Multiscale Carhart Four-Factor Pricing Model: Application to the French Market

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This paper focuses on a methodology aimed at analyzing the Carhart multifactor model (Carhart, 1997) over various time horizons in the French Stock Market. The suggested approach exploits the decomposition scheme inherent to the wavelet-based Multiresolution Analysis, *allowing one to investigate the time scale relationships between stock returns and risk factors. The empirical results show that the explanatory power of the wavelet-based four factor model is scale-sensitive. The market factor is highly significant over the range of time scales and positively effects intermediate and long-term investment horizons. Besides, the size factor is found to be negative for the portfolios constructed by small capitalization assets. The size risk also becomes negative for big portfolios at the largest time scale. The value proxy HML, which is rejected for the unitary (single) scale model is, however, significant over a large array of resolution levels (investment periods). Finally, it is found that the momentum factor, within the multiscale framework, has a significant impact on the expected stock returns.*

Introduction

Over the last 15 years, criticism addressed to the single-factor Sharpe-Lintner Capital Asset Pricing Model (CAPM) has led to the development of multifactor asset pricing models, motivated by Ross's (1976) Arbitrage Pricing Theory (APT). The basic methodology has been to consider supplementary risk factors in addition to the systematic risk, in order to explain the expected excess returns. Fama and French (1993) propose three factors as surrogates for risk: the market, the size and the value factors. Adding a fourth factor which is the momentum anomaly, Carhart (1997) introduces an alternative multifactor pricing model.

Though these models have been the subject of theoretical and empirical investigation and have shown practical importance, they fail, however, to take into account, the tendency of investors to hold stocks over different periods of time. The reason is that classical pricing models are restricted in a single scale perspective.

In this paper, we exploit the time scale localization property of wavelet functions to develop a multiscale Carhart four-factor model, aimed at analyzing the relationships between stock returns and risk factors at different investment periods within the French Stock Market.

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This paper is structured as follows: First, it presents a short overview on the Carhart four-factor pricing model, followed by the delineation of wavelet techniques in time-series analysis, an introduction of the multiscale Carhart four-factor model, data description, empirical results drawn from the implementation of the single-scale and the multiscale pricing models, before arrising at the conclusion.

Carhart Four-Factor Model

The Carhart multifactor asset pricing model (1997) extends the Fama-French model by adding a fourth factor which takes into consideration, the momentum anomaly (Jegadeesh and Titman, 1993).

The resulting model is consistent with a market equilibrium model, with four risk factors which are: the Fama-French risk factors, i.e., the value-weighted market proxy *Mkt* (High minus Low), the size risk *SMB* (Small Minus Big), and the book-to-market factor *HML* in addition of the momentum anomaly proxied by *WML* which is short for (Winner Minus Loser). The Carhart four-factor model is as follows:

$$R_{it} - R_{ft} = \alpha_i + \beta_i Mkt_t + \gamma_i SMB_t + \delta_i HML_t + \theta_i WML_t + \varepsilon_{it}$$

It should be noted here that in the equation below, *Mkt* is the "market premium": $(R_{mt} - R_{ft})$, where R_{mt} and R_{ft} stand respectively for the market return and the free rate. β, γ, δ and θ are the factor sensitivities to the state variables obtained from the multiple regression of $R_{it} - R_{ft}$ on $R_{mt} - R_{ft}$, *SMB*, *HML* and *WML*, respectively.

The Fama-French factors are constructed using six portfolios based on size and $\frac{BE}{BM}$ ratio. Similarly, the Carhart (momentum) factor exploits six portfolios based on size and momentum.

In order to build *SMB* and *HML* factors, the following methodology is repeated at the end of June, every year. Firstly, two groups are formed based on size and labeled Small (S) and Big (B). Here, we use the median market capitalization at the end of June of that year as the size breakpoint.

Secondly, we use the $\frac{BE}{BM}$ ratio for the year *t*, which corresponds to the Book Equity (BE) for the fiscal year ending in $(t - 1)$, divided by the Market Equity (ME) for December of $(t - 1)$ to construct three groups of stocks. These groups, denoted by Low (L), Medium (M) and High (H), result from dividing the cumulative stocks, with respect to the 30^{th} and 70^{th} percentiles, respectively.

Then, the intersection of the five groups described below, allows us to obtain six portfolios denoted by *SL*, *SM*, *SH*, *BL*, *BM* and *BH*, for which we calculate the corresponding returns labeled R_{SL}, R_{SM}, R_{SH}, R_{BL}, R_{BM} and R_{BH}.

