Default, Liquidity and Credit Spread: Empirical Evidence from Structural Model

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Amongst the important issues related to credit risk are the factors which affect yield spreads of corporate bonds. In recent literature, the yield spread is regarded as a measure of a comprehensive risk premium to compensate investors for a number of risks associated with corporate bonds. Using a first passage model in which the default occurs when corporate asset values hit a predefined default barrier, it is concluded that the credit spreads associated with Tunisian bonds are highly defined by default risks. It is to be noticed that residual spreads are sensible to the dynamics of default barrier, depending on the drift and volatility of a firm's assets values. Moreover, this paper also explores the role of liquidity risks in the pricing of corporate bonds. This liquidity risk is a priced factor for the yield spread of risky corporate bonds and the associated liquidity risk premia helps to explain the credit spread puzzle.

Introduction

In the literature, there are two theoretical approaches to the pricing of risky debt. Both approaches take into account, credit spreads as a central component in their pricing models and attempt to describe default processes. The credit spread is defined as the difference between the yield of corporate bond and the associated yield on Treasury bonds with the closest matching maturity.

In the structural approach,[1] one makes explicit assumptions about the dynamics of a firm's assets, its capital structure, as well as its debt and shareholders. It is then assumed that the firm defaults, if its assets are not sufficient to pay off the due debt. In this situation, corporate liabilities can be considered as contingent claims on the firm's asset. The structural literature on credit risk starts with the paper by Merton (1974), which applies the option pricing theory developed by Black and Scholes (1973) to the modelling of a firm's debt. Notice that in this first category of models, defaults can only happen at the maturity of zero coupon debt.

The paper by Black and Cox (1976) is the first of the so-called first passage models.[2] This category of models specify default as the first time, the firm's assets values hit a lower barrier, allowing default to take place at any time. Black and Cox (1976) and Longstaff and Schwartz (1995), among others, assume that default barrier can be taken exogenously and

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[1] This class includes, among others, the models of Nielson *et al.* (1993), Shimko *et al.* (1993), Leland (1994), Longstaff and Schwartz (1995), and Saâ-Requejo and Santa-Clara (1999).

[2] For an extensive review of first passage models, see Bielecki and Rutkowski (2002).

it acts as a safety covenant to protect bondholders. Alternatively, it can be endogenously fixed as a result of stockholders' attempt to choose the default threshold which maximizes the value of firm (Leland, 1994).

Other recent studies depart from Merton (1974) to include, for example, stochastic interest rates (Longstaff and Schwartz, 1995; and Briys and de Varenne, 1997), stationary leverage ratios (Collin-Dufresne and Goldstein, 2001) and the information contained in stock market returns (Demchuk and Gibson, 2006). Also, Anderson and Sundaresan (1996) examine violations of the absolute priority rule and strategic default. Zhou (2001) introduces jumps in the asset value process. Although the second category of structural models leads to more realism in the pricing of defaultable bonds, it introduces serious drawbacks and analytical complexities.

Catchart and El-Jahel (1998) propose a defaultable bond model in which, default occurs when some signaling process hits a lower constant default barrier. This model assumes the signaling process for each firm that determines the occurrence of default rather than the value of the assets of the firm. While Catchart and El-Jahel's paper does not identify what the signaling variable is, Lo and Hui (2002) suggest that the signalling variable could be some macroeconomic variables such as foreign exchange rates in the valuation model. It is to be noticed that, Lo and Hui (2002) model the default threshold as a stochastic process.[3]

Reduced form models,[4] which are not the focus of this paper, treat default as a random event that has a positive probability of occurrence for any firm at any time. In the basic reduced form model, the default is not causally modelled in terms of the firm's assets and liabilities, but is typically given exogenously. In fact, the default occurs completely unexpectedly. The stochastic structure of default is directly prescribed by an intensity or the compensators' process. Due to the unpredictability of defaults, the implied credit spread properties are empirically quite plausible.

