Price Exploration and Financial Market Efficiency

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Efficiency has been a very important issue for financial markets all over the world ever since trading began in the stock exchanges. The most ardent market observers believe that although short-term inefficiencies remain, the financial markets are efficient in the long run. However, this question has not been investigated in terms of a theoretical model in detail. This issue has been addressed considering the two financial assets in order to obtain a rigorous definition of market efficiency. Through simulation and graphical verification, it is shown that the market is efficient in the long run, albeit in a simplistic setting. It explores alternative value generating processes by considering models with discrete jumps, heavy-tailed probability models and a regime-switching model.

Introduction

One of the problems that has drawn the interest of all the market forces involved in trading, and which has a very high potential of affecting the academic thinking on investments, is that of market efficiency. The issue of efficiency is a very fundamental one, and is something which most of the market players would like to settle forever, so that they can make long-term decisions based on the notion of the market efficiency. Unless they do reckless speculation and trading momentum, the belief is that the market is efficient in the long run, whereas short-term inefficiencies still remain. The problem is giving a concrete shape to validate the existing beliefs about the short-term and long-term efficiencies or to disprove the conjecture.

Although the issue of market efficiency is a crucial one, and a lot of econometric and empirical research has been done on this topic (Fama, 1965; Fama and French, 1993; and Campbell *et al*., 1997), there is very little theoretical literature on it and no attempt has been made to formulate the problem mathematically or deal with it computationally. This paper attempts to define market efficiency rigorously and perform an analytical and numerical study of a simple model of trading. In particular, the questions that arise are: What is an optimal model for R&D in the financial market and what are the structural specifications of the stock market that help in gaining efficiency? Mukherjee and Ghosh (2006) have discussed the problem with regard to one financial asset. This paper generalizes it to two assets. A further generalization to multiple assets would be immediate, but algebraically tedious.

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The formulation of the model, including the traders' trading, R&D activities, and the trading rules in the market is discussed, based on the studies of Hull (2004) and Reilly and Brown (2003). The basic analytical derivations are then presented. Subsequently, the paper takes up the issue of defining market efficiency in a formal way and proposes alternative definitions in this regard. The results for the basic model are verified and presented through simulation studies. The paper discusses the results for several generalizations of the base line model. In particular, it discusses scenarios where the stock price is subject to discrete shocks at random points of time, when the intrinsic value of the stock is governed by heavy-tailed processes like the lognormal or the gamma distribution rather than the normal distribution. The paper also discusses the case when there are alternative regimes in the value generating process and random switching occurs among the regimes. Finally, it offers the conclusion.

Formulating the Problem

The Traders and the Market

It is assumed that there are two securities or financial assets such as, stocks that are traded in the market and previously determined 'fundamental real value' functions, which for our purpose can be modeled as Brownian motions. Traders do not know the functions or the drift and volatility parameters of the Brownian motions.

Discrete time, $t \in N$ is considered. It is assumed that the fundamental value for the two assets are given by stochastic processes $\{u_t\}_{t \in N}$ and $\{v_t\}_{t \in N}$, where $u_t = u(e_t, e_{t-1}, \ldots)$, $v_t = v\ (e_t, e_{t-1}, \ldots)$, and $\{e_i: i \leq t\}$ is the history of fundamental economic variables.

It is assumed that there are many players in the market, which could be the large investment banks, financial institutions or individual investors. Each market player has a market prediction, a function of public information on previous trades, prices and economic variables and a private unknown called 'processing capacity'. The processing capacity can be thought of as the intrinsic capacity of a firm to do high-end research, so as to maximize its profit by improving prediction accuracy or to take advantage of arbitrage opportunities in the market as and when they arise. We assume that there is no borrowing or short selling to start with, and all the commitments are honored so that there is no risk of noncompliance in a deal.

The market price of the two assets are perfectly observable, which changes only through trade. The market price is a stochastic process $\{p_t^u, p_t^v\}_{t \in N}$, whose value is governed by the demand-supply law of economics, i.e., if more traders put a 'buy' on the asset or the stock then the market price increases, whereas if more traders put a 'sell' on the asset then its market price decreases. We assume that the market price is determined on a first-come-first-serve basis, i.e., if the first trader to trade that asset at a particular point of time quotes the market price to be higher (i.e., wants to buy) or lower (i.e., wants to sell) than the price at the previous point of time, then his quoted price is taken to be the market price for that point of time.

R&D by Traders

To process the information and decide on trading (buy/sell), the trader engages in research. The trader i at time t starts with initial wealth w_t^i or shareholding $q_t^i = \left(q_{t,u}^i, q_{t,v}^i\right)$ (he would always have one or the other as he always uses all his wealth). Each trader has his own prediction for the fundamental real value function. The outcome of the prediction process for person i is a valuation, subject to some random noise. To be precise, the trader i observes an estimate $\left(\hat{u}_t^i, \hat{v}_t^i\right)$ of (u_t, v_t), which is governed by the relationship:

$$\hat{u}_t^i = u_t + \sigma_t^i \epsilon_{1,t}^i$$

and

$$\hat{v}_t^i = v_t + \sigma_t^i \epsilon_{2,t}^i .$$

Therefore, the prediction is a noisy estimate of the fundamental value. σ_t^i is the Standard Deviation (SD), (inverse of precision) of the estimate, given by one of the following two schemes:

R&D Scheme 1: Suppose cost incurred in R&D is c_t^i. This implies a resultant variance,

$$\left(\sigma_t^i\right)^2 = \sigma^2\left(c_t^i, \rho^i\right) \text{ such that, } \frac{\partial\left(\sigma_t^i\right)^2}{\partial c_t^i} < 0, \frac{\partial\left(\sigma_t^i\right)^2}{\partial \rho^i} < 0, \left(\sigma_t^i\right)^2 (0,0) < \infty$$

Here, ρ^i is the processing capacity of the i^{th} trader and c_t^i is the amount of money invested in R&D by the i^{th} trader during the t^{th} point of time. For obvious reasons, the accuracy of the i^{th} trader's prediction, $\left(\sigma_t^i\right)^2$ is a non-increasing function of ρ^i and c_t^i.

Also, $\begin{pmatrix} \epsilon_{1,t}^i \\ \epsilon_{2,t}^i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}\right)$. So, the noises are random but correlated (as they are generated out of the same R&D process).

We take $r = 0.25$ or 0.5 for our numerical applications.

