Ernest Koenigsberg
University of California, Berkeley

Uncertainty, Capacity, and Market Share in Oligopoly: A Stochastic Theory of Product Quality

The behavior of a duopoly is examined in terms of a stochastic model in which firms have three control parameters: price, capacity, and an estimate of quality. Customers choose a source firm and, if their estimate of delay at source is too large, they switch ("jockey") to the alternate source. Arrivals and production are random. For a given set of control variables the model computes market shares. Given the form of the cost functions of the firms, one can compute the optimal values of the parameters in the short run and in the long run.

Introduction

The markets represented by the textbook definitions of "monopoly," "oligopoly," and "monopolistic competition" tend to merge into one another in real life. Customers (the demand side of the market) are constrained by technological requirements, or by the conviction that one supplier's product is superior to that of another, but only up to a certain point. They may, for several reasons, take their business elsewhere or even change their purchasing requirements permanently, even though the market structure for the product or component is not characterized as being competitive. Hotelling (1929), Stigler (1968), and others have discussed the role of nonprice factors in the decisions of firms and customers. More recently, De Vany (1976) has treated the role of service quality in a monopoly by defining the service offered by the firm to include the customer's cost of waiting for the product.

"Monopoly" in this model is not absolute: a customer may cancel an order if, on arrival, he finds the expected delay too long. In the short run it makes no difference to the monopolist whether the cancelling customer has a substitute service or whether he leaves the market altogether.

De Vany models the monopoly market as a

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single-server queueing system. Customers requiring single units arrive at a rate λ, and the monopolist provides service at a rate μ. Customers who arrive when the number in the system is B will not join the line, but will seek an alternate source. Because customers balk, the monopolist will lose a portion of the custom and the effective arrival rate will be $\lambda' < \lambda$. The arrival rate λ is a function of price, p, and the firm's capacity is measured by the service rate, μ. The balking number B is a function of the difference between the customer's total cost (price plus the cost of waiting) at this firm and the same quantity for some alternate source. The customer who balks knows that he must wait until the B customers in front of him are served if he joins the monopolist's queue; he only knows the *expected* waiting time at the alternate source.

In this model the monopolist has some competition which is characterized by both price and nonprice factors. While De Vany considers only waiting time as the nonprice factor, the model could just as well include others. The monopolist has two control parameters, price p and capacity (service rate, μ). Given the form of $\lambda(p)$ and the cost function $c(\mu,\lambda')$, the monopolist will choose the profit-maximizing values of p and μ.

The model has features analogous to many others that are well known. For example, Dorfman and Steiner (1954) have treated image-building or advertising expenditures by firms to preserve or increase their market share. The total cost of production is increased by the amount spent on advertising. The firms hope that this added cost can be recovered from the increase in demand brought about by the improved image that the advertising produces.

In this paper, we extend the queue model to apply in the case of oligopoly. We consider a duopoly in which the competing firms have three control parameters: price, capacity, and quality. "Quality" can mean waiting time (or delivery lead time) or some other quality-of-service factors. The total cost to the customer is the sum of price, waiting cost, and other "quality" costs. The customer is assumed to have perfect information on price and his position in the order book (the number of customers in front of him), but imperfect information on the expected line lengths at, or the capacity of, the firms. Each customer makes the choice between the two competing firms and decides which queue to join.

Customers arrive at the firms at rates λ_i and join their queue. After joining the queue and learning the number of customers in front of him, a customer may switch (jockey) from line i to j. The probability of switching is proportional to the line length, and inversely proportional to a cost function which includes relative price and anticipation of the service quality at the alternate firm. The use of a probability of switching, rather than single balking number, can be considered as analogous to the case in which the customers collectively represent a distribution of balking numbers.

Representing the customers for each firm as a queue provides a mechanism for explaining certain observed features of a duopolistic market. For example, it is possible to demonstrate the behavior underlying the kinked demand curve and also the breakdown of cartels due to market forces.

