Richard A. Elnicki
University of Florida

Warren R. Hughes
University of Waikato, Hamilton, New Zealand

An Empirical Analysis of Demands for Priority Computing Goods by Externally and Internally Funded Researchers*

> Priority mechanisms have received wide attention in the literature on managerial and operations-research problems. In contrast, they have been virtually ignored by economists . . . , the only way for the computer center to discover if all users value waiting and access equally is to ask them—that is, to establish a market in which preferences may be expressed. [Singer, Kanter, and Moore 1968]

Priority pricing mechanisms are commonly used in university and other computing facilities with substantial demands from researchers. These mechanisms permit users to obtain faster (slower) turnaround on batch jobs by paying a higher (lower) price. This study shows how researchers at the University of Florida's computer facility utilized a five-class priority system. Users allocated significant proportions of their computing incomes to the discount and premium classes. The analysis permits preference rankings of the classes indicating results consistent with traditional demand theory. The income allocations did not indicate that researchers wasted internal or university-budgeted monies when compared with external or grant and contract monies.

I. Introduction

During the decade following the above suggestions by Singer, Kanter, and Moore, the use of priority pricing structures has become common in university computing centers.[1] However, we did not find any empirical studies of how university researchers expressed preferences in a priority market. And we did not find any empirical studies on how different kinds of funding affected preferences. The literature on computer service pricing has increased substantially over

* This work was supported in part by National Science Foundation grant no. MCS77-07985.

1. For example, from July 1975 through January 1978 the writers' computer center received 45 newsletters from academic computing centers that contained new charging algorithms. Forty—89%—of the algorithms included various types of schemes that reduced charges for computing during low-demand periods or for slower turnaround and increased charges for faster turnaround.

(*Journal of Business*, 1980, vol. 53, no. 1)

0021-9398/80/5301-0004$01.51

the decade. But all this literature is prescriptive, and it falls into two categories. One category bases prescriptions on resource allocation considerations (Greenberger 1966; Bauer and Hill 1967; Kanter, Moore, and Singer 1968; Marchand 1968; Nielsen 1968; Singer et al. 1968; Smidt 1968*a*, 1968*b*; Gill and Samet 1969; Hootman 1969; Sharpe 1969; Nielsen 1970; Sobczak 1974; H. S. Berg 1975; Cotton 1975; Ghanem 1975; Zmud 1975; Babad and Modiano 1976; Bernard et al. 1977; Maruyama and Tang 1977), while equity considerations are the basis of the second (Hetrick 1971; Jacobs 1972; Rettus 1972; Wiorkowski and Wiorkowski 1973; S. V. Berg 1975; Gladney 1975).

In this paper we analyze the behavior of faculty researchers at one university computing center during a 41-month period when external (grant and contract) computing incomes varied, internal (university budget) computing incomes varied, and computing-good prices were constant except for the prices of priorities.[2] Since all prices but those for priorities were constant, we analyze the priorities as goods, following the suggestion by Singer et al., rather than as quality differences. The researchers consumed these goods subject to very heterogeneous groups of peer-pressure, superior-subordinate, and worktime–leisure time relationships and very heterogeneous individual drives and desires. There was no possible way to measure inputs to a research production function (see Marchard 1968). We postulate that the faculty researchers were consumers maximizing a utility function subject to budget or income constraints.

The objective of our analysis was to determine whether the goods were systematically consumed in a manner consistent with economic theory. Such results (1) might be useful in managing resources allocated to computing in universities with similar computing structures and (2) would suggest that similar research in different computing structures might also contribute to our knowledge of computer resource management in academic settings. In Section II, we describe the computing structure—the marketplace—we studied: the seller, the consumers, and the goods under consideration. Section III develops the utility model applied to individual users and derives expressions for demand curves, substitution and income effects, and price and income elasticities. The implications of aggregations over many users are discussed in Section IV. The estimation procedures are discussed in Section V, and the results are presented in Section VI. Policy implications regarding the management of academic computing resources are presented in our concluding section. The results suggest that the users studied did not waste internally funded, "soft" monies, as compared to

2. The computing good prices that were constant for the periods studied are for tape mounts, tape drive minutes, tape input and/or output (I/O), plot inches, print pages, cards punched, disk I/O, disk drive minutes, disk setups, disk rentals, jobs and steps, unit record I/O, memory 2-K core minutes, and CPU job-time seconds.

their consumption patterns with externally funded, "hard" monies. The results also suggest that priority-pricing schemes can move demands from peak-demand times to low-demand times.

II. The Marketplace

The seller is a computer center owned by the State University System (SUS) of Florida located on the University of Florida (UF) campus in Gainesville. The center serves all research, instruction, and administrative users at the UF; its teaching hospital and clinics; the University of North Florida in Jacksonville; and about 30 other government institutions in the state. During the period studied, August 1973 through December 1976, the center had an IBM 370/165 (OS/MVT/HASP), 13–16 remote batch terminals, and 150–200 various types of interactive terminals. The center charges the same prices to all its users regardless of funding sources, all users have access to all the center's services, and state law requires that the center not have cash flow deficits. All funds considered here were processed through UF or UNF accounts to pay the center for research computing.

