes is a decreasing (or, at least, nonincreasing) function of s_B. A typical search-reaction function for A is shown in figure 1.

It is easy to show that as g_A increases, A will increase (or at least not decrease) s_A for every value of s_B. Thus, an increase in g_A will shift A's search reaction to the northeast in figure 1.

Since similar arguments apply to B's search decision, B's search-reaction function can also be plotted as in figure 2.

As figure 2 is drawn, the intersection of the two reaction functions

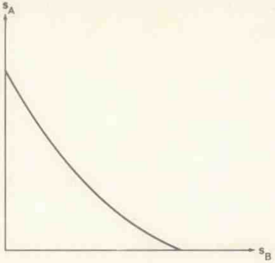

FIG. 1.—A's optimal search as a function of s_B

will be a stable Cournot-Nash equilibrium. This point is denoted s_A^* and s_B^* in the figure.

The stability condition requires that B's reaction function have a smaller slope (in algebraic terms) than A's reaction function at the point of intersection. This condition would not be met if we reversed the labels on the two reaction functions. When the labels are reversed, there are two points of Cournot-Nash equilibrium: one is s_B^{**} (where $s_A = 0$) and the other is s_A^{**} (where $s_B = 0$). In either case all of the search is undertaken by only one of the two traders.

Another situation in which the search is undertaken exclusively by one trader is shown in figure 3. In figure 3, A searches at s_A^* where A's reaction function intersects the vertical axis. At this level of search by

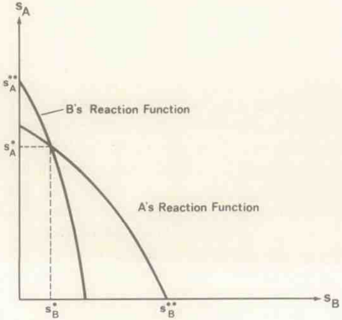

FIG. 2.—Search reaction functions of A and B

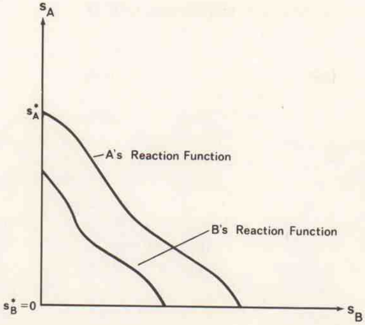

FIG. 3.—Search reaction functions where A is the only searcher

A, B's optimal level of search is $s_B^* = 0$. Thus, A is the exclusive searcher, and in this case the Cournot-Nash equilibrium is unique.

IV. Some Determinants of Market Making

Returning to figure 2, we can see how market-making behavior responds to changes in underlying parameters. Suppose, for example, that the trading gain to A, g_A, increases. The result will be that s_A^* will increase and s_B^* will decrease. The same result will obtain if the trading gain to B decreases, ceteris paribus.[5]

We are not able to say that the largest amount of search will be undertaken by the individual with the largest gain because search will also be affected by the derivative π_A (or π_B), which measures how effective the individuals are at searching. However, the effect of changes in the amount of search in response to changes in gains readily can be determined as seen in proposition 1.

PROPOSITION 1: In a stable Cournot-Nash equilibrium involving some search by both traders

$$\frac{ds_A}{dg_A} > 0, \text{ and } \frac{ds_B}{dg_A} < 0.$$

PROOF: This follows directly from the results in the Appendix with $dg_B = 0$.

The last proposition also shows how the market may fail to exist at all if the gains from search to both parties are sufficiently small. With

[5] When the labels on the reaction functions are reversed in figure 2, the intersection is no longer stable, so that either A or B will be the exclusive searcher. As the gain of the exclusive searcher increases, that searcher will increase the amount of search. If the gain to the other searcher is increased enough, their roles could be reversed.

very small values of g_A and g_B, the response functions of A and B will be collinear with the horizontal and vertical axis, respectively, of figure 2. In this situation, the Cournot-Nash point is at the origin.

It should be noted that proposition 1 only says something about the inputs to the market-making process, s_A and s_B. It says nothing about how the probability of trading, π, is affected by the change. The reason for this is that the effect of an increase in g_A is to increase π through the increase in s_A but to decrease π because of the decrease in s_B. The net effect on π is, therefore, ambiguous. This is the content of proposition 2.

