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A Price Theoretic Approach to the Specification and Estimation of the Sales-Advertising Function

This paper emphasizes the usefulness of modeling the sales-advertising function based explicitly on a theory of individual consumer choice. The proposed model of consumer choice rests on two assumptions: that the consumer maximizes utility net of information costs, and that advertising lowers these information costs. This model is used to generate the precise form of an estimable sales-advertising function. The function allows for the systematic measurement of own and competitive advertising effects. Also, the consumer choice model has implications that can be empirically tested.

I. Introduction

In spite of the great attention that marketing researchers have given to it, the determination of the sales-advertising function remains primarily an exercise in fitting statistical models to aggregate data sets. The approach of numerous studies to modeling the sales-advertising function has been to let the market data determine the "final" form of the model.[1] Although such interesting phenomena as cumulative effects of advertising, the two-way causality between sales and advertising, and competitive advertising effects have been modeled in past studies, there has been little effort to explicitly link estimable sales-advertising functions to a theory of consumer behavior.

Because the major use of the sales-advertising function is to set the firm's optimal level of advertising, it is necessary to obtain a model that adequately represents the true relationship between advertising and consumer demand for the

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1. Notable examples are the studies by Palda (1964), Bass (1969), Bass and Parsons (1969), Bass and Clarke (1972), Beckwith (1972), Schmalensee (1972), Clarke (1973), Wildt (1974), and Helmer and Johansson (1977).

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advertised brand. Statistical testing with small samples (a typical situation in studies of the sales-advertising function), and in the absence of an appropriate theory, however, often leaves the issue of model adequacy unresolved. To illustrate, consider a recent survey by Clarke (1976). He compared several distributed lag models in the literature to determine the duration of cumulative advertising effects on sales. Based upon studies reporting "statistically significant" results, Clarke found that the estimated duration of cumulative advertising effectiveness displayed significant sensitivity to the data interval used in the model: the estimated duration of cumulative advertising effectiveness based upon annual data was 17 times greater than that based upon monthly data. This signals a major problem. It is likely that the parameter estimates of the models are biased as a result of intertemporal aggregation, but the possibility that the models are functionally misspecified remains open.

There is an increasing effort among marketing researchers to study theoretically the impact of advertising on individual consumer behavior, prior to empirical analysis at the aggregate level (e.g., Blattberg and Jeuland 1979). This approach should answer how (in a predictive sense) and why (in an explanatory sense) advertising affects consumer choices. At the empirical stage, the theory should imply the form of the sales-advertising function as well as prove useful in interpreting the results. In this paper, first, the role of advertising in individual consumer choices is modeled. Second, an estimable sales-advertising function is derived based on this model.

An approach, same as this in spirit, is employed in an early contribution by Kuehn (1961). Based on a stochastic model of consumer brand choice, Kuehn derived an aggregate sales-advertising function. His study represents consumer choices by a first-order Markov model where the probability of a consumer shifting to the advertised brand, having bought another brand previously, is increased by advertising. Using assumptions and procedures similar to Kuehn's, Telser (1962), and recently Horsky (1977), developed a model of market-share response to advertising. The consumer choice models in these studies, though instructive, fail to give the mechanism through which advertising affects consumer brand choices. This paper utilizes well-established tools of price theory to model individual consumer choices and, as a result, provides this mechanism.

The paper is organized as follows. The impact of advertising on consumer behavior within the context of utility maximization is analyzed in Section II. The household production model[2] is extended to include consumer's search for information; advertising, given ex-

2. The household production model is due to Becker (1965) and Muth (1966). See Michael and Becker (1973) for an exposition and a justification of this model.

ogenously to the consumer, lowers information costs. In Section III, the model of Section II is used to generate the relationship between demand for a given brand and advertising levels and prices of all brands in the product class. It is shown that some specific assumptions on the household production functions and on the consumer utility function imply an estimable sales-advertising function. This function is linear in the unknown parameters, and the interpretation of the parameters is explicit. Also, estimation aspects are discussed briefly. Section IV develops some implications for advertising levels across different markets. Finally, the summary and some concluding remarks are presented in Section V.

II. Consumer Response to Advertising

Discussion

We begin with the neoclassical model of consumer behavior, as given in the economics literature. The consumer is assumed to maximize a well-defined utility function that is continuous, nondecreasing, and quasi concave with continuous second order partials. This maximization is subject to the usual constraints. Under these conditions, the allocation of the consumer's total expenditures is unique. To induce a reallocation of these expenditures, advertising must change the consumer utility function, or some of the constraints in the consumer's problem. Since we lack a theory that systematically predicts the effect of advertising on consumer tastes, it would be completely ad hoc to model the effect of advertising via the utility function. We assume that the utility function remains fixed.[3]

Some developments in economics emphasize the information role of advertising. An early study by Stigler (1961) demonstrated the consumer's incentive to search for a low price when the market is characterized by a homogeneous product quality[4] but some price variation among sellers. Given that there are costs to searching, optimal search may leave the consumer somewhat ignorant. Advertising that features product price and the seller's identity is equivalent to consumer's own search for a low-priced seller. Such advertising lowers the product's expected cost to the consumer and therefore provides valuable information.[5]

3. The issue here is not whether tastes actually change due to advertising, but whether it is efficient to invoke the change-in-tastes hypothesis while analyzing the effect of advertising on consumer behavior. For an excellent discussion, see Stigler and Becker (1977).

4. The words "product" and "brand" are used interchangeably to represent a given bundle of attributes. It is assumed that consumer perceptions of attribute bundles are homogeneous and that "quality" refers to a distinct bundle of attributes.

5. This analysis applies to most of retail-level advertising.

Recently, Nelson (1974) considered markets where both price and quality vary.[6] He distinguished two strategies that consumers follow in obtaining product-attribute information: the method of searching, that is, inspecting product attributes prior to purchase; and the method of evaluating product attributes by actually experiencing the product. Nelson then classified products as either search or experience, depending on the strategy that is optimally chosen. The basic notion underlying this classification is that, for experience products, attributes cannot be effectively evaluated by the consumer prior to consumption of the product.

Using this distinction between search and experience products, Nelson argued that advertising is information. In the case of search products, advertising provides direct information about product attributes. For experience products, advertising provides indirect information; this is simply the amount that the product advertises. Since higher quality brands face a higher probability of repeat purchase, the present value of inducing a trial purchase through advertising is higher for such brands. These effects result in a positive relationship between product quality and the level of advertising. Thus, the more heavily advertised brands make better buys, implying that information about a brand's advertising level is valuable to the consumer.

Irrespective of the merits of Nelson's argument, his analysis strongly suggests that advertising may be usefully treated as information. More recently, essays by Stigler and Becker (1977) and Rosen (1978) also favor this view. In this paper, I assume that advertising, through its role of informing consumers, affects some of the constraints in the consumer's problem. I have chosen the household production model to represent the consumer's behavior.

The consumer obtains utility from the consumption of commodities. These commodities are produced by the consumer with the input of products purchased in the market and own time. Since consumers differ in their preferences, abilities to produce commodities (household technology), and time values, they will also differ in their choice of attribute bundles. Efficient matching of desired attribute bundles to those available in the market requires information about attributes embodied in various products and about the corresponding prices. Then this information must be analyzed. The process of gathering and analyzing information, that is, producing information relevant to the production of a commodity, is costly to the consumer. The production and consumption of such information is endogenous to the consumer's behavior, and in general, information is supplied in the same manner as household commodities.[7]

6. See also Nelson (1970, 1978).

7. Examples of market products and services used to produce information are newspapers, magazines, and television services. The time input includes the time spent in reading and watching advertisements, and in personal search.