The consumers considered here are faculty researchers that had externally funded research grants and contracts and/or internally funded research monies.[3] About one-third of the researchers had both kinds of monies. Faculty could divert external monies to noncomputing uses subject to the regulations and approvals of contracting or granting agencies.[4] They could not divert internal funds to noncomputing uses. The internal research funds were allocated through a complex procedure that started at the SUS level. An academic department could allocate previously budgeted operating expense monies for computing if its initial computing allocation was expended, but this rarely occurred. "Soft" monies for computing could not be created in the internal funding procedure.[5] However, users' internal fund balances, positive and negative, were erased at the end of budget years. Therefore, the internal and external funds—consumer incomes for computing—considered here were similar in that they were limited in total in various time frames. They differed in that external funds could be diverted to noncomputing uses.

3. In November 1976, the center had 655 open research accounts with 3,738 authorized user numbers, and 1,044 open instruction and administration accounts. About one-third of the research users had internal and external funding, and almost all also had control of instruction computing monies that are not considered here. While university regulations prohibited diversion of instruction funding to research uses, it was known this type of diversion did take place. We did not attempt to measure such diversion for inclusion in this analysis.

4. We did not measure the extent to which diversion to noncomputing uses occurred or the extent to which regulations restricted diversion to noncomputing uses.

5. Smidt (1968*b*) discusses computing monies created to limit computing demands.

The goods considered in this marketplace are batch job priorities. No interactive computing was considered.[6] Demands were high weekdays during the daytime; they were low weekday evenings and nights and even lower on weekends and holidays. At all job entry locations, the user could determine his account balance and the size of the job queue prior to job entry, he could determine the relative position of his job(s) immediately after entry, and he could cancel a job in queue and resubmit it at a different priority.

We assumed a number of user norms based on casual observations of, and discussions with, research users. These behavioral assumptions are as follows.[7] The typical user allocated his computing income to three priority classes because computing capacities were limited relative to demands during some periods and excessive during other periods. One priority class was "normal," which the user chose for weekday, daytime work when there were no demands on him for immediate research results. The second class gave "fast" computing which he chose during high total system demand periods and when there were demands on him for immediate results or pressing deadlines: he paid premium prices for this fast computing, or turnaround. The third class gave "slow" computing at discount prices which the user chose at the end of his daytime work for execution evenings or nights. When faced with a large production job, the user compiled and executed with test data daytimes at normal or premium prices, if necessary, and ran the actual job evenings at discount prices.

In addition, we assumed that if the user's income was "low" relative to demands on him for research results and he chose to give up leisure time to achieve research goals, he did more computing evenings, nights, and weekends at discount prices. If the price of the normal priority increased but the priority execution rule did not change, he reduced his normal computing and increased his computing with the fast and slow classes. The increased use of slow computing permitted his income to support a quantity of work not possible with his prior mix of priorities. The reduced daytime, weekday normal computing resulted in more situations where premium-priced, fast computing was purchased to get immediate results or meet deadlines. His final adjustment to price (and income) changes was not determined instantaneously: his initial adjustments to a higher normal price increase, for example, might have been too large or small, requiring iterations to achieve the balance of income and leisure time costs he preferred. Reactions of all users determined turnaround times in this demand-

6. Jobs written to the system's TCP (terminal control program) facility and entered as batch jobs through a RJE (remote job entry) facility are included in this study.

7. In section III we discuss the inferences related to this assumption given the utility model used in the analysis, and in Section IV we present the case for considering the typical user.

based system. The formal behavior model implicit in these behavior assumptions is developed in Section III.

The specific priority goods we analyze in this study are as follows: normal enters the queue in a position determined by requested CPU time and input and output over the range 3–9. The price is \$2.00 per computing unit (CU). All specific services used are scaled to CU's (see n. 2), for example, 1 CPU sec costs \$0.10, so it is 1/20th of a CU. The fast-priority class includes high and urgent. High enters the queue two positions higher than normal and costs \$3.00 per CU. Urgent enters at position 14, the highest, costing \$6.00 per CU. Normal and high are "aged," that is, moved up one position for each hour a job is in queue. A slow-classed priority, idle, costing \$1.60 per CU, existed during the first 27 months of the period we studied. It entered at position 1. Standby, at \$0.50 per CU and entering the queue at position 0, existed during the last 14 months of the period we studied. Standby had service restrictions which we will discuss in Section VI. Low, costing \$1.00 per CU, entered the queue at position 0 when idle existed and at 1 when idle was dropped and standby was added. The slow-class priorities did not age in queue.