R&D Scheme 2: As an alternative, let us now suppose that the total R&D cost is to be distributed as $c_t^i = c_{1,t}^i + c_{2,t}^i$ (Distribution may be done according to marginal returns, equating $\frac{\partial \sigma_1}{\partial c_1} = \frac{\partial \sigma_2}{\partial c_2}$). The resultant SDs now are, $\sigma_{1,t}^i = \sigma_1\left(c_{1,t}^i, \rho^i\right)$ and $\sigma_{2,t}^i = \sigma_2\left(c_{2,t}^i, \rho^i\right)$ where $\epsilon_{1,t}^i$ and $\epsilon_{2,t}^i$ are independent.

We take $\left(\sigma_{1,t}^i\right)^2 = 0.25 - 0.06 c_{2,t}^i \rho^i$ (a multiplicative form) for our illustration.

For our initial empirical purpose, we assume $u_t \sim N(bt, t)$ and $v_t \sim N(at, t)$, where a, b are known constants.

The vital question that crops up is: While holding share, how is c_t^i funded? To circumvent this problem, we make the following assumption:

Let c be the minimal amount of money that a trader always wants to keep aside for R&D. Suppose the bank pays an interest at the rate *of* r% per annum on the amount deposited by a trader in the bank. We assume that initially, each trader deposits the amount $\frac{100c}{r}$ in the bank, so that he is always assured of the amount c, for pursuing his research even when he is holding shares and has no cash in hand.

An almost immediate question that follows is: Does the trader always spend the amount c on R&D or does he spend a fraction of it and if so, what fraction does he spend? This question has been discussed later. Suppose, for the moment, we have an optimum amount c_t^i for the i^{th} trader to spend on R&D at the t^{th} point of time. Suppose $q_{t,u}^i$ and $q_{t,v}^i$ are the number of stocks of the two assets owned by the i^{th} trader at time t.

The Trading Rule

A simple trading rule is as follows:

Whenever a person's prediction is above the current market price, he buys the asset using all his wealth at a price marginally above the current price. Similarly, whenever a person's prediction is below the current market price, he sells the asset. This price differential which is assumed to be a constant and a small amount, is the market impact of his trade.

In short, if at time t, the market price is p_t and the first trader to make a trade puts a 'buy' on the asset, then $p_{t+1} = p_t + \delta$, where δ is a known *constant*. Similarly, if the first trader to make a trade puts a 'sell' on the asset, then $p_{t+1} = p_t - \delta$.

At time t, the following variables are known to us:

w_t^i = The amount of wealth the i^{th} trader has at time t.

$q_{t,u}^i$, $q_{t,v}^i$ = Number of stocks of the two assets owned by the i^{th} trader at time t.

$p_{t+1} = \left(p_{t+1}^u, p_{t+1}^v\right)$ = The market price that will prevail at time $t + 1$.

Next, the alternative cases that may arise are analyzed.

Case I: $\left\{\left(\hat{u}_t^i > p_t^u + \delta\right) \text{ and } p_{t+1}^u = p_t^u + \delta\right\} \text{and} \left\{\left(\hat{v}_t^i > p_t^v + \delta\right) \text{ and } p_{t+1}^v = p_t^v + \delta\right\}$

If the prediction of the i^{th} trader is assumed to be approximately correct, then the market has undervalued the shares and the price is expected to rise further. The trader should buy more shares now, so that he can sell them off later, when the prices have peaked so as to earn a profit. So, he buys the asset which offers higher margin, i.e., among $\left(\hat{u}_t^i - p_t^u\right)$ and $\left(\hat{v}_t^i - p_t^v\right)$. Therefore, if u offers higher margin, then,

$$q_{t+1,u}^i = q_{t,u}^i + \frac{\left(w_t^i + c\right) - c_{t+1}^i}{\left(p_t + \delta\right)}, q_{t+1,v}^i = q_{t,v}^i \text{ and } w_{t+1}^i = 0$$

Case II: $\left\{\left(\hat{u}_t^i > p_t^u + \delta\right) \text{ and } p_{t+1}^u = p_t^u + \delta\right\} \text{and} \left\{\left(\hat{v}_t^i < p_t^v - \delta\right) \text{ and } p_{t+1}^v = p_t^v + \delta\right\}$

Again assuming the prediction of the i^{th} trader to be correct, the market has overvalued the v-shares and the price is expected to fall further. Also, the u-shares are undervalued. The trader should sell off all his v-shares to take advantage of the overvaluation and buy as many u-shares as possible. Therefore,

$$q_{t+1,v}^i = 0 \text{ and } w_{t+1}^i = 0 \text{ and } q_{t+1,u}^i = q_{t,u}^i + \frac{w_t^i + c - c_{t+1}^i + q_{t,v}^i\left(p_t^v - \delta\right)}{\left(p_t^u + \delta\right)}.$$

Case III: $\left\{\left(\hat{u}_t^i < p_t^u - \delta\right) \text{ and } p_{t+1}^u = p_t^u - \delta\right\} \text{and} \left\{\left(\hat{v}_t^i < p_t^v - \delta\right) \text{ and } p_{t+1}^v = p_t^v - \delta\right\}$

Again assuming the prediction of the i^{th} trader to be correct, the market has overvalued both the shares and the price is expected to fall further. The trader should sell off all his shares to take advantage of the overvaluation. Therefore,

$$q_{t+1,u}^i = 0, q_{t+1,v}^i = 0 \text{ and } w_{t+1}^i = w_i^t + q_{t,u}^i\left(p_t^u - \delta\right) + q_{t,v}^i\left(p_t^v - \delta\right)$$

In this case, the amount c earned as interest will be used for R&D, since q_{t+1}^i or w_{t+1}^i (and hence, the expected utility) is not a function of c_{t+1}^i and therefore, there is no question of obtaining an optimum c_{t+1}^i by maximizing utility.

Case IV: $\left(p_t^u - \delta \leq \hat{u}_t^i \leq p_t^u + \delta \text{ and } p_t^v - \delta \leq \hat{v}_t^i \leq p_t^u + \delta\right)$

In this case, the market prediction p_t's and the trader's prediction $\left(\hat{u}_t^i, \hat{v}_t^i\right)$ are incompatible with trade. $p_{t+1} = p_t + \delta$, implies that more traders are interested in buying rather than selling and hence prices will increase, whereas $\hat{v}_t^i \leq p_t + \delta$ implies that the market has overvalued the shares and the prices will fall. Similar is the case when $\left(\hat{v}_t^i \geq p_t - \delta\right)$ and $p_{t+1} = p_t - \delta$. Hence, in this case, the trader does not take a stand and neither buys nor sells shares. So, $q_{t+1,u}^i = q_{t,u}^i$, $q_{t+1,v}^i = q_{t,v}^i$ and $w_{t+1}^i = w_t^i$.