Formally, the commodity production in this study is expressed by the following two equations:

$$f_i : C_i = f_i(X_i, T_i, I_i) \tag{1}$$

$$e_i : I_i = e_i(G_i, H_i \mid A_i), \tag{2}$$

where C_i is the ith commodity; I_i is information relevant to the production of C_i; X_i, G_i $(X_i \neq G_i)$ are inputs of products and services purchased into C_i, I_i, respectively; T_i, H_i are consumer's time inputs into C_i, I_i, respectively; and A_i is the number of standardized information messages supplied by the firm through advertising. The production of C_i is not accomplished by the input of market products and time only; it requires the input of self-produced information, I_i, also. Moreover, the full price of I_i differs among consumers. This differential is caused by heterogeneous time values and information processing abilities across consumers.

Let A_i affect I_i as an exogenous technical change, such that

$$[d\log MP(G_i)/dA_i] > 0 \quad \text{and} \quad [d\log MP(H_i)/dA_i] > 0, \tag{3}$$

where $MP(G_i)$, $MP(H_i)$ are the marginal products of G_i, H_i, respectively. This assumption implies that for a fixed level of inputs, output of information and advertising are positively related. Advertising makes the production of information more efficient, which results in a lower full price of information to the consumer. Two objections may be raised. First, since the value of product-attribute information depends on the attributes desired by the consumer, (3) may not hold in general. Second, if advertising is misinformation, rather than information, then advertising will raise, not lower, the full price of information. To evaluate these objections, it is necessary to consider the complete market.

Consider a market where both the consumers and the sellers behave optimally. If there is diversity in consumer desires for attribute bundles, or if firms specialize in producing different attribute bundles, then the competitive equilibrium results in firms locating themselves at different points along the product-attribute spectrum (Rosen 1974). Assuming that consumers and firms find it uneconomical to untie attribute bundles (i.e., combine two attribute bundles to form a third distinct variety), different product varieties in the market will be sold through distinct, but closely knit, markets. Moreover, the attribute bundle that a consumer buys must satisfy the condition that the slope of the price-attribute indifference surface equals that of the market hedonic price function. Consumers with similar price-attribute indifference surfaces will tend to buy similar attribute bundles, and the net result is the formation of market segments.[8] Firms will cater to the

8. Consider, for example, fashion products like clothing, food, furniture, etc. High income consumers will tend to cluster around the point that, in the price-attribute space,

demand of specific segments, and advertising will necessarily feature only those product attributes that are desired by the given segment; thus, the first objection is invalid. Incidentally, these arguments are consistent with the notion of "benefit segmentation" (see Haley 1968).

The issue of false advertising is easily resolved in the case of search products. There will tend to be no false claims about product attributes since, for search products, the true attributes may be cheaply determined by the consumer prior to purchase (Nelson 1974). For experience products, according to Nelson, rational consumers will disregard any claims about product attributes that are made by the seller. Clearly, there is no room for false advertising. The only role of advertising is to inform consumers how much the brand advertises relative to other brands.

This argument about experience products is unsatisfactory. The positive relationship between product quality and advertising levels implied by Nelson's analysis relates only to the proposition that, *ceteris paribus,* higher product quality implies greater advertising. There is no assurance that the converse holds. But Nelson requires the converse proposition to conclude that advertising levels provide valuable information to the consumer for experience products. This problem arises basically because Nelson draws this conclusion in the absence of any formal market model.

A more fruitful approach is a modification of Rosen's argument (see Rosen 1978). Consider a competitive market where products of various quality are sold, and assume that the firm can raise the perceived quality of its product by misinforming consumers. Assume also that the attribute bundles are fully specified by one variable, quality, denoted by q. For a given level of quality, product price (p) is exogenous to an individual firm's decisions. The equilibrating price-quality schedule (the hedonic price function), $p(q)$, is the locus of all feasible price-quality combinations, such that this function is strictly increasing in quality.[9] A firm maximizes profits by optimally choosing product quality and output.

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is high in the fashion attribute. To see this formally, assume, as Stigler and Becker (1977) do, that fashion products produce social distinction and that net social distinction is the sum of fashion products consumed and the environment. Then the income elasticity of fashion products will be greater than one, even if social distinction has an average income elasticity.

9. In constructing the market hedonic price function, Rosen maximizes the consumer utility function in the product space, where this maximization is subject to the usual income constraint. I have defined the utility function in the commodity space, but this causes no real problem since utility in the commodity space may be translated into the product space. Household production technology is a map from each set of inputs (products, consumer's time, etc.) onto a set of feasible commodities. Define the utility function in the product space by letting the utility for a set of inputs equal the maximum utility in the commodity space, such that the commodities range over the feasible set of commodities implied by the given input set. This utility function in the product space

At the initial equilibrium, let firm X be located at (p_1,q_1) on the hedonic price function. If X falsely advertises that it sells product quality q_2, $q_2 > q_1$, rather than q_1, then X relocates itself to (p_2,q_2), where $p_2 > p_1$. In pursuing this strategy, X's costs remain intact since the (p_1,q_1) position incorporates optimal expenditures on advertising, but the higher price enables X to collect economic profits. Such profit opportunities in the market attract competition, which indulges in the same activity as X, except that the competition sells q_2 at a price slightly below p_2. It follows that competition will drive X back to (p_1,q_2) position. The new market equilibrium will consist of all firms producing the lowest possible quality, all charging prices compatible with this quality, but all advertising the highest possible quality. In this scenario, however, the concept of product quality becomes meaningless.

The assumption that consumers can be easily fooled is now dropped. If consumers know the true product quality after the initial purchase, false advertising in the current period essentially raises the future costs of informing consumers. Put differently, X will collect negative economic profits in the future. Thus, X will indulge in false advertising until, at the margin, profits gained in the current period equal the present value of those lost in the future. In some cases, a positive amount of false advertising may be optimal.

But rational consumers will disbelieve advertising messages in precisely the cases where a positive amount of false advertising is optimal for the firm. In such cases, consumers will seek other sources of information (e.g., *Consumer Reports*), implying that advertising will be ineffective. Nevertheless, outside parties are not always the most efficient sources of information. In equilibrium, the identity of the information source (or sources) will depend on the exact nature of the experience product and on the ability of consumers to analyze information. Though not unrelated, the issue here, given that the firm supplies information at the least cost, is the form of cost minimizing transfer of information to the consumer and the role of advertising in it.

An efficient market solution, under these circumstances, could be for the firm to enter into an (informal) agreement with consumers.[10] Usually the most efficient form of such an agreement is the firm's "reputation."[11] This agreement internalizes the costs of false advertising and

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may now be maximized subject to the income and time constraints, resulting in the usual demand equations for products (and time). Also, for a neoclassical production technology with constant returns to scale, a quasi-concave utility function in the commodity space implies a quasi-concave utility function in the product space. Details are given in Pollak and Wachter (1975).