The system first looked at jobs in position 14 in the queue and took all position 14 jobs for which it had resources available, then it looked at position 13, etc., down to position 0. In peak-load periods, usually weekday afternoons during academic quarters, no discounted, slow-class job was executed. Weekday evenings and nights during academic quarters had lower total demands; normal would execute about as fast as high would execute during the daytime. Throughout weekends, low and standby would turnaround about as fast as urgent and high on weekdays. This priority system was structured to reflect total demand levels. There were no guaranteed turnaround times related to the priorities—invariant goods—that could be explicitly related to the priority prices. However, users developed expectations based on experience regarding turnaround to plan when they would compute and, as noted above, could determine the level of total demand prior to job entry and could determine their jobs' positions immediately after entry. We analyzed the consuming patterns based on monthly expenditure data to reflect the choices made by users given their preferences over, not within, the daily, weekly, and holiday cycles exhibited by total computing demands.

III. Individual User Utility Maximization

Economic analysis of consumer behavior specifies a minimal number of a priori constraints. The least restrictive assumption is that consumers can rank-order preferences over various baskets of goods, the ranking is transitive, and hence the resultant (unspecified) utility function is invariant under any monotonic increasing transformation. The only condition on this utility function is that it be concave, so that the utility-maximizing consumer subject to a budget constraint is assured of achieving a unique income allocation. The following constitute the basic data for the computer user's utility maximization problem:

$q_i, i = 1, 2, \ldots, n$	the quantities (computing units) purchased each month of the n available goods.
$p_i, i = 1, 2, \ldots, n$	the corresponding unit prices of each good.
$m = \sum_{i=1}^{n} p_i q_i$	monthly expenditure or income of the user.

Here monthly income equals total expenditure by assumption. Typically, a user has a stock of wealth from which he draws as needed, and not a monthly income as such. However, we are examining the implications arising out of his chosen allocation of this monthly expenditure among the competing goods offered, so monthly income and expenditure can be considered the same quantity. This is the same approach as that taken by Houthakker and Taylor (1970).

We take the utility function maximized by the user to be the Klein-Rubin utility function (Klein and Rubin 1947–48):

$$U(q_1, q_2, \ldots, q_n) = \sum_{i=1}^{n} \mu_i \ln(q_i - \lambda_i). \tag{1}$$

The optimization problem for the individual user is then:

$$\text{Maximize } q_1, q_2, \ldots, q_n \quad \sum^{n} \mu_i \ln(q_i - \lambda_i) - \lambda(\sum^{n} p_i q_i - m). \tag{2}$$

It can be shown that the total expenditure on a good is a linear function of income, m, and all prices:

$$p_i q_i = p_i \lambda_i + \mu_i (m - \sum^{n} p_i \lambda_i), \text{ all } i. \tag{3}$$

We interpret equation (3) as follows for this application. The LHS is the total expenditure on good i. In each month the user buys *at least* λ_i units of the ith good, which exhausts

$$\sum^{n} p_i \gamma_i$$

of discretionary or supernumerary income. This discretionary income is allocated to each of the n goods in proportions $\mu_i, i = 1, 2, \ldots, n$. Hence the μ_i's are defined as the marginal shares. Dividing (3) by p_i (which is fixed) yields the conventional demand curve for good i as a function of income and all prices.

Using the fundamental matrix equation of consumer theory (Barten

1964), the effects of a price change in good j on the demand for good i can be decomposed into substitution and income effects. For the Klein-Rubin utility function to be applicable to our market, we require that the negative income effects dominate the substitution effects so that a price rise for good j leads to an increase in the quantity consumed of all other goods. This conclusion is based on interpreting the $\gamma_j, j = 1, 2, \ldots, n$ as minimum purchases with $\gamma_j > 0$ for all j, and hence excludes consideration of inferior goods. The conclusion is consistent with the user behavior discussed in Section II.

In the analysis, we will estimate own price and income elasticities for different goods and for each user group. Price and income elasticities are defined as follows:

$$\frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{q_i} = -\mu_i \cdot \frac{p_j \gamma_j}{p_i q_i} \tag{4}$$

$$\frac{\partial q_i}{\partial m} \cdot \frac{m}{q_i} = \frac{\mu i}{\omega i}, \tag{5}$$

where $\omega_i = p_i q_i / m$ is defined as the budget share of the ith good.

IV. Parameter Estimates Resulting from Aggregation over Users

In the previous section we derived in (3) the equation relating total expenditure on any good for a single user to his income and unit prices based on the Klein-Rubin utility function. Before estimating equation (3) using market data, we need to aggregate the individual equations and observe the resulting form so that we can correctly interpret the parameters estimated.

If we assume that each user has the same utility function with identical parameters, summing (3) over the $l = 1, 2, \ldots, N$ users gives:

$$\sum_{l=1}^{N} p_i q_{il} = p_i(N\gamma_i) + \mu_i \left[\sum_{l=1}^{N} m_l - \sum_{i=1}^{n} p_i(N\gamma_i) \right]. \tag{6}$$

Here q_{il} and m_l are the quantity purchased of good i and the income of user l, respectively. Clearly equation (6) has the same form as equation (3) where $N\gamma_i$ is the market minimum purchase level for good i. It seems unduly restrictive to assume that each user has exactly the same utility function with identical parameters. In analyzing the computer market data our objective will be to estimate market parameters which are the expected values for a typical user.