A number of empirical studies demonstrate that structural models are less successful, compared to reduced-form models, in terms of fitting the empirically observed credit yield spreads[5] (see, for example, Huang and Huang, 2003; and Eom *et al*., 2004). However, the structural approach provides a solid theoretical framework regarding the determinants of corporate credit spreads. According to this approach, credit spreads are driven by an interest-rate and an asset-value factor.

[3] *See* Nielson *et al.* (1993), Briys and de Varenne (1997), and Saâ-Requejo and Santa-Clara (1999) for models with stochastic default barrier.

[4] The name comes from the reduction of the credit economics behind the probability of default. The reduced form approach is widely used for credit risk analysis in both, academic and real world research. This class of models include, among others, the works of Jarrow and Turnbull (1995), Jarrow *et al.* (1997), Elton *et al.* (2001), Lando and Skodeberg (2002), Duffie *et al.* (2005), and Koopman *et al.* (2005).

[5] This deficiency has long been recognized and diffusion-based models are commonly criticized for this characteristic. For example, the theoretical yield curves reported in Kim *et al.* (1993) and Longstaff and Schwartz (1995) report zero credit spreads for maturities upto four years and remain below 50 basis points upto 10 years for high-rated firms. The classical reference for low spreads in this literature is Jones *et al.* (1984).

Since highly-rated corporates have little probability of default, the credit spread seems too large to be explained solely by default risk premium. Additionally, some studies reveal a low explanatory power of the default risk for the level as well as for the changes in the US corporate yield spread.

Elton *et al.* (2001)[6] estimate a state tax premium on the order of 40 basic points for investment grade corporate bonds. Houweling *et al.* (2003) estimate a liquidity premium on the order of 20 basic points. Delianedis and Geske (2001), using a structural model,[7] conclude that the default risk only explains a small fraction of the credit spreads, and the rest is attributable to taxes, jumps, liquidity and market risk factors. Huang and Huang (2003) calibrate several structural models to historical default probabilities. Applying standard estimates of the equity risk premium, they reach a similar conclusion with a structural model for credit risk evaluation. They verify that, for investment grade bonds (Baa and higher rating), only 20% of the spread is explained by default risk. Ericsson and Renault (2006) indicate that a large component of corporate credit spreads may be attributed to the effects of taxes and liquidity. Gebhardt *et al.* (2005) show that corporate bond returns are affected by systematic risk factors, but after controlling both systematic factors and firm characteristics, only 44% of the variation is explained. Longstaff *et al.* (2005) estimate that default risk accounts for more than 50% of the credit default swap spread. Driessen (2005) estimates a reduced form model and uncovers a significant jump risk premium. Yu (2005) and Zhou (2005) find that corporate yield spreads are influenced by a degree of asymmetric information. Finally, Helwege and Kleiman (1997) and Hackbartha *et al.* (2006) indicate that credit spreads are explained by macroeconomic conditions.

In this paper, we extend the previous studies by identifying the signalling variable from the firm's asset value. The dynamics of the default barrier depend on the volatility and the drift of the corporate asset values. This extension seeks to propose a structural model for the valuation of defaultable bonds in a closed form. The key assumption of the paper is that, default occurs when the firm's asset value process hits a stochastic default barrier.

We use credit spreads of corporate bonds issued by Tunisian leasing firms to compare the modelled default spreads. In this study, default spread is estimated using extended structural model. We define the difference between the observed corporate credit spreads and the structural approach based estimates of default spreads as the residual spread.

Valuation Model of Defaultable Bonds

Hypotheses

This paper presents two main hypotheses:

H_1: *The total assets value of the firm is assumed to follow a stochastic process, which in a risk neutral world is of the form:*

[6] For the default component of the credit spread, they use two measures of an actual rating transition matrix and default vector, one from Moody's and one from S&P.

[7] Contrary to Delianedis and Geske (2001), we use a sophisticated model that incorporates a default barrier, with dynamics depending on the volatility and drift of the firm's assets values.

$$dV = \alpha\, Vdt + \sigma_V V dz_V \qquad ...(1)$$

where α and σ_v are the drift and the volatility of V, respectively and are time independent.

