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Computerized Double-Auction Markets: Some Initial Experimental Results*

In 1975 the Securities and Exchange Commission received a congressional mandate to move toward the creation of a national stock trading system. A potentially major step toward the development of such a system is the Cincinnati Stock Exchange's highly controversial pilot project which is currently testing an "all electronic" computerized trading mechanism. This paper reports on the design and analysis of a series of laboratory experiments which explore the behavior of computer-automated "double-auction" markets where face-to-face buyer-seller interaction has been eliminated. The experiments also serve to demonstrate the use of the PLATO computer as a research tool for social scientists interested in laboratory experimental techniques.

I. Introduction

Over the last 2 decades there has evolved a considerable volume of literature describing the results of various experimental games in market decision making as well as their implications concerning economic theories of market price and quantity determination (see, e.g., Smith 1962, 1964, 1965, 1976*b*; Miller, Plott, and Smith 1977; Plott and Smith 1978; Williams 1979). The relevance of using controlled laboratory experiments to study resource allocation mechanisms has recently been addressed formally by V. L. Smith (1980), who has also provided fine summaries of the more important experimental results derived from contract price observations generated under various market institutions (Smith 1976*b*, 1980).

The potential for increased efficiency obtainable through various electronic auction mechanisms has been recognized for many years (Cassidy 1967). However, recent discussions of automating and computerizing auction markets have generally been in relation to the creation of a congressionally mandated national securities market (e.g., *Wall Street Journal* 1978*a*, 1978*b*, 1978*c*). Charged with overseeing the develop-

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ment of the National Market System (NMS), the Securities and Exchange Commission is currently evaluating the relative merits of two separate approaches toward centralization: the Cincinnati Stock Exchange's NMS pilot project, and the New York Stock Exchange–sponsored Intermarket Trading System (ITS). The Cincinnati NMS project is testing a fully automated, computerized trading mechanism, while the more conservative ITS establishes only an information linkage between major U.S. stock exchanges.

The most obvious bonus of automation, common to both laboratory and "real world" markets, is the potentially enormous saving of manpower. Data generated by an electronic market are quickly and easily collected and retained in the computer's memory for later analysis or accounting purposes. Considering strictly data-processing arguments, the cost of automating large-scale markets is surely warranted. A much more subtle issue is whether alterations of a basic trading institution brought about by automation can significantly alter the institution's stability or trading efficiency. An example of this is speculation concerning the effect of eliminating face-to-face buyer-seller interaction on the trading floor of major stock exchanges.

This study reports on the first series of computer-automated double-auction experiments. The institutional constraints on information transfer and contractual agreement of the double auction are exemplified by those of major security and commodity markets such as the New York Stock Exchange and the Chicago Commodity Exchange. In the experiments, both buyers and sellers are permitted to enter a price quote for (bid to buy or offer to sell) one unit of an undefined homogenous commodity. Any buyer (seller) can accept the price quote of any seller (buyer) resulting in the formation of a binding contract. Individual unit valuations and hence aggregate market supply and demand arrays are experimentally controlled using a cash-reward structure in accordance with the theory of induced valuation formally discussed by Smith (1976*a*). It is stressed that unit valuations are strictly private information. Bids, offers, and subsequent contracts are the only public information. Previous research has shown that experimental oral double-auction markets with such information specifications generally converge rapidly to near the theoretical price-quantity equilibrium.

The problem of modeling the rapid-convergence properties of the double auction has so far proven to be intractable. This, combined with the debate over the potential effects of automation, provides the rationale for choosing the double auction as the object of extensive computerized experimental testing. Computerized data records can be much more accurate, complete, and quickly analyzed than in noncomputerized procedures. The first step toward the realization of such an experimental study was to tackle the technically challenging problem of designing a flexible, economically feasible, computerized "written-visual" transformation of the "oral-auditory" double auction. The mechanism must be easy to comprehend and operate without *any* experience in the use of computers. This task was accomplished using the PLATO computer as the passive medium of public-information transfer and private-information display.

PLATO handles all aspects of the experiment except the recruiting of subjects and their payment of earnings in cash at the market's conclusion. The computer automatically (1) randomizes the subjects into market functionaries (and individual valuation or cost conditions), (2) presents a uniform set of instructions at an individually controlled speed, (3) waits for all the subjects to finish the instructions, and then (4) executes the experiment, strictly enforcing all of the institutional and technical rules of the game. Experimental data are stored on magnetic disk for later analysis and can be monitored immediately, while the experiment is in progress. A priori, it was reasoned that a PLATO double auction (PDA) which captured the institutional characteristics of the oral double auction would exhibit similar convergence characteristics in experimental testing.

II. The PLATO Double-Auction Mechanism

Figure 1 illustrates the basic screen display for a seller in a PDA experiment. Figure 2 gives the basic screen display for a PDA buyer. In order to keep the operation of the basic mechanism as simple as possible to operate, extensive use was made of PLATO's touch-sensitive (infrared light grid) display panel.

Bidding Procedure

To enter an offer (bid) to sell (buy) one unit of the commodity a subject types in an entry, which appears on the screen to the right of the arrow, and then taps the box labeled "Seller x Offers" ("Buyer x Bids"). The entry then passes through a series of checks for any violations of the rules of the market. Subjects have the option of erasing and retyping entries after the arrow as frequently as desired. The price quote is processed, and subsequently rejected or entered into the market, only after the "Bid Box" ("Offer Box") has been touched.

The following institutional and technical constraints are always in effect:

Rule 1. Any price quotation must yield the maker nonnegative potential profit (excluding the "commission").[1]

Rule 2. Price quotes must lie in the range \$0.01–\$9.99 inclusive, the maximum offer price being a very rarely binding technical constraint

1. Covering a 1–5-cent loss with the commission is not allowed since it is assumed that the commission is a reimbursement only for transaction costs.

which tremendously reduces the data storage requirement by allowing all bids, offers, and hence contracts, to be expressed as a maximum of 10 binary "bits" of information.

If either of these constraints is violated the entry is rejected and an appropriately descriptive error message is generated. The subject may then erase the rejected entry and try again if desired.

Combining rule 1 and rule 2 constrains bids to lie in the range \$0.01 through $\max_i(mv_i^u)$ and offers to lie in the range $\min_j(mc_j^u)$ through \$9.99 inclusive. Hence, contracts must occur in the range $\min_j(mc_j^u)$ through $\max_i(mv_i^u)$ inclusive, where mv_i^u is the marginal valuation of buyer i's first untraded demand unit ($i = 1, \ldots,$ number of buyers) and mc_j^u is the marginal cost of seller j's first untraded supply unit ($j = 1, \ldots,$ number of sellers).

In addition to rule 1 and rule 2 three variations of a third institutional rule are examined.

Rule 3a. Any price quote is rejected if it is entered before the previous quote has been "standing" open to acceptance in the market for a predetermined minimum number of seconds. There is only one—the most recent—bid or offer standing displayed to the market at any point in time. A bid or offer remains standing until it is either accepted or another bid or offer is made. There are no rules governing the bid-offer sequence. If rejected, the maker receives the following message: "Current entry still standing. Try again if you wish."

The minimum time is a variable initialized prior to the experiment. Actual values used ranged from 2 to 3 seconds. Postexperiment reaction by participants to the experimenter's comment that a longer minimum standing time might be adopted in the future was almost unanimously negative, the general feeling being that 3 seconds was sufficient time to allow acceptance of the standing quote if anyone so desired. The necessity of having some minimum standing time for each price quote is easily seen if one considers the consequences of a dominant "bumping" strategy where subjects try to rapidly displace the current standing bid or offer with their own. In the absence of a human auctioneer-experimenter to slow things down and maintain order in the market, such behavior would render the act of accepting a *particular* price quote very difficult. Contract prices might tend to have a high degree of variation as haphazard and panic acceptance occurred.