Finally, we compute the *SMB* risk factor as the difference between the monthly average return on the three small portfolios and the monthly average return on the three big portfolios:

$$SMB = \overline{R}_S - R_B = \frac{1}{3}\left[R_{SL} + R_{SM} + R_{SH}\right] - \frac{1}{3}\left[R_{BL} + R_{BM} + R_{BH}\right]$$

and *HML* as the difference between the monthly average return on the two portfolios, within the high group and the monthly average return on the two portfolios with low $\frac{BE}{BM}$ ratio:

$$HML = \overline{R}_H - \overline{R}_L = \frac{1}{2}\left[R_{SH} + R_{BH}\right] - \frac{1}{2}\left[R_{SL} + R_{BL}\right]$$

So as to obtain the *WML* factor, we opt for the following methodology: we use Big (B) and Small (S) portfolios based on size decomposition and Winner (W), Minus (M) and Loser (L) portfolios based on the cumulative past stock returns. The last three portfolios are obtained from the decomposition of the sorted stocks in function of the 12 cumulative past returns where, the breakpoints are chosen to be the 30^{th} and 70^{th} percentiles. Thus, the 30% of stocks which have the most important cumulative past returns form the Winner (W) group, the next 40% represent the Minus group (M) and the remaining 30% constitute the Loser group (L).

The intersection of these five groups allow us to obtain six portfolios, denoted by *SW*, *SM*, *SL*, *BW*, *BM* and *BL*. For each portfolio, we calculate the corresponding returns, labeled hereafter R_{SW}, R_{SM}, R_{SL}, R_{BW}, R_{BM} and R_{BL}, respectively.

The *WML* factor is the difference between the monthly average return of the two winner portfolios and the monthly average return of the two loser portfolios:

$$WML = \bar{R}_W - \bar{R}_L = \frac{1}{2}\left[R_{SW} + R_{BW}\right] - \frac{1}{2}\left[R_{SL} + R_{BL}\right]$$

Wavelet Analysis

Wavelets are mathematical functions used to split a given function into different scale components. Each scale component corresponds to a specific frequency range, which can then be analyzed at a precise resolution level. A wavelet transform is a time scale representation of a function by wavelets. The wavelet transform outperforms the traditional Fourier transform in terms of 'time-frequency' localization and is, therefore, well suited for representing functions that have discontinuities and sharp peaks, as well as for accurately handling non-stationary time series.

Formally, wavelets $\psi_{j,k}$, $j, k \in Z$ are dilated (by a factor 2^j) and/or translated (by a factor k) versions of a unique function ψ such that, the family $\left\{\psi_{j,k} = 2^{j/2}\psi\left(2^j x - k\right), j, k \in Z\right\}$ is an orthonormal basis in $L^2(IR)$. This allows expressing any function $f \in L^2(IR)$ as:

$$f(x)=\sum_{j\in Z}\sum_{k\in Z}\gamma_{jk}\,\psi_{jk}(x)$$

where $\gamma_{jk}=\int f(x)\psi_{jk}(x)dx$ or equivalently,

$$f(x)=\sum_{k\in Z}c_k\,\varphi_{0k}(x)+\sum_{j=0}^{\infty}\sum_{k\in Z}d_{jk}\,\psi_{jk}(x) \qquad \text{...(1)}$$

where $\varphi_{jk}(x)=2^{j/2}\varphi\left(2^j x-k\right)$. The function φ is called the 'scaling function'.

The approximation coefficients $c_k\int f(x)\varphi_{0k}(x)dx, k\in Z$ capture the general form of the function f at level $j = 0$.

The detail coefficients $d_{jk}=\int f(x)\psi_{jk}(x)dx, j\in Z_+$ depict the local details of f at a resolution level j.

It should be stressed here that the scaling function is such that:

$$\varphi(x)=\sqrt{2}\sum_k h_k\varphi(2x-k)$$

where

$$h_k=\sqrt{2}\int_{\square}\varphi(x)\varphi(2x-k)dx, k\in Z$$

Here $\{h_k, k\in Z\}$ is a low-pass filter satisfying $\sum_n h_n h_{n+2j}=0$ for $j \neq 0$ and $\sum_n h_n^2=1$.

The wavelet function ψ is related to the scaling function φ by the following relationship:

$$\psi(x)=\sqrt{2}\sum_k g_k\varphi(2x-k)$$

where $g_k=(-1)^{k+1}h_{1-k}$. The series $\{g_k, k\in Z\}$ is a high-pass filter verifying the quadrature mirror relationship: $\sum_n h_n g_{n+2j}=0$, for all $j\in Z$.