Let $\alpha = \mu - d$

where μ and d are constants.

H_2: *Contrary to Longstaff and Schwartz (1995), default occurs when firm's asset values reach a stochastic default barrier. The dynamics of default barrier depend on the volatility and the drift of the corporate asset values.*

Default event simultaneously affects all discount bonds. Hence, coupons bonds can be valued as simple portfolios of discount bonds.

Model

The partial differential equation governing the discount defaultable bond is:

$$\frac{\partial P}{\partial t} = \frac{1}{2}\sigma_V{}^2 V^2 \frac{\partial^2 P}{\partial V^2} + \frac{1}{2}\sigma_r{}^2 r \frac{\partial^2 P}{\partial r^2} + \alpha V \frac{\partial P}{\partial V} + \kappa(t)\left[\theta(t) - r\right]\frac{\partial P}{\partial r} - rP \qquad ...(2)$$

where,

$P(V,r,t)$ is the price of discount defaultable bond.

For a special case of constant α and σ_V , the moving barrier will simply be reduced to the form:[8]

$$H(t) = V_0 \exp\left[-\beta\left(\alpha - \frac{\sigma_V^2}{2}\right)t\right] \qquad ...(3)$$

β is a real number to adjust the rate of the drift. When the term $\left(\alpha - \frac{\sigma_V^2}{2}\right) < 0$, the default barrier increases with time to maturity for a positive β. Contrary, for β negative, the barrier level decreases with time to maturity. Notice that, when $\beta = 0$, the case of a constant barrier is obtained (Cathcart and El-Jahel, 1998).

The payoffs to bondholders are specified by:

$$P(V = H, r, t) = WFQ(r,t) \qquad t < 0\ ;\ W < 1 \qquad ...(4)$$

where $Q(r, t)$ is the default free bond function, according to the CIR model and F is the bond face value.

$$P(V, r, t = 0) = F \qquad H(t) < V \qquad ...(5)$$

The solution of the partial differential equation, subject to the boundary condition and the final payoff condition is:

[8] In this paper, we use a special case of constant α and σ_V.

$$P(V,r,t)=FQ(r,t)\left\{W+(1-W)\left[N(d_1)-\left(\frac{V}{V_0}\right)^{\eta_1}\exp(\eta_2)N(d_2)\right]\right\} \quad \text{...(6)}$$

where,

$$d_1=\frac{\ln\left(\frac{V}{V_0}\right)+\left(\alpha-\frac{\sigma_V^2}{2}\right)t}{\sigma_V\sqrt{t}}, \qquad d_2=-d_1-2(\beta-1)\left(\frac{\alpha-\frac{\sigma_V^2}{2}}{\sigma_V}\right)\sqrt{t}$$

$$\eta_1=-2(1-\beta)\left(\frac{\alpha-\frac{\sigma_V^2}{2}}{\sigma_V^2}\right), \qquad \eta_2=\eta_1\beta\left(\alpha-\frac{\sigma_V^2}{2}\right)t \quad \text{...(7)}$$

Notice that coupons bonds can be valued as simple portfolios of discount bonds. The detailed derivation of the solutions in equations is given in the Lo and Hui's paper.

Methodology

The empirical tests conducted in this paper address two main questions: First, what is the relation between default spreads and default scenarios? Second, is the default risk, the only determinant of credit spreads observed on Tunisian bonds market?

Sample

Our data is composed of the secondary market bonds spanning the period April 23, 2004 to December 26, 2008. This database contains two rating categories: investment grade (rating A) and speculative grade (rating BBB). For each observation in our database, credit rating information is provided. We use North Africa Fitch's rating. We also consider financial bonds, namely, bonds issued by four leasing firms: ATL, TLS, CIL and GL.

Bond Maturity

The sample is divided into two categories with respect to the time-to-maturity of each bond. Maturity ranges of <3 years and +3 years are defined. Notably, the maturity subsamples are well-balanced.

To estimate the residual spreads, one needs the data to measure both firm default spreads and credit spreads.

The default spreads calculated herein, require the data for the structural model. The credit spread data is given for the two rating classes and is quoted as the spread above an equivalent maturity treasury rate.