10. This point is due to Darby and Karni (1973).

11. Interestingly, the persistence of family brand names in some industries is evidence for the existence of such "informal agreements." Two conditions, in the case of experience products, favorable to a positive amount of false advertising being optimal are a high interest rate and a low frequency of purchase. Notice the widespread practice of

thus eliminates the incentive for false advertising. Since reputation does not accrue to the firm costlessly, a reputable firm must be compensated in the form of a higher price for its product. Also, reputation is an economic good because it provides product quality information; thus, consumers are willing to pay for it. Indeed, a firm's investment in reputation is analogous to investment in other forms of capital. Advertising still provides direct information, which is valuable to the consumer, to the extent that it features product price, seller's identity (brand or company name), and search attributes of the product.

Even though this discussion does not completely resolve the issue of misinformation, it does show the assertion that advertising is misinformation to be rather presumptuous.

The Basic Model

Let the consumer face the following problem:

Maximize $U(C_1, \ldots, C_n)$

$$\{C_1, \ldots, C_n\} \tag{4}$$

Subject to:

$$C_i \geq 0, i = 1, \ldots, n, \tag{4a}$$

$$\sum_{i=1}^{n} (P_{Xi}X_i + P_{Gi}G_i) \leq WT_0 + V, \tag{4b}$$

$$\sum_{i=1}^{n} (T_i + H_i) \leq T - T_0, \tag{4c}$$

$$f_i : C_i = f_i(X_i, T_i, I_i), \tag{4d}$$

$$e_i : I_i = e_i(G_i, H_i \mid A_i), i = 1, \ldots, n, \tag{4e}$$

where T_0 is time spent working at a constant wage rate W, T is the total time resource available, P_{Xi} (P_{Gi}) is the competitive market price of X_i (G_i), V is nonwage income, and $U(\cdot)$ represents the consumer's utility function. Both production functions, e_i and f_i, are assumed to be positive, nondecreasing, positively linear homogeneous, and concave over the positive orthant in the input space. The utility function is

family branding in the case of experience durables—think of tires, appliances, motor vehicle parts, motor vehicles, clocks and watches, etc. The brand-extension strategy is also along the same lines, except that it enables the firm to cater to several segments within the same product class. Similarly, in markets with no significant repeat purchase, e.g., tourist areas, informal agreements often exist in the form of national franchises. In both cases, these agreements are costly to firms. Family branding implies uniform quality standards for all products under the family name, and thus the lost opportunities of selling to consumer segments that desire lower product quality. In the case of franchising, the firm's management costs rise sharply relative to those for a number of individually owned firms selling to the same market.

assumed to satisfy the conditions stated in the previous subsection. Notice that the variable A_i itself is not a direct input into the production of I_i, but instead it affects production by changing the productivity of G_i and H_i. The variable A_i, assumed to be parametric for the consumer's decisions, is then an "environmental variable" because its use by one consumer does not affect its availability to another.

In maximizing utility derived from commodity consumption, the consumer faces some constraints. The income constraint, (4b), and the time constraint, (4c), define the feasible set of inputs. The production technology constraints, (4d) and (4e), reflect the consumer's efficiency in producing household commodities. Therefore, these constraints restrict the feasible set of commodities for a given set of inputs. The constant returns to scale assumption on the production functions e_i and f_i, $i = 1, \ldots, n$, allows us to identify the unit cost function $\Pi(C_i)$ as being the full price of one unit of the commodity C_i.

The optimum combination of inputs in the production of C_i satisfies the conditions: $MP(X_i)/MP(T_i) = P_{Xi}/W$, $MP(X_i)/MP(I_i) = P_{Xi}/\Pi(I_i)$, and $MP(T_i)/MP(I_i) = W/\Pi(I_i)$, where $\Pi(I_i)$ is the unit cost function defined by e_i. Also, the optimum commodity bundle satisfies the equality between the marginal rates of substitution and the full price ratios, $(\partial U/\partial C_i)/(\partial U/\partial C_j) = \Pi(C_i)/\Pi(C_j)$, for $i, j = 1, \ldots, n$.

An exogenous increase in advertising lowers the inputs of G_i and H_i required to produce a given output of I_i. The net result is a reduction in the full price of information:

$$d\log \Pi(I_i)/dA_i = -\{S_{Gi}[d\log MP(G_i)/dA_i] + S_{Hi}[d\log MP(H_i)/dA_i]\}, \quad (5)$$

where S_{Gi} is the factor share of G_i and $S_{Hi} = 1 - S_{Gi}$. A fall in $\Pi(I_i)$ further induces a full price reduction for C_i. This price reduction can be simply written as:[12]

$$d\log \Pi(C_i)/dA_i = S_{Ii}[d\log \Pi(I_i)/dA_i], \quad (6)$$

where S_{Ii} is the factor share of I_i. Now equations (5) and (6) can be combined to give the change in $\Pi(C_i)$ in terms of changes in the marginal products of G_i and H_i.

Recall that the basic restriction on this model is that the production functions exhibit constant returns to scale. This restriction is essential for the competitive utility maximization analysis to be applicable. But if consumers are not indifferent between alternative uses of their time (a reasonable assumption), then $\Pi(C_i)$ will depend on the particular commodity bundle that the consumer chooses (Pollak and Wachter 1975). Such a dependence causes several complications and greatly limits the usefulness of this model.

12. See the Appendix for a derivation.

If $\Pi(C_i)$ depends on the commodity bundle chosen, then $\Pi(C_i)$ is a function of input prices, household technology, and tastes as well. A change in $\Pi(C_i)$, due to a change in A_i, will induce the consumer to reallocate consumption opportunities as well as change the optimal mix of inputs in the production of C_i. The sum of these two effects will induce a change in consumer demand for X_i. But a change in the consumption bundle further changes $\Pi(C_i)$, and this again results in a change in the demand for X_i. This adjustment goes on until the commodity full price ratios are once again equated to the marginal rates of substitution. Thus, the effect of advertising, in terms of changing consumer demand for X_i, will depend on tastes.

These conditions adversely affect the predictive ability of this model. Consider, for example, two consumers facing identical production technologies, input prices, and full incomes ($\mu = WT + V$), such that they consume exactly the same commodity bundle when $A_i = A_i'$. Now let A_i be changed to A_i''. At this point it is perfectly possible that one consumer's demand for X_i increases, while another's is reduced, depending purely on their tastes. Clearly no unambiguous prediction based on marketing variables or economic factors can be made. Also, since real consumption opportunities cannot be treated as a predetermined variable in the estimation of the sales-advertising function, the applicability of the model is restricted.

This situation is remedied when there is no joint production.[13] Commodity prices, $\Pi(C_i), i = 1, \ldots, n$, are independent of the consumer's commodity bundle if and only if the production technology exhibits both constant returns to scale and no joint production.[14] The rest of the paper assumes the absence of joint production in the use of all inputs.

Now it is straightforward to measure the total change in the derived demand for X_i. Advertising lowers two kinds of relative prices, the relative full price of C_i and the relative full price of I_i. Therefore the demand for X_i is affected by three forces: the production substitution effect, the pure substitution effect in consumption, and the expansion (real full income) effect in consumption. The net change in the demand for X_i is:[15]

$$d\log X_i/dA_i = S_{Ii}\sigma_{XiIi}[d\log \Pi(I_i)/dA_i] + (d\log C_i/dA_i), \qquad (7)$$

13. Joint production occurs when an input simultaneously produces more than one commodity. For example, shopping around and searching for the preferred attribute bundle can produce information as well as entertainment. In this case, then, $C_i = f_i(X_i, T_i, I_i)$, where $I_i = e_i(G_i, H_i)$ and $C_j = H_i$, such that $i \neq j$.