In this case also, the amount c will be allocated for R&D for the same reasons as in Case III.

One can similarly analyze the other possible combinations. The trading rules are summarized in Table 1.

Table 1: Trading Rules			
Trade Scheme	$\hat{u} > p_t^u + \delta$	$p_t^u - \delta \le \hat{u} \le p_t^u + \delta$	$\hat{u} < p_t^u - \delta$
$\hat{v} > p_t^v + \delta$	I	Buy v	Sell u, Buy v
$p_t^v - \delta \le \hat{v} \le p_t^v + \delta$	Buy u	Do nothing	Sell u
$\hat{v} < p_t^v - \delta$	Sell v, Buy u	Sell v	Sell Both
Note: I – Buy the one with higher margin $\hat{u} - \left(p_t^u + \delta\right)$ or $\hat{v} - \left(p_t^v + \delta\right)$.			

The Optimization Problem

Now, we get back to the question of trying to find an optimum amount for pursuing R&D so as to maximize the returns. We try to maximize the expected utility of the traders. The utility is a function of the amount of tangible wealth, a trader has. Suppose, we want to find an optimum value c_{t+1}^i , i.e., the amount of money to be spent on R&D by the i^{th} trader at the $(t + 1)^{\text{th}}$ unit of time. The total amount of tangible wealth the i^{th} trader has at the $(t + 1)^{\text{th}}$ unit of time is: $w_{t+1}^i + q_{t+1,u}^i u_{t+1}^i + q_{t+1,v}^i v_{t+1}^i$.

Since u_{t+1} and v_{t+1} are unknown, we will use $\hat{u}_{t+1}^i$ as a proxy for u_{t+1} and $\hat{v}_{t+1}^i$ as a proxy for v_{t+1} for the i^{th} trader. Hence, we assume that the utility is a function of $w_{t+1}^i + q_{t+1,u}^i \hat{u}_{t+1}^i + q_{t+1,v}^i \hat{v}_{t+1}^i$,

$$U = U\left(w_{t+1}^i + q_{t+1,u}^i \hat{u}_{t+1}^i + q_{t+1,v}^i \hat{v}_{t+1}^i\right)$$

A question that arises is: What should be the functional form of the utility function? Intuition tells us that it should be such that $U(x)$ should have a slope decreasing in x, i.e., as x increases, the rate of change of the utility should not increase rapidly. Therefore, $u(x) = 1 - e^{-x}$ is a reasonable form of the utility function. Here, for the sake of convenience in notation, we drop the suffix i. Then,

$$\text{Expected Utility } = EU\left(w_{t+1} + q_{t+1,u}\hat{u}_{t+1} + q_{t+1,v}\hat{v}_{t+1}\right)$$

Now,

$$w_{t+1} + q_{t+1,u}\hat{u}_{t+1} + q_{t+1,v}\hat{v}_{t+1} =$$

$$2\left\{q_{t,u} + \frac{(w_t + c) - c_{t+1}}{p_t^u + \delta}\right\}\hat{u}_{t+1} + 2q_{t,v}\hat{v}_{t+1} \quad (\text{buy } u)$$

$$+2\left\{q_{t,v} + \frac{(w_t + c) + c_{t+1}}{p_t^v + \delta}\right\}\hat{v}_{t+1} + 2q_{t,u}\hat{u}_{t+1} \quad (\text{buy } v)$$

$$+\left\{q_{t,u}+\frac{w_t+c-c_{t+1}+q_{t,v}\left(p_t^v-\delta\right)}{p_t^u+\delta}\right\}\hat{u}_{t+1}\quad (\text{sell } v \text{ buy } u)$$

$$+\left\{q_{t,v}+\frac{w_t+c-c_{t+1}+q_{t,u}\left(p_t^u-\delta\right)}{p_t^v+\delta}\right\}\hat{v}_{t+1}\quad (\text{sell } u \text{ buy } v)$$

For the buy *v* and buy *u* case, depending on the higher marginal profit, the case 'buy *v*' or 'buy *u*' will occur. The tangible wealth in these cases have already been found.

We assume,

1. $u_{t+1} \sim N\left(a(t+1),(t+1)\right), v_{t+1} \sim N\left(b(t+1),(t+1)\right)$

2. $\hat{u}_t^i = u_t + \sigma_t^i \epsilon_{1,t}^i$ and $\hat{v}_t^i = v_t + \sigma_t^i \epsilon_{2,t}^i$

   and $\begin{pmatrix}\epsilon_{1,t}^i \\ \epsilon_{2,t}^i\end{pmatrix} \sim N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & r\\ r & 1\end{pmatrix}\right)$

3. σ_{t+1}^2 is a function of ρ and c_{t+1}

The Analysis

Now, we write the bivariate normal distribution,

$$\hat{u}_{t+1} \sim N\left(a(t+1),(t+1)+\sigma_{t=1}^2\right)$$

$$\hat{v}_{t+1} \sim N\left(b(t+1),(t+1)+\sigma_{t+1}^2\right)$$

p or the coefficient of correlation = 0.5.

The density function for bivariate normal is:

$$\frac{1}{2\pi\sqrt{1-\rho^2}\left\{(t+1)+\sigma_{t+1}^2\right\}}\exp\left\{\frac{-1}{2\left(1-\rho^2\right)}\left[\frac{\left(\hat{u}_{t+1}-a(t+1)\right)^2}{t+1+\sigma_{t+1}^2}\right.\right.$$

$$\left.\left.+\frac{\left(\hat{v}_{t+1}-b(t+1)\right)^2}{t+1+\sigma_{t+1}^2}+\frac{\left(\hat{u}_{t+1}-a(t+1)\right)\left(\hat{v}_{t+1}-b(t+1)\right)}{t+1+\sigma_{t+1}^2}\right]\right\}$$

The utility function is, $U(x) = 1 - e^{-x}$ and we consider all possible cases as stated earlier. Expected utility is: $\int U(x)dx$. We assume $\hat{u}_{t+1} = x$ and $\hat{v}_{t+1} = y$. Now, the elementary utility function becomes:

1 – exp [2(case for buy u) + 2(case for buy v) + (case for sell u and buy v) + (case for buy u and sell v)]

$$=1-\exp\left[-\left\{2\beta_1 x+2q_v y+2q_u x+2\alpha_1 y+\alpha_2 y+\beta_2 x\right\}\right] \qquad \ldots(1)$$

(dropping the notation t) where,

$$\alpha_1 = q_{t,v}+\frac{(w_t+c)-c_{t+1}}{p_t^v+\delta},$$

$$\alpha_2 = q_{t,v}+\frac{(w_t^{'}+c)-c_{t+1}}{p_t^v+\delta}\text{ where } w_t^{'}=w_t+q_{t,u}\left(p_t^u-\delta\right)$$

$$\beta_1 = q_{t,u}+\frac{(w_t+c)-c_{t+1}}{p_t^u+\delta}\text{ and}$$

$$\beta_2 = q_{t,u}+\frac{(w_t^{''}+c)-c_{t+1}}{p_t^u+\delta}\text{ where } w_t^{''}=w_t+q_{t,v}\left(p_t^v-\delta\right)$$

Now, this expression gets multiplied with the bivariate normal expression, producing two terms, i.e., we are performing,

{1 – exp[.]}{bivariate normal}.

Considering the second term after multiplication and assuming that,

$$Const = \frac{1}{2\pi\sqrt{1-\rho^2}\left\{t+1+\sigma_{t+1}^2\right\}}$$

$$t+1+\sigma_{t+1}^2=\epsilon$$

and

$$a(t+1)=\theta, b(t+1)=\phi,$$

Equation (1) becomes:

$$\exp\left[-(2q_u)x-(2\beta_1)x-(\beta_2)x-(2q_v)y-(2\alpha_1)y-(\alpha_2)y\right]$$

$$\times \quad (const)\exp\left[\frac{-2}{3}\left\{\frac{(x-\phi)^2}{\epsilon}+\frac{(y-\theta)^2}{\epsilon}-\frac{(x-\phi)(y-\theta)}{\epsilon}\right\}\right] \qquad \ldots(2)$$

Putting $\rho = 0.5$, we get, $\dfrac{1}{2\left(1-\rho^2\right)}=\dfrac{2}{3}$

Now, we put $2q_u+2\beta_1+\beta_2=2z_1$ and $2q_v+2\alpha_1+\alpha_2=2z_2$

Thus, Equation (2) becomes:

$$(const)\exp\left[-(2z_1)x-(2z_2)y-\frac{2}{3\epsilon}(x-\theta)^2-\frac{2}{3\epsilon}(y-\phi)^2+\frac{2}{3\epsilon}(x-\theta)(y-\phi)\right] \qquad ...(3)$$

Simplifying and assuming $(3\epsilon z_2)y+(3\epsilon z_1)\theta+(y-\theta)^2-\frac{(2\lambda)^2}{4}k$, where $2\lambda=y-\phi-3\epsilon z_1$, Equation (3) becomes,

$$(const)\exp\left[\frac{-2k}{3\epsilon}-\frac{2}{3\epsilon}(x-\theta-\lambda)^2\right] \qquad ...(4)$$

(initially we integrate only with respect to x),

$=(const)\exp\left[\frac{-2k}{3\epsilon}\right]\exp\left[\frac{-2}{3\epsilon}m^2\right]$ we assume $x - \lambda - \theta = m$ and limits of the integration will remain same, that is from $-\infty$ to $+\infty$.

Now, integrating with respect to m within the limit $-\infty$ to $+\infty$, Equation (4) becomes:

$$=\int (const)\exp\left[\frac{-2k}{3\epsilon}\right]\exp\left[\frac{-2m^2}{3\epsilon}\right]dm$$

$$=\frac{\Gamma\left(\frac{1}{2}\right)}{\sqrt{2}}\sqrt{3\epsilon}\,(const)\exp\left(\frac{-2k}{3\epsilon}\right)$$

Assuming $\Gamma\left(\frac{1}{2}\right)\sqrt{\frac{3\epsilon}{2}}(const)=\sigma$, the Equation becomes: $\sigma\exp\left(\frac{-2k}{3\epsilon}\right)$.

Now,

$$k=\frac{(-2\lambda)^2}{4}+(y-\phi)^2+3\epsilon z_1\theta+3\epsilon z_2 y$$

$$=\frac{-(y-\phi-3\epsilon z_1)^2}{4}+(y-\phi)^2+3\epsilon z_1\theta+3\epsilon z_2 y$$

Hence, $\frac{-2k}{3\epsilon}=\frac{1}{2\epsilon}\left[-(y-\{\phi-(\epsilon z_1+2\epsilon z_2)\})^2+4\epsilon^2 z_1^2+4\epsilon^2 z_2^2+4\epsilon^2 z_1 z_2+4\epsilon(z_1\theta+z_2\phi)\right]$

Now assuming, $y-\{\phi-(\epsilon z_1+2\epsilon z_2)\}=w$, dy becomes dw and integration limits remain the same, $-\infty$ to $+\infty$.

Also assuming, $4\epsilon^2 z_1^2+4\epsilon^2 z_2^2+4\epsilon^2 z_1 z_2+4\epsilon(z_1\theta+z_2\phi)=\kappa_2$, we have (integrating from $-\infty$ to $+\infty$),

$$\int \exp\left(\frac{-2k}{3k}\right)dy = \int \exp\left[\frac{w^2}{-2\epsilon} + \frac{(\kappa_2)}{2\epsilon}\right]dw = \left(\sqrt{2\epsilon}\right)\Gamma\left(\frac{1}{2}\right)\exp\left(\frac{\kappa_2}{2\epsilon}\right) \quad ...(5)$$

σ is also multiplied with this as by integrating, with respect to x we get $\sigma \exp\left(\frac{-2k}{3\epsilon}\right)$ and now we integrate $\exp\left(\frac{-2k}{3\epsilon}\right)$ with respect to y.

The second term is obtained by multiplying the elementary utility function and the bivariate normal density and then by integrating with respect to x and y, we get the expected utility as follows:

$$= \sigma\left(\sqrt{2\epsilon}\right)\Gamma\left(\frac{1}{2}\right)\exp\left(\frac{\kappa_2}{2\epsilon}\right)$$

Substituting the value of $\sigma = \Gamma\left(\frac{1}{2}\right)\sqrt{\frac{3\epsilon}{2}}(const)$, expected utility becomes:

$$\left[\Gamma\left(\frac{1}{2}\right)\right]^2(\epsilon)\left(\sqrt{3}\right)(const)\exp\left(\frac{\kappa_2}{2\epsilon}\right)$$

Assuming $\left[\Gamma\left(\frac{1}{2}\right)\right]^2(\epsilon)\left(\sqrt{3}\right)(const) = K$, the expected utility is given as: $K\exp\left(\frac{\kappa_2}{2\epsilon}\right)$.