Suppose each user maximizes a Klein-Rubin utility function where the parameters γ_{il}, μ_{il} for all i differ over the $l = 1, 2, \ldots, N$ users. Our objective is to utilize estimation procedures such that the estimate we derive, γ_i, is the expected value for typical user l $E\{\gamma_{il}\}$. The right-hand side of equation (3) can be written as a linear function of the variables

$m, p_1, p_2, \ldots, p_n$. We now present properties of the market estimates using a linear one-variable model to illustrate a property of the estimator using aggregated data.

Let the individual user equation be:

$$q_l = \beta_l m_l + \epsilon_l, \tag{7}$$

where q_l, m_l are variables peculiar to user l as is the parameter β_l and the error ϵ_l. Averaging equation (7) over N users, it can be shown that the expected value of the coefficient of $\overline{m}$ is β (Theil 1975, p. 145), assuming either that each m_l is fixed in repeated samples or the conditional distribution of β_l given the m_l is independent of m_l. Further, the variance is

$$\text{var}\, \Sigma \frac{\beta_l m_l}{N} = \frac{1}{N^2} \sigma_\beta^2 \sum_1^N m_l^2, \tag{8}$$

which approaches zero when $N \rightarrow \infty$. So, $E(\overline{q}) = \beta\overline{m}$ and (7) *averaged* over all users,

$$\overline{q} = \beta\overline{m} + \frac{1}{N} \sum^N \epsilon_l, \tag{9}$$

has exactly the same form as the individual user equation (7).

Multiplying (9) by N and using aggregate market data leads to the same estimate for β, so we do not need to know N. Furthermore, since we will be analyzing data from the total population of users, we need not be concerned that our estimates of the expected parameter values for a typical user result from an atypical sample. Thus, the estimates we derive will be the population means of the respective distributions of user parameters which can be interpreted as the expected parameter value for any given user.

V. Estimation Procedures

For the computer goods market under consideration, $n = 5$, the expenditure equation (3) to be estimated has the form:

$$p_i q_i = p_i \gamma_i + \mu_i (m - \overset{5}{\Sigma} p_i \gamma_i) - \epsilon_i, \quad i = 1, 2, \ldots, 5, \tag{10}$$

where $\epsilon_i \sim N(0, \sigma_i^2)$ and the explanatory variables p_i, $i = 1, 2, \ldots, 5$, and m are nonstochastic.

If we estimate the equations:

$$p_i q_i = c_i + \mu_i m + \epsilon_i, \quad \text{all } i, \tag{11}$$

we deduce that:

$$c_i = p_i \gamma_i - \mu_i \overset{5}{\Sigma} p_i \gamma_i. \tag{12}$$

Using equation (12) and the fact that $p_1, p_2, \ldots, p_5$ are fixed values, we derive the system:

$$\begin{bmatrix} (1-\mu_1)p_1 & -\mu_1 p_2 & -\mu_1 p_3 & \cdots & -\mu_1 p_5 \\ -\mu_2 p_1 & (1-\mu_2)p_2 & -\mu_2 p_3 & \cdots & -\mu_2 p_5 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\mu_5 p_1 & \cdots & \cdots & -\mu_5 p_4 & (1-\mu_5)p_5 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}. \quad (13)$$

Since the rank of the coefficient matrix of (13) is less than five (adding rows 1–4 yields the negative of row 5), this system cannot be solved to yield a unique set of estimates for the γ_i, $i = 1, 2, \ldots, 5$. Accordingly, another constraint needs to be imposed on the data. Since we interpreted the γ_i as minimum purchase levels, we know with constant prices we must have

$$\sum^{5} p_i \gamma_i = m_m \leqq \min(m), \quad (14)$$

where m_m is not greater than the minimum monthly income occurring over the data period. By fixing equation (14) as a strict equality, we can replace one of the equations in (13) by (14). In using (14) to fix our estimates for the γ_i, we are only determining approximate minimum purchase levels. In fact, changing the value of m_m does not preserve relatives in the changed γ_i, although the ranking remains the same for quite substantial changes in m_m. An example of this is given in note 4 to table 2 for internal money users. We can therefore only rank goods in order of importance as far as minimum purchase levels are concerned. Since the γ_i are used to determine own price elasticities, these must also be considered as approximate.

If we sum both sides of equation (10) for $i = 1, 2, \ldots, 5$ and use the constraint

$$\sum^{5} \mu_i = 1,$$

we have that:

$$\sum^{5} \epsilon_i = 0. \quad (15)$$

Summing (11) over $i = 1, 2, 3, 4$, and using $p_5 q_5 = m - \sum^{4} p_i q_i$, we have for each month:

$$\begin{aligned} p_5 q_5 &= -\sum^{4} c_i + \left(1 - \sum^{4} \mu_i\right)m - \sum^{4} \epsilon_i \\ &= \sum^{4} c_i + \mu_5 m + \epsilon_5. \end{aligned} \quad (16)$$

Note that we could estimate the first four equations in (11) and then use (16) to get the parameter estimates for the fifth equation. Since the parameter μ_5 does not appear in the first four equations, the constraint

$$\sum^{5} \mu_i = 1$$

becomes ineffective and OLS yields best linear unbiased estimates (see Theil 1975, sec. 5.2). Alternatively, all five equations can be estimated simultaneously to give exactly the same point estimates for the parameters (Theil 1975, sec. 7.2). By estimating each equation individually, however, we can derive parameters such as the coefficient of determination which is of interest in determining how each good's purchases are explained by prices and income using a Klein-Rubin utility function.