14. See Pollak and Wachter (1975) for a proof.

15. Since the production of I_i and C_i is linear homogeneous, the demand for X_i may be written as $X_i = d_i[P_{Xi}, W, \Pi(I_i)]C_i$. It follows that $d\log X_i/dA_i = \omega_{XiIi}[d\log \Pi(I_i)/dA_i] + d\log C_i/dA_i$, where $\omega_{XiIi} = [d\log X_i/d\log \Pi(I_i)]$, at a fixed level of C_i. This results in eq. (7) when Allen's partial elasticity of substitution is defined as $\sigma_{XiIi} = \omega_{XiIi}/S_{Ii}$.

where σ_{XiIi} is Allen's partial elasticity of substitution in production between X_i and I_i. The change in C_i can be decomposed into the substitution and expansion effect:

$$d\log C_i/dA_i = -\Big(\eta_i(d\log \Pi/dA_i) + \epsilon_i\{[d\log \Pi(C_i)/dA_i] - (d\log \Pi/dA_i)\}\Big), \quad (8)$$

where η_i and ϵ_i are the full income and pure price elasticities of C_i, respectively, and Π is the commodity price index. For a particular choice of Π,

$$\Pi = \prod_{i=1}^{n} \Pi(C_i)^{v_i},$$

where the v_i's are fixed weights with

$$\sum_{i=1}^{n} v_i = 1,$$

equations (6), (7), and (8) combine to give:

$$d\log X_i/dA_i = -S_{Ii}\,[v_i\eta_i + (1 - v_i)\,\epsilon_i - \sigma_{XiIi}]\,[d\log \Pi(I_i)/dA_i]. \quad (9)$$

In view of equations (5) and (9), the demand for X_i, with respect to A_i, will rise, fall, or be unchanged as $[v_i\eta_i + (1 - v_i)\epsilon_i]$ is greater than, less than, or equal to σ_{XiIi}. Put differently, if information and the advertised product are sufficiently close substitutes in commodity production, then advertising will lower, not increase, demand for the product.

At this point, it is important to predict the sign of σ_{XiIi}. In general, the complex nature of information makes it difficult to predict whether X_i and I_i will be complements, substitutes, or independent (i.e., $\sigma_{XiIi} \lesseqgtr 0$). But information conveyed through advertising is unique in at least this aspect: the advertiser has an incentive not to provide information that is a substitute for the product's use.[16] Therefore, $\sigma_{XiIi} \leq 0$, and we can unambiguously predict that (for a normal commodity) advertising increases consumer demand for the advertised product. Although this conclusion is not exactly surprising, the important point is that the conclusion does not require the assumption that advertising changes consumer tastes. Implications of this model are explored further in Section IV.

16. This argument imposes a restriction on the other two elasticities of substitution: If σ_{XiIi} is negative, then σ_{XiTi} and σ_{IiTi} must both be positive. This follows from the fact that, for a production function with three inputs, at most one partial elasticity of substitution can be negative. But the implication that both σ_{XiTi} and σ_{TiIi} are positive seems perfectly reasonable.

III. The Sales-Advertising Function with Competitive Advertising

The General Demand Equation

Except for observation and approximation errors, demand equation (9) completely specifies changes in product demand as a result of changes in advertising. In markets characterized by brands that are closely related in terms of their attributes, however, advertising and prices of all brands can affect the demand for any given brand. In such markets, equation (9) is inadequate for estimating advertising effects with commonly available time series data at the aggregate level. In this section, within a natural extension of the framework developed earlier, the precise mechanism through which competitive advertising affects the brand demand function is analyzed. A general demand equation that relates changes in brand demand to changes in consumer's wage rate, and to changes in advertising and prices of all available brands is derived. Later, special forms for the consumer utility function and the household production functions are assumed to facilitate the econometric estimation of this demand equation.

Let X_{ij} represent the jth brand in the ith product class, and let C_{ij} be the commodity produced by the input of X_{ij}. If m is the total number of product classes,

$$n = \sum_{i=1}^{m} n(i),$$

where $n(i)$ is the number of brands in the ith product class, then the consumer maximizes the following utility function:

$$U[C_{11}, \ldots, C_{1n(1)}, \ldots, C_{i1}, \ldots, C_{in(i)}, \ldots, C_{m1}, \ldots, C_{mn(m)}].$$

The constraints in this maximization problem are identical to those given in equations (4a)–(4e), except the subscript i is replaced by the subscript ij. The restrictions on the utility function, and on the production functions for commodities and information, at this point, are the same as the corresponding restrictions imposed earlier. Conceptualize an experiment such that only prices and advertising levels of brands in the ith product class, P_{xil} and A_{il}, for all l, vary. Commodities in the ith group, C_{il}, $l = 1, \ldots, n(i)$, will tend to be close substitutes in consumption, implying that derived demand for X_{ij} will vary systematically with commodity full prices, $\Pi(C_{il})$'s. Brand prices, via their direct effects on commodity full prices, and advertising levels, via their effects on information full prices and hence on commodity full prices, affect the demand for brand j. If advertising messages are separable—in the sense that attribute information supplied through advertising of one brand provides no useful information about another—then competitive advertising changes demand for X_{ij} by reducing relative full prices of the competing commodities in the ith group, C_{il}, with $l \neq j$. Effects of P_{Xil} and A_{il} are measured in detail below.

Since the demand for X_{ij} can be written as $X_{ij} = F_{ij}[P_{Xij}, W, \Pi(I_{ij})]C_{ij}$, where C_{ij} depends on full income, μ, and on all commodity full prices, $\Pi(C_{ij})$, for all i and j, it follows that:

$$
\begin{aligned}
\partial \log X_{ij}/\partial A_{il} &= S_{Iij}\sigma_{XijIij}[\partial \log \Pi(I_{ij})/\partial A_{ij}] \\
&\quad - \eta_{ij}(\partial \log \Pi/\partial A_{ij}) \\
&\quad - \epsilon_{ij}\{\partial \log[\Pi(C_{ij})/\Pi]/\partial A_{ij}\}, \quad \text{if } l = j, \\
&= -\eta_{ij}(\partial \log \Pi/\partial A_{il}) \\
&\quad - \epsilon_{ijl}\{\partial \log[\Pi(C_{il})/\Pi]\partial A_{il}\}, \quad \text{if } l \neq j,
\end{aligned} \tag{10}
$$

where the quantities with the *ij* subscript have the obvious meaning and $\epsilon_{i,jl}$ is the cross price elasticity between C_{ij} and C_{il}, $l \neq j$. Equation (10) measures the net effect of A_{il}, for all l, on the demand for X_{ij}. Own advertising of brand j induces substitution among inputs in the production of a fixed level of C_{ij}, substitution in consumption of C_{ij}, and the full income effect. As shown in the previous section, all three effects increase demand for X_{ij}. On the other hand, the substitution effect in production induced by competitive advertising leaves demand for X_{ij} unchanged. Since increased advertising by a competing brand lowers the relative full price of the corresponding commodity, the derived demand for X_{ij} declines. For a fixed percentage increase in advertising by a competing brand, X_{ik}, this decline in demand for X_{ij} varies directly with the cross price elasticity between C_{ij} and C_{ik}. Stated differently, to the extent that the cross price elasticity between commodities measures the similarity between the corresponding brands, the effect of competitive advertising in absolute magnitude, *ceteris paribus*, and the similarity between the competing brands are positively related—an intuitive result. The full income effect resulting from increased competitive advertising increases demand for X_{ij}. This increase in demand, given a fixed percentage increase in a competitor's advertising, varies in proportion to the full income elasticity of C_{ij}.