Rule 3b. There is no minimum time that each price quotation must remain standing, but price quotes must progress so as to reduce the bid-ask spread. The current "best" bid to buy *and* offer to sell are displayed to the entire market. Thus, a bid (offer) must be higher (lower) than the currently standing best bid (offer) to be admissible. Price quotes which satisfy this constraint are displayed to the entire market, replacing the previous best bid or offer. The subject is then free to enter another quote at any time. If an entry is rejected for not being better, an appropriate error message is generated. The subject may then erase the rejected entry and try again if desired. When a contract occurs a new auction for one commodity unit begins. The first bid and offer in each auction is constrained only by rules 1 and 2. Any attempt to enter a quotation which, if accepted, would yield a profit less than that which would occur if the subject were to accept the current standing bid (offer) is rejected and a message prompting the subject to accept the current standing bid (offer) is generated. Market organization incorporating a bid-ask spread-reduction rule is similar to that employed in trading in the New York Stock Exchange (Leffler and Farwell 1963, pp. 186–90).

Combining rules 1, 2, and 3*b* yields

$$mv^{ij} \geqslant b_{t+1}^{ij} \begin{cases} \geqslant \$0.01, t = 0 \\ \geqslant b_t, t > 0 \end{cases}$$

$$mc^{ij} \leqslant a_{t+1}^{ij} \begin{cases} \leqslant \$9.99, t = 0 \\ \leqslant a_t, t > 0 \end{cases}$$

where mv^{ij} (b^{ij}) is the ith buyer's marginal evaluation of (bid for) his jth unit, mc^{ij} (a^{ij}) is the ith seller's marginal cost of (offer for) his jth unit, and $b_t(a_t)$ is the tth bid (offer) in the current auction.

Rule 3c. Each price quote entered stands displayed to the market for a minimum number of seconds as in rule 3a. However, if quotes come in more rapidly than this, rather than being rejected, they are placed in a queue according to the time of entry (i.e., first in, first out). If queued, the message "#*x* in line" appears above the maker's typed entry (fig. 1). The subject may not enter another price quote until the one in the queue is displayed to the entire market. After a quote stands for the minimum time it is automatically replaced by the entry at the front of the queue (no. 1 in line) and the xth queued entry becomes the $(x - 1)$th. All participants are given continuously updated (approximately every 1 second) queue-length information as shown in figure 1. As in rule 3*a*, there is no restriction on the bid-ask sequence, and only the most recent bid or offer is standing in the market open to acceptance.

Acceptance Procedure

To accept the standing buyer's bid (seller's offer) any seller (buyer) simply taps the box labeled "ACCEPT" (fig. 1). The only constraint on acceptance is that the resulting contract generates a nonnegative profit (excluding commission) for the accepter.

This being the case, "PLEASE CONFIRM" will then begin flashing above the box labeled "CONFIRM." The accepter must confirm the contract within 5 seconds by tapping the confirm box, at which time "CONTRACT CONFIRMED" appears above the box. If confirmation

RECORD SHEET for SELLER 1

Trading Period		1	2	3	4	5
1	Selling Price of 1st Unit	3.50				
2	Cost of 1st Unit	3.00	3.00	3.00	3.00	3.00
3	Profit (row 1 - row 2)	0.50				
4	Profit + $.05 Commission	0.55				
5	Selling Price of 2nd Unit	3.80				
6	Cost of 2nd Unit	3.80	3.80	3.80	3.80	3.80
7	Profit (row 5 - row 6)	0.00				
8	Profit + $.05 Commission	0.05				
9	Total Profit (row 4 + row 8)	0.60				

BUYER 1 BIDS $2.56 SELLER OFFERS $

SELLER 1 OFFERS | #3 in line. ▶$ 4.00 | ACCEPT | CONFIRM

There are now 3 people waiting in line.

Contracts: none so far in period 2.

Trading Period 2 now in progress. TIME REMAINING: 293 seconds.
0 of 8 people have voted to end market period 2: LAB to vote.

FIG. 1.—Basic screen display for a seller

does not occur, "NOT CONFIRMED" appears and the market proceeds as though the accept box had not been tapped.

Upon contract confirmation the contract price and resulting profit (plus 5¢ commission) are automatically logged in both the buyer's and seller's record sheets.

The following contract information is transmitted to all but the "accepting" subject as soon as confirmation is requested:

CONTRACT at $x.xx: Buyer x; Seller x
Waiting to Confirm

which appears on the second and third lines from the bottom of the screen just below the three boxes. "Waiting to Confirm" is subsequently replaced by either "CONTRACT CONFIRMED" or "CONTRACT NOT CONFIRMED". The screen display is then returned to that depicted in figure 1 or 2 with the contract information line appropriately updated if confirmation occurred.

RECORD SHEET for BUYER 1

Trading Period		1	2	3	4	5
1	1st Unit Resale Value	4.00	4.00	4.00	4.00	4.00
2	Purchase Price	3.50				
3	Profit (row 1 - row 2)	0.50				
4	Profit + $.05 Commission	0.55				
5	2nd Unit Resale Value	3.80	3.80	3.80	3.80	3.80
6	Purchase Price					
7	Profit (row 5 - row 6)					
8	Profit + $.05 Commission					
9	Total Profit (row 4 + row 8)					

BUYER 1 BIDS $2.55 SELLER 1 OFFERS $4.00

BUYER 1 BIDS ▶$ ACCEPT CONFIRM

Contracts: 3.50

Trading Period 1 now in progress. TIME REMAINING: 269 seconds.
0 of 8 people have voted to end market period 1: LAB to vote.

FIG. 2.—Basic screen display for a buyer

Stopping Rules

The experiment takes place over a series of trading periods each lasting a maximum of 300 seconds. The period length and number of trading periods are variables initialized prior to the experiment. If acceptance occurs with less than 5 seconds remaining, the trading period is extended in order to allow a full 5 seconds for confirmation. Every 5 periods constitute a "trading week." Subjects receive a new personal record sheet at the beginning of each week. The unit values on the new record sheets may be changed by the experimenter or allowed to remain the same.

The 300-second stopping rule may be shortened through a unanimous vote to end the trading period. Subjects may vote to end a period by pressing the key labeled "-LAB-." After voting to end a period the subject may still actively participate in the market unless he has transacted for all of his commodity units in the period. No group of subjects in any of the experiments reported herein has unanimously voted out a trading period. Information on time remaining in the period and the group's vote to end the period are continuously updated (approximately every second) and presented as shown in figures 1 and 2.

III. Initial Experimental Results

The series 1 PDA experiments presented in this section represent results obtained while the computer program was in its initial phase of field testing and development. In the experiments which follow each subject had two units potentially traded per period. Unless otherwise noted, all were University of Arizona graduate and undergraduate students who were "inexperienced" in the sense that none had participated in a previous PDA experiment.

Comparison of an Oral and a PLATO-Automated Double Auction

Figure 3 displays the underlying supply and demand arrays for an oral double-auction market experiment run "by hand." The S and D arrays are followed on the right by a charting of sequential contract prices. The rules of the game parallel those presented in the preceding section for PDA experiments run without the bid-ask spread reduction rule. The presence of a data-recording experimenter-auctioneer to maintain order eliminates the necessity of having an explicitly stated minimum standing time rule. Trading periods were ended after 5 minutes with a bell sounding after 4 indicating that there was 1 minute remaining in that period. The record sheets handed out at the beginning of the experiment encompassed only trading periods 1–5. At the end of period 5 new record sheets were distributed containing the induced values for the second "trading week" (periods 6–10).