The De Vany Model (Balking)

Customers (orders) arrive at a monopolist's plant at a rate $\lambda(p)$ where p is the price. The monopolist has a capacity rate μ for a single output process. The distribution of the time between successive arrivals and the time between successive departures is given by independent Poisson processes with mean values $1/\lambda(p)$ and $1/\mu$, respectively. Customers who wait to be served here have a waiting cost per unit time w, and their expected waiting cost if they arrive when $n - 1$ customers are ahead of them is $w(n - 1)/\mu$. Customers have an alternate source with a cost p_a and an expected waiting cost wL_a/μ_a. On the basis of expectations each customer chooses the "balking" number B and leaves the line if B customers are already in front of him when he arrives.

The De Vany system can be represented as a single-server queue with a finite holding space ($N_{\max} = B$). The well-known queue results are given by De Vany; we only note here the major results for discussion and comparison purposes.

De Vany (Balking)	*Unrestricted Queues*	
$P_0 = (1 - \rho)/(1 - \rho^{B+1})$	$P_0 = 1 - \rho$	(1)
$L = \rho/(1 - \rho) - (B + 1)\rho^{B+1}/(1 - \rho^{B+1})$	$L = \rho/(1 - \rho)$	(2)
$r = (1 - \rho^{B})/(1 - \rho^{B+1})$	$r = 1,$	(3)

where $\rho = \lambda/\mu$, P_0 is the probability that the system is empty, and r is the market share (the ratio of the number of customers served to the number of customers who arrive) obtained by the monopolist. L is the mean number of customers in the line.

The monopolist has two decision variables: the market price p that he sets, and the capacity μ. It makes no difference to the analysis whether the lost customer is actually served by his alternate supplier or not served so long as B is known.

Probabilistic Departures from the Queue (Reneging)

To introduce the concept of probabilistic departures used in the oligopoly model, we will consider a monopoly model very similar to the model above. Arrival and service distributions and disciplines are identical to those already described. All customers who arrive join the queue. Given n in the queue at time zero, the probability that a customer will leave without being served at time t is $[1 - e^{-k(n-1)t}]$ (i.e., the customer being served cannot leave the system by this process). In the queueing literature this departure process is known as "reneging." The results for the system have been reported elsewhere (see Barrer 1957*a*, 1957*b*; Haight 1957; Anker and Gafarian 1963); we include the important terms for comparison

$$P_0 = [{}_1F_1(1, b_1; {}_0y_1)]^{-1} \tag{4}$$

$$L = P_0\rho \; {}_1F_1(2, b_1 + 1; y_1) \tag{5}$$

$$r = (1 - P_0)/\rho = [1 - 1/{}_1F_1(1, b; y)]/\rho, \tag{6}$$

where $\rho = \lambda/\mu$, $b = \mu/k$, $y = \lambda/k$. ${}_1F_1(a, b; y)$ is the confluent hypergeometric function (Abramowitz and Stegun 1965), given by

$${}_1F_1(\alpha, \beta; y) = \sum_{n=0}^{\infty} \frac{\Gamma(a + n)\Gamma(b)}{\Gamma(a)\Gamma(b + n)}(y^n/n!), \tag{7}$$

and $\Gamma(z)$ is the gamma function given by

$$\Gamma(z + 1) = \int_0^{\infty} u^z \, e^{-u} \, du = z\Gamma(z). \tag{8}$$

When z is an integer, $\Gamma(z + 1) = z! = z\Gamma(z)$.

The reneging rate can be examined in economic terms. A customer arriving when $(n - 1)$ other customers are ahead of him can expect to wait a time $(n - 1)/\mu$ before he is served. Thus his expected waiting cost is $w(n - 1)/\mu$ and his decision process considers this as added to the monopolist's price p. If the customer chooses the alternate source he faces a total expected cost of $p_a + wL_a/\mu_a + C$, where C is the cost of transferring to the alternative source. We can define the reneging rate k as

$$k \propto \frac{w}{(p_a - p) + wL_a/\mu_a + C}, \tag{9}$$

so that the customer compares his waiting cost at the monopolist with the expected cost if he reneges and seeks his alternate source. We note that the customer in the queue has no information on the actual total cost at the alternate source; he knows p_a and has only an estimate (which may or may not be correct) of the expected waiting time at the alternate source.