The proportionate effect on X_{ij} due to changes in P_{Xil}, for all l, can be similarly measured by adding the proportionate change in C_{ij} and the total substitution effect in production:

$$
\begin{aligned}
\partial \log X_{ij}/\partial P_{Xil} &= -(S_{Iij}\sigma_{XijIij} + S_{Tij}\sigma_{XijTij})/P_{Xil} \\
&\quad - \eta_{ij}(\partial \log \Pi/\partial P_{Xij}) \\
&\quad - \epsilon_{ij}\{\partial \log[\Pi(C_{ij})/\Pi]/\partial P_{Xij}\}, \text{ if } l = j, \\
&= -\eta_{ij}(\partial \log \Pi/\partial P_{Xil}) \\
&\quad - \epsilon_{i,jl}\{\partial \log[\Pi(C_{il})/\Pi]/\partial A_{il}\}, \text{ if } l \neq j.
\end{aligned} \tag{11}
$$

Equations (10) and (11) both contain the proportionate change in the commodity price index, Π. Since Π is not observed, it must be measured in terms of other variables. Therefore, consider the indirect utility function, $U = U(\underline{C}_{ij},\underline{\Pi}_{ij}) = U_t(\mu,\underline{\Pi}_{ij})$, where $\underline{C}_{ij} = \underline{C}_{ij}(\mu,\underline{\Pi}_{ij})$, is the vector of commodity demand equations obtained on solving the utility maximization problem defined earlier in this section. The vector $\underline{C}_{ij}$ is written as $[C_{11}, \ldots, C_{1n(1)}, \ldots, C_{m1}, \ldots, C_{mn(m)}]$, and $\underline{\Pi}_{ij}$, the vector of commodity full prices, is written similarly. $U_t(\cdot)$ specifies the maximum level of utility that can be attained by the consumer when full income is μ and full prices are $\underline{\Pi}_{ij}$. Now a fixed level of utility, say U^*, implies a critical level of μ when the full prices are $\underline{\Pi}_{ij}$. At time t, let $\mu = \Pi_t(U^*,\underline{\Pi}_{ij})$. This is nothing but the lowest level of full income that is compatible with U^* and $\underline{\Pi}_{ij}$. If the commodity price index is defined by $\Pi_t(U^*,\underline{\Pi}_{ij})$, then the following approximation holds:[17]

$$\begin{aligned} &\log \Pi_t(U^*,\Pi_{ij,t}) - \log \Pi_{t-1}(U^*,\Pi_{ij,t-1}) \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n(i)} k_{il,t}\,[\log \Pi(C_{il,t}) - \log \Pi(C_{il,t-1})] + E, \end{aligned} \tag{12}$$

where $k_{il,t}$ is the average budget share of C_{il} between times t and $(t - 1)$. Two points deserve attention here. First, notice that the commodity price index defined by $\Pi_t(U^*,\underline{\Pi}_{ij})$ is the true commodity price index. This index evaluates changes in the commodity price index at the fixed utility level U^*. Second, equation (12) allows us to approximate proportionate changes in the true commodity price index by a weighted average of proportionate changes in the individual full prices, while making an error of only third degree, E.

Assume that advertising affects information production as a Hicksian factor neutral technological change, that is, let $I_{il}(G_{il,t},H_{il,t} \mid A_{il,t}) = a_{il}A_{il,t}^{\theta_{il}}I'_{il}(G_{il,t},H_{il,t})$. Then discrete time approximations of equations (10), (11), and (A3) (see Appendix), together with equation (12), imply the following sales-advertising function:

$$\begin{aligned} \Delta\log X_{ij,t} = &- S_{Iij}\sigma_{XijIij}\theta_{ij}(\Delta\log A_{ij,t}) + \epsilon_{ij}(1 - k_{ij,t}) \\ &\times S_{Iij}\theta_{ij}(\Delta\log A_{ij,t}) + \eta_{ij}k_{ij,t}S_{Iij}\theta_{ij}(\Delta\log A_{ij,t}) \\ &- \{[\eta_{ij}k_{ij,t} + \epsilon_{ij}(1 - k_{ij,t})]S_{Xij} + S_{Iij}\sigma_{XijIij} \\ &+ S_{Tij}\sigma_{XijTij}\}(\Delta\log P_{Xij,t}) + \eta_{ij}\sum_{l\neq j} k_{il,t}S_{Iil}\theta_{il} \\ &\times (\Delta\log A_{il,t}) + \sum_{l\neq j} \epsilon_{i,jl}(1 - k_{il,t})S_{Iil}\theta_{il}(\Delta\log A_{il,t}) \\ &- \sum_{l\neq j}[\eta_{ij}k_{il,t} + \epsilon_{i,jl}(1 - k_{il,t})]S_{Xil}(\Delta\log P_{Xil,t}) + E', \end{aligned} \tag{13}$$

where E' is a constant that incorporates approximation errors, and $\Delta f_t = f_t - f_{t-1}$, for any function f_t.

In spite of its formidable appearance, equation (13) is not difficult to

17. See the Appendix for a proof.

interpret.[18] The net effect of own advertising equals the sum of coefficients of the first three terms on the right-hand side. Each of these coefficients is positive; thus, the theory predicts a positive coefficient of $\Delta \log A_{ij,t}$, that is, the own advertising elasticity is positive.[19] The coefficient of $\Delta P_{Xij,t}$ represents the own price elasticity of X_{ij}. Even though the component $S_{Iij}\sigma_{XijIij}$ counters the combined negative effect of the other components in the coefficient of $\Delta P_{Xij,t}$, this coefficient is predicted to be negative since X_{ij} is necessarily a substitute for the other two inputs as a whole (i.e., $S_{Iij}\sigma_{XijIij} + S_{Tij}\sigma_{XijTij} > 0$). Competitive advertising causes two opposing effects on the demand for X_{ij}: the full income effect and the consumption substitution effect. The sum of these effects for all the competitive brands is measured by the fifth and sixth term, respectively, and the sign of the cross advertising elasticity is ambiguous. In the last term, the coefficient of $\Delta P_{Xil,t}$, for each $l \neq j$, measures the cross price elasticity of X_{ij}. This elasticity is also ambiguous in sign. But for an average income effect, and in the face of strong substitution between commodities in the same group, the cross advertising elasticity is expected to be negative and the cross price elasticity to be positive.

Equation (13) treats the income and the substitution effects of competitive advertising separately. This separation provides a precise meaning to the notion of "primary" and "competitive" effects of advertising. These effects have been noted before (Bass and Parsons 1969; Clarke 1973; Schultz and Wittink 1976), yet no existing methodology can separately measure these effects. Clearly, a separate measurement of these effects provides useful managerial information, and equation (13) allows directly for such measurement. But it is not without its cost since data on the commodity budget shares are also required for this measurement. Budget share data (the weights in the true commodity price index) cannot be computed directly from data on prices, advertising levels, sales, and so on; the data require indirect statistical estimation. This topic is beyond the scope of this paper. Here, I shall simply proceed with the estimation of own and cross advertising elasticities, that is, the net effectiveness of own and competitive advertising.