Estimation of the Short Interest Rate Model

To estimate the defaultable bonds price given by Equation (6), one needs estimates of the Cox, Ingersoll and Ross model parameters. This is done using discrete time approach which is of the form:

$$r_{t+1} - r_t = \kappa(\theta - r_t) + \sigma_r \sqrt{r_t}\,\varepsilon_{t+1} \qquad \text{...(8)}$$

where the short-term interest rate is mean reverting to long-run mean θ at speed k, and the stochastic term has a standard deviation proportional to $\sqrt{r}$.

Empirical studies like Chan *et al*. (1992) use the generalized method of moments. In this paper, we use likelihood maximum approach.[9]

The likelihood function is presented as follows:

$$\ln(r_t / r_{t-1}\,;\,\kappa\,;\theta\,;\sigma_r) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\left(\sigma_r^2 r_{t-1}\right) - \frac{1}{2}\left(\frac{r_t - (1-\kappa)r_{t-1} - \kappa\theta}{\sigma_r^2 r_{t-1}}\right)^2 \qquad \text{...(9)}$$

and

$$\max L_T\,(\kappa\,;\,\theta;\,\sigma_r) = \sum_{t=1}^{T} \ln\left(r_t / r_{t-1}\,;\,\kappa;\,\theta;\,\sigma_r\right)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\ln\left(\sigma_r^2 r_{t-1}\right) - \frac{1}{2}\sum_{t=1}^{T}\frac{(r_t - (1-\kappa)r_{t-1} - \kappa\theta)^2}{\sigma_r^2 r_{t-1}} \qquad \text{...(10)}$$

We examine the stationarity of risk free interest rate using the Augmented Dickey Fuller (ADF) test statistic. Table A1 (see Appendix) provides the results of the stationarity test of interest rate. The evidence suggests that the sample spanning period from June 1997 to December 2006 is stationary.

Table A2 provides the results of the estimation of the interest rate parameters. Notice that all interest rate model parameters are significant at 5% level. Table A3 reports the estimated parameters of volatility and the drift parameters of the dynamics of the assets value of the firm, for the period spanning June 30, 1997 to December 18, 2006.

Results

To study the impact of the dynamics of the default barrier on the default spreads, different scenarios of default, $\beta = -1$, $\beta = 0$ and $\beta = 1$; different figures of volatility and different maturities are used to calculate the model default spreads.

Tables A4 and A5 confirm a positive relation between default spreads estimated and β, where the magnitude of the increases is sensitive to the volatility of the firm's assets

[9] *See* Winne (1998) for a detailed discussion.

values and the time to maturity. This demonstrates that the levels of the default barrier with different β imply different early default risk.

The average difference between the two maturity class are 3.61 and 3.68 basis point, respectively, for $\beta = -1$ and for $\beta = 0$. When the time to maturity increases, the price of risk free bond decreases and the default probability increases. The combination of these two effects lead to a decrease of defaultable bond value.

The average difference between σ_vCIL and σ_vGL, for $\beta = 1$, is 3.52 basis point, while the corresponding average difference for $\beta = 0$ is 3.39 basis point and 3.24 basis point for $\beta = -1$. This finding is consistent with barrier structure which moves above V_0 at higher rates when σ_v is higher and β is positive.

To summarize, the empirical results show that the dynamics of the default barrier has material impacts on the corporate credit spreads, observed in Tunisian secondary market. The model incorporating the risk adjustable default barrier that depends on volatility and the drift of the firm's asset values, is capable of producing diverse shapes of term structures of credit spreads.

This paper contributes to the extant literature by empirically analyzing the fraction of the corporate credit spread due to default risk.

We assume that the credit spread is composed by default fraction and residual spread. Hence, the credit spread is presented as follows:

$$CS = DS + RS \qquad ...(11)$$

In this study, the default spread is estimated by the structural model presented above. The residual spread is attributable to risks, other than default risk.

We use credit spreads of corporate bonds issued by Tunisian leasing firms to compare modelled default spreads.