Finally, we note that there appeared to be monthly seasonality in the total systems demands related to academic quarters. Higher priority goods were more (less) valuable in busy (slow) months, for example, May, a busy month at the end of the spring quarter, and September, a slow month typically including 7 or fewer academic-quarter days. Users expectations reflected these variations in total system demands, but the small amount of data available for this analysis precluded estimates of seasonal parameters. Systematic measures of seasonal peaks would be important for computing resource management in research settings where the governing body can provide capacities to meet peak demands. The governing body of the center considered in this study did not have the funds necessary for capacities to meet peak demands.

VI. Parameter Estimates

We had available 41 months of data containing monthly expenditures on five priority goods, all prices, and total income. In the period August 1973 through October 1975 (27 months, period 1) the computer utility supplied five priority goods—urgent, high, normal, idle, and low—at prices \$6.00, \$3.00, \$2.00, \$1.60, and \$1.00 per computing unit, respectively. From November 1975 through December 1976 (14 months, period 2) the five goods were urgent, high, normal, low, and standby at prices \$6.00, \$3.00, \$2.00, \$1.00, and \$0.50 per unit, respectively. We will use U, H, N, I, L, and S as subscripts to identify a particular price or parameter, U for urgent, etc.

Since the estimation methodology is based on the assumption that prices were constant, we have two periods in which to estimate the parameters. This results in only 14 observations for the period following November 1975, which we will refer to in this Section as period 2. Accordingly, the conclusions reached on the parameters for period 2

TABLE 1 Means, Standard Deviations, and Proportions of Monthly Expenditures by Priority and Type of Money

	External Money Users			Internal Money Users		
Priority	Mean	SD	Proportion	Mean	SD	Proportion
Period 1 (August 1973–October 1975):						
Urgent	$ 2,281	$1,501	.12233	$ 1,659	$ 2,000	.06530
High	$ 416	$ 276	.02231	$ 601	$ 821	.02366
Normal	$11,792	$4,419	.63238	$16,263	$ 5,986	.64015
Idle	$ 62	$ 56	.00332	$ 212	$ 538	.00834
Low	$ 4,096	$2,207	.21966	$ 6,670	$ 4,640	.26255
Total	$18,647	$7,494	1.00000	$25,405	$10,769	1.00000
Minimum	$ 7,184	...	...	$16,358	...	...
Period 2 (November 1975–December 1976):						
Urgent	$ 3,635	$1,962	.13110	$ 1,065	$ 944	.04871
High	$ 2,165	$ 999	.07809	$ 545	$ 531	.02493
Normal	$16,039	$2,246	.57848	$13,804	$4,309	.63142
Low	$ 3,893	$1,121	.14041	$ 3,614	$1,416	.16531
Standby	$ 1,994	$ 822	.07192	$ 2,834	$1,341	.12963
Total	$27,726	$4,691	1.00000	$21,862	$7,420	1.00000
(Minimum)	$20,697	...	...	$14,375	...	...

need to be tempered by the realization that only a small quantity of data were available. Table 1 shows the means, standard deviations, and proportions of monthly expenditures by priority and type of money for the two periods. For example, in period 1 external money users spent an average of $2,281 per month on urgent goods at $6.00: this gave them 760 "computing units," or $760 of computing at the $2.00 normal price. In period 2, internal money users spent $2,834 on standby goods at $0.50: this gave them 11,336 computing units, or $11,336 of computing at the $2.00 normal price.[8]

Our first task was to identify any differences between the behavior of external and internal money users.[9] Second, we could with hindsight

8. All research users had net discounts every month observed, that is, the sums of positive premiums and negative discounts were negative. Hence the total computing at normal prices was greater than expenditures. The mean expenditures and computing at normal prices by month and type of money user are as follows. In period 1, external funded users spent $18,647 for $21,099 in computing goods at normal prices and internal funded users spent $25,405 for $29,722 in goods. In period 2, external funded users spent $27,726 for $34,461 in goods, while internal funded users spent $21,862 for $33,586 in goods.

9. We refer to internal and external money users in this discussion as though they were distinct groups. As noted in Section I, about one-third of the users had both types of research monies. It should also be noted here that most researchers also had control of instruction computing monies that are not considered here. And many had computing monies from organizations that paid the center directly; these are also not considered here.