Now, the first term obtained by multiplying the bivariate normal density and the elementary utility function, when integrated gives a constant, whose value is: $(const)\left(\sqrt{3}\right)(\epsilon\pi)$, which is immaterial. Now the expected utility becomes: $(const)\left(\sqrt{3}\right)(\epsilon\lambda) - K\exp\left(\frac{\kappa_2}{2\epsilon}\right)$

Thus, to maximize this function, we have to minimize, $\left(\frac{\kappa_2}{2\epsilon}\right)$. Now,

$$\frac{\kappa_2}{2\epsilon} = 2\epsilon\, z_1^2 + 2\epsilon\, z_2^2 + 2\epsilon\, z_1 z_2 + 2\left(z_1\theta + z_2\phi\right).$$

Substituting $2z_1 = 2q_u + 2\beta_1 + \beta_2$ and $2z_2 = 2q_v + 2\alpha_1 + \alpha_2$ as had been assumed earlier, we have:

$$\frac{\kappa_2}{2\epsilon} = \frac{\epsilon}{2}\left[2q_u + 2\beta_1 + \beta_2\right]^2 + \frac{\epsilon}{2}\left[2q_v + 2\alpha_1 + \alpha_2\right]^2 + \frac{\epsilon}{2}\left[2q_u + 2\beta_1 + \beta_2\right]\left[2q_v\, 2\alpha_1 + \alpha_2\right]$$

$$+\left[(2q_u+2\beta_1+\beta_2)\theta+(2q_v+2\alpha_1+\alpha_2)\phi\right]$$

Now,

$$2z_1=2q_u+2\beta_1+\beta_2$$

$$=2q_u+2\left(q_u+\frac{w_t+c-c_{t+1}}{p_t^u+\delta}\right)+\frac{q_u+w_t^{'}+c-c_{t+1}}{p_t^u+\delta}$$

$$=5q_u+\frac{2(w_t+c)}{p_t^u+\delta}+\frac{q_u+w_t^{'}+c-c_{t+1}}{p_t^u+\delta}-\frac{3c_{t+1}}{p_t+\delta} \quad ...(6)$$

We have assumed $w_t^{'}=w_t+q_u\left(p_t^u-\delta\right)$.

Now, let us also assume: $5q_u+\frac{2(w_t+c)}{p_t^u+\delta}+\frac{q_u+w_t^{'}+c-c_{t+1}}{p_t^u+\delta}=K_1$

So, Equation (6) becomes: $K_1+\frac{3c_t+1}{p_t^u+\delta}$.

Similarly,

$$2z_2=2q_v+2\alpha_1+\alpha_2=5q_v+\frac{2(w_t+c)}{p_t^v+\delta}+\frac{w_t^{''}+c}{p_t^v+\delta}-\frac{3c_{t+1}}{p_t^v+\delta}\,//\,w_t^{''}=w_t+q_v\left(p_t^v-\delta\right) \quad ...(7)$$

Now, here also let us assume, $5q_v+\frac{2(w_t+c)}{p_t^v+\delta}+\frac{w_t^{''}+c}{p_t^v+\delta}=K_2$

Thus, Equation (7) becomes: $K_2-\frac{3c_t+1}{q_t+\delta}$

Now, we assume $\epsilon=t+1+\sigma_{t+1}^2$ and putting $\sigma_{t+1}^2=0.25-0.06c_{t+1}\rho$, we have $\epsilon=t+1.25-0.06c_{t+1}\ \rho$.

Putting, $c_{t+1}=x$, we have $\frac{\epsilon}{2}=0.5t+0.625-0.03\rho\,c_{t+1}$ ($\frac{\epsilon}{2}$ is found because in $\frac{\kappa_2}{2\epsilon}$, which we are minimizing, and where $\frac{\epsilon}{2}$ is multiplied with various terms).

Now, assuming $A = 0.5t + 0.625$, $B = 0.03\rho$ and $c_{t+1} = x$ (since we are differentiating $\frac{\kappa_2}{2\epsilon}$ with respect to c_{t+1}). Therefore, $\frac{\epsilon}{2}=A-Bx$.

Thus, $\frac{\kappa_2}{2\epsilon}$ becomes:

$$(A-Bx)\left(K_1-\frac{3x}{p_t^u+\delta}\right)^2+(A-Bx)\left(K_2-\frac{3x}{p_t^v+\delta}\right)^2+(A-Bx)\left(K_1-\frac{3x}{p_t^u+\delta}\right)\left(K_2-\frac{3x}{p_t^v+\delta}\right)$$

$$+\left(K_1-\frac{3x}{p_t^u+\delta}\right)\theta+\left(K_2-\frac{3x}{p_t^v+\delta}\right)\phi.$$

Let us assume, $(A-Bx)\left(K_1-\frac{3x}{p_t+\delta}\right)\left(K_2-\frac{3x}{q_t+\delta}\right)=\Phi, \frac{3}{p_t+\delta}=\lambda_1$ and $\frac{3}{q_t+\delta}=\lambda_2$

Now differentiating with respect to x we get,

$$\frac{d\left(\frac{\kappa_2}{2\epsilon}\right)}{dx}=2(A-Bx)(\lambda_1 x-K_1)\lambda_1-B(K_1-\lambda_1 x)^2+2(A-Bx)(\lambda_2 x-K_2)\lambda_2$$

$$-B(K_2-\lambda_2 x)^2-\lambda_1\theta-\lambda_2\phi+\frac{d\Phi}{dx}$$

where,

$$\frac{d\Phi}{dx}=(-3B\lambda_1\lambda_2)x^2+(2BK_1\lambda_2+2BK_2\lambda_1+2A\lambda_1\lambda_2)x-A(K_1\lambda_2+K_2\lambda_1)-BK_1K_2$$

Thus, the entire term, that is, $\frac{d\left(\frac{\kappa_2}{2\epsilon}\right)}{dx}$ becomes:

First term: $(-2B\lambda_1^2)x^2+(2BK_1\lambda_1+2A\lambda_1^2)x-2AK_1\lambda_1$

Second term: $(-B\lambda_1^2)x^2+(2BK_1\lambda_1)x-BK_1^2$

Third term: $(-2B\lambda_2^2)x^2+(2BK_2\lambda_2+2A\lambda_2^2)x-2AK_2\lambda_2$

Fourth term: $(-B\lambda_2^2)x^2+(2BK_2\lambda_2)x-2BK_2^2$

Fifth term:

$$\frac{d\Phi}{dx}=(-3B_1\lambda_1\lambda_2)x^2+(2BK_1\lambda_2+2BK_2\lambda_1+2A\lambda_1\lambda_2)x-A(K_1\lambda_2+K_2\lambda_1)-BK_1K_2$$

Adding the terms and equating to zero for minima, we get a quadratic equation whose, coefficient of $x^2=3B\lambda_1^2+3B\lambda_2^2+3B\lambda_1\lambda_2$, coefficient of $x = 2A\left(\lambda_1^2+\lambda_2^2+\lambda_1\lambda_2\right)+2B\left(2K_1\lambda_1+2K_2\lambda_2+K_2\lambda_1+K_1\lambda_2\right)$ and a *constant* C whose value is:

$$-2A\left(K_1\lambda_1+K_2\lambda_2\right)-B\left(K_1^2+K_2^2\right)-A\left(K_1\lambda_2+K_2\lambda_1\right)-BK_1K_2-\lambda_1\theta-\lambda_2\phi$$

So, finally, we have to solve the quadratic equation to get the optimum value of c_{t+1}. We call it $\hat{c}_{t+1}$. If the quadratic has no real solution then $f(c_{t+1})$ is unbounded below and hence, we take $\hat{c}_{t+1}$ = c, where c is the money received as interest from the bank. Otherwise, let x_1 and x_2 be the two solutions, and if $f(x_1) \le f(x_2)$ and $x_1 \ge 0$, then $\hat{c}_{t+1}$ = min (x_1, c). Likewise, if $f(x_1) > f(x_2)$ and $x_2 \ge 0$ then $\hat{c}_{t+1}$ = min (x_2, c). On the other hand, if both x_1 and x_2 are negative then $\hat{c}_{t+1}$ = 0. Therefore, we have arrived at an optimum amount $\hat{c}_{t+1}$ which is used for R&D.

Defining Market Efficiency

We now consider the crucial question of this paper: What is market efficiency? Or what should be a reasonable definition of market efficiency? The following definitions are proposed:

Definition 1: A market is inefficient if $\exists\ \epsilon > 0$ such that, the set where the absolute relative difference of the market price and the fundamental value function exceeds ϵ, has positive Lebesgue measure, i.e, a market is inefficient if $\exists\ \epsilon > 0$ such that, $\lambda\left(\left\{s:\left|\frac{u(s)-p^u(s)}{u(s)}\right|>\epsilon\right\}\right)>0$ or $\lambda\left(\left\{s:\left|\frac{v(s)-p^v(s)}{v(s)}\right|>\epsilon\right\}\right)>0$ where λ denotes the Lebesgue measure on $\mathbb{R}$.

A market is efficient if it is not inefficient.

We define a measure of market efficiency by,

$$\chi_\epsilon^u \underline{\underline{def}}\, \lambda\left(\left\{s:\left|\frac{u(s)-p^u(s)}{u(s)}\right|>\epsilon\right\}\right) \qquad \text{...(8)}$$

Since we restrict ourself to only discrete time case, we use counting measure on time space instead of Lebesgue measure. Similarly, we also define χ_ϵ^v.

Let N be the number of times the stock has been traded. Then, we can redefine the above measure for our purpose as follows:

$$\chi_\epsilon^u \underline{\underline{def}}\, \frac{\#\left\{n:\left|\frac{u(n)-p^u(n)}{u(n)}\right|>\epsilon\right\}}{N} \qquad \text{...(9)}$$

and similarly for χ_{ϵ}^{v}. As we must allow the market to evolve, for the price process to correct and match the fundamental value process, we start counting after $n > 200$. Then market is efficient in our sense if,

$$\chi_{\epsilon}^{u} \to 0 \text{ and } \chi_{\epsilon}^{u} \to 0 \text{ as } N \to \infty \, \forall \epsilon > 0 .$$

Simulation Studies

We have assumed that there are $m = 100$ traders in the exchange. Let N represent total number of rounds of trade in the stock. To simulate $u_1, u_2, \ldots, u_N$ and $v_1, v_2 \ldots, v_N$, we simulate N i.i.d. (independent and identically distributed) $N(a, 1)$ and $N(b, 1)$ random variables, say, $X_1, X_2, \ldots, X_N$ and $Y_1, Y_2, \ldots, Y_N$. Then, $u_i = \sum_{j=1}^{i} X_i$ and $\sum_{j=1}^{i} Y_i$. We have chosen $a = 0.25$, and $b = 0.5$. To simulate the processing-capacities of the m traders, we generate m independent $U(0, 1)$ numbers and allocate them to be the processing capacity of the m traders.

The question raised here is: how do we choose p_1^u, p_1^v, the initial market prices, because the rate of convergence of the market price process to the fundamental value process will depend on the proximity of p_1^u, p_1^v to u_1, v_1. We take $p_1^u = I^+$ where $I \sim N(a, 10)$, independent of $\{X_i\}_{i \geq 1}$ in our simulation. Similarly, $p_1^v = J^+$ where $J \sim N(b, 10)$.

Now increments or decrements for the price process occurs in steps of δ. Should δ be equal to a or b, the mean of valuation process? Notice that if δ is very large compared to a or b, it could lead to an incorrect decision during the next trade even when the price process and the fundamental value process are at the same level, during the present round of trading.

On the other hand, suppose an incorrect decision is made at the i^{th} round of trading. Consider $\{u_t\}$ first. For the price process to catch up with the value process within the shortest possible time of trading, δ must be approximately equal to $3a$. Similarly, to make up for two consecutive wrong decisions in minimum span of time, δ has to be approximately equal to $5a$. Hence, there is a trade-off involved here regarding the magnitude of δ as compared to that of a. Simulation studies give us the best result when $\delta = 4a$, which is shown in detail later. Similarly, for $\{v_t\}$, the optimal choice would be $\delta = 4b = 2$. Therefore, we have chosen $\delta = 1.5$.