Assume that the utility function is strongly separable, such that:

$$U = K \prod_{i=1}^{m} C_i^{\phi_i}, \quad \text{and,} \quad C_i = \left[\sum_{j=1}^{n(i)} \psi_{ij} \, C_{ij}^{-\rho_i} \right]^{-1/\rho_i}, \tag{14}$$

where $\psi_{ij} > 0$, $-1 < \rho_i < \infty$, $\phi_i > 0$, and $\sum_{i=1}^{m} \phi_i = 1$,

18. In general, factor shares and the partial elasticities of substitution are not constant—they depend on P_{Xil}, A_{il}, and ω. Factor shares are constant if and only if the partial elasticities of substitution equal one. These facts are used in choosing a functional form for the production functions later in this section.

19. It is assumed throughout that $\eta_{il} > 0$, $l = 1, \ldots, n(i)$.

for $i = 1, \ldots, m$, and $j = 1, \ldots, n(i)$. This function is essentially Cobb-Douglas in C_i, $i = 1, \ldots, m$, and Constant-Elasticity-of-Substitution in $C_{ij}, j = 1, \ldots, n(i)$. Under (14), the demand for X_{ij} is independent of commodity prices not in the ith group. Also, since the quantity index, C_i, for the ith group is linear homogeneous in $C_{ij}, j = 1, \ldots, n(i)$, the two-stage maximization procedure is consistent,[20] and the price index for the ith group, Π_i, exists, such that $\Pi_i C_i$ equals total expenditures on the ith group.[21]

These properties are very useful in estimating brand demand equations with aggregate time series data. The only variables that affect the demand for X_{ij} are input prices corresponding to commodities in the ith group. Also, the true commodity price index defined earlier is now independent of the utility level at which it is defined. Therefore, equation (12) provides as good an approximation as the index evaluated at a utility level other than the one at geometric mean income and prices (see the Appendix). Moreover, Π_i is only a function of $\Pi(C_{il})$, $l = 1, \ldots, n(i)$, and consequently, changes in real income that affect the demand for X_{ij} can be evaluated independently from changes in commodity prices not in the ith group.

Furthermore, assume that information is produced and supplied more efficiently by the firm, at all relevant levels, than by the consumer via self-production. Abstracting from the time costs of consuming advertising messages, the level of information is now given exogenously to the consumer. Put differently, $f_{il} = C'_{il}(X_{il}, T_{il}) I^0_{il}(A_{il})$, where $\partial I^0_{il}/\partial A_{il} > 0$. Let $I^0_{il}(A_{il}) = \bar{a}_{il} A_{il}^{\tau_{il}}$, $\bar{a}_{il} > 0$, $0 < \tau_{il} < 1$, where $\bar{a}_{il}$ incorporates other "environmental variables," such as consumer's age, education level, etc., that affect the consumer's ability to analyze information. Also, let C'_{il} be of the Cobb-Douglas variety, that is, $C'_{il} = \bar{C}\, X_{il}^{r_{il}} T_{il}^{(1-r_{il})}$, for $\bar{C} > 0$ and $0 < r_{il} < 1$.

Under these special forms for the consumer utility function and the household production functions, and if the wage rate changes over time, equation (13) becomes[22]

$$
\begin{aligned}
\Delta \log X_{ij,t} = B_0 &+ \sum_{l=1}^{n(i)} B_l \Delta \log A_{il,t} \\
&+ \sum_{l=1}^{n(i)} B'_l \, \Delta \log P_{Xil,t} + B'' \Delta \log W_t,
\end{aligned}
\tag{15}
$$

20. Consistency of the two-stage maximization procedure implies that the consumer maximizes overall utility by following a two-stage budgeting procedure. First, full income is allocated to groups of commodities, i.e., commodities corresponding to product classes. In the second stage, given expenditures on a group are allocated among commodities within that group. These commodities correspond to brands within a product class. Such a budgeting procedure is intuitively appealing in the context of consumer brand choice.

21. In fact, these results hold if and only if C_i is linear homogeneous in $C_{ij}, j = 1, \ldots, n(i)$. See Green (1964) for a proof.

22. The dependence of k_{il}, $l = 1, \ldots, n(i)$, on commodity prices is ignored. This essentially amounts to using the base period values of k_{il} as the weights in equation (12).

where W_t is the wage rate at time t, $B_0 = E'$, $B_l = \tau_{ij}J(ij)$ for $l = j$, $B_l = \tau_{il}J(il)$ for $l \neq j$, $B_l' = -r_{ij}J(ij) - (1 - r_{ij})$ for $l = j$, $B_l' = -r_{il}J(il)$ for $l \neq j$, and $B'' = r_{ij} - (1 - r_{ij})J(ij) - \Sigma_{l \neq j}(1 - r_{il})J(il)$; with $J(ij) = [k_{ij} + (1 - k_{ij})\epsilon_{ij}]$ and $J(il) = [k_{il} + (1 - k_{il})\epsilon_{i,jl}]$.

Estimation

The exact systematic part of the sales-advertising function is given by model (15). This model is linear in the unknown parameters, and the variables are directly observable. Combined with an appropriate error structure, model (15) can be estimated by well-known econometric techniques. It is worth emphasizing that this form of the sales-advertising function is the first difference of the commonly estimated log linear sales-advertising function. With the intercept term being the exception, parameters in this model are interpreted exactly as those in log linear models, namely, as constant elasticities. The model implies increasing, constant, or diminishing returns to advertising as the estimated value of B_j is greater than, equal to, or less than one.

In contrast with the coefficient of income term,[23] which in log linear models is expected to be positive, the coefficient of $\Delta \log W_t$ is ambiguous in sign. Assuming zero nonwage income, an exogenous increase in the wage rate, W_t, causes two income and two substitution effects. W_t increases real full income directly, but also reduces it indirectly by increasing the commodity full prices. The net income effect and the production substitution effect each influence demand for X_{ij} positively. But the consumption substitution effect can be negative or positive, depending on relative time intensities of commodities in the ith group; thus, the sign of B'' is uncertain.

Model (15) was estimated for three major brands that belong to an infrequently purchased product class. The three brands are distributed nationally through retail food outlets, and they account for almost all the industry sales. The data consist of a 7-year time series, composed of bimonthly observations, by the firm, on unit sales, dollar sales, and several competitive decision variables. Based on preliminary tests, the sales-advertising function implied by the theory is acceptable. Nevertheless, this estimation ignores two important considerations: the possible lack of identification of the sales-advertising function, and the presence of the well-known cumulative effects of advertising. A profit maximizing firm will jointly set its level of sales, advertising, and price, and may have to take account of sales-advertising functions that constrain the activities of competing firms. This can cause the sales-advertising function to be unidentified. The absence of cumulative effects of advertising, in this model, is a problem of omitted variables.

23. For example, Bass (1969), in estimating advertising effects for filter and nonfilter cigarettes, uses a log linear model that includes per capita disposable income divided by the price index.

In either case, the resulting parameter estimates are likely to be unsatisfactory. These problems are dealt with elsewhere (Verma, in preparation).

IV. Extensions

An increase in advertising shifts the product demand curve to the right, and this enables the firm to charge a higher price for its product at any level of sales. Clearly, marginal revenue from advertising is greater, the greater the shift in the demand curve for an incremental change in advertising. The exact amount of this shift is given by equation (9). Here, under the assumption that C_i is produced by a fixed coefficient technology, two more implications of the model are considered: how the firm's incentive to advertise varies with the degree of competition in the market, and with the product's information category (i.e., experience or search product).