PROPOSITION 2: In a stable Cournot-Nash equilibrium involving some search by both traders, an increase in g_A has ambiguous effects on π. That is, $\partial\pi/\partial g_A$ can be positive, negative, or zero.

PROOF: The derivative of π with respect to g_A is

$$\frac{\partial \pi}{\partial g_A} = \pi_A \frac{ds_A}{dg_A} + \pi_B \frac{ds_B}{dg_A}.$$

Substituting the expressions for ds_A/dg_A and ds_B/dg_A from the Appendix and rearranging terms, we obtain

$$\frac{\partial \pi}{\partial g_A} = \frac{\pi_A}{\Delta g_A}(\pi_B\pi_{AB} - \pi_A\pi_{BB}),$$

where $\Delta = \pi_{AA}\pi_{BB} - \pi_{AB}^2$. We know from the stability conditions that $\Delta > 0$. However, the bracketed term contains a positive and a negative term. Thus, the sign of the partial derivative of π with respect to g_A is ambiguous as asserted.

A somewhat more surprising result is that an increase in both g_A and g_B may lead to a reduction in π. This is the content of proposition 3.

PROPOSITION 3: The effect on π of an increase in both g_A and g_B is ambiguous.

PROOF: The differential of π is

$$d\pi = \frac{\partial \pi}{\partial g_A} dg_A + \frac{\partial \pi}{\partial g_B} dg_B.$$

Substituting values of $\partial\pi/\partial g_A$ and $\partial\pi/\partial g_B$ from the Appendix, we obtain

$$d\pi = \frac{\pi_A}{\Delta g_A}(\pi_B\pi_{AB} - \pi_A\pi_{BB})dg_A + \frac{\pi_B}{\Delta g_B}(\pi_A\pi_{AB} - \pi_B\pi_{AA})dg_B.$$

Both parenthetical terms in this last expresson are ambiguous as to sign. If, for example, the first expression is negative and the second positive, the relative sizes of dg_A and dg_B may be such that d_π is negative.[6]

[6] When $\Delta > 0$, as required by stability considerations, it is not possible for the bracketed expressions to be negative simultaneously.

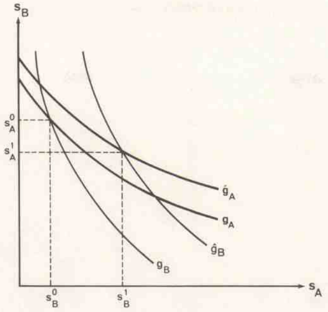

FIG. 4.—Shifts in response curves

The result discussed in proposition 3 is shown for small changes in g_A and g_B, but the intuition underlying can be shown for large changes in gains as illustrated in figure 4. In figure 4, g_A and g_B are used to label the response functions of A and B before the increase. After g_A and g_B are increased, the response curves shift to $\hat{g}_A$ and $\hat{g}_B$. The effect of the increase is to raise B's search from s_B^0 to s_B^1 and reduce A's search from s_A^0 to s_A^1. The value of π will increase unambiguously only if both s_A and s_B increase. In figure 4 the reduction in A's searching activity may more than balance the increase in B's search activity, so there is a net reduction in π.

V. Allocation of Gains between the Traders

If we assume that the traders are dealing in a transferable good or commodity, we can specify the total gains from trade as the sum of the gains of the individual traders. Suppose the total gain g is fixed and that g_A and g_B represent the shares of g going to A and B, respectively; then[7]

$$g_B = g - g_A \qquad \text{and} \qquad 0 \leq g_A \leq g. \tag{4}$$

[7] A more general specification would be

$$g_A = f(g_B)$$

with

$$0 \leq g_B \leq \bar{g}_B = f^{-1}(0)$$

and

$$0 \leq g_A \leq f(0).$$

We forego this generality for the present paper.