We have chosen the following two functional forms of σ^2:

$$\left(\sigma_t^2\right)^i = 0.25 - 0.6c_t^i - 0.005\rho^i \qquad \ldots(10)$$

$$\left(\sigma_t^2\right)^i = 0.25 - 0.6c_t^i \rho^i \qquad \ldots(11)$$

Now, we simulate the values of χ for $N = 1000$, $N = 5000$ and $N = 10000$. Plots clearly shows the decreasing nature of $\left|\frac{v(i)-p(i)}{v(i)}\right|$ in i and hence, the convergence of the price process towards the fundamental value process as $N \to \infty$ in the sense of χ is clear.

What are the factors on which the rate of convergence of the market price process to the fundamental value process depends? Clearly the convergence rate depends on the market differential δ and a, the mean increment of the value process at each step. We have already argued that there is a trade-off involved in the magnitude of δ relative to a for the market to be efficient. What should be the value of the ratio $\frac{\delta}{a}$ so that optimum efficiency is attained? The answer to the above question is provided in Table 1.

Table 1: The Best δ in the Bivariate Case

ϵ \ δ	1	1.5	2
0.01	h_u = 0.9225 h_v = 0.9563	h_u = 0.9700 h_v = 0.9912	h_u = 0.9125 h_v = 0.9912
0.005	h_u = 0.6963 h_v = 0.8550	h_u = 0.6550 h_v = 0.9263	h_u = 0.6388 h_v = 0.8550
0.001	h_u = 0.2338 h_v = 0.3088	h_u = 0.1638 h_v = 0.3850	h_u = 0.1525 h_v = 0.2913

Note: h_u and h_v are for the two separate assets in bivariate case.

Table 1 shows that in both the cases, the price discovery process is the most efficient for $\{u_t\}$ when $\delta = 4a$ and $\delta = 4b$ for $\{v_t\}$. Hence, we have chosen the middle value $\delta = 1.5$ in all the later simulations, unless specifically mentioned otherwise.

We illustrate our results in contrast to the earlier univariate results as in Mukherjee and Ghosh (2006) in Table 2.

In both the cases, we get an increasing pattern except for the inherent randomness in the process. The rate of increase of χ decreases with the intervals moving lower (i.e., when N is larger), which is to be expected. Thus, we conclude that the market is indeed efficient in the long run according to our definition of efficiency.

Table 2: Comparing the Univariate and Bivariate Results			
For $\delta = 1$			
N \ ϵ	0.01	0.005	0.001
1,000	$h_1 = 0.9200$ $h_u = 0.9137$ $h_v = 0.9300$	$h_1 = 0.7262$ $h_u = 0.6637$ $h_v = 0.8300$	$h_1 = 0.2275$ $h_u = 0.2275$ $h_v = 0.3038$
5,000	$h_1 = 0.9700$ $h_u = 0.9827$ $h_v = 0.9940$	$h_1 = 0.9381$ $h_u = 0.9544$ $h_v = 0.9631$	$h_1 = 0.6417$ $h_u = 0.6583$ $h_v = 0.7665$
10,000	$h_1 = 0.9847$ $h_u = 0.9891$ $h_v = 0.9947$	$h_1 = 0.9651$ $h_u = 0.9780$ $h_v = 0.9860$	$h_1 = 0.7969$ $h_u = 0.8038$ $h_v = 0.8664$
Note: h_1 is for univariate case.			

For $\delta = 1.5$			
N \ ϵ	0.01	0.005	0.001
1,000	$h_1 = 0.8388$ $h_u = 0.9487$ $h_v = 0.9725$	$h_1 = 0.6713$ $h_u = 0.7512$ $h_v = 0.9200$	$h_1 = 0.1313$ $h_u = 0.1713$ $h_v = 0.3337$
5,000	$h_1 = 0.9779$ $h_u = 0.9944$ $h_v = 0.9985$	$h_1 = 0.9306$ $h_u = 0.9613$ $h_v = 0.9873$	$h_1 = 0.6248$ $h_u = 0.6579$ $h_v = 0.8354$
10,000	$h_1 = 0.9874$ $h_u = 0.9902$ $h_v = 0.9988$	$h_1 = 0.9701$ $h_u = 0.9791$ $h_v = 0.9959$	$h_1 = 0.8233$ $h_u = 0.8236$ $h_v = 0.9226$

Extensions and Generalizations

Now, we consider several generalizations of our baseline model keeping in mind, the alternative suggestions put forth by the empirical literature for modeling financial asset prices.

Discrete Shock

The first departure from our baseline model is in allowing for random shocks to be applied to the system. These could be the external events that impact the stock price process. To consider this situation, we assume that the value processes u and v are subject to multiplicative random shocks. To be precise, we assume that with a small probability (assumed to be 0.1 here), the values get multiplied by independent randon quantities J_1, J_2 following uniform distribution over [0.9, 1.1]. That is, we are assuming that the size of the shock is at most 10% on either side.

$$\text{So, } u_t \text{ (shock) } = \begin{cases} u_t \times J_1 & w.p.\ 0.1 \\ u_t & w.p\ \ 0.9 \end{cases}$$

$$\text{Similarly, } v_t \text{ (shock) } = \begin{cases} v_t \times J_2 & w.p.\ 0.1 \\ v_t & w.p\ \ 0.9 \end{cases}$$

Under this value generating process, we repeat our simulation and χ-computation exercise, and the results obtained are presented in Table 3.

Table 3: Results for Discrete Random Shocks (δ = 1.5)

N \ ϵ	0.01	0.005	0.001
1,000	h_u = 0.6488 h_v = 0.4938	h_u = 0.5425 h_v = 0.4375	h_u = 0.5350 h_v = 0.4313
5,000	h_u = 0.3244 h_v = 0.1635	h_u = 0.0910 h_v = 0.1133	h_u = 0.1194 h_v = 0.1113
10,000	h_u = 0.3141 h_v = 0.1879	h_u = 0.3536 h_v = 0.1362	h_u = 0.0082 h_v = 0.0153

The results of Table 3 stand in stark contrast to those found in Tables 1 and 2. In our baseline model, efficiency certainly improves with time, but now we see strong violations to that. Considering the (h_u, h_v) values for larger N (for any choice of ϵ), it is obvious that efficiency is not monotonically increasing with trading periods. In fact, the reverse seems to be the case most often. This indicates that the market is prone to remain unstable if it is subjected to external shocks.

Considering Heavy-Tailed Stable (Non-Normal) Distributions for the Value Processes

It is often surmised that the financial asset prices do not follow a normal distribution, instead it is more accurate to model it as a stable, but heavy-tailed process (Campbell *et al.*, 1997 and Embrechts *et al.*, 1997). Common examples of such processes would be lognormal, gamma, exponential (a special case of gamma) and Weibull distributions. The purpose of this subsection is to explore the impact on the price discovery process, if the asset prices follow such heavy-tailed distributions.