TABLE 2 Period 1: Parameter Estimates: August 1973–October 1975

	External Money Users							Internal Money Users						
	c	μ	R^2	γ	ω	I_e*	P_e†	c	μ	R^2	γ‡	ω	I_e	P_e
Urgent	−925	.17195 (.02056)	.74	52	.12233	1.40563 (.16807)	−.0161	−1914	.14063 (.02428)	.57	64	.06530	2.15360 (.37182)	−.1034
High	12	.02164 (.00597)	.34	56	.02231	.96983 (.26759)	−.0045	− 989	.06257 (.008725)	.67	12	.02366	2.64455 (.36877)	−.0127
Normal	1,250	.56535 (.03361)	.92	2656	.63238	.89400 (.05315)	−.1752	3179	.51502 (.04181)	.86	5809	.64015	.80453 (.06531)	−.5608
Idle	79	−.000876 (.00147)	.01	45	.00332	na§	na	− 512	.02850 (.00820)	.33	−37	.00834	na	na
Low	−414	.24189 (.03361)	.67	1323	.21966	1.10120 (.15301)	−.0424	235	.25330 (.06971)	.35	4378	.26255	.96477 (.26551)	−.2849

NOTE.—The standard error is given below the coefficient in parentheses.

*Income elasticity as defined by eq. (4). The standard error is derived from the standard error of the corresponding μ and ω.

†Own price elasticity as defined by eq. (5) using the average of the last 3-month expenditures which were as follows. External money users: \$3,309, \$799, \$17,142, \$60, \$7,549; internal money users: \$525, \$171, \$10,669, \$15, \$3,893.

‡Calculated using the second lowest monthly total expenditure, \$16,358, since the lowest expenditure, \$12,255, was substantially lower than the remaining total expenditures and occurred in a month when administrative procedures delayed the funding of internal research accounts. The γ vector for \$12,255 was (−32, −74, 4742, −97, 3340). The γ_i rankings are N, S, U, H, and L for both monthly total expenditures.

§Estimates were not applicable since the good exhibited negative marginal utility or was considered inferior by the users.

examine the rationale for, and the effects of, the priority price change effected in November 1975. Table 2 shows the regression results for both types of users in period 1: the intercepts and marginal shares for each good are c and μ, respectively. Also shown for each type of money user are minimum purchase levels (γ), budget shares (ω), income elasticities (I_e), and own-price elasticities (P_e). All five equations were estimated, but equation (14) could have been used to derive the parameter estimates for one equation.

In period 1 the external money results show significant (at .01) positive μ values for all but the idle good where $\mu_I = -.000876$ ($t = -.594$). The small negative μ_I value, not significantly different from zero, indicates this good had very little, if any, utility for external money users. But γ_I was negative, indicating that the idle good was regarded as inferior by internal money users. The proportions of income, ω, allocated to the idle good in period 1, .0033 by external and .0083 by internal users, support the inferior good conclusion. Accordingly, there are clear indications this good was not meeting the requirements of the users, and the computer center was justified in removing it from the market.

In comparing external and internal money results, we note that in terms of the marginal shares (μ), both external and internal money users ranked the goods normal, low, urgent, and high (ignoring idle). Thus, both types of users appear to favor a lower-priced (slower-turnaround) good after the normal good. This conclusion is further supported when we note that minimum purchase levels (γ) follow almost the same ranking for both types of users, the only difference being that external money users interchange urgent with high.

The income elasticities indicate some differences between the two user groups. Internal money users clearly view the urgent and high goods as luxuries (Theil 1975, p. 9) with point estimates of their income elasticities exceeding unity. Furthermore, a 99% confidence interval[10] for these elasticities excludes values 1.0 or less. Conversely, the income elasticities of external money users for these same goods are lower with a point estimate less than unity for the high good. However, since a 95% confidence interval for the income elasticity of the urgent good excludes values less than unity, it appears external money users did regard the urgent good as a luxury.

A 99% confidence interval for the income elasticity of internal money users with respect to the normal good excludes values greater than unity, indicating they regarded this good as a necessity. While the point estimates and standard errors of the income elasticities of both user groups for the remaining goods do not permit firm conclusions, the high

10. All confidence intervals are two-tailed.

value for the low good exhibited by external money users indicates they may have regarded this good as a luxury.

Both types of users show the urgent and high goods to be relatively price inelastic when compared with the normal and low own price elasticities.[11] Internal money users exhibit relatively higher-price elasticity with respect to the normal and low goods than do the external money users.

If the price of low good were lower, in retrospect, the best point estimates of price elasticity suggest a large reallocation of expenditures favoring the lower-priced goods for internal money users. The reallocation would have been less marked for external money users: the low good P_e of internal money users was 6.72 times the P_e of external money users ($-.28489$ v. $-.04240$). This indication, together with the significant luxury appraisals of the urgent and high goods by the internal money users, are the only obvious difference between the two groups in period 1. There is no evidence that internal money users exhibited irrational consuming behavior such as excessive consumption of the more costly goods. Only 8.9% of total expenditure by internal money users over this period is accounted for by the urgent and high goods, as compared to 14.5% for external money users.