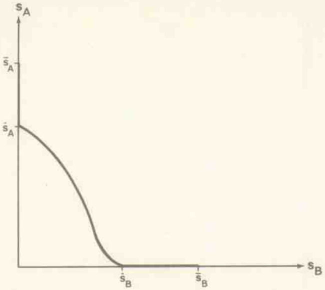

FIG. 5.—Values of s_A and s_B as g_A ranges from g to 0

Applying (4) to the search problems of A and B, we can specify both s_A and s_B as a function of the gain g_A. This will generate a locus of points in the (s_A, s_B) space that represents all possible combinations of search by A and B. Using (4) and the results in the Appendix, we find:

$$\frac{ds_A}{dg_A} = -\frac{1}{\Delta}\left(\frac{\pi_A\pi_{BB}}{g_A} + \frac{\pi_B\pi_{AB}}{g - g_A}\right) > 0,$$
$$\frac{ds_B}{dg_A} = \frac{1}{\Delta}\left(\frac{\pi_A\pi_{AB}}{g_A} + \frac{\pi_B\pi_{AA}}{g - g_A}\right) < 0, \tag{5}$$

where $\Delta = \pi_{AA}\pi_{BB} - \pi_{AB}^2$.

It follows from (5) that as g_A ranges from g to 0, s_A will fall and s_B will increase. Actually, depending on the value of g, s_A and s_B may be collinear with the horizontal and vertical axes, respectively, over some range. This is shown in figure 5. As may be seen in figure 5, s_A has a maximum of $\bar{s}_A$ when $g_A = g$. When g_A decreases, s_A falls along the vertical axis until the point $\hat{s}_A$, where B begins to search. As g_A falls further, s_A continues to decrease until it hits the horizontal axis at $\hat{s}_B$. From this point on s_A stays at zero, but s_B continues to rise until it reaches $\bar{s}_B$, where $g = 0$ $(g_B = g)$.

Recall that $\pi(s_A, s_B)$ depends only on the values of s_A and s_B. This means that isoprobability lines can also be shown in the (s_A, s_B) space. The slope of the isoprobability line with π fixed at some values, say π^0, is easily found:

$$d\pi^0 = 0 = \pi_A ds_A + \pi_B ds_B,$$

or

$$\left.\frac{ds_A}{ds_B}\right|_{\pi = \pi^0} = -\frac{\pi_B}{\pi_A} < 0.$$

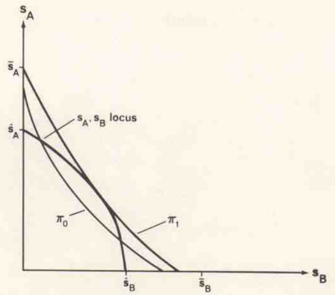

FIG. 6.—Locus of s_A and s_B and two isoprobability lines

Thus the isoprobability lines have negative slope. Moreover, since for fixed values of s_A, $(d\pi/ds_B) > 0$ and vice versa, isoprobability lines to the northeast will have higher values of π. Combining these results we obtain figure 6.

Tracing the s_A, s_B locus in figure 6, it can be seen that as s_A goes from $\bar{s}_A$ to $\hat{s}_A$, the value of π will decrease. Over this range $s_B = 0$, so the decreases in s_A must necessarily reduce π. When s_B becomes positive,

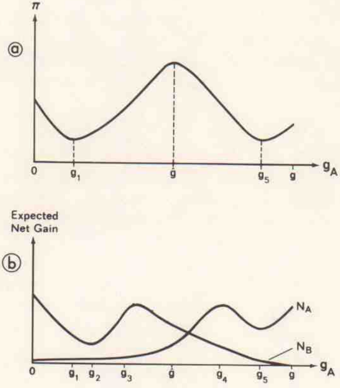

FIG. 7.—π, N_A, and N_B as a function of g_A

π begins to rise and continues to do so until the maximum value is achieved at h, where the s_A, s_B locus is tangent to π_1. As s_A continues to fall (and s_B continues to rise), π falls until the point $\hat{s}_B$ is reached. As s_B continues to rise between $\hat{s}_B$ and $\bar{s}_B$, π again rises, since over this range $s_A = 0$.

By using these results, it is possible to determine π as a function of g_A. Because g_A also determines s_A and s_B, it is also possible to show the expected net gains to A and B as a function of g_A, where expected net gain is defined as:

$$N_A = \pi g_A - s_A,$$

and

$$N_B = \pi(g - g_A) - s_B.$$

These functions are shown in figure 7. Figure 7*a* shows π as a function of g_A. When g_A is less than g_1, only B is searching but does so less and less as g_A rises. Thus π falls to g_1 in figure 7*a*. In figure 7*b* this means that, over the range from 0 to g_1, N_B is falling and N_A is rising. When g_A exceeds g_1, A begins to search, and this is enough to offset the continuing drop in s_b up to $\hat{g}$ (point h in fig. 6). This increasing π may cause N_B to rise over some range as indicated by the increase in N_B between g_2 and g_3 in panel figure 7*b*.[8] In the range from g_2 to g_3, N_A is also increasing. From g_3 to $\hat{g}$, π continues to rise but N_B falls because the increases in π are not enough to compensate for the losses in gains to B as g_A rises. The N_A continues to rise over this range. The probability of trading, π, reaches a maximum at $\hat{g}$. From this point on, the net gain to B continues to fall. Somewhere to the right of $\hat{g}$ the net gain to N_A achieves a local maximum (point g_4 in fig. 7*b*). For reasons similar to those described for N_B over the range from 0 to g_3, N_A hits a local minimum at g_5 and then increases over the range from g_5 to g.[9]

VI. Employment, Net Gains, and Market Policies

Figure 7 provides a means of assessing possible consequences of various market policies. The model presented here is very simple, and this limits its usefulness for this kind of assessment. Nonetheless, it has some features that may provide some general insights. For example, consider a policy of resale price maintenance. In this case trader A could be the retailer and trader B could be the consumer.

Suppose that without a policy of resale price maintenance the price is such that the gain to the retailer (g_A) is in the range from g_2 to g_3. If the

[8] The rise in N_B over this range will occur only if the increase in π and decrease in s_B are enough to offset the reduction in g_B.

[9] The local minimum of N_A may be to the left of g_5.

manufacturer raises the retail price to a larger value such that, say, g_3 is the gain to the retailer, the net gain to all parties will increase. This is because at the higher gain the retailer searches more and, being relatively more efficient at searching in this range than the customer, π increases. Both N_A and N_B increase and the manufacturer's net gain also increases since π is larger.[10]

The usual economic analysis of monopoly concludes that monopolization will result in higher prices and lower output. This result assumes, of course, that there is no cost to making the market. Referring again to figure 7, it is easy to see that in some cases monopolization could increase output in the transactional sense. Suppose B is the producer and A is the consumer of the good in question. Suppose that when B has no monopoly power, N_B is $g - g_5$ and N_A is g_5. If the ability to monopolize allows B to shift the gains to $N_B = g - g_4$, the probability of successful trades, π will rise. In this case the net gains to each party will also rise.

Most economists agree that as a theoretical matter minimum-wage laws will increase unemployment. The analysis of this paper indicates that an increase in unemployment need not be a logical consequence of such laws. Just as in the case of resale price maintenance and monopoly, the gains could be shifted in a manner such as to motivate more search by workers of a kind that causes π (a measure of the employment rate) to increase.

For the same reasons a policy of wage subsidies of the kind currently being discussed by various economists may not increase employment. Actually this is a direct consequence of proposition 2 above.

VII. Search Complementarity

In Sections III–VI it was assumed that $\pi_{AB} < 0$; that is, search by A reduced the marginal effectiveness of search by B and vice versa. Scenario 2 discussed the possibility that search by A may enhance the marginal effectiveness of search by B (and vice versa) in the sense that $\pi_{AB} > 0$. If this kind of complementarity exists, what effect would it have on the above analysis and propositions?

We know from equations (A5) and (A6) of the Appendix that the slope of the reaction functions becomes positive when $\pi_{AB} > 0$. In this case it can be shown that intersection of the reaction functions will be a stable Cournot-Nash equilibrium if the slope of A's reaction function is smaller than B's at the point of intersection. This is illustrated in figure 8. Using the results of the Appendix, it is easy to see how propositions

[10] Telser (1960) has argued that resale price maintenance could have these effects. Offering promotional allowances to retailers is not a viable alternative to resale price maintenance when the free-rider problem cannot be effectively policed.

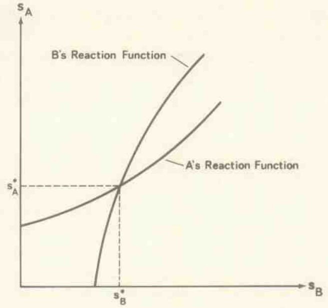

FIG. 8.—Search reaction functions of A and B when search is complementary.

1, 2, and 3 are affected. The modified propositions are restated here as propositions 1′, 2′, and 3′.