Table 1 presents descriptive summary statistics for each of the 10 trading periods. The mean, standard deviation, and number of contract prices are given by $\bar{p}$, s, and q. The "coefficient of convergence,"

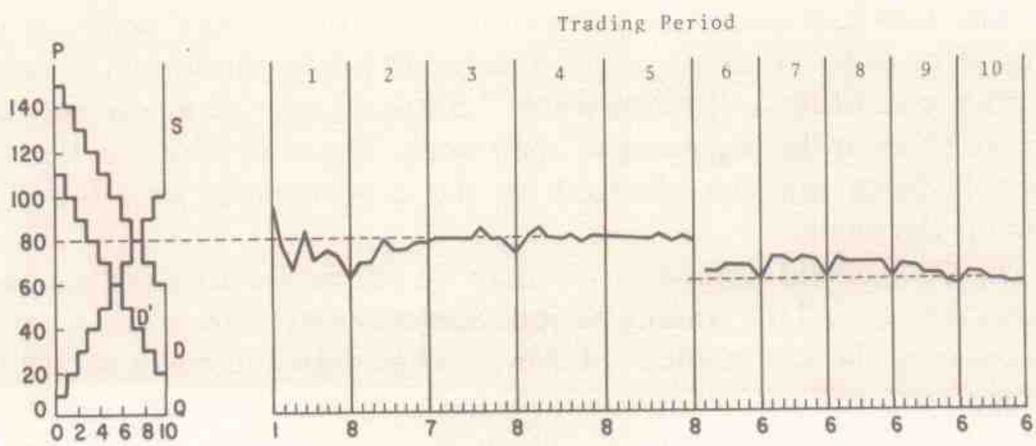

FIG. 3.—Sequential contract prices for oral double auction

TABLE 1 Summary Statistics for Oral Double Auction

Period	$\bar{p}$	s	q	α	E
1	.752	.1164	8	.1257	100.00
2	.749	.0344	7	.0619	100.00
3	.795	.0304	8	.0308	100.00
4	.803	.0192	8	.0194	100.00
5	.794	.0086	8	.0106	100.00
6	.650	.0283	6	.0574	100.00
7	.678	.0358	6	.0861	100.00
8	.670	.0321	6	.0770	100.00
9	.632	.0353	6	.0474	100.00
10	.605	.0150	6	.0158	100.00

NOTE.—P_e (week 1) = 0.80, P_e (week 2) = 0.60, Q_e (week 1) = 8, Q_e (week 2) = 6.

given by α, is the standard deviation of contract prices around the theoretical equilibrium price (P_e) rather than the actual mean contract price. Thus s and α will tend toward equality as $\bar{p}$ tends toward P_e. An index of market efficiency, given by E, is the ratio of actual period profits (excluding commissions) to the maximum available, expressed as a percentage. Inefficiencies occur when extramarginal units (marginal supply valuations above P_e or marginal demand valuations below P_e) are traded and when intramarginal units (marginal supply valuations below P_e or marginal demand valuations above P_e) fail to be traded. Hence, 100% efficiency ($E = 100$) occurs if and only if subjects receive the maximum available consumer plus producer surplus (regardless of total rent distribution). Notice that a sufficient, but not necessary, condition for 100% efficiency is that Q_e (number of exchanges at the theoretical equilibrium) units be traded within the "efficiency band," defined as the price range bounded on top by the first extramarginal supply unit and on the bottom by the first extramarginal demand unit.

Figure 3 and table 1 show the distinct tendency for contract prices to approach the predicted equilibrium by the final trading period in each "week" (periods 5 and 10). Note that after the week 2 demand reduction contract prices approach the new equilibrium price from the direction of the previous week's equilibrium price. Similar "hysteresis" effects have been observed in other experimental double-auction markets (see Smith 1977, pp. 10–15). With the exception of period 7 compared with period 6, α declines progressively in periods 1–5 and periods 6–10. The market is 100% efficient in all trading periods, with the number of transactions equal to the equilibrium quantity (Q_e) in all but period 2 when the marginal supply and demand units were not traded.

These results are typical of oral double-auction experiments and are consistent with two empirical propositions presented by Smith (1976*b*,

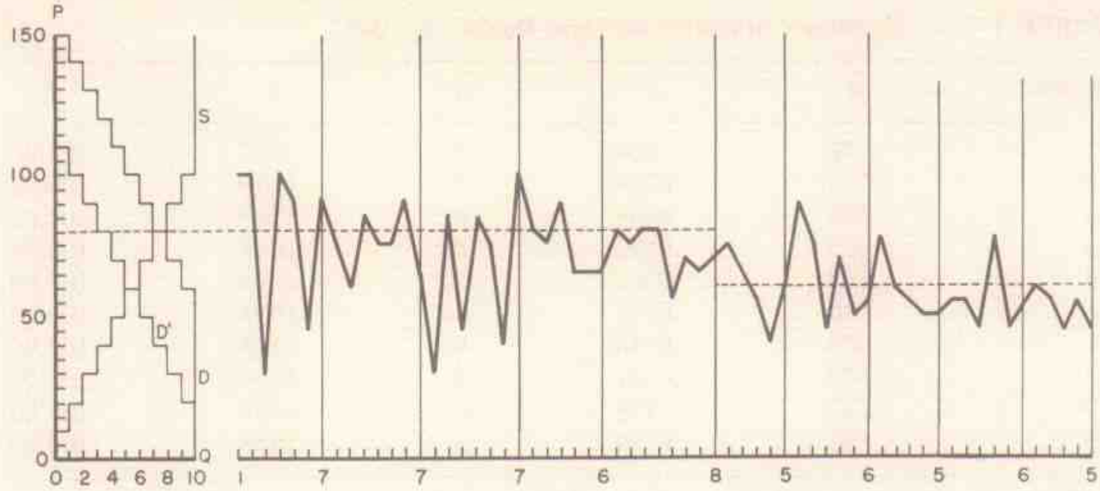

FIG. 4.—Sequential contract prices for experiment 1

pp. 48–55) based on his experimental work with oral double auctions: (1) "Contract prices converge to 'near' the theoretical (Supply-Demand) equilibrium level usually within the first twenty to thirty transactions," and (2) "Quantities exchanged per period rarely differ from the theoretical ($S = D$) equilibrium by more than a single unit in any trading period."

The first PDA experiment was run using rule 3*c* under the stipulation that while queued a subject cannot accept any price quote. This constraint existed in order to initially avoid the additional programming complexity required to allow acceptance while queued. The rule was explicitly stated in the instructions, where it was implied that the subject should check the queue length before entering a quote. Since this attaches a rather high opportunity cost to being queued, it was expected that the result would be short queues.

The market structure of this first PDA (hereafter, experiment 1) was identical with the oral double auction just presented. Subjects in both experiments were recruited from "principles of macroeconomics" classes. Figure 4 charts sequential contract prices for experiment 1; table 2 presents descriptive summary statistics for each trading period.

Experiment 1 is characterized by much larger variance of contract prices in all trading periods. Convergence toward the equilibrium price is much slower and less distinct than in the oral double auction. Using contract price variance ratios ($F = \hat{\sigma}_1^2/\hat{\sigma}_2^2$), one for each trading period, to compare price stability in experiment 1 with the oral double auction yields F = 6.44*, 7.86*, 68.93**, 25.36**, 86.67**, 17.43**, 19.12*, 10.11*, 9.23*, 16.67**, for periods 1–10, respectively, where * and ** indicate rejection of the null hypothesis of homoscedasticity at $p = .05$ and .01, respectively.

Casual observation of the queue size indicated that, although the queue was generally short in the initial trading periods, queues of four

TABLE 2 Summary Statistics for Experiment 1

Period	$\bar{p}$	s	q	α	E
1	.775	.2883	6	.2894	85.71
2	.750	.0964	7	.1086	100.00
3	.657	.2499	7	.2878	92.86
4	.733	.0943	6	.1155	92.86
5	.720	.0802	8	.1132	100.00
6	.590	.1158	5	.1162	100.00
7	.642	.1566	6	.1620	96.67
8	.584	.1001	5	.1014	100.00
9	.548	.1075	6	.1192	100.00
10	.520	.0600	5	.1000	90.00

NOTE.—P_e (week 1) = 0.80, P_e (week 2) = 0.60, Q_e (week 1) = 8, Q_e (week 2) = 6.

or more were not unusual thereafter. Thus subjects may have been frozen out of the market, unable to "accept" if a desirable price quote came up. However, an inspection of a sequential listing of bids and offers does not directly indicate that favorable (high) bids or (low) offers were consistently passed over. It appeared that subjects were deriving sufficient utility from the mechanism itself (using the touch panel to enter price quotes) to offset the costs of queuing. In relation to this it is interesting to note that the number of bids and offers per period in experiment 1 ran about three times the number entered in the oral double auction (approximately 90:30). To the extent that such nonmonetary utility considerations affect individuals' behavior in the market, the experimenter's control on the underlying supply and demand conditions is lessened.

For whatever reasons, it became evident after the first PDA experiment that the convergence properties displayed in oral double-auction experiments were not necessarily embodied in a computerized version of the basic procedural mechanism even though it seemed to possess the same informational exchange characteristics.