There are various families of wavelet bases. Daubechies (1992) constructed a special class of wavelet filters $\{h_l, l = 0, ..., L-1\}$ of even length L, denoted by $D(L)$. These filters are orthogonal, compactly supported, and have a number of vanishing moments (see Daubechies (1992) for more details).

Next, we describe a stationary version of the wavelet transform known as the Maximal Overlap Discrete Wavelet Transform (MODWT). This Transform will be used in the paper.

Let X be a length N vector, containing a real-valued time series $\{X_t,\ t = 0, \ldots, N-1\}$ and let J_0 be a positive integer. The MODWT of level J_0 transforms the vector X from the time domain to the wavelet time scale domain yielding $J_0 + 1$ vectors: $\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_{J_0}$ and $\widetilde{v}_{J_0}$, each of length N. Here $\widetilde{w}_j, j=1,\ldots,J_0$ are vectors composed of the MODWT wavelet coefficients associated with changes on scale $\tau_j = 2^{j-1}$ while v_J contains the MODWT scaling coefficients associated with averages on scale $\iota_{J_0} = 2^{J_0}$. With matrix notation, the MODWT coefficients are obtained via: $w_j = \widetilde{W}_j X, j=1,\ldots,J_0$ and $v_J = \widetilde{V}_{J_0} X$ where the matrices $\widetilde{W}_j$ and $\widetilde{V}_{J_0}$ contain the MODWT wavelet and scaling filter coefficients.

Actually, matrices $\widetilde{W}_j$ and $\widetilde{V}_{J_0}$ are not explicitly constructed and the MODWT is performed via a pyramidal algorithm.

Renormalizing the wavelet filters $\{h_l\}_{l=0}^{L-1}$ and $\{g_l\}_{l=0}^{L-1}$ as follows: $\widetilde{h}_l = h_l/2^{1/2}$ and $\widetilde{g}_l = g_l/2^{1/2}$ and (starting with) setting $X_t = \widetilde{w}_{0,t}$, the MODWT coefficients are obtained through the following filtering steps:

$$\widetilde{w}_{j,t} = \sum_{l=0}^{L-1} \widetilde{h}_l \, \widetilde{v}_{j-1,t-2^{j-1}l \bmod N} \quad \text{and} \quad \widetilde{v}_{j,t} = \sum_{l=0}^{L-1} \widetilde{g}_l \, \widetilde{v}_{j-1,t-2^{j-1}l \bmod N}, \ t = 0, \ldots, N-1$$

If we write $\{w_{j,t} := 0,\ldots,N_j - 1\}$ as $\widetilde{w}_j$ and $\{v_{j,t} : t=0,\ldots,N_j-1\}$ as $\widetilde{v}_j$, then, after J_0 iterations, the pyramidal algorithm yields $\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_{J_0}$ and $\widetilde{v}_{J_0}$.

It should be stressed that we can get an additive decomposition of a time series $\{X_t, t=0,\ldots,N-1\}$. In fact, we have:

$$X = \sum_{j=1}^{J_0} \widetilde{W}_j^T w_j + \widetilde{V}_{J_0}^T v_{J_0} = \sum_{j=1}^{J_0} \widetilde{d}_j + \widetilde{s}_{J_0}$$

The latter equation defines a Multiresolution Analysis (MRA) of X. The j^{th} level wavelet detail $\widetilde{d}_j = \widetilde{W}_j^T w_j$ describes changes at a scale $\tau_j = 2^{j-1}$, corresponding to frequency bands $[1/2^{j+1}, 1/2^j]$, $\widetilde{s}_{J_0}$, is referred to as the smooth and is associated with averages at a scale $\iota_{J_0} = 2^{J_0}$. The smooth represents the trend component of a time series, while the wavelet details capture deviations from that trend.

It is noteworthy that each wavelet detail $\widetilde{d}_j$ corresponds to a frequency band $[1/2^{j+1}, 1/2^j]$ and thus to $[2^j, 2^{j+1}]$ time period, e.g., for monthly data, the level-one wavelet detail $\widetilde{d}_1$ is associated with oscillations of two-four months period.

For an exhaustive overview of wavelet techniques in time series analysis, we refer to Percival and Walden (2000) and Gençay *et al.* (2001).