The commodity price elasticity, ϵ_i, and the product price elasticity, ϵ_{Xi}, are related as $\epsilon_i = \epsilon_{Xi}/S_{Xi}(1 - \nu_i)$. This relationship between the elasticities and equation (9) implies that the shift in the product demand curve, for an incremental change in advertising, is greater, the greater the product price elasticity. In other words, and contrary to conventional wisdom, marginal revenue from advertising (and thus the optimal level of advertising), *ceteris paribus,* is greater, the more competitive is the product market.[24] The notion of advertising being compatible with competition is not entirely new (Telser 1964), but the implication of this model is much stronger. Recently, Stigler and Becker (1977) also arrived at this important conclusion; their model is substantially different from this one (but consistent with it), yet their basic result is also that the shift in the product demand curve is directly related to elasticity of the demand curve.

To investigate the difference between advertising effects for search and experience products, consider Nelson's classification discussed in Section II. Nelson assumed that in the case of search products, consumers "must search by random sampling," and for experience products, "consumers either sample at random from among all brands or from among those brands in the price range the consumer deems appropriate for himself" (Nelson 1970, p. 313). Though the distinction among products based on information costs is very useful, this classification seems objectionable on at least two grounds.

First, in general, products contain a combination of both search and experience attributes. Second, experience attributes also can be optimally evaluated by search. The primary difference lies in the type of

24. This result is unchanged for a general production function since $\epsilon_i = (\epsilon_{Xi} - \sigma_{XiIi}S_{Ii} - \sigma_{XiTi}S_{Ti})/S_{Xi}(1 - \nu_i)$.

search; "guided" or "informative" search can be an effective means of evaluating experience attributes. Shopping among the reputable retail outlets, searching among the reputable brands, and following recommendations of *Consumer Reports* and newspaper and magazine ratings are moderately good examples of guided sampling. But guided sampling is costly relative to random sampling since guidance itself is not free—reputable stores charge higher prices, reputable brands sell at higher prices (see the earlier discussion), and there is a cost to obtaining information from magazines. Moreover, we expect greater guidance to be used, the more experience-intense the product. If the (pure) search cost of sampling is fixed, then the cost of information per unit of the commodity will vary directly with the experience-intensity of the commodity. Translated into this model, if α_i, β_i, and γ_i are the fixed input-output coefficients corresponding to X_i, T_i, and I_i, respectively, then the shadow price of information, $\gamma_i\Pi(I_i)$, will rise as we move along the search-experience spectrum. Let $\delta = (\alpha_i P_{Xi} + \beta_i W)/\gamma_i\Pi(I_i)$. The quantity $(\alpha_i P_{Xi} + \beta_i W)$ measures the shadow price of the commodity given free information. Since the distinction between search and experience affects the analysis only because it affects the cost of information, this quantity is independent of the product's location in the search-experience spectrum. Therefore, as experience-intensity of the product rises, δ falls.

Since $S_{Ii} = 1/(1 + \delta)$, equation (9), at least in the case of Constant-Elasticity-of-Substitution preferences, implies that the shift in the product demand curve, given an incremental change in advertising, is greater, the greater the experience intensity of the product. Put in more familiar terms, marginal revenue from advertising is greater for experience products than for search products. Nelson's model also predicts greater advertising for experience products relative to that for search products, and Nelson finds empirical support for this prediction (see Nelson 1974). In Nelson's model, however, the prediction is critically based on the unsatisfactory argument that for experience products, highly advertised brands make better buys. In this model, experience products are advertised more than search products because commodities using experience-intense products as inputs are also information intense, and the shift in the product demand curve varies directly with the information intensity of the commodity.

The basic model in this paper can be extended in several directions. For example, if consumers are segmented based on the usual demographic variables, then equation (9) can be used to predict differences in the marginal response to advertising by various segments. Demographic variables like wage rates, educational levels, age, family size, and so forth, all affect some of the relevant "shadow prices" in this model. These predictions, if found empirically valid, imply optimal segmentation strategies with respect to advertising.

V. Summary and Concluding Remarks

In this paper, consumer behavior is formalized by extending the household production model. This extension recognizes that costly information about attributes embodied in market products and services also constrains consumer choices. Advertising, via its role as information, induces a cost savings in information production for the consumer. The proposition that advertising increases demand for the advertised product follows without any reliance on the alleged taste-changing effects of advertising.

The consumer behavior model provides an analytical framework for estimating own and competitive advertising effects. An estimable sales-advertising function that specifies changes in demand for a given brand with respect to changes in advertising and prices of all brands, and to changes in the consumer's wage rate, is derived under some fairly harmless assumptions. The interpretation of the parameters is unambiguous. But cumulative advertising effects fail to appear in the sales-advertising function, and the function may be econometrically unidentified. The absence of cumulative advertising effects is caused by the static nature of the consumer behavior model. Since this paper deals only with the demand side of the market, the issue of identification is left unsettled. Clearly, future work in the area must account for these problems.

The assumption that consumers maximize utility subject to resource constraints has been frequently criticized for implying "too much rationality" on the part of consumers. The analysis presented here does employ this assumption, however, the emphasis is placed on consumer decision making under the optimal amount of ignorance. Operationally, the distinction between consumer choices inconsistent with rationality and those made under ignorance is rather blurred—a point made earlier by Michael and Becker (1973). In any event, the final acceptance of any model depends on its usefulness. Besides providing the formal apparatus for modeling the sales-advertising function, fortunately, this consumer behavior model has predictions for the observable variables (see the discussion following eq. [13] and Sec. IV). Given further investigation and validation of these predictions, the consumer behavior model can substantially contribute to the development and applicability of marketing theory.

Appendix

1. To derive equation (6), consider a change in the price vector from the point $[P_{Xi}^{(1)}, W^{(1)}, \Pi(I_i)^{(1)}]$ to $[P_{Xi}^{(2)}, W^{(2)}, \Pi(I_i)^{(2)}]$. By taking a linear Taylor's approximation of $\Pi(C_i)$ around the point $[P_{Xi}^{(1)}, W^{(1)}, \Pi(I_i)^{(1)}]$, and by evaluating this approximation at the point $[P_{Xi}^{(2)}, W^{(2)}, \Pi(I_i)^{(2)}]$, we get

$$\Delta\Pi(C_i) = [\partial\Pi(C_i)/\partial P_{Xi}]\Delta P_{Xi} + [\partial\Pi(C_i)/\partial W)\Delta W + (\partial\Pi(C_i)/\partial\Pi(I_i)]\Delta\Pi(I_i), \tag{A1}$$

where "Δ" represents a change from $[P_{Xi}^{(1)}, W^{(1)}, \Pi(I_i)^{(1)}]$ to $[P_{Xi}^{(2)}, W^{(2)}, \Pi(I_i)^{(2)}]$, and all the derivatives are evaluated at the point $[P_{Xi}^{(1)}, W^{(1)},\Pi(I_i)^{(1)}]$. Since the production process of C_i is linear homogeneous, we have $X_i = k_1(\cdot)C_i, t_i = k_2(\cdot)C_i$ and $I_i = k_3(\cdot)C_i$, where "$\cdot$" represents the price vector $[P_{Xi}, W, \Pi(I_i)]$. These demand equations imply that $\Pi(C_i) = P_{Xi}k_1 + Wk_2 + \Pi(I_i)k_3$, which gives

$$\partial\Pi(C_i)/\partial P_{Xi} = k_1 + P_{Xi}(\partial k_1/\partial P_{Xi}) + W(\partial k_2/\partial P_{Xi}) + \Pi(I_i)\ (\partial k_3/\partial P_{Xi}). \tag{A2}$$

On taking the total differential of the production function for C_i, and on combining the cost minimizing conditions along with (A2), we get that $\partial\Pi(C_i)/\partial P_{Xi} = k_1$. Similarly, $\partial\Pi(C_i)/\partial W = k_2$ and $\partial\Pi(C_i)/\partial\Pi(I_i) = k_3$. Equation (A1) may now be written as:

$$\Delta\Pi(C_i)/\Pi(C_i) = S_{Xi}(\Delta P_{Xi}/P_{Xi}) + S_{Ti}(\Delta W/W) + S_{Ii}[\Delta\Pi(I_i)/I_i]. \tag{A3}$$

Equation (6) follows on letting $\Delta W = \Delta\Pi(I_i) = 0$, and by using the chain rule.