The Bid-Ask Spread Reduction Rule as an Experimental Treatment Variable

Experiments 2*a*, 2*b*, 2*c* and 3*a*, 3*b*, 3*c* were designed to provide an opportunity to reevaluate a third empirical proposition presented by Smith: "A variation on the double-auction rules in which a bid (offer) is not admissible unless it provides better terms than the previous bid (offer) does not appear to provide any significant increase in the convergence rate of contract prices" (1976*b*, p. 55).

A charting of the underlying supply and demand arrays followed by sequential contract prices for experiments 2*a*, 2*b*, and 2*c* are presented in figures 5–7. These three experiments were run using rule 3*a* (no bid-ask reduction rule, no queue). The similar chartings for experiments 3*a*, 3*b*, and 3*c* are presented in figures 8–10. This set of three

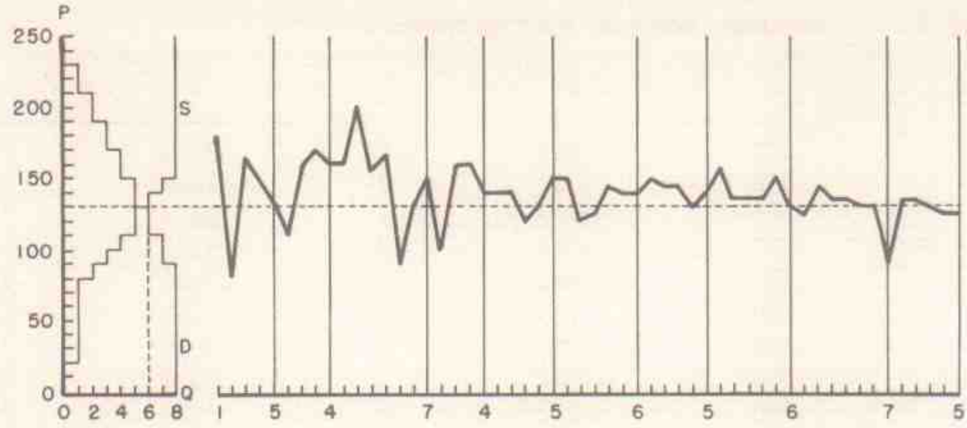

FIG. 5.—Sequential contract prices for experiment 2*a*

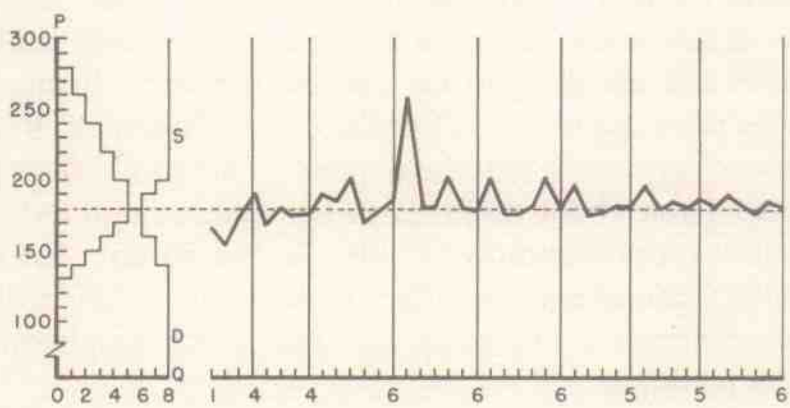

FIG. 6.—Sequential contract prices for experiment 2*b*

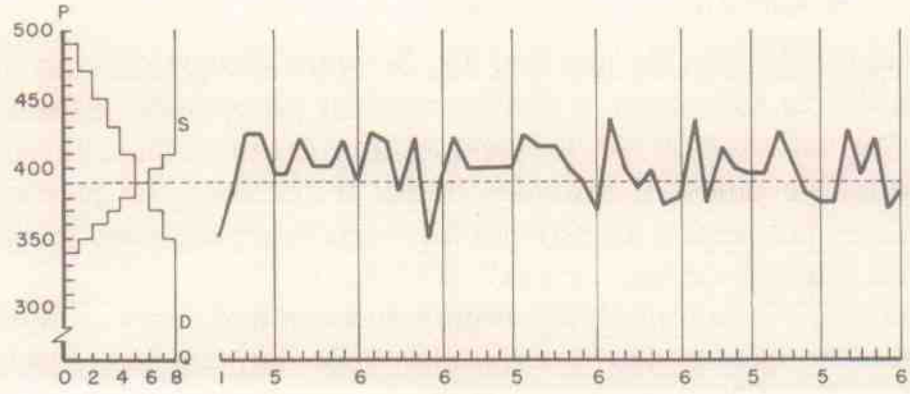

FIG. 7.—Sequential contract prices for experiment 2*c*

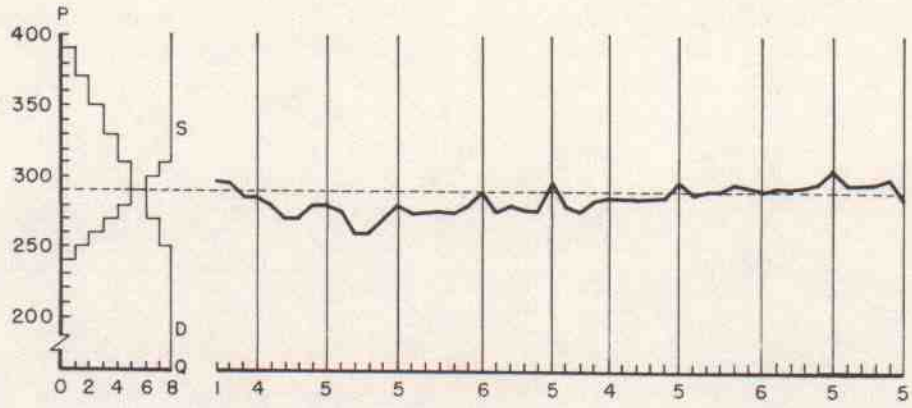

FIG. 8.—Sequential contract prices for experiment 3*a*

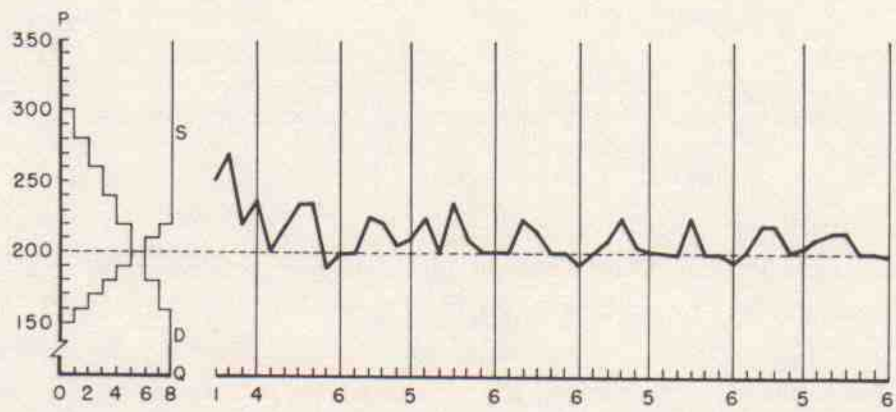

FIG. 9.—Sequential contract prices for experiment 3*b*

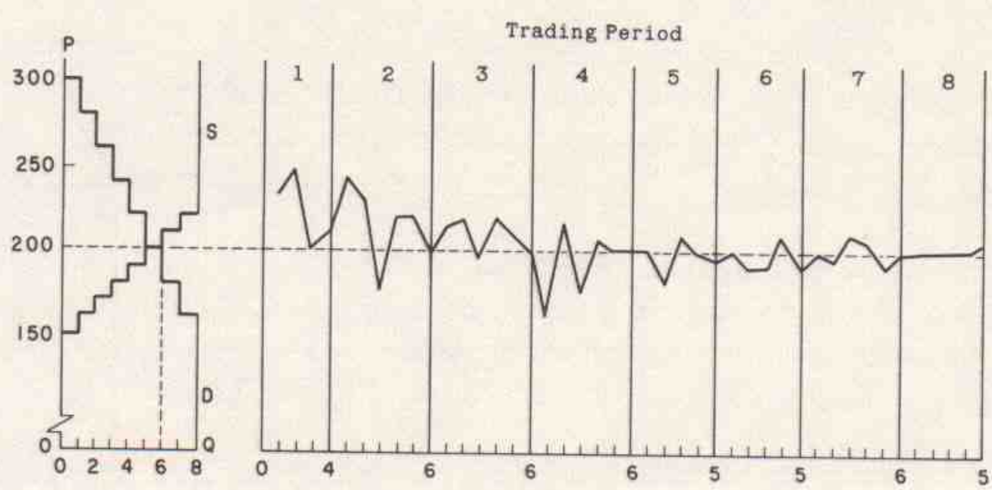

FIG. 10.—Sequential contract prices for experiment 3*c*

TABLE 3 **Comparison of Observations of $(P - P_e)$ for Experiments 2 and 3**

	Mean $(P - P_e)$		Quantity Exchanged		$\hat{\sigma}_1$	$\hat{\sigma}_2$			
Period	Exp. 2 (1)	Exp. 3 (2)	Exp. 2 (3)	Exp. 3 (4)	Exp. 2 (5)	Exp. 3 (6)	F $\hat{\sigma}_1^2/\hat{\sigma}_2^2$ (7)	t (8)	Z_u (9)
1	.035	.229	14	12	.299	.247	1.47	−1.78	−1.63
2	.101	.056	14	17	.187	.221	.71	.60	.74
3	.115	.009	19	16	.263	.178	2.19	1.36	1.48
4	.136	−.021	15	18	.229	.181	1.61	2.19*	2.24*
5	.076	−.017	17	16	.152	.117	1.70	1.95	1.79
6	.045	−.012	17	14	.147	.108	1.87	1.20	1.34
7	.101	.001	15	17	.138	.083	2.78*	2.54*	2.81**
8	.056	.035	17	16	.122	.068	3.17*	.62	.95
1–2	.068	.128	28	29	.247	.243	1.03	− .92	− .62
3–4	.124	−.006	34	34	.245	.177	1.91*	2.52*	2.53*
5–6	.060	−.015	34	30	.148	.111	1.80	2.26*	2.19*
7–8	.077	.017	32	33	.129	.077	2.83**	2.29*	2.68**
3–8	.087	−.001	100	97	.183	.129	2.00**	3.91**	4.03**
1–8	.083	.029	128	126	.198	.170	1.35*	2.35**	3.08**

NOTE.—Experiment 2 = 2*a*, 2*b*, 2*c*; experiment 3 = 3*a*, 3*b*, 3*c* in all tables.
*Reject H_0, $p = .05$ (direction not predicted).
**Reject H_0, $p = .01$ (direction not predicted).

experiments was run using rule 3*b* (the bid-ask spread reduction rule). In experiment 3*c*, the subjects were nonstudents from the community of Tucson. The market structure is identical across the two treatment groups with a consumer-producer surplus ratio of 2:1.[2]

The results of these six experiments support the conclusion that PDA experiments using inexperienced subjects lack the rapid convergence characteristics of experimental oral double-auction markets described by Smith's propositions cited previously. Further, they provide support for the proposition that price convergence will tend to be from above when consumer's surplus is greater than producer's surplus (Smith 1976*b*, p. 56).

Table 3 displays statistics derived from observations of contract price deviations from the appropriate theoretical equilibrium, $(P - P_e)$. The observations are pooled, period by period, for the two treatment groups. The bottom six rows present the results of various interperiod poolings. The square root of the unbiased estimate of the population variance used to calculate the variance ratios in column 7 is $\hat{\sigma}$. Column 8 presents t-ratios, followed in column 9 by Z_U, a nonparametric alternative to the t-ratio which is derived from a normal approximation of the sampling distribution of the Mann-Whitney U-statistic.[3]

2. Note that P_e may be changed without altering a market's structure by adding a constant to all unit valuations. This rescaling is done in order to disguise P_e when replicating a particular experiment.

3. See, e.g., Blalock 1972, pp. 255–60, or Siegel 1956, pp. 116–27.