PROPOSITION 1′: In a stable Cournot-Nash equilibrium involving some search by both traders

$$\frac{ds_A}{dg_A} > 0 \qquad \text{and} \qquad \frac{ds_B}{dg_A} > 0.$$

COMMENT: The sign of ds_B/dg_A is now positive because the increased search by A raises the marginal effectiveness of search by B. Thus, an increase in the gains to trade for any party motivates an increase in the amount of search by all parties.

PROPOSITION 2′: In a stable Cournot-Nash equilibrium involving some search by both traders, an increase in g_A increases π, unambiguously.

COMMENT: This result is an obvious consequence of proposition 1′. This proposition suggests a reinterpretation of Adam Smith's famous assertion that the division of labor is limited by the extent of the market. When market making is perceived as an endogenous activity, it can be seen that the extent of the market may be limited by the division of labor. In other words, suppose that exogenous technological change made further division of labor feasible. If that is understood to mean an increase in g_A or g_B or both, then proposition 2′ says that there will be more effort directed to making a market and that more trading (i.e., larger π) will result.

PROPOSITION 3′: The effect of an increase in both g_A and g_B is to increase π.

COMMENT: This follows from proposition 1′ and proposition 2′ in an obvious fashion.

The analysis of Section VI is also affected by complementarity in search effectiveness. We see from (5) that when $\pi_{AB} > 0$, it is not necessary that s_A monotonically increases as gains from trade are shifted from B to A. Over some range the reallocation may lead to an increase in search by both parties. In terms of figure 5, this means the s_A/s_B locus could have positively sloped segments.

VIII. The Externalities of Market Making

The above analysis makes clear that market making involves external effects which sometimes have unexpected consequences. The externality that is involved may be seen by considering the joint global maximization problem

$$\max_{s_A, s_B} \pi g - (s_A + s_B),$$

where the total gain g is assumed to be the sum of the individual gains as in Section V. The first-order conditions for this problem are

$$\pi_A g = 1$$

or

$$\pi_B g = 1.$$

These conditions differ from the earlier ones (see eq. [3]) because in the earlier ones neither A nor B takes into consideration the effects of his search on the additional gains to the other party. In other words, if A searches ϵ more, the gain to A will be

$$(\pi_A g_A - 1)\epsilon,$$

but there will also be a gain to B of

$$\pi_B g_B \, \epsilon$$

that A does not take into account. Because this is also true of B, it would appear obvious that the externality results in less search activity than would be in the joint interest of A and B. Closer inspection shows that the obvious is not true in this case. It is possible that A and B spend more on search than is in their joint interest. This is shown in the following proposition.

Proposition 4: Let $s_T^{**} \equiv s_A^{**} + s_B^{**}$ be the solution to the joint global maximum problem

$$\max_{s_A, s_B} (\pi g - s_A - s_B),$$

where $g \equiv g_A + g_B$, and let s_A^* and s_B^* be the solutions arising from individual maximization by A and B (i.e., the Cournot-Nash solutions discussed earlier). If $\pi_{AB} > 0$, then $s_T^{**} > s_A^* + s_B^*$. However, if $\pi_{AB} < 0$, then the relationship of s_T^{**} and $s_A^* + s_B^*$ is ambiguous; it is possible that $s_T^{**} < s_A^* + s_B^*$.

PROOF: The joint global maximization problem has first-order conditions[11]

$$\pi_A g - 1 = 0,$$

$$\pi_B g - 1 = 0,$$

where $g \equiv g_A + g_B$. The Cournot-Nash problem has first-order conditions

$$\pi_A g_A - 1 = 0,$$

and

$$\pi_B g_B - 1 = 0.$$

Comparing these sets of conditions, it is obvious that the joint global problem is equivalent to an increase in the gain to both parties relative to the individual (Cournot-Nash) problem. Thus, we wish to know the effect of simultaneous increases in g_A and g_B on total search expenditures $s_T \equiv s_A + s_B$ in the Cournot-Nash problem. Specifically, we are interested in the behavior of the differential

$$ds_T = \frac{\partial s_T}{\partial g_A} dg_A + \frac{\partial s_T}{\partial g_B} dg_B = \left(\frac{\partial s_A}{\partial g_A} + \frac{\partial s_A}{\partial g_A}\right) dg_A + \left(\frac{\partial s_A}{\partial g_B} + \frac{\partial s_B}{\partial g_B}\right) dg_B.$$