The formula for k shows that the reneging rate increases as the unit waiting cost w increases, decreases as the price differential $(p_a - p)$ increases, and decreases as the expected waiting line at the alternate source and the cost of transferring C increase.

In this system, as in De Vany's (1976), all customers who enter in the market are served by the monopolist or by the alternate source. Again, the monopolist has two decision variables, price and capacity.

Sample results for the unrestricted, balking, and reneging systems for the case with potential system utilization $\rho = 2/3$ are given in

TABLE 1 Comparison of Balking and Reneging

B	P_0	r	L	$W = L/\lambda'$
Balking:				
1	.600	.600	.400	.667
2	.474	.790	.737	.933
4	.384	.924	1.428	1.545
6	.354	.969	1.614	1.661
8	.342	.968	1.772	1.796
10	.337	.994	1.874	1.885
∞	.333	1.000	2.000	2.000
	P_0	r	L	$W = L/\lambda$
Reneging results:				
3/2	.513	.730	.666	.666
1	.493	.761	.747	.747
1/2	.456	.871	.912	.912
1/10	.384	.924	1.378	1.378
1/20	.365	.953	1.571	1.571
1/50	.348	.978	1.769	1.769
1/100	.341	.988	1.867	1.867
0	.333	1.000	2.000	2.000

NOTE.—$\lambda = 1$, $\mu = \frac{3}{2}$, $\rho = \frac{2}{3}$, $\lambda' = \lambda r$.

table 1. We note here that reneging and balking yield similar results, that is, market share decreases as B decreases or as k increases.

The Oligopoly Model (Jockeying)

Here we consider a duopoly with two suppliers with initial arrival rates λ_1 and λ_2 and capacities μ_1 and μ_2. The firms offer an essentially identical good at market prices p_1 and p_2. We assume that all customers who arrive will obtain service from one of the suppliers. The customers know the prices p_1 and p_2 and have an estimate of the mean line lengths, L_1 and L_2, which may or may not imply a full knowledge of the capacities μ_1 and μ_2. Further, we assume that a customer arriving at supplier i knows his position in line but has no information on the instantaneous line length at the other supplier.

A customer in line i with $n - 1$ other customers in front of him may remain in line i or switch (jockey) to line j. The probability that if there are n customers in line i at time zero a customer will leave at time t without being served is $(1 - e^{-k_i(n_i - 1)t})$ where k_i, the jockeying rate, is proportional to the ratio of the unit waiting time cost to the expected cost if he jockeys to line j; that is,

$$k_i \propto \frac{w}{(p_j - p_i) + wL_j/\mu_j + C} \tag{9a}$$

where $k_i \geq 0$. When $k_i > 0$ and $k_j > 0$, jockeying can occur in either direction.

This system was described as a "route-changing" jockeying system by Koenigsberg (1966). The results are as follows:

$$P_0^1 = \{1 + \rho_1 + [\rho_1 y_1/(b_1 + 1)]\ {}_2F_1(1, \rho_1/\beta_1 + 1, b_1 + 2; x_1)\}^{-1} \tag{10}$$

$$\begin{aligned} L_1 = P_0^1\{\rho_1 + [\rho_1 y_1/(b_1 + 1)][\ {}_2F_1(1, \rho_1/\beta_1 + 1, b_1 + 2; x_1) \\ + {}_2F_1(1, \rho_1/\beta_1 + 1, b_1 + 2; x_1)]\} \end{aligned} \tag{11}$$

$$r_1 = \mu_1(1 - P_0^1)/(\lambda_1 + \lambda_2), \tag{12}$$

where ${}_2F_1\ (\alpha, \beta, \gamma: x)$ is the hypergeometric function (see Abramowitz and Stegun 1965) given by