Inspection of table 3 indicates that contract price mean deviation from P_e is larger for experiments 2*a*, 2*b*, 2*c* (rule 3*a*) in trading periods 2–8. The variance ratio exceeds unity in all except period 2. With a single exception in each treatment group, $\hat{\sigma}$ declines progressively in successive trading periods.

The *t*-ratios for the interperiod poolings indicate that, with the exception of the initial periods, we may generally reject the null hypothesis of equal mean price deviations from the theoretical equilibrium.[4] Previous research indicates that it is not unreasonable to assume that early trading periods serve as simply a training interval for inexperienced subjects and will not reflect subtle differences in the basic mechanism. Smith (1976*b*, pp. 57–58) reports a similar finding in a comparison of double, offer, and bid auctions. Plott and Smith (1978) also report this finding with respect to a comparison of the posted-bid and oral-bid auctioning institutions.

To the extent that interperiod contract price observations are characterized by different underlying distributions, a comparison of the treatment groups using a matched-pairs procedure is justified. Matching the mean price deviations from P_e presented in table 3 by trading periods is undesirable since the information on variances is lost in the matching procedure. A matching of the mean absolute deviation would be a more appropriate reflection of relative convergence since negative deviations cannot offset positive deviations yielding an unrepresentatively low mean deviation.

Table 4 presents the results of period-by-period poolings using such observations. Note that contract price mean absolute deviation from P_e for experiments 3*a*, 3*b*, 3*c* (rule 3*b*) declines progressively in periods 1–8 and is smaller than the mean value for experiments 2*a*, 2*b*, 2*c* (rule 3*a*) in periods 3–8. Application of the Wilcoxon Matched Pairs Signed Ranks Test (see, e.g., Siegel 1956, pp. 75–83) to the means in table 4 yields $T = 3$. We reject the null hypothesis that there is no difference between period means for the two treatment groups with a .025 level of significance (direction predicted); $T < 2$ is needed for rejection at the .01 level. Applying a matched pairs *t*-test (see, e.g., Blalock 1972, pp. 233–35) yields $t = 3.05$, barely rejecting the null hypothesis at the .01 level. The power efficiency of the nonparametric procedure is ap-

4. The two-sample *t*-ratio is calculated using a pooled estimate of the population variance under the assumption that $\sigma_1 = \sigma_2$. The *F*-ratios in col. 7 of table 3 indicate that this assumption is not tenable in some instances. With heteroscedastic samples the proper form of the test statistic and associated degrees of freedom is somewhat debatable and is generally known as the Behrens-Fisher problem. Substituting separate variance estimates for the pooled estimate yields a variable which can be shown to possess an approximate *t*-distribution with the associated degrees of freedom given by a rather elaborate formula which does not necessarily yield an integer (see, e.g., Hoel 1971, pp. 262–65, or Blalock 1972, pp. 220–28). Calculations were performed to compare the results of this procedure with the standard *t*-test. With fairly large and approximately equal sample sizes the two methods yield strikingly similar results.

TABLE 4 Comparison of Observations on abs($P - P_e$) for Experiments 2 and 3

	Mean abs($P - P_e$)		$\hat{\sigma}_1$	$\hat{\sigma}_2$
Period	Exp. 2 (1)	Exp. 3 (2)	Exp. 2 (3)	Exp. 3 (4)
1	.244	.244	.165	.053
2	.161	.185	.134	.126
3	.217	.146	.183	.095
4	.179	.131	.195	.123
5	.129	.088	.108	.075
6	.113	.085	.101	.063
7	.121	.055	.119	.060
8	.092	.039	.096	.066
1–2	.203	.209	.153	.176
3–4	.200	.138	.187	.109
5–6	.121	.087	.104	.069
7–8	.106	.047	.107	.063
3–8	.143	.091	.143	.091

proximately 95% of the t-test when the assumptions of the latter hold (Blalock 1972, p. 266).

We may reject the null hypothesis ($p < .01$) that contract price mean absolute deviation from P_e is the same for the two treatment groups for a pooling of periods 3–8 ($t = 3.02$, $Z_U = 2.51$) as well as a pooling of periods 7–8 ($t = 2.71$, $Z_U = 2.54$).