Substituting for $\partial s_A / \partial g_A$, $\partial s_B / \partial g_A$, $\partial s_A / \partial g_B$, and $\partial s_B / \partial g_B$ from (A3) and (A4) of the Appendix, we obtain

$$ds_T = \left[\frac{\pi_A(\pi_{AB} - \pi_{BB})}{\Delta}\right] \frac{dg_A}{g_A} + \left[\frac{\pi_B(\pi_{AB} - \pi_{AA})}{\Delta}\right] \frac{dg_B}{g_B}, \qquad (6)$$

where, as before, $\Delta = \pi_{AA}\pi_{BB} - \pi_{AB}^2$ and is positive because of the stability assumption. We are interested in the sign of (6) when dg_A and dg_B both are positive. The sign of (6) then depends on the sign of π_{AB}. If $\pi_{AB} > 0$, then $ds_T > 0$ when dg_A and dg_B are positive. Thus, when search is complementary, the joint global maximum involves more search in total than A and B provide at the Cournot-Nash equilibrium. Indeed, we know from earlier results that both s_A and s_B will be larger in the joint global solution than in the Cournot-Nash solution when $\pi_{AB} > 0$. If $\pi_{AB} < 0$ (search substitutability), the sign of both bracketed terms in (6) is ambiguous. Either or both could be negative or zero.

The possibility that the global joint maximizing search solution may involve a smaller amount of total search than the individuals provide in the individual maximization is of some independent interest. In some respects this model parallels the economic theory of inventive activity.

[11] This proof applies to cases where the joint global maximization problem has an interior solution rather than a corner solution.

In that area it is often asserted that, because of incomplete assignments of property rights in inventions, the amount of total inventive activity is smaller than would prevail if expropriability of inventions could be prevented.[12] This externality is represented by the fact that individual A does not consider B's gain in making a search choice (and vice versa). However, when substitutability of search (or invention) exists in the sense of this paper, another externality is present. Each searcher (or researcher) will take into account the search (research) activities of others. If the activities of others reduce the marginal probability of success with respect to search (i.e., if $\pi_{AB} < 0$), then it may be possible that the deterrent effects from expropriability are substantial enough for one party so as to lead the other party to increase search at the point where total search exceeds level of the joint global optimum.

The existence of these externalities means there may be gains to a third party who brings A and B together. For example, a broker whose commission depends upon the gains to both parties would, to some degree, take these externalities into account. In addition, there may be technological economies of scale that a broker could realize by virtue of being in the market for longer periods of time than any given trader. Thus, it may be easier for a known and experienced broker to find traders who are only temporarily in the market than it is for such traders to find each other.

IX. Conclusions

The simple model developed here is obviously far too limited to answer all of the questions raised earlier in this paper. It would be useful to know more about how markets develop in cases involving more than two traders. That analysis is complicated by the fact that the gain to any trader will be itself a stochastic variable that depends on the particular set of other traders that are located in the search process. Because of these complications, the results of this paper must be regarded as only tentative. Nonetheless, intuition strongly suggests that several characteristics of the simple model will continue to hold in more general situations.

Appendix

If both A and B are searching at the Cournot-Nash equilibrium, then s_A and s_B simultaneously satisfy the equations

$$\pi_A(s_A, s_B)g_A = 1 \tag{A1}$$

$$\pi_B(s_A, s_B)g_B = 1. \tag{A2}$$

[12] For example, see Kenneth Arrow (1962); Harold Demsetz (1969) and Jack Hirshleifer (1971) have raised some important objections to this view.

The relationships between changes in s_A, s_B and changes in g_A, g_B can be found by solving the differential of this system. The result is

$$ds_A = \frac{\pi_{AB}\pi_{BB}}{\pi_{AA}\pi_{BB} - \pi_{AB}^2}\left(\frac{-\pi_A}{\pi_{AB}g_A}dg_A + \frac{\pi_B}{_{BB}g_B}dg_B\right) \tag{A3}$$

$$ds_B = \frac{\pi_{AB}\pi_{AA}}{\pi_{AA}\pi_{BB} - \pi_{AB}^2}\left(\frac{\pi_A}{\pi_{AA}g_a}dg_A - \frac{\pi_B}{\pi_{AB}g_B}dg_B\right). \tag{A4}$$