Multiscale Carhart Four-Factor Model

We discuss a multiscale version of the Carhart multifactor pricing model in order to investigate the relationship between the risk factors and the portfolio returns, over different time horizons. It should be noticed here that the proposed model is an extension of the works of Kim and In (2006, 2007) and is line with In (2004).

The underlying motivation is that financial time series display different dynamics, each associated with a given time horizon. In order to deal with this multiscaling phenomenon, we make use of the time scale decomposition of the wavelet MRA. In fact, a time series can be expressed as the sum of the smooth component and the wavelet details, each pertaining to a given frequency band and hence, to a given time period allowing one to analyze the sensibilities of the investment strategies and make a decision regarding different resolution scales.

The suggested scheme consists then in carrying out a MODWT-based MRA, curtailed at a resolution level J_0 for the series of excess portfolio returns as well as the four risk factors. After that, at each resolution scale $\lambda_j = 2^{j-1}, j = 1, \ldots, J_0$, we consider the following time series regression scheme, hereafter, called the multiscale Carhart four-factor model:

$$\begin{aligned} R_{it}(\lambda_j) - R_{ft}(\lambda_j) = {} & \alpha_i(\lambda_j) + \beta_i(\lambda_j) Mkt_t(\lambda_j) + \gamma_i(\lambda_j) SMB_t(\lambda_j) + \delta_i(\lambda_j) HML_t(\lambda_j) \\ & + \theta_i(\lambda_j) WML_t(\lambda_j) + \varepsilon_{it}(\lambda_j) \end{aligned}$$

where $R_{it}(\lambda_j), j = 1, \ldots, J_0$, is the excess return over the risk free rate on a buy-and-hold portfolio i at time t and scale $\lambda_j = 2^{j-1}$. $Mkt_t(\lambda_j), SMB_t(\lambda_j), HML_t(\lambda_j)$ and $WML_t(\lambda_j)$ are the risk factors of the Carhart pricing model at time *t* and scale $\lambda_j = 2^{j-1}$. The intercept $\alpha_i(\lambda_j)$ is the measure of the abnormal performance at scale λ_j. The residual $\varepsilon_{it}(\lambda_j)$, is a zero mean abnormal portfolio return, unexplained by common risk factors at scale λ_j. $\beta_i(\lambda_j), \gamma_i(\lambda_j), \delta_i(\lambda_j)$ and $\theta_i(\lambda_j)$ are the assigned loadings at scale λ_j for the market factor, the size factor, the value factor, and the momentum factor, respectively.

Data Description

The dataset used to test the single-scale and the multiscale Carhart Models are the monthly returns of equities quoted on the French Stock Market over the period—January 1985 to October 2006. The total number of equities is 90. The dataset are derived from DataStream database. Returns of the risk-free asset are estimated from the 13-week (91-day) Treasury Bill series whereas, the Cac40 is considered as the Market Index.

Table 1: Statistical Description

	SW	SM	SL	BW	BM	BL	Mkt	SMB	HML	WML
Mean	–0.0530	–0.0539	–0.0553	–0.0508	–0.0536	–0.05358	–0.0501	0.0005	0.0026	0.0027
Median	–0.0496	–0.0463	–0.0548	–0.0429	–0.0449	–0.04480	–0.0446	0.0032	0.0011	0.0029
Maximum	0.0547	0.0589	0.1072	0.0919	0.0868	0.11620	0.1383	0.0772	0.1189	0.0557
Minimum	–0.1778	–0.2772	–0.1876	–0.2904	–0.2916	–0.22380	–0.3246	–0.0835	–0.0924	–0.0666
Std. Dev.	0.0444	0.0457	0.0443	0.0502	0.0453	0.04830	0.0671	0.0271	0.0297	0.0184
Skewness	–0.1801	–0.7578	–0.0813	–0.6356	–0.7407	–0.29030	–0.3147	–0.0932	0.3900	–0.3427
Kurtosis	2.4456	4.6315	3.3824	4.5368	5.5361	3.32920	3.7364	3.4333	4.5240	3.8119
J-B	4.4432	50.4149***	1.7553	40.4394***	87.7019***	4.53020	9.5397***	2.2620	29.7958***	11.4794***
Q(20)	650.6800***	559.5800***	494.1600***	383.1600***	450.3800***	430.98000***	163.9300***	22.8630	55.5020***	24.5380

Note: J-B is the value of Jarque-Bera test statistics of the return residuals; Q(20) is the Box-Pierce test statistics for the return residuals for upto 20th order serial correlation; ***, ** and * indicate rejection at 1%, 5% and 10% significance level, respectively.