$${}_2F_1(\alpha, \beta, \gamma; x) = \sum_{r=0}^{\infty} \frac{\Gamma(\alpha + r)\ \Gamma(\beta + r)\ \Gamma(\gamma)}{\Gamma(\alpha)\ \Gamma(\beta)\ \Gamma(\gamma + r)} (x^r/r!) \tag{13}$$

and

$$\rho_i = \lambda_i/\mu_1,\ b_i = \mu_i/k_i,\ y_i = \lambda_i/k_i,$$

$$x_i = k_j/k_i,\ \beta_i = k_j/\mu_i.$$

Since all customers are served and service occurs when the servers are busy, at rates μ_i,

$$\lambda_1 + \lambda_2 = \Lambda = \mu_1\ (1 - P_0^1) + \mu_2(1 - P_0^2). \tag{14}$$

The market share of server i is then:

$$r_i = \mu_i(1 - P_0^i)/\Lambda. \tag{15}$$

We note here that the monopoly reneging case is a special form of the jockeying case above when $k_i > 0$, $k_j = 0$, and $\lambda_j = 0$. If $0 \neq \lambda_j \ll \lambda_i$, when $k_j = 0$ we have the case in which one firm dominates the market but the smaller firm (firm j) can obtain "overflow" customers from the dominant firm. For this to occur we must have $p_j \gg p_i$ and $\mu_j < \mu_i$—the smaller firm must have the higher price.

Market share r_i is very sensitive to k_i (p_i, p_j, L). Figure 1 shows the behavior of market share as a function of k_i/k_j. Small differences in the vicinity of $k_i/k_j = 1$ result in a large portion of the potential market share change. Small price changes or small changes in the quality of service expectations can be significant in this model. We note that, as k_i and k_j both approach infinity, the system approaches the perfect competition model described by De Vany and Saving (1977). In that case, $(p_i - p_j) \to 0$ and μ_i and $\mu_j \to 0$.

Economic Analysis I—Monopoly (Balking and Reneging)

De Vany (1976) considers the demand rate λ as a function of p only [i.e., $\lambda = \lambda(p)$], with $\lambda_p = (d\lambda/dp) < 0$. The actual number of customers served, using De Vany's notation, is

$$\lambda' = r\lambda(p) = \lambda'(p, \mu, B). \tag{16}$$

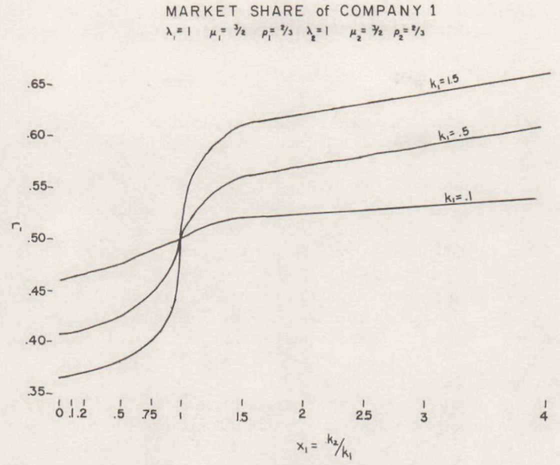

FIG 1.—Market share of company 1 as a function of the ratio of jockeying rates (k_2/k_1) for several values of k_1—equal demands and capacities.

For the reneging case,

$$\lambda' = r\lambda(p) = \lambda'(p, \mu, k). \tag{16a}$$

The derivatives of λ' have the following directions: for both cases, $\lambda'_p < 0$ and $\lambda'_\mu > 0$; for balking, $\lambda'_B > 0$; and for reneging, $\lambda'_k < 0$.

The cost function $C\ (\mu, \lambda')$ is defined for all λ' and μ such that $\lambda' < \mu$. The derivative of cost with respect to λ', $C_{\lambda'} > 0$ is the marginal cost of expected output. The derivative of cost with respect to capacity μ, $C_\mu > 0$ is the marginal cost of a higher level of planned capacity.