In table 3, we duplicate the results for period 2. Idle was eliminated. Standby, at $0.50 per computing unit, was added by the center for two reasons. The first was to enable research users to buy more computing units with a dollar. The second was to shift demands from HASP- and CPU-saturated 11:00 A.M. to 5:00 P.M. weekday hours to evenings and weekends. Standby was made technically inferior to the four higher-priced goods: (1) Only one setup (tape or disk mount) was permitted Sunday evenings through Friday. (2) In times of peak demands, standby jobs were not permitted to enter the queue, and were cancelled if in the queue when demands peak (as was the case for low jobs). (3) Standby jobs were run after 6:00 P.M. weekdays or at the operators' discretion when the system was not busy. (4) Job regions were restricted to class large, 384K bytes or smaller (the system maximum was class enormous, at 1,536K bytes).

The conclusion we reached for period 1—that the next-to-lowest priced good had very little, if any, utility for external money users—is also supported in period 2: μ_L is negative and not significantly different from zero. However, internal money users had significant positive utilities for all goods. Ranking by marginal shares, external money users exhibit the order normal, urgent, high, and standby. For internal money users the order is normal, low, urgent, standby, and high, which

11. In commenting on price elasticities we need to recall that there was no price variation over the period to test consumer reaction. Accordingly, the elasticity estimates can only be regarded as indicative of consumer assessment.

TABLE 3 **Period 2: Parameter Estimates: November 1975–December 1976**

	External Money Users							Internal Money Users						
	c	μ	R^2	γ	ω	I_e	P_e*	c	μ	R^2	γ	ω	I_e	P_e*
Urgent	−5,899	.34385 (.06879)	.68	203	.13110	2.62281 (.52471)	−.2595	−1,295	.10798 (.01943)	.72	43	.04871	2.21679 (.39889)	−.0738
High	−2,328	.16205 (.03988)	.58	342	.07809	2.07517 (.51069)	−.1120	− 795	.06129 (.01070)	.73	29	.02493	2.45848 (.42929)	−.0175
Normal	3,856	.43941 (.05499)	.84	6,475	.57848	.75959 (.09506)	−.3783	1,300	.57194 (.02910)	.97	4,761	.63142	.90580 (.04609)	−.4712
Low	4,888	−.03588 (.06823)	.02	4,145	.14041	na	na	217	.15539 (.03200)	.66	2,450	.16531	.93999 (.19358)	−.1262
Standby	−516	.09055 (.04329)	.27	2,715	.07192	1.25904 (.60192)	−.0443	574	.10338 (.04280)	.33	4,119	.12963	.79750 (.33017)	−.0802

*Average monthly expenditures for the last three periods were as follows. External money users: $1,613, $1,484, $15,042, $4,376, $2,775; internal money users: $277, $302, $11,556, $3,016, $2,656.

is consistent with their ranking in the previous period. Using minimum purchase levels, external money users rank the goods as normal, standby, high, and urgent, with low apparently also a necessity despite its negative marginal utility.[12] Internal money users rank the goods as normal, standby, low, urgent, and high, which matches their order for the previous period with standby now inserted in the runner-up position.

Based on the results tabulated, it follows that internal money users are heavier users of the lower-priced goods, and they are much more price sensitive to these goods than external money users. There is no evidence, in either period, to support the hypothesis that luxury goods were wasted by internal money users.

VII. Conclusions

The evidence suggests that there were significant differences in the behavior of external and internal money users with respect to the consumption of computer services. Internal money users exhibit higher price elasticities with respect to the lower-priced priority goods, whereas external money users show greater price elasticity for the higher-priced priority goods (at least in period 2). Furthermore, the minimum purchase levels for the lowest-priced good were significantly higher for internal money users in both periods. The income elasticities of both groups are similar and follow the pattern suggested by economic theory of declining values with declining priority. The one anomaly here is the persistent luxury rating accorded the lowest-priced goods by external money users.

The new pricing structure introduced in November 1975 appeared to substantially change consuming habits of both groups. External money users demanded a greater proportion of the higher-priced goods. The marginal shares of the two most expensive goods increased by factors of 2 and 8, respectively, for external money users. Conversely, for the internal money users these shares dropped slightly. The marginal shares are important since they indicate where proportions of increased income will be spent. This is valuable information for a computer facility that is approaching capacity at times of peak demand. The results suggest that prices can be used to increase demands during periods of low facility utilization, thereby extending the useful life of a given hardware configuration. Moreover, the income elasticities following the priority price change are more in accordance with economic

12. This may be accounted for by the fact that low is the lowest-priced good with no setup restrictions. For example, it is the lowest-priced good that will permit two or more tape and/or disk mounts. The center continuously received complaints about the one-setup restriction on standby.

theory, suggesting that the price change may have made users more price conscious. A pricing scheme to which users do not react is a waste of computer center resources (Sobczak 1974).

We noted above that there is no evidence to support the hypothesis that luxury goods were wasted by internal money users. The combined shares of the two lowest-priced goods were higher for internal money users than for external money users in both periods. Since the income elasticities of the internal money users were similar to those of the external money users, and they follow the pattern suggested by economic theory, there is a clear implication that the internal allocation scheme employed gives rational economic results. Perhaps the "limited funds" characteristic of both the internal and external monies discussed in Section II is adequate to produce these results. The "ability-to-divert" characteristic may not be necessary. Our implication for managing internal research monies is that users may consume computing goods rationally if clear, finite funding limits are imposed, regardless of the actual form or source of the monies.