Basic descriptive statistics for the six portfolios and the four-factors series are shown in Table 1. While examining the key features of the portfolios' returns, we observe that all sample means are negative with values between –0.0553 (SL) to –0.0508 (BW). Moreover, the six portfolios share almost the same degree of volatility as reflected by the estimates of the standard deviation. Except for the portfolio *SW,* all kurtosis magnitudes are greater than three indicating leptokurtic distributions while, negative skewness characterize all the returns. The normality assumption is rejected for the return series of three portfolios, namely *SM*, *BW* and *BM* whereas, the Jarque-Bera test fails to reject the null hypothesis of Gaussianity at conventional statistical levels for the other portfolios. For the six portfolios, the Box-Pierce test statistics upto 20th order reveal the existence of linear and non-linear dependencies.

Observing the summary statistics for the four risk factors given in columns 8-11 of Table 1, we see that, in contrast to the other risk proxies, the *Mkt* variable exhibits negative mean and median values and appears to be more volatile with larger standard deviation estimate. Moreover, while all the series seem to have leptokurtic distributions, the Jarque-Bera test rejects the normality assumption for three of them, i.e., *Mkt*, *HML* and *WML* and fails to reject the null

Table 2: Correlation Between Factors				
	Mkt	*SMB*	*HML*	*WML*
Mkt	1.0000	0.179932 (0.004814)	0.077453 (0.2280)	0.065168 (0.3106)
SMB	–	1.0000	–0.036885 (0.5664)	–0.007790 (0.9036)
HML	–	–	1.0000	–0.281937 (7.733e-06)
WML	–	–	–	1.0000
Note: *p*-values for the two-sided Pearson test are given in parentheses.				

hypothesis for the size factor *SMB*. The Box-Pierce test indicates that the 20th order serial correlations for the excess market returns and the value factors are highly significant. The Portmanteau test fails to reject the null hypothesis of independence for *SMB* and *WML* series.

Table 2 reports the Pearson's correlation coefficients between the independent variables used in this study. One of the most salient features is that *HML* and *WML* factors are negatively correlated with a correlation coefficient value of –0.28, which is significant at 1% level. Moreover, the correlation coefficient between the size and the market risk proxies, although small-valued, is highly significant. This coefficient is positively valued, reflecting a positive association between the two factors.

Single-Scale Carhart Model Estimation Results

The estimation results are obtained from the application of the unit-scale four-factor pricing Model to the French Stock Market dataset. The estimation results are reported in Table 3.

The primary inference drawn is that the considered risk factors fail to capture common variation in stock returns. Indeed, the adjusted R square $\left(\overline{R}^2\right)$ values are very weak, varying between 0.1321 for the SM portfolio and 0.2165 for the SW portfolio. Consequently, we can state that the single-scale Carhart four-factor model is unable to properly explain the cross-sectional variation of the returns in the French Stock Market.

We notice that the intercept value α_i is statistically significant at the conventional level and keeps the same value –0.04, showing that all portfolios have a negative abnormal returns when the four pricing factors are set equal to zero.

When considering the market risk, we see that all estimated coefficients $\hat{\beta}$ are statistically significant at 1% level, exhibiting positive magnitude. It should be stressed here that the market has almost the same positive effect across the constructed portfolios with coefficients' values restricted in the range going from 0.2069 (*BM*) to 0.2376 (*SW*).

Table 3: Single-Scale Carhart Model Estimation Results						
	Intercept	*Mkt*	*SMB*	*HML*	*WML*	$\bar{R}^2$
SW	–0.041880 (–12.869)	0.237679 (6.168)	–0.422321 (–4.457)	–0.060475 (–0.679)	0.549684 (3.833)	0.2165
SM	–0.043033 (–12.221)	0.229661 (5.508)	–0.262271 (–2.558)	–0.047444 (–0.492)	0.345010 (2.224)	0.1321
SL	–0.041970 (–12.580)	0.231649 (5.864)	–0.399308 (–4.110)	–0.027105 (–0.297)	–0.531976 (–3.619)	0.1729
BW	–0.041473 (–11.085)	0.231568 (5.227)	0.264643 (2.429)	0.103665 (1.012)	0.688397 (4.176)	0.1895
BM	–0.044548 (–12.817)	0.206900 (5.027)	0.239222 (2.364)	0.108272 (1.138)	0.325802 (2.127)	0.1409
BL	–0.041382 (–11.168)	0.237598 (5.415)	0.241630 (2.239)	0.070296 (0.693)	–0.229943 (–1.408)	0.1394

Note: *t*-statistics are given in parentheses.