Table A6 shows that default risk is not the only determinant of corporate credit spreads observed in Tunisian bonds market.

With initial sample, we find that 48.35% to 51.65% of credit spreads is explained by default risk. For <3 years maturity class (+3 years maturity class), empirical results show that default risk contributes by 49.44% (48.35%) to 50.54% (51.65%) in explaining the credit spreads. The rest is attributable to other risks. We can conclude that:

- Tunisian investors are highly averse to default risk rather than other risks; and
- Contrary to Delianedis and Geske (2001) and Elton *et al.* (2001), default spreads estimated by our structural model are very near to credit spreads observed in the corporate bonds market.

The liquidity of the corporate bond market has been of interest to researchers, practitioners and policy makers. Many studies have mentioned the influence of liquidity on credit spreads (Houweling *et al.*, 2003 and Edwards *et al.*, 2004).

The purpose of our study is to examine the effects of liquidity on corporate yield spreads. The paper investigates whether liquidity conditions are related to residual spreads and therefore, contribute to variations in the corporate bond yield spreads.

We use two different proxies for liquidity in order to show that our results do not depend on the specific proxy used.

The issue size of a bond reflects the availability of the security in the secondary market. A larger issue size should be associated with greater liquidity and therefore, lower yield spreads. The issue size is measured by the natural logarithm of the amount of bonds at issuance in millions of dinars.[10]

If corporate bonds exhibit a similar pattern as the Treasury bonds in that, on-the-run bonds are much more liquid than off-the-run ones, the bond age can then be used as a proxy for corporate bond liquidity.[11] Hence, the younger the bond, the greater the liquidity and the lower the spread. To measure the bond age, expressed in years, the difference between the issuance date and the transaction quote is taken.

Table A7 provides descriptive statistics of residual spreads observed in Tunisian secondary market and all the explanatory variables for the three scenarios of default.

In our analysis, we regress credit spreads on individual bond liquidity measures as follows:

$$RS_i = \beta_0 + \beta_1 Age_i + \beta_2 Size_i + \varepsilon \qquad \text{...(12)}$$

where dependent variable RS_i is the corporate residual spread; Age_i and $Size_i$ are two proxies for liquidity for corporate bond.

We find that all liquidity measures are mostly significantly related to residual spreads and hence to corporate credit spreads. The regression results for the impact of liquidity variables on residual spreads, $\beta = -1$, $\beta = 0$ and $\beta = 1$ are presented in Tables A8 to A10. Surprisingly, while the coefficients on age and size are statistically significant, the signs are not in accordance with intuition.

A general conclusion that can be drawn from the analysis is that the Tunisian yield spreads are significantly affected by default risk and liquidity risk. Importantly, we find that the liquidity risk is a priced factor for the yield spread of risky corporate bonds, and the associated liquidity risk premiums help to explain the credit spread puzzle.

[10] Fisher (1959) was among the first academicians who proposed the issued amount as a proxy variable. Also *see*, Sarig and Warga (1989) and Edwards *et al.* (2004).

[11] *See* Warga (1992) and Perraudin and Taylor (2002).

Conclusion

This paper extends the defaultable bond valuation developed by Lo and Hui (2002). Corporate asset values are used to identify the signalling variable. Hence, a first passage model is obtained. The extended model incorporates a default barrier with dynamics depending on the volatility and the drift of the firm's assets values.

The empirical results show that dynamics of the default barrier have an important impact on default spreads. Given, $\left[\left(\alpha-\frac{\sigma_s^2}{2}\right)\right]$, it can be concluded that the default spreads estimated increase with positive β and the magnitude of the increase is sensitive to the volatility of the firm's assets values and the time to maturity.

Additional study based on the decomposition of corporate credit spreads show that an important fraction of credit spreads observed on Tunisian bonds market is explained by default spreads. The rest is attributable to other risks. This paper contends that liquidity conditions should have an impact on residual spreads and hence on credit risk. The findings suggest that actual corporate yield spreads may incorporate a liquidity premium that is ignored by traditional corporate bond