These conclusions suggest the need for similar analyses with this type of model in at least two settings. The first is where externally funded users can freely divert computing monies to other uses, that is, without funding agency constraints, and internally funded users cannot divert monies, with both types having time limits on spending. The second setting is where externally and internally funded users have time and diversion constraints, with internal money users having influence on the creation of computing funds.

References

Babad, J. M., and Modiano, M. M. 1976. Joining policies in a multipriority multiclass batch computer system. *Communications of the ACM* 19, no. 3 (March): 127–36.

Barten, A. P. 1964. Consumer demand functions under conditions of almost additive preferences. *Econometrica* 32:1–38.

Bauer, W. F., and Hill, R. H. 1967. Economics of time-shared computers. *Datamation* 13, no. 11 (November): 48–55; and 13, no. 12 (December): 41–49.

Berg, H. S. 1975. Computer accounting algorithm, computer operations division Lockheed-California company. In *Share Project: Computer Measurement and Evaluation* 3 (December 1973–March 1975): 370–73.

Berg, S. V. 1975. Planning for computer networks: the trade analogy. *Management Science* 21, no. 2 (August): 1458–65.

Bernard, D.; with Emery, J.; Nolan, R.; and Scott, R. 1977. *Charging for Computer Services: Principles and Guidelines*. EDUCOM, The inter-University Communications Council.

Cotton, I. W. 1975. Microeconomics and the market for computer services. *Computing Surveys* 7, no. 2 (June): 95–111.

Ghanem, S. B. 1975. Computer center optimization by a pricing-priority policy. *IBM Systems Journal*, no. 3, pp. 272–91.

Gill, S., and Samet, P. A. 1969. Charging for computer time in universities. *Computer Bulletin* 13, no. 1 (January): 14–16.

Gladney, H. M.; Johnson, D. L.; and Stone, R. L. 1975. Computer installation accounting. *IBM Systems Journal*, no. 4, pp. 314–39.

Greenberger, M. 1966. The priority problem and time sharing. *Management Science* 12, no. 11 (July): 888–906.
Hetrick, R. C., Jr. 1971. Systems management and allocation of resources technique—SMART. Paper presented at the Sixteenth Annual College on University Machine Records Conference, Dallas, May 1971.
Hootman, J. R. 1969. The pricing dilemma. *Datamation* 15, no. 8 (August): 61, 66.
Houthakker, H. S., and Taylor, L. D. 1970. *Consumer Demand in the United States: Analyses and Projections*. 2d ed. Cambridge, Mass.: Harvard University Press.
Jacobs, D. 1972. Equitable machine accounting in a multi-programming environment. *Data Management* (November), pp. 21–25.
Kanter, H.; Moore, A.; and Singer, N. 1968. The allocation of computer time at university computer centers. *Journal of Business* 41, no. 3 (July): 375–84.
Klein, L. R., and Rubin, H. 1947–48. A constant-utility index of the cost of living. *Review of Economic Studies* 15:84–87.
Marchand, M. 1968. Priority pricing with application to time-shared computers. *Proceedings AFIPS Fall Joint Computer Conference*, pp. 511–19.
Maruyama, K., and Tang, D. T. 1977. Discrete link capacity and priority assignments in communications networks. *IBM Journal of Research Developments* (May), pp. 254–63.
Nielsen, N. R. 1968. Flexible pricing: an approach to the allocation of computer resources. *Proceedings AFIPS Fall Joint Computer Conference*, pp. 521–31.
Nielsen, N. R. 1970. The allocation of computer resources—is pricing the answer? *Communications of the ACM* 13, no. 8 (August): 467–74.
Rettus, R. C., and Smith, R. A. 1972. Accounting control of data processing. *IBM Systems Journal*, no. 1, pp. 74–92.
Sharpe, W. F. 1969. *The Economics of Computers*. New York: Columbia University Press.
Singer, N.; Kanter, H.; and Moore, A. 1968. Prices and the allocation of computer time. *Proceedings AFIPS Fall Joint Computer Conference*, pp. 493–98.
Smidt, S. 1968*a*. Flexible pricing of computer services. *Management Science* 14, no. 10 (June): B581–B600.
Smidt, S. 1968*b*. The use of hard and soft money budgets, and prices to limit demand for centralized computer facility. *Proceedings AFIPS Fall Joint Computer Conference*, pp. 499–509.
Sobczak, J. J. 1974. Pricing computer usage. *Datamation* 20, no. 2 (February): 61–64.
Theil, H. 1975. *Theory and Measurement of Consumer Demand, Vol. 1* Amsterdam: North-Holland.
Wiorkowski, G. K., and Wiorkowski, J. J. 1973. A cost allocation model. *Datamation* (August), pp. 60–65.
Zmud, R. W. 1975. Two enhancements to a flexible pricing control system for allocating computer resources. *Computer Journal* 18, no. 2 (May): 112–14.