For the size factor represented by the *SMB* variable, significant positive relationships can be observed for the Big portfolios (*BW, BM* and *BL*), with an average coefficient of 0.248, whereas negative coefficients characterize Small portfolios (*SW*, *SM* and *SL*).

Besides, it is found that the value risk proxied by the explanatory variable *HML* is not significant at the conventional statistical level, indicating that the book-to-market ratio (value risk) has no significant effect on the expected stock returns.

Finally, for the momentum anomaly, all the regression coefficients are significant at the 10% level, except the *BL* portfolio. The coefficients' estimates display negative values for the losers' portfolios, i.e., *SL* and *BL* and positive values for the other portfolios' return series.

Multiscale Risk Factor Estimation Results for the Carhart Model

Given that the intercept values are not statistically significant at conventional statistical levels, Table 4 reports the estimation results pertaining to the multiscale Carhart pricing model without a constant term.

For the excess market returns which mimic the systematic risk, the estimated coefficients $\hat{\beta}(\lambda_j)$ are all statistically significant at 1% level for all portfolios at all resolution scales. Thus, we can affirm that the market factor plays an important role in explaining the cross-sectional variation of the six constructed portfolios over the analyzed wavelet scales.

Table 4: Multiscale Carhart Model Estimation Results

	d_1					d_2					d_3		
	Mkt	*SMB*	*HML*	*WML*	$\bar{R}^2$	*Mkt*	*SMB*	*HML*	*WML*	$\bar{R}^2$	*Mkt*	*SMB*	*HML*
SW	–0.25145 (–8.828)	–0.24729 (–4.058)	0.04455 (0.779)	0.49355 (5.918)	0.4055	0.21901 (6.476)	–0.26614 (–4.182)	–0.32186 (–4.540)	0.35578 (3.050)	0.2575	0.51193 (23.641)	–0.23153 (–4.768)	0.37851 (5.933)
SM	–0.29439 (–9.627)	–0.03564 (–0.545)	0.02784 (0.454)	0.19800 (2.212)	0.3130	0.22902 (6.201)	–0.13002 (–1.871)	–0.32440 (–4.190)	0.17605 (1.382)	0.1834	0.54390 (18.761)	–0.02275 (–0.350)	0.19821 (2.320)
SL	–0.24855 (–7.770)	–0.14339 (–2.095)	0.06761 (1.053)	–0.58031 (–6.196)	0.3209	0.18759 (5.513)	–0.31946 (–4.989)	–0.28808 (–4.039)	–0.70244 (–5.984)	0.2422	0.49591 (21.200)	–0.42904 (–8.180)	0.30290 (4.395)
BW	–0.32058 (–9.151)	0.55480 (7.402)	0.17887 (2.544)	0.59241 (5.776)	0.3655	0.21567 (5.553)	0.36913 (5.051)	–0.18422 (–2.263)	0.49014 (3.658)	0.2778	0.64067 (20.147)	0.30063 (4.216)	0.40254 (4.296)
BM	–0.27960 (–9.175)	0.45113 (6.920)	0.20804 (3.402)	0.18799 (2.107)	0.3021	0.21034 (5.591)	0.37019 (5.229)	–0.29892 (–3.790)	0.14649 (1.129)	0.2533	0.47517 (14.417)	0.43604 (5.900)	0.49005 (5.047)
BL	–0.32348 (–9.982)	0.45091 (6.504)	0.15581 (2.396)	–0.33373 (–3.517)	0.3285	0.24710 (6.166)	0.42245 (5.601)	–0.21800 (–2.595)	–0.45164 (–3.267)	0.2484	0.65669 (21.175)	0.49814 (7.164)	0.47815 (5.233)

	d_3		d_4					d_5				
	WML	$\bar{R}^2$	*Mkt*	*SMB*	*HML*	*WML*	$\bar{R}^2$	*Mkt*	*SMB*	*HML*	*WML*	$\bar{R}^2$
SW	0.59440 (6.570)	0.7060	0.36098 (16.288)	–0.19113 (–2.149)	0.16802 (3.805)	0.38358 (4.620)	0.5747	0.72027 (26.822)	–1.23795 (–13.993)	–0.06718 (–1.853)	0.21507 (2.489)	0.7959
SM	0.38850 (3.208)	0.6095	0.33281 (11.926)	–0.31872 (–2.846)	0.51217 (9.212)	0.55392 (5.299)	0.4967	0.83102 (23.375)	–1.08800 (–9.289)	0.38571 (8.038)	0.99286 (8.681)	0.7451

(Cont.)