The expected profit $E(\Pi)$ is given by

$$E(\Pi) = p\lambda' - C\ [\lambda', \mu]. \tag{17}$$

De Vany (1976) shows that the Kuhn-Tucker conditions for balking are

$$p: \quad (p - C_{\lambda'})[r\lambda_p + \lambda(p)r_p] + r\lambda \leqslant 0 \tag{18}$$

$$\mu: \quad (p - C_{\lambda'})\ \lambda(p)r_\mu - C_\mu \qquad \leqslant 0. \tag{19}$$

The same equations hold for reneging, as do the other results discussed below. The first equation can be defined as the short-run condition which can be written as

$$p(1 + 1/\epsilon) = C_{\lambda'}, \tag{20}$$

where ϵ is the price elasticity of effective demand, $\epsilon = \lambda'_p\, p/\lambda'$. The second equation is defined by De Vany as the long-run condition, which can be written as

$$p = C_{\lambda'} + C_\mu/\lambda'_\mu, \tag{21}$$

which states that price equals the long-run marginal cost of capacity. Long run here implies that capacity changes are long-term commitments.

De Vany also shows that if λ and μ increase so as to preserve the ratio $\lambda/\mu = \rho$, r increases if B (or k in the case of reneging) remains the same. Thus the effective utilization of capacity increases.

One can relate reneging to balking in the case of monopoly. The De Vany model assumes that all customers have the same balking number B. If we assume a distribution of the balking numbers of individual customers, then we might consider the situation as one of reneging. As an alternative, we can consider the case in which each customer has a distribution of balking numbers as being equivalent to reneging. Comparable results are given in table 1.

B or k may be considered as being characteristic of the industry. A small B (or large k) could indicate that the monopolist's product has many close substitutes. A large B (or small k) could indicate a product with few substitutes, or one which can be replaced by an expensive technological change.

Economic Analysis—Duopoly (Jockeying)

In jockeying, the initial arrival rate at firm i is assumed as

$$\lambda^i = \lambda^i\,(p_i, p_j, Q_i, Q_j). \tag{22}$$

We can consider Q_i as the customers' image of the service quality offered by firm i. Image could be represented as the anticipated line length L_i or anticipated waiting time W_i (L_i/μ_i), but the exact values μ_i and μ_j may not be known by the customers. The total input to both firms is

$$\Lambda(p_i, p_j, Q_i, Q_j) = \lambda^i + \lambda^j. \tag{23}$$

The probability of jockeying from firm i to firm j is

$$k^i = k^i\,(p_i, p_j, Q_j). \tag{24}$$

The derivatives of λ^i and k^i are assumed to have the following properties:

$$\begin{array}{ll} \lambda^i_{p_i} < 0 & k^i_{p_i} > 0 \\ \lambda^i_{p_j} > 0 & k^i_{p_j} < 0 \\ \lambda^i_{Q_i} < 0 & k^i_{Q_j} < 0 \\ \lambda^i_{Q_j} > 0. & \end{array}$$

Thus a larger value of Q_i implies an anticipation of poorer service; a larger value of k^i implies that the probability of jockeying from firm i is larger. Λ^i is the number of customers served per period by firm i,

$$\Lambda = \Lambda^i + \Lambda^j = \lambda^i + \lambda^j, \tag{23a}$$

and

$$\Lambda^i = r_i\Lambda = \Lambda^i(p_i, p_j, \mu_i, Q_i, Q_j). \tag{25}$$

The derivatives of Λ^i have the following properties:

$$\Lambda^i p_i < 0 \qquad \Lambda^i_{Q_i} < 0$$
$$\Lambda^i p_j > 0 \qquad \Lambda^i_{Q_j} > 0$$
$$\Lambda^i_{\mu_i} > 0.$$

Firm i has a cost function C^i, given by

$$C^i = C^i[\mu_i, \Lambda^i, Q_i], \tag{26}$$

that is, the cost depends on capacity offered, the number of customers served per period, and the costs (if any) associated with the customers' image of the quality of service. The derivatives of the cost function have the following properties:

$$C^i_{\mu_i} > 0,\ C^i_{\Lambda^i} > 0, \text{ and } C^i_{Q_i} < 0.$$

The firm has the objective of maximizing expected profits, where the expected profit $E(\Pi_i)$ can be written as

$$E(\Pi_i) = p_i\Lambda^i(p_i, \mu_i, Q_i) - C^i[\mu_i, \Lambda^i, Q_i]. \tag{27}$$