2. Equation (12) can be proved as follows. The indirect utility function, $U = U_t[\Pi(\mu,\underline{\Pi}_{ij}),\underline{\Pi}_{ij}]$, implies that

$$(\partial U_t/\partial\mu)(\partial\Pi/\partial\mu) = 1, \text{ and } (\partial U_t/\partial\mu)[\partial\Pi/\partial\Pi(C_{ij})] + [\partial U_t/\partial\Pi(C_{ij})] = 0. \tag{A4}$$

Since $U = U_t[\underline{C}_{ij}(\mu,\underline{\Pi}_{ij})] = U_t(\mu,\underline{\Pi}_{ij})$, we obtain

$$(\partial U_t/\partial\mu) = \sum_i \sum_l (\partial U/\partial C_{il})(\partial C_{il}/\partial\mu) = \lambda \sum_i \sum_l \Pi(C_{il})(\partial C_{il}/\partial\mu), \tag{A5}$$

where the second equality follows on observing that $(\partial U/\partial C_{il}) = \lambda\Pi(C_{il})$ at the point of constrained utility maximization, and λ is the value of the Lagrange multiplier at that point. Recall that a weighted average of the full income elasticities equals one, the weights being the budget shares. Therefore, (A5) may be written as:

$$(\partial U_t/\partial\mu) = \lambda. \tag{A6}$$

Now on differentiating the full income constraint,

$$\sum_i \sum_l \Pi(C_{il})C_{il} = \mu,$$

and the utility function in the indirect form, we have that

$$[\partial U_t/\partial\Pi(C_{kj})] = \sum_i \sum_l (\partial U/\partial C_{il})[\partial C_{il}/\partial\Pi(C_{kj})], \text{ or}$$

$$[\partial U_t/\partial\Pi(C_{kj})] = \lambda \sum_i \sum_l \Pi(C_{il})[\partial C_{il}/\partial\Pi(C_{kj})] = -\lambda\ C_{kj}, \tag{A7}$$

$$k = 1, \ldots, m, \quad \text{and} \quad j = 1, \ldots, n(k).$$

Also, (A4), (A6), and (A7) combine to give

$$\frac{\partial \log \Pi(U,\underline{\Pi}_{ij})}{\partial \log \Pi(C_{ij})} = k_{ij} = \frac{\Pi(C_{ij})C_{ij}}{\Pi(U,\underline{\Pi}_{ij})}. \tag{A8}$$

A result on local quadratic approximation shows that (see Theil 1976) $f(\mathbf{x} + \mathbf{h}) - f(\mathbf{x}) = \frac{1}{2}h'(f_{\mathbf{x}} + f_{\mathbf{x+h}}) + 0_3$, where $\mathbf{h}$ is a vector of increments on $\mathbf{x}$, $f_{\mathbf{x}}$ and $f_{\mathbf{x+h}}$ are gradients of $f(\mathbf{x})$ and $f(\mathbf{x} + \mathbf{h})$, respectively, and 0_3 is an error of third degree. Using this result and (A8) gives

$$\log\left[\frac{\Pi_t(U_t^*,\underline{\Pi}_{ij,t})}{\Pi_{t-1}(U_t^*,\underline{\Pi}_{ij,t-1})}\right] = \sum_i \sum_t \frac{k_{it}(U_t^*,\underline{\Pi}_{ij,t}) + k_{it}(U_t^*,\underline{\Pi}_{ij,t-1})}{2} \times \log\left[\frac{\Pi(C_{it,t})}{\Pi(C_{it,t-1})}\right] + E. \tag{A9}$$

Equation (A9) would imply equation (12) if we could show that

$$k_{it}(U_t^*,\underline{\Pi}_{ij,t-1}) + k_{it}(U_t^*,\underline{\Pi}_{ij,t}) = k_{it}(U_t,\underline{\Pi}_{ij,t}) + k_{it}(U_{t-1},\underline{\Pi}_{ij,t-1}). \tag{A10}$$

Define $U_t^* = U_t(\mu_t^*,\underline{\Pi}_{ij,t}^*)$, where $\mu_t^* = (\mu_{t-1}\mu_t)^{1/2}$ and $\underline{\Pi}_{ij}^* = (\underline{\Pi}_{ij,t-1}\underline{\Pi}_{ij,t})^{1/2}$. Consider a change from the point $(U_t^*,\underline{\Pi}_{ij,t}^*)$ to $(U_t^*,\underline{\Pi}_{ij,t})$. A linear Taylor's expansion gives

$$k_{it}(U_t^*,\underline{\Pi}_{ij,t}) = k_{it}(U_t^*,\underline{\Pi}_{ij,t}^*) + \sum_i \sum_t \left.\frac{\partial k_{it}(U,\underline{\Pi}_{ij})}{\partial \log \Pi(C_{it})}\right|_{\substack{U = U_t^* \\ \underline{\Pi}_{ij} = \underline{\Pi}_{ij,t}^*}} \times \frac{1}{2}\log\left[\frac{\Pi(C_{it,t})}{\Pi(C_{it,t-1})}\right] + E_1,$$

and a change from $(U_t^*,\underline{\Pi}_{ij,t}^*)$ to $(U_t^*,\underline{\Pi}_{ij,t-1})$ gives

$$k_{it}(U_t^*,\underline{\Pi}_{ij,t-1}) = k_{it}(U_t^*,\underline{\Pi}_{ij,t}^*) + \sum_i \sum_t \left.\frac{\partial k_{it}(U,\underline{\Pi}_{ij})}{\partial \log \Pi(C_{it})}\right|_{\substack{U = U_t^* \\ \underline{\Pi}_{ij} = \underline{\Pi}_{ij,t}^*}} \times \frac{1}{2}\log\left[\frac{\Pi(C_{it,t-1})}{\Pi(C_{it,t})}\right] + E_2.$$

On adding the above two equations we find that the left-hand side of (A10) equals $2k_{it}(U_t^*,\underline{\Pi}_{ij,t}^*)$. It can be shown along similar lines that the right-hand side of (A10) equals $2k_{it}(U_t^*,\underline{\Pi}_{ij,t}^*)$ (see Theil [1976] for similar developments in the context of the Rotterdam model), which completes this proof. Notice that the proof is without loss of generality, except that changes in $\Pi(U,\underline{\Pi}_{ij})$ are evaluated at geometric mean full income and full prices. Theil (1976) shows that only the geometric means satisfy the four desirable axioms on which any true price index should be based.

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