We consider two common examples: gamma and lognormal distributions with parameters chosen so that, the mean and variance match with our earlier choices a = 0.25, b = 0.5 and σ^2 = 1.

Gamma Distribution: The density function is given by:

$$G(x;\alpha,\beta)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{\frac{x}{\beta}}$$

For fitting a gamma distribution to the process $\{u_t\}$, we need $\alpha = \alpha\beta = 0.25$ and $\sigma^2 = \alpha\beta^2 = 1$, which yields $\alpha = 1/16$ and $\beta = 4$. The corresponding values for $\{v_t\}$ are, $\alpha = 1/4$ and $\beta = 2$.

Lognormal Distribution: If $x \sim$ lognormal with parameters μ and σ^2 then log x has a $N(\mu, \sigma^2)$ distribution.

For fitting $\{u_t\}$, we need $\mu = \log\left(\frac{0.25^2}{\sqrt{1+0.25^2}}\right)$ and $\sigma = \sqrt{\log\left(1/0.25^2+1\right)}$

For v, $\mu = \log\left(\frac{0.5^2}{\sqrt{1+0.5^2}}\right)$ and $\sigma = \sqrt{\log\left(1/0.5^2+1\right)}$

The simulated efficiency results for these cases are presented in Tables 4a and 4b.

Table 4a: Results for Gamma Distribution

N \ ϵ	0.01	0.005	0.001
1,000	$h_u = 0.8888$ $h_v = 0.9487$	$h_u = 0.8225$ $h_v = 0.8912$	$h_u = 0.1437$ $h_v = 0.3125$
5,000	$h_u = 0.9729$ $h_v = 0.9946$	$h_u = 0.9083$ $h_v = 0.9771$	$h_u = 0.6335$ $h_v = 0.7902$
10,000	$h_u = 0.9853$ $h_v = 0.9922$	$h_u = 0.9602$ $h_v = 0.9874$	$h_u = 0.7645$ $h_v = 0.8860$

Table 4b: Results for Lognormal Distribution

N \ ϵ	0.01	0.005	0.001
1,000	$h_u = 0.8788$ $h_v = 0.9925$	$h_u = 0.7275$ $h_v = 0.9000$	$h_u = 0.1625$ $h_v = 0.2788$
5,000	$h_u = 0.9521$ $h_v = 0.9958$	$h_u = 0.9396$ $h_v = 0.9821$	$h_u = 0.6031$ $h_v = 0.8050$
10,000	$h_u = 0.9897$ $h_v = 0.9985$	$h_u = 0.9701$ $h_v = 0.9886$	$h_u = 0.8131$ $h_v = 0.9021$

From the tables, it is observed that the efficiency results are robust with respect to the distribution of the intrinsic value process. This has interesting implications for the market analysts. As R&D is always performed in the financial markets under the normality assumption, it is an important question as to whether the lack of normality in the underlying processes invalidates such R&D exercises altogether or not. Our analysis provides a partial, but strongly encouraging answer to this question. Even if the underlying value process is generated by a distribution different from normal, it is still possible to forecast the prices closely, using a Gaussian estimation strategy.

Regime Switching

We finally consider another important variation of the benchmark model—the case when the value generation process switches between the alternative regimes. This possibility has been considered in the empirical literature in different ways, looking at the mean/drift functions (threshold models, etc.) or the variance function (stochastic volatility models). In this paper, we consider only the mean switching situation. We assume that there are three alternative regimes. The innovation processes are given by,

$$u_{t+1} = u_t + \epsilon_u \text{ and } v_{t+1} = v_t + \epsilon_v$$

where $\epsilon_u \sim N(a_i, 1)$ and $\epsilon_v \sim N(b_i, 1)$, $i = 1, 2$ and 3.

The values for a_i and b_i as well as the independent transition probabilities from one state to the other are given in Tables 5a and 5b.

Table 5a: Independent Transition Probabilities from One State to the Other

a	–1	0.25	1
–1	0.5	0.5	0
0.25	1/16	7/8	1/16
1	0	0.5	0.5

Table 5b: Independent Transition Probabilities from One State to the Other

b	–2	0.5	2
–2	0.5	0.5	0
0.5	1/16	7/8	1/16
2	0	0.5	0.5

Initially, we start from state 2, i.e., when $a = a_2 = 0.25$ and $b = b_2 = 0.5$. The simulation results are presented in Table 6.

Table 6: Results for the Regime-Switching Model

N \ ϵ	0.01	0.005	0.001
1,000	$h_u = 0.8063$ $h_v = 0.9363$	$h_u = 0.4675$ $h_v = 0.7338$	$h_u = 0.1825$ $h_v = 0.1963$
5,000	$h_u = 0.9117$ $h_v = 0.9160$	$h_u = 0.9242$ $h_v = 0.9402$	$h_u = 0.4483$ $h_v = 0.6162$
10,000	$h_u = 0.9806$ $h_v = 0.9892$	$h_u = 0.9291$ $h_v = 0.9668$	$h_u = 0.6818$ $h_v = 0.8044$

The results obtained indicate that the price exploration algorithm considered in this paper is robust with respect to the regime switching variation.

Conclusion

In this paper, we have attempted to simulate the trading scenario in a stock exchange, using a set of heuristically justified assumptions. We arrived at a reasonable and workable definition of market efficiency, which is the χ measure Equation (9). The convergence of the price process to the fundamental value process was subjected to a battery of diagnostic and graphical tests. The optimum ratio between the differential impact of the trade and the mean increment of the fundamental value process is obtained so as to achieve maximum market efficiency.

We conclude that the market is indeed efficient in the long run. Therefore, after the market has evolved for a significant amount of time, the price process can be used as a proxy for the value process. The amount of time depends on how closely we want the value process to be approximated by the price process.

We have also considered several departures from our benchmark model by considering value processes subject to random shocks and situations where the value process is governed by a heavy-tailed probability distribution rather than the normal distribution. Finally, we have also considered a situation where the value generating process randomly switches between three alternative regimes. We show that our efficiency results carry over to the heavy-tailed distribution and the regime-switching cases, but similar results do not hold for the case when the value process is subject to random shocks.

Overall, an attempt has been made to identify the situations where a Gaussian price exploration algorithm would achieve efficiency, and among the possibilities considered here, it is observed that except for the random-switching case, the results are very encouraging. ❖

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