We have written the profit function in this form to emphasize the parameters under the control of the firm: p_i, μ_i, and Q_i. The conditions for optimization for firm i, acting independently of firm j, are

$$p_i{:}(p_i - C^i_{\Lambda^i})\quad \Lambda^i p_i + \Lambda^i \leqslant 0 \tag{28}$$

$$\mu_i{:}(p_i - C^i_{\Lambda^i})\ \Lambda^i_{\mu_i} - C^i_{\mu_i} \leqslant 0 \tag{29}$$

$$Q_i{:}(p_i - C^i_{\Lambda^i})\ \Lambda^i_{Q_i} - C^i_{Q_i} \leqslant 0. \tag{30}$$

The first of these equations is equivalent to the short-run condition for a monopoly, that is,

$$p_i(1 + 1/\epsilon^i) = C^i_{\Lambda^i}. \tag{31}$$

where $\epsilon^i = \Lambda^i_{p_i}p_i/\Lambda^i$ is the price elasticity of effective demand. The second equation is equivalent to the long-run condition on capacity as defined by De Vany:

$$p_i = C^i_{\Lambda^i} + C^i_{\mu_i}/\Lambda^i_{\mu_i}, \tag{32}$$

which states that the price equals the long-run marginal cost of capacity.

The third equation relates to the quality of service, Q_i,

$$p_i = C^i_{\Lambda^i} + C^i_{Q_i}/\Lambda^i_{Q_i}. \tag{33}$$

This implies that price equals the cost of improving the customers' image of quality of service. To show this, rewrite the right-hand side as

$$\frac{C^i_{\Lambda^i}\Lambda^i_{Q_i} + C^i_{Q_i}}{\Lambda^i_{Q_i}}.$$

The first term in the numerator is the short-run cost of marginal output times the increase in output induced by an improvement in service quality. The second term is the marginal cost of service quality improvement. The increment in quality necessary to induce a unit increase in output is $1/\Lambda^i_{Q_i}$.

The analysis here assumes that Q_i is independent of μ_i; that is, the customer has no direct information on capacity. If Q_i is a function of μ_i and another quality factor, then additional terms appear in equations (29) and (30).

The system examined here can be considered as one of competitive oligopoly, that is, one in which the firms can attempt to adjust price, capacity, and image in order to optimize their performance. Some competition may still exist in highly regulated industries in which some or most of the firms' control variables are restricted.

One example of rigid regulation is the international airline industry. In the sixties IATA regulated price, frequency of service (i.e., capacity on a route), and part of the quality of service (seat spacing, meals, charges for drinks, etc.). Only a portion of the quality-of-service factors remained open for competition (ground service, meal quality, etc.). In this case changes in market share could only arise from competition in service quality.

Let us consider a duopoly industry in which price is regulated but there are no restrictions on capacity or quality. Further we assume that the firms use similar technologies. Initially let both firms have the same capacity ($\mu_i = \mu_j$) and similar quality images ($Q_i = Q_j$). At equilibrium both firms have half the market ($\lambda_i = \lambda_j = \lambda/2$; $\Lambda^i = \Lambda^j = \lambda/2$).

Now, if firm i is lax in providing perceived service quality, Q_i increases so that $Q_i > Q_j$. The equations show that this results in $\Lambda^i < \Lambda^j$. To regain its market share, firm i must increase μ_i or decrease Q_i. There is a unit cost $C^i_{\mu_i}$ or $C^i_{Q_i}$ for changing μ_i or Q_i. Firm j may read the increase in Λ^j as a call for additional capacity if it is to maintain its service quality. Thus it incurs an additional cost $C^j_{\mu_j}$ per unit of capacity added. If so, firm i would have to counter with added capacity. Thus, any deviation from the previous equilibrium must result in added cost and reduced profit overall when equilibrium is attained once again.