The difference between the two treatment groups is illustrated by the cumulative frequency ogives presented in figure 11. The contract prices generated in markets employing rule 3b would seem to be less sensitive to the asymmetric rent distribution especially in the three later poolings. This conclusion is further supported by the periods 7–8 empirical frequency histograms presented in figure 12.

It should be noted that rank-sum statistics such as Z_U assume that the underlying distributions are continuous. Thus ties are assumed to occur strictly due to crudities of measurement. The calculation of Z_U includes a correction factor for the number of ties (see n. 3 above). This is important in the application of this nonparametric procedure to PDA contract price data. As indicated by figure 11, contracts tend to occur on "nickel nodes" clustering closer to P_e as the markets stabilize in later trading periods. The use of abs($P - P_e$) observations further aggravates the problem. The assumption of continuity may be so grossly violated, as in period 3–8 comparison of abs($P - P_e$) when 178 of 197 observations are involved in intergroup tie-runs, that the nonparametric procedure should be applied with caution and only in conjunction with a parametric procedure.

Market Efficiency Comparison

Table 5 presents a comparison of market efficiency for the two treatment groups (three observations per period, per group). The mean efficiency is greater under the bid-ask reduction rule in five of the eight

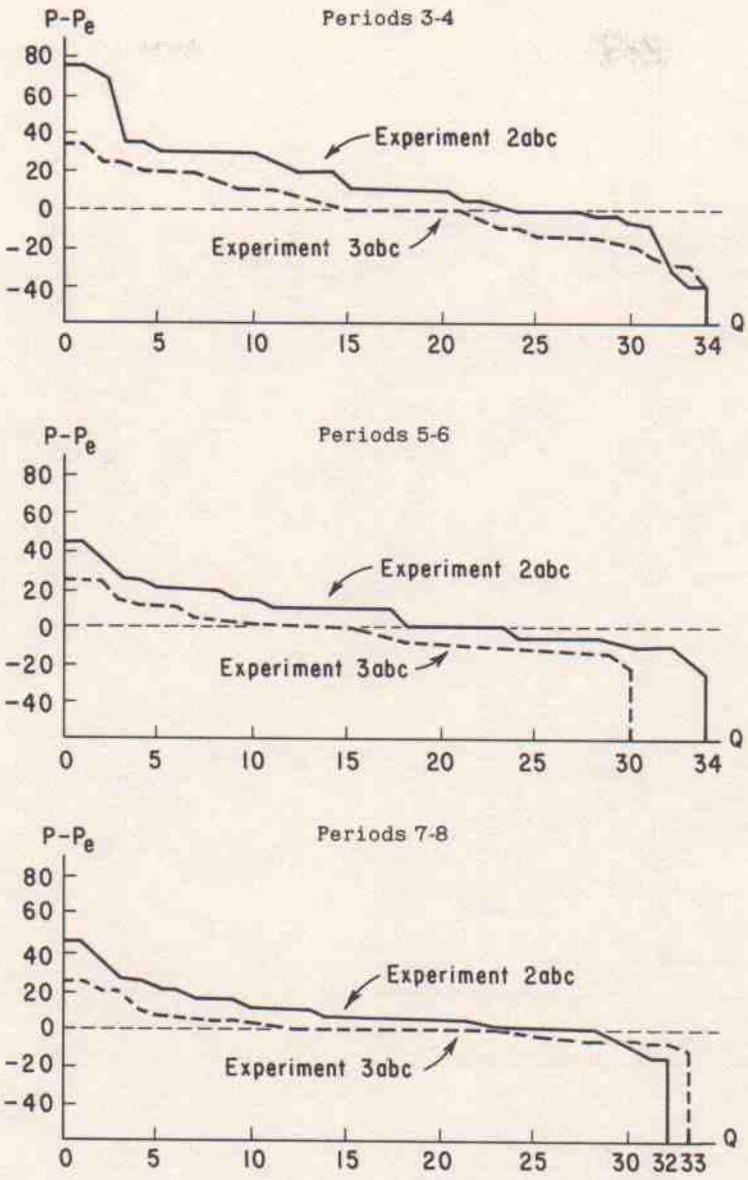

FIG. 11.—Cumulative frequency ogives

individual trading periods and in all the interperiod poolings. A matched-pairs t-test applied to the eight individual periods yields $t_{(7)} = 1.457$. Note that in the two-period poolings the difference of means increases with time. None of the poolings yield t or (Mann-Whitney) U sufficient to require rejection at the .05 level of confidence. The variance ratios for the period 7–8 and period 3–8 poolings yield $F_{(5,5)} = 30.41$ and $F_{(17,17)} = 3.549$, respectively; both require rejection at a .01 confidence level. We must therefore conclude that, although the bid-ask reduction rule tends to have a stabilizing effect on market efficiency, no statistically significant difference in actual efficiency between the two treatment groups seems to exist.

The Importance of Trading Experience in PDA Experiments

In conducting the first series of PDA experiments it became apparent that the ocular-motor skills required to function well in PDA markets

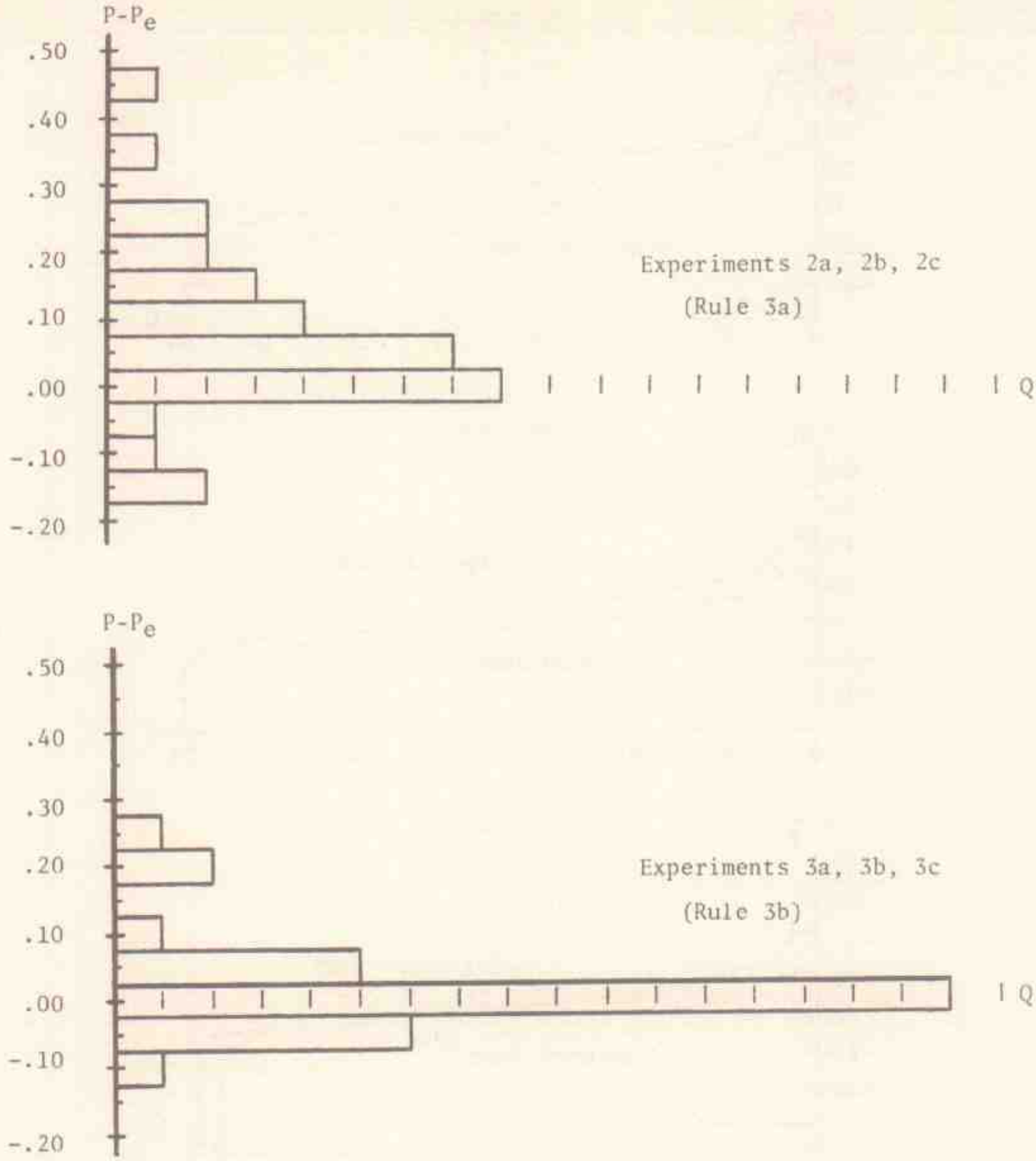

FIG. 12.—Empirical contract price frequency histograms for trading periods 7–8.