Table 4: Multiscale Carhart Model Estimation Results

	d_3		d_4					d_5				
	WML	$\bar{R}^2$	*Mkt*	*SMB*	*HML*	*WML*	$\bar{R}^2$	*Mkt*	*SMB*	*HML*	*WML*	$\bar{R}^2$
SL	–0.55961	0.6710	0.39253	–0.42152	0.21245	–0.77537	0.6175	0.81865	–1.35078	0.19600	–0.68986	0.7058
	(–5.726)		(16.328)	(–4.369)	(4.436)	(–8.610)		(23.907)	(–11.973)	(–11.973)	(–6.262)	
BW	0.72436	0.6747	0.42027	–0.04926	0.56996	0.57030	0.5916	0.83451	–0.63106	0.57951	1.13133	0.7296
	(5.452)		(14.386)	(–0.420)	(9.792)	(5.211)		(21.879)	(–5.022)	(11.256)	(9.219)	
BM	0.43446	0.5547	0.24638	0.46662	0.64239	0.41640	0.6059	0.83855	–0.69074	0.47355	0.75835	0.7087
	(3.155)		(9.387)	(4.429)	(12.284)	(4.235)		(21.878)	(–5.470)	(9.153)	(6.150)	
BL	–0.12163	0.7202	0.38872	0.18113	0.52554	–0.27075	0.6046	0.73613	–0.51823	0.31634	0.03626	0.7632
	(–0.939)		(13.909)	(1.615)	(9.438)	(–2.586)		(25.993)	(–5.554)	(8.275)	(0.398)	

Note: *t*-statistics are given in parentheses.

Yet, it is noteworthy that the market effect is positive at all resolution levels, except for the highest frequency scale d_1 corresponding to an investment period of 2-4 months. For the latter level, the $\hat{\beta}(\lambda_j)$ shows negative magnitudes, with values ranging from –0.32348 (BL) to –0.24855 (SL).

It is noteworthy that the market effect is scale sensitive. In fact, in the time scale evolution, the market premium is as follows: the $\hat{\beta}(\lambda_j)$ coefficients show an increase between the scales d_1 and d_3, decrease at the level d_4 and finally, ascend, again at the largest wavelet scale d_5 showing the highest magnitudes (0.72027 to 0.83855).

Focusing on the size factor, we observe that the factor loadings are statistically significant at the 10% level for all the series and at all resolution scales. Exceptions are the estimates associated with the scales d_1, and d_3 for the portfolio *SM*, and those related to the scale d_4 for the *BW* and *BL* which fail to reject the null hypothesis $\gamma(\lambda_j) = 0$ at the 10% level of statistical significance.

An important remark to be drawn here is that, for the scales d_1, d_2, d_3 and d_4, the size risk has a negative impact on small stocks' returns whereas, for big caps, coefficients $\hat{\gamma}(\lambda_j)$ are positively valued reflecting a positive effect of the *SMB* factor on the excess returns. At the largest scale d_5, i.e., in the long-run, the point estimates, displaying the highest values, are of negative sign for big as well as for small stocks.

Moving to the *HML* estimation results, we observe that the value risk effect is rejected for small portfolios—*SW*, SM and SL at the resolution scale d_1, i.e., at the shortest investment period. On the contrary, for the higher scales d_2, d_3, d_4 and d_5, all coefficients' estimates are statistically significant, indicating that the book-to-market factor is a key variable that explains the cross-sectional variations of the portfolios' returns over intermediate and long-term investment horizons. It is to be noted that these empirical results for the value factor contrast sharply with those yielded by the single-scale Carhart pricing model where, no significant effect of the book-to-market ratio is found for the series of study.

Interestingly, while looking at the sign of the $\hat{\delta}_j$ coefficients, we see that the value factor has almost a positive effect for all portfolios at all frequency levels except for the intermediate wavelet scale d_2 corresponding to a 4-8 month period, for which all coefficients are negatively valued.

When considering the *WML* factor, we observe that loadings $\hat{\theta}(\lambda_j)$ are significant at conventional statistical levels for almost all portfolios' returns with very few exceptions. This points out that the momentum factor has a strong impact on the expected stock returns within the French Market. It is to be noted that the momentum effect is negative for loser stocks *SL* and *BL* and positive for all other constructed portfolios. Hence, for *SW*, *SM*, *BW* and *BM* stocks with high recent performance continue to earn higher average returns over the next 3-12 months than stocks with low recent performance while the opposite holds for loser stocks.