In a competitive duopoly, we would expect that firms would react to changes in price, quality, and capacity in order to retain market share.

In a price-sensitive industry (i.e., one in which price is the dominant factor in the customer's selection of a source), firm i's reaction to a price reduction by firm j would be to meet or beat the price reduction. Because increases in capacity or improvements in quality of service are subject to time lags, the price reaction is the dominant one in the short run. Thus, we can see the firms as being faced with a kinked demand curve in the short run.

Unbalanced Initial Market Shares

Stidham (personal communication, July 1975) has treated the problem of the distribution of input to a multiqueue system without jockeying in which each server has a different service rate μ_j. If the price differential is zero, and customers seek to minimize their waiting time, then for cooperative equilibrium,

$$\lambda_i(\Lambda) = \mu_i - \frac{\sqrt{\mu_i}}{\sum_{m=1}^{2} \mu_i} \left(\sum_{m=1}^{2} \mu_m - \Lambda \right), \qquad i = 1, 2; \tag{34}$$

for competitive equilibrium,

$$\lambda_i(\Lambda) = \mu_i - \tfrac{1}{2}\left(\sum_{m=1}^{2} \mu_m - \Lambda \right), \qquad i = 1, 2. \tag{35}$$

In cooperative equilibrium, customers are distributed so that the total waiting time for all customers is minimized. Competitive equilibrium refers to the case in which the expected waiting time is the same for all customers.

For the case $\mu_1 = 2$, $\mu_2 = 1$, and $\Lambda = 2$, we have for cooperative equilibrium, $\lambda_1 = 1.4142$, $\lambda_2 = .5858$; for competitive equilibrium, $\lambda_1 = 1.50$, $\lambda_2 = .500$.

In the cooperative equilibrium case the expected total time in the system, averaged over all customers, is 1.914, while in the competitive equilibrium case the expected time in the system for all customers is 2.00.

In either case, the firm with the higher capacity (larger μ_i) obtains a market share which is greater than the ratio of its capacity to the total capacity. In queue terms this result obtains because larger capacity implies shorter waiting time and shorter total time in the system. In industry we also find larger firms operating at higher utilization than smaller firms in the industry because there are economies of scale in the production of one quality component, waiting time. De Vany (1976) uses hospital bed utilization as an example of this phenomenon.

Figure 2 shows the market share, $r\Lambda^i$, as a function of k_2/k_1 under the assumption that the original inputs are those given by the results of

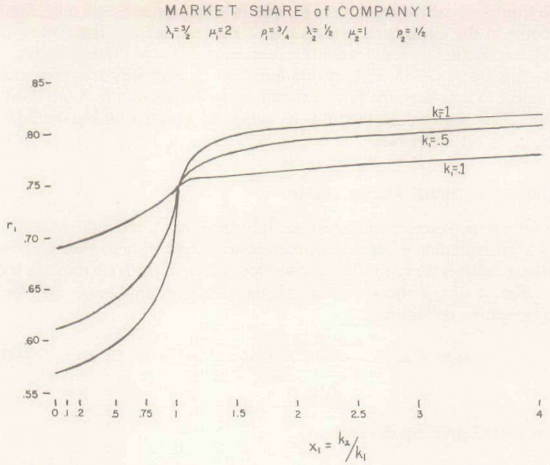

FIG 2.—Market share of company 1 as a function of the ratio of jockeying rates (k_2/k_1) for several values of k_1—unbalanced capacities and initial demands for competitive equilibrium.

competitive equilibrium. We can consider this as a case in which price is fixed and companies compete in the short run only by service quality—which can be taken as the expectation of the time cost of waiting and being serviced. Thus k_i, the switching probability, is a function of anticipated service quality at firm j. The results are also presented in table 2.