generally developed after a few periods of trading but seemed to totally elude some people. Experiments 4 and 6 which follow were run using "experienced subjects" in the sense that all had participated in a previous PDA experiment and had not displayed unusual difficulty in grasping the basic PDA mechanism. Experienced subjects thus had the opportunity to proceed through the instructions a second time with the perspective of having already participated in a PDA market.

Figure 13 charts S and D arrays followed by sequential contract prices for experiment 4 (using experienced subjects). Rule 3*b*, requiring bid-ask spread reduction, is in effect. Table 6 displays summary statistics for experiment 4. Contract price variance is much smaller than in previous PDA experiments, especially in the early trading periods. Prices stabilize rapidly to a mean price 7–10 cents below P_e and appear to be moving toward P_e at the end of period 5. Adjustment to the week

TABLE 5 **Comparison of Market Efficiency for Experiments 2 and 3**

	Mean E		Standard Deviation	
Period	Exp. 2 (1)	Exp. 3 (2)	Exp. 2 (3)	Exp. 3 (4)
1	86.67	85.93	13.37	9.25
2	93.33	96.30	4.44	3.39
3	96.60	97.78	4.63	2.22
4	97.78	100.00	3.85	0
5	86.67	97.04	4.44	3.40
6	98.52	94.07	1.28	8.42
7	88.55	98.52	12.92	2.56
8	98.79	98.52	2.10	1.28
1–2	90.00	91.11	9.61	8.43
3–4	97.04	98.89	3.89	1.86
5–6	92.59	95.55	7.12	5.96
7–8	93.67	98.52	9.10	1.81
3–8	94.43	97.65	7.25	3.85
1–8	93.33	96.02	7.92	5.90

2 demand reduction is very rapid, with prices tending to be biased in the direction of the previous week's equilibrium price. The market is 98.93% efficient in week 1 and 100% in week 2.

Experiment 5 replicates week 1 of experiment 4 using inexperienced subjects. Figure 14 charts sequential contracts; descriptive statistics are presented in table 7. The market converges somewhat more rapidly than in previous PDA experiments using inexperienced subjects. Most contracts occur within the \$0.20 "efficiency band" around P_e beginning in period 3; mean price deviates from P_e by more than \$0.04 only in periods 1 and 2. The market is 93.57% efficient in week 1 and 95.97% over period 1–8.

A comparison of periods 1–5 for experiments 4 and 5 yields variance ratios of 27.7**, 7.58**, 2.70, 226**, 6.35*, and, for an overall pooling,

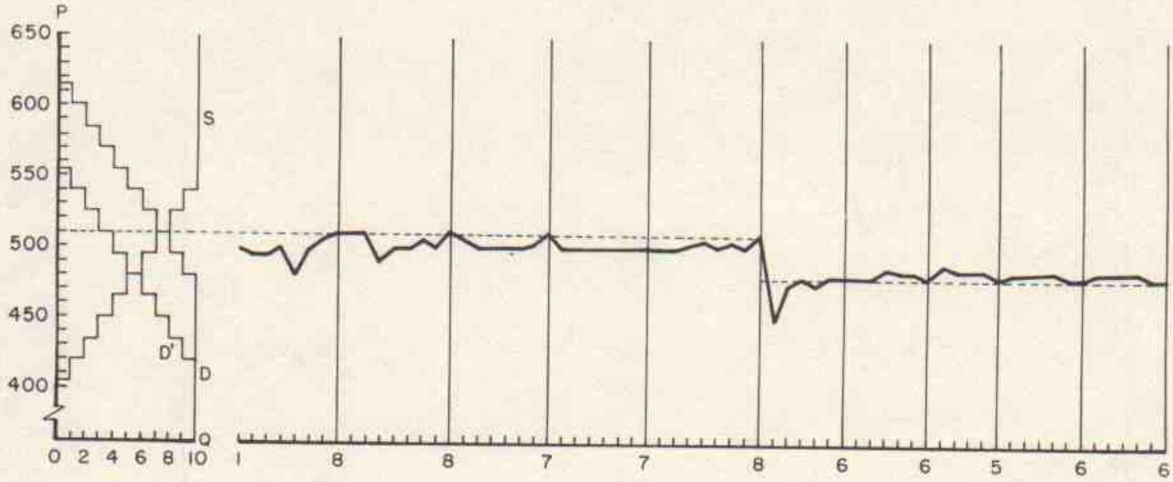

FIG. 13.—Sequential contract prices for experiment 4

TABLE 6 **Summary Statistics for Experiment 4**

Period	$\bar{p}$	s	q	α	E
1	4.980	.0825	8	.1456	100.00
2	5.030	.0656	8	.0959	100.00
3	5.016	.0444	7	.0952	96.43
4	4.999	.0064	7	.1016	98.21
5	5.029	.0333	8	.0787	100.00
6	4.737	.1073	6	.1246	100.00
7	4.823	.0292	6	.0374	100.00
8	4.848	.0291	5	.0564	100.00*
9	4.835	.0214	6	.0410	100.00
10	4.835	.0214	6	.0410	100.00

NOTE.—P_e (week 1) = 5.10, P_e (week 2) = 4.80, Q_e (week 1) = 8, Q_e (week 2) = 6.

*A low probability software bug occurred in period 8 allowing two sellers to accept a single buyer's bid. The removal of the additional supply unit from the market prohibited the marginal demand unit from being traded.

11.84** (** reject at $p = .01$, * reject at $p = .05$). A test of equality of mean $(P - P_e)$ for a pooling of periods 1–5 calls for rejection of the null hypothesis at $p = .01$ ($t = 3.91$, $Z_U = 3.86$). However, using observations on abs$(P - P_e)$ we may not reject H_0 at $p = .05$.

Week 1 of experiment 6 replicates the 2/3 buyer–1/3 seller skewed rent design of experiment 2*a*, 2*b*, 2*c* and 3*a*, 3*b*, 3*c* using experienced subjects; rule 3*b* is in effect. Supply is increased and demand decreased for periods 6–10 such that P_e falls from 4.20 to 3.50 and the rent distribution is skewed in favor of sellers, 2/3–1/3; Q_e remains constant at six units. The relative positioning of week 2 unit values are such that the week 1 expected high (low) profit buyer (seller) becomes the week 2 expected low (high) profit buyer (seller).