On examining the $\overline{R}^2$ values, we see that the quality fit of the Carhart four-factor depends strongly on the regression scale and thus on the investment period.

Actually, for the small scales d_1 and d_2, depicting the short-term dynamics (2-4 months and 4-8 months, respectively), the estimated $\overline{R}^2$ are relatively low-valued. The coefficients of determination lie between 30.21% and 40.55% at resolution level d_1 whereas at scale d_2, the four risk variables explain between 18.34% and 27.78% of the portfolios' returns built on size and momentum strategy. Thus at the high frequency levels, we can state that the four considered regressors show relatively limited power in explaining the common variation of the stock returns in the French market.

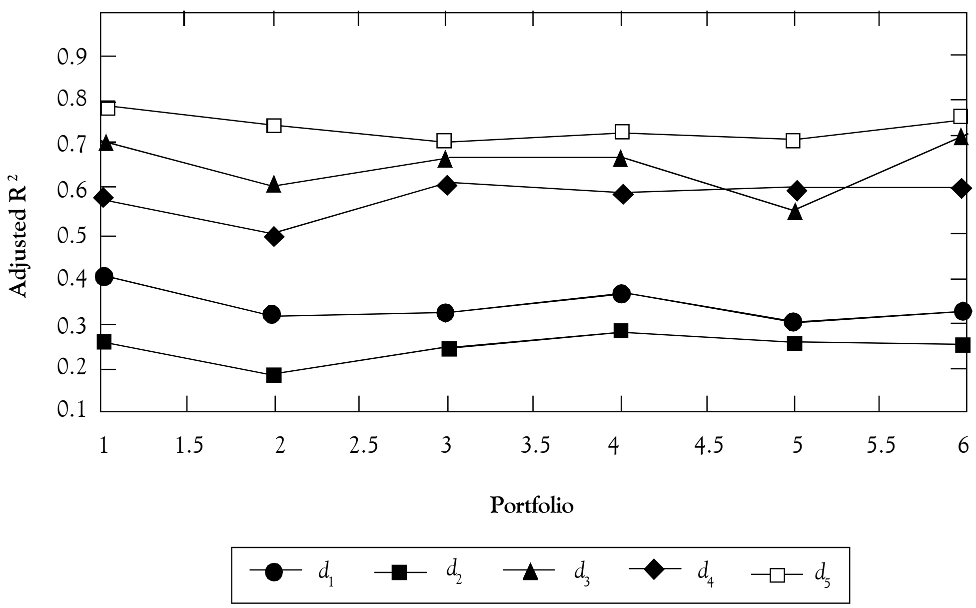
However, at the medium scale d_3 covering the period 8-16 months, the $\overline{R}^2$ values show a significant increase varying between 55.47% for *BM* and 72.02% for *BL*. Thus at this wavelet decomposition level, the explanatory variables *Mkt, SMB*, *HML* and *WML* explain the portfolio returns better.

Nevertheless, at scale d_4 (16-32 month period), the quality fit of the four-factor model deteriorates as the $\overline{R}^2$ values decrease compared to those at scale d_3 for all constructed portfolios except for the *BM* series.

Finally, at the largest wavelet scale d_5 corresponding to the longest investment period, the goodness of fit of the Carhart multifactor model improves considerably as the adjusted coefficients of determination $\overline{R}^2$, attaining their highest magnitudes with a maximal value of 79.59% for *SW* portfolio. Hence in the long-run, the Carhart pricing model adequately explains the returns of the French stocks.

Figure 1 reports the adjusted coefficients of determination $\overline{R}^2$ for the six portfolios at each wavelet scale.

Figure 1: Variations of R^2 in the Function of Constructed Portfolios Over Various Time Scales



Conclusion

In this paper, the relationship between expected returns and the Carhart four risk factors have been investigated over different investment horizons (holding periods) for the French Equity Market. This has been done using a multiscale version of the Carhart multifactor pricing model based on the MRA induced by the MODWT. In contrast to the single scale model which is rejected, empirical results drawn from the implementation of the wavelet-based model show that the four risk factors explain the cross-section of returns in the French Market quite well for the medium and long-run investment horizons. Besides, it has been shown that the impact of the market effect, the size risk, the value factor and the momentum anomaly are scale-sensitive, depending on the targeted time-period. In consideration of the obtained results, the multiscale framework offers academics as well as financial practitioners a new insight into investment strategies and risk management. ❖

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