Here we have

$$\lambda_1 = 3/2, \quad \mu_1 = 2, \quad P_0^1 = 3/4, \quad r_0^1 = 3/4, \tag{36a}$$

$$\lambda_2 = 1/2, \quad \mu_2 = 1, \quad P_0^2 = 1/2, \quad r_0^2 = 1/4, \tag{36b}$$

where r_0^i is the initial market share, the market share without jockeying.

The market share is extremely sensitive to the values of k_i and k_j. In the example here, we can consider low values of k_i implying a high degree of customer loyalty or a high degree of customer confidence in the "quality" offered by a firm. Figure 2 demonstrates that firms can lose a significant portion of market share if their k_i is significantly larger than that of their competitors. Again, even small changes in the ratio of the k_i's can lead to large changes in market share.

TABLE 2 **Results for Duopoly with Jockeying (Market Share of Company 1)**

$k_1 = 1/10$		$k_1 = ½$		$k_1 = 1$	
$x_1 = k_2/k_1$	r_1	$x_1 = k_2/k_1$	r_1	$x_1 = k_2/k_1$	r_1
10.000	.8021	10.000	.8235	10.000	.8280
4.000	.7832	4.000	.8116	4.000	.8204
2.500	.7727	2.500	.8016	2.500	.8131
2.000	.7676	2.000	.7953	2.000	.8080
1.500	.7608	1.500	.7848	1.500	.8034
1.100	.7584	1.100	.7658	1.100	.7798
1	*.7500*	*1.00*	*.7500*	*1.00*	*.750*
.833	.7328	.960	.7195	.950	.6835
.667	.7211	.875	.6929	.900	.6628
.500	.7118	.750	.6703	.800	.6380
.200	.6985	.500	.6429	.600	.6097
.100	.6947	.200	.6220	.500	.6000
0	.6912	.100	.6134	.400	.5920
...	...	0	.6116	.200	.5792
...	...	...	...	.100	.5799
...	...	...	...	0	.5692

NOTE.—$\lambda_1 = \frac{3}{2}$, $\mu_1 = 2$, $\rho_1 = \frac{3}{4}$; $\lambda_2 = \frac{1}{2}$, $\mu_2 = 1$, $\rho_2 = \frac{1}{2}$.

For a fixed initial market share (λ_i, λ_j), Λ^i is a function of price (p_i and p_j), capacity μ_i, and quality (Q_i and Q_j). De Vany (1976) considers anticipated waiting time as the single quality factor. The model can consider perceived quality as consisting of several factors. In a sense the assumptions we have made in equation (9a) about the form of k_i are equivalent to the assertion that the customer gives a dollar value to each component of quality and makes decisions about joining a line or switching to another line on the basis of total cost (out-of-pocket price plus the cost of waiting plus the value of service in the quality factors).

Let us suppose that the two firms in the example above have agreed to apportion their production in the ratio of three to one and to establish a cartel price, $p_i = p_j = p$. If the ratio, $k_i/k_j \neq 1$, the market shares realized by the companies would not be in the ratio of three to one, and profits would not be those anticipated by the firms. If side payments are not made, one firm would gain at the expense of the other, thus threatening the stability of the cartel. The deviation from the agreed cartel shares occurs because the customers' estimates of qualities of service do not match the estimates made by the firms. This would imply that, without controlling the customer's perception of quality, cartels cannot exist for long in industries whose products are quality or image sensitive.

Many of the characteristics of the markets that have been described are well known, but the queue models give an actual mechanism by which these characteristics are made to occur. In particular, the introduction of a quality parameter to represent quality as perceived by the customer allows an understanding of the process and the effects of customer responses to action or inaction by firms.

We have not exhausted the potential of the monopoly and duopoly models. For example, introducing a priority system to the queue will allow classifying customers by their waiting-time cost, and introducing more firms in the oligopoly market would provide some insight into the dividing line between oligopoly markets and perfectly competitive markets. Lastly, the model should be extended, perhaps by considering period-to-period transitions by firms and customers. That is, firms react to their attained market share in a period by changing their control variables (p_i, μ_i, Q_i) for the next period, and customers react to their waiting time and quality experience in choosing their alternate source.

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