Figure 15 charts the S and D arrays as well as sequential contracts for experiment 6; summary statistics are given in table 8. The market stabilizes quickly in week 1 to a mean price 8 cents above P_e (periods

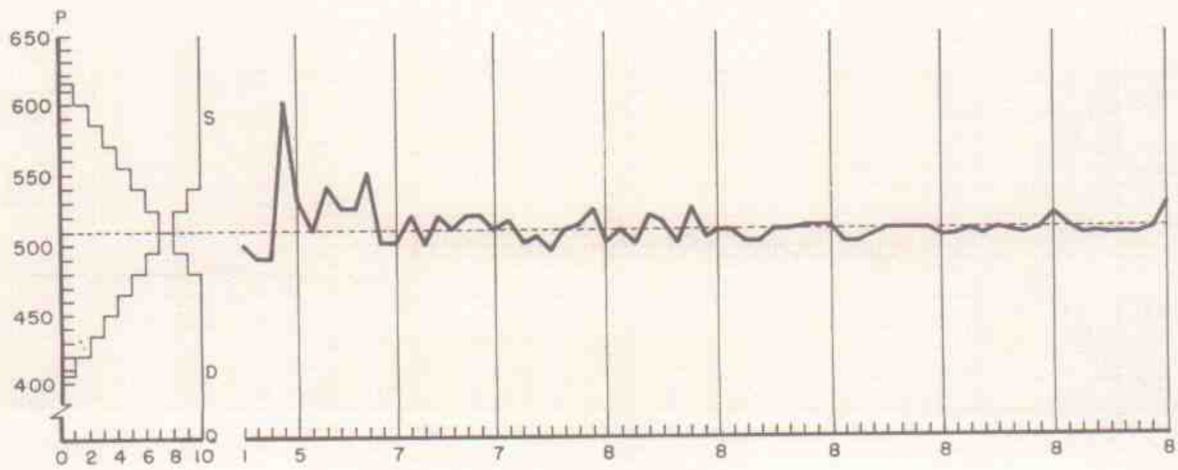

FIG. 14.—Sequential contract prices for experiment 5

TABLE 7 **Summary Statistics for Experiment 5**

Period	$\bar{p}$	s	q	α	E
1	5.222	.4151	5	.4327	89.29
2	5.214	.1787	7	.2121	83.93
4	5.143	.0728	7	.0845	100.00*
5	5.072	.0971	8	.1009	96.43
6	5.105	.0840	8	.0841	98.21
7	5.080	.0469	8	.0510	100.00
8	5.065	.0424	8	.0550	100.00
9	5.094	.0447	8	.0451	100.00

NOTE.—P_e (week 1) = 5.10, P_e (week 2) = 5.10, Q_e (week 1) = 8, Q_e (week 2) = 8.
*Period 3 was aborted after one contract due to hardware problems. Fig. 14 and statistical tests use periods 4–9 as 3–8.

3–5). A comparison of contract price stability in experiments 6 and 3*a*, 3*b*, 3*c* yields period 1–5 variance ratios of 4.19, 39.07**, 18.02**, 8.02*, and 7.81*; a period 1–5 pooling yields $F = 9.85^{**}$.

In week 2, after the supply and demand shifts, the market again stabilizes quickly but at a level somewhat below the new equilibrium price, slowly approaching a mean price 5 cents below P_e in periods 9 and 10. It would seem that the reversal of the rent bias was sufficient to offset the hysteresis effect displayed by the oral double auction (fig. 3) and to a lesser extent by experiment 4 (fig. 13).

Summary and Conclusions

This first series of PDA experiments certainly exposes the fact that some essential characteristic(s) of the experimenter-run oral double auction is absent in the PLATO computerized version when inexperienced subjects are used. In the context of the asymmetric rent distribution used in experiments 2*a*, 2*b*, 2*c* and 3*a*, 3*b*, 3*c* the inclusion of an

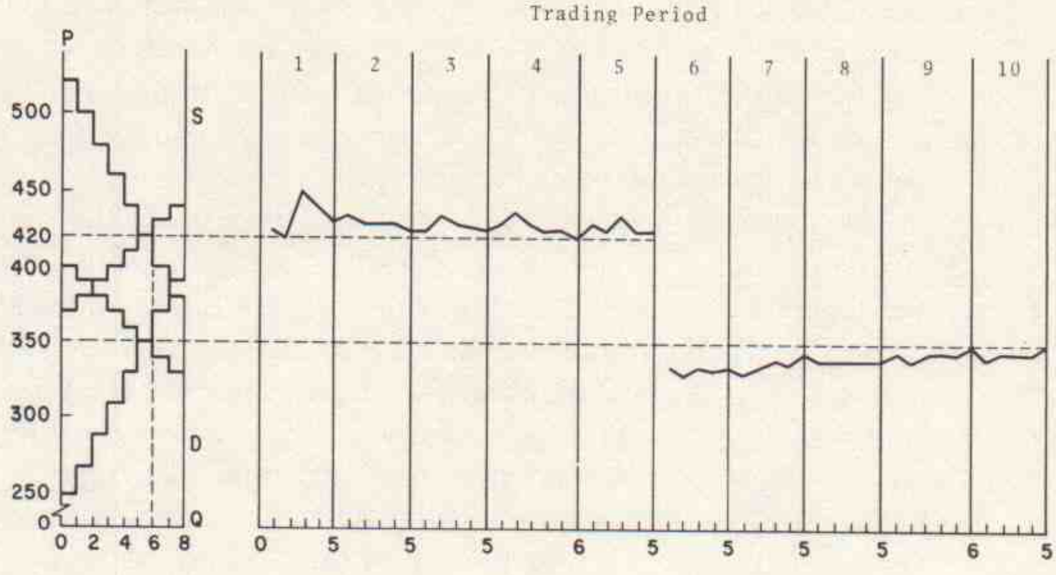

FIG. 15.—Sequential contract prices for experiment 6

TABLE 8 Summary Statistics for Experiment 6

Period	$\bar{p}$	s	q	α	E
1	4.330	.1077	5	.1688	95.56
2	4.300	.0316	5	.1049	91.11
3	4.284	.0377	5	.0921	97.78
4	4.283	.0585	6	.1018	100.00
5	4.284	.0372	5	.0919	100.00
6	3.334	.0206	5	.1673	95.56
7	3.378	.0471	5	.1308	84.44
8	3.400	.0000	5	.1000	100.00
9	3.447	.0298	6	.0611	100.00
10	3.450	.0290	5	.0578	100.00

NOTE.—P_e (week 1) = 4.20, P_e (week 2) = 3.50, Q_e (week 1) = 6, Q_e (week 2) = 6.

institutional rule requiring new bids and offers to provide better terms somewhat improves the stability and convergence properties of the PDA mechanism. The proposition that participants may require a training period to be adequately prepared to cope with the task required by the PDA mechanism is tentatively confirmed by the results of experiments 4 and 6. Only when experienced subjects are used do the results of PDA markets begin to approach the degree of price stability displayed by comparable oral double auctions.

The most obvious difference between the oral and PLATO-automated double auctions is that the latter provide visual rather than verbal feedback on bids, offers, and contracts. The importance of this oral element of the double auction is unclear. Certainly verbalization is a simpler task than typing in an entry or tapping a computer display screen, just as listening to auditory informational inputs may be easier for participants to digest than reading written information. The isolation of participants at individual PLATO terminals during PDA experiments could retard the usual learning through listening and observing which occurs in oral double auctions run by hand.

Two other, possibly important, elements of oral auctions are eliminated in the PLATO version. First, participants who orally enter or accept a price quote which is unusually favorable to the other side of the market often spur their fellow functionaries to express their displeasure through frowns, moans, groans, sighs, and other less subtle vocalizations. Such occurrences are against the rules of the experimental market but seem to be unavoidable in the face-to-face interaction of the oral double auction. In PDA experiments, where isolated participants interact silently through PLATO, vocalization of feelings is rare. Second, bids (offers) from buyers (sellers) which are unusually high (low) often generate multiple acceptance responses from sellers (buyers). This verbal feedback is public information in the oral double auction, with a coin toss or some other random device used to determine the accepter if no one was clearly the first to accept. Such feedback may help to bring deviant price makers back into line with others in the market. In PDA markets the possibility of tied acceptance is eliminated since PLATO will recognize only the first accepter (to within milliseconds). Other participants who attempt to accept the price quote are given the following private message: "You have to be quick! Someone else accepted first." The maker receives no feedback on the fact that multiple acceptance was attempted.

It should be noted that all of the statistical calculations employed in the analysis of the preceding sections were performed by PLATO via data storage, retrieval, and analysis options available as part of the PDA computer package. The desired experimental data can be quickly isolated and read from disk storage into variables which may then be analyzed statistically by either experimenter-programmed routines or automatically transferred to a statistical package available as part of the PLATO system.

The series 1 PDA experiments demonstrate that the PLATO system can be a very effective laboratory resource in the study of interactive experimental games where simple written messages are the medium of informational exchange. It is hoped that the incredible flexibility of the PLATO system will challenge the creativity and resourcefulness of theorists and PLATO programmers in the design of future computerized experiments.

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