When the assumptions of scenario 1 apply (i.e., $\pi_{AB} < 0$), A's reaction function will be a decreasing (or nonincreasing) function of s_B and B's reaction function will be a decreasing (or nonincreasing) function of s_A. This can be seen formally by considering the differential of (A1), namely,

$$\pi_{AA}g_A ds_A + \pi_{AB}g_A ds_B = 0.$$

Solving this expression we get the slope of A's reaction function

$$\frac{ds_A}{ds_B} = -\frac{\pi_{AB}}{\pi_{AA}}. \tag{A5}$$

By a similar procedure using (A2) we obtain the slope of B's reaction function

$$\frac{ds_B}{ds_A} = -\frac{\pi_{AB}}{\pi_{BB}}. \tag{A6}$$

When $\pi_{AB} < 0$, both (A5) and (A6) are negative. Stability of the Cournot-Nash point requires that the slope (A5) be smaller (in algebraic terms) than (A6). Thus, at equilibrium, the stability condition implies

$$\pi_{AA}\pi_{BB} - \pi_{AB}^2 > 0. \tag{A7}$$

When $\pi_{AB} > 0$, both (A5) and (A6) are positive. Stability in this case requires that the slope (A5) be smaller (algebraically) than (A6) at equilibrium. Thus, the stability condition for $\pi_{AB} > 0$ (scenario 2) also implies (A7).

Return to (A3) and (A4) we see that the term in the denominator of the fraction outside the brackets (i.e. [A7]) is positive for both scenario 1 and scenario 2. Since π_A, π_B, π_{AA}, and π_{BB} have the same sign in both scenarios, the sign of ds_A and ds_B depends on the sign of π_{AB}. The following table summarizes the differences:

		With Respect to	
Sign of		dg_A	dg_B
ds_A	$\pi_{AB} < 0$	+	−
	$\pi_{AB} > 0$	+	+
ds_B	$\pi_{AB} < 0$	−	+
	$\pi_{AB} > 0$	+	+

References

Alchian, A. A. 1970. Information cost, pricing, and resource unemployment. In E. S. Phelps et al., *Microeconomic Foundations of Employment and Inflation Theory*. New York: Norton.

Arrow, K. J. 1962. Economic welfare and the allocation of resources for invention. In *The Rate and Direction of Inventive Activity*. Princeton, N.J.: Universities National Bureau of Economic Research Conference Series.

Butters, G. R. 1977. Equilibrium distributions of sales and advertising prices. *Review of Economic Studies* 44 (October): 465–91.

Carlton, D. W. 1978. Market behavior with demand uncertainty and price inflexibility. *American Economic Review* 68 (September): 571–87.

Demsetz, H. 1969. Information and efficiency: another viewpoint. *Journal of Law and Economics* 12 (April): 1–22.

De Vany, A. S. 1976. Uncertainty, waiting time, and capacity utilization: a stochastic theory of product quality. *Journal of Political Economy* 84 (June): 523–41.

Gould, J. P. 1970. Diffusion processes and optimal advertising policy. In E. S. Phelps et al., *Microeconomic Foundations of Employment and Inflation Theory*. New York: Norton.

Gould, J. P. 1978. Inventories and stochastic demand: equilibrium models of the firm and industry. *Journal of Business* 51 (January): 1–42.

Hirshleifer, J. 1971. The private and social value of information and the reward to inventive activity. *American Economic Review* 61 (September): 561–74.

Kormendi, R. C. 1979. Dispersed transaction prices in a model of decentralized pure exchange. In S. Lippman and J. McCall (eds.), *Studies in the Economics of Search*. Amsterdam: North-Holland.

Nelson, P. 1974. Advertising as information. *Journal of Political Economy* 82 (July): 729–54.

Nerlove, M., and Arrow, K. J. 1962. Optimal advertising policy under dynamic conditions. *Economics* 29 (May): 124–42.

Schmalensee, R. 1972. *The Economics of Advertising*. Amsterdam: North-Holland.

Stigler, G. J. 1961. The economics of information. *Journal of Political Economy* 69 (June): 213–25.

Telser, L. G. 1960. Why should manufacturers want fair trade? *Journal of Law and Economics* 3 (October): 86–105.

Telser, L. G. 1964. Advertising and competition. *Journal of Political Economy* 72 (December): 537–62.

Telser, L. G., and Higinbotham, H. N. 1977. Organized futures markets: costs and benefits. *Journal of Political Economy* 85 (October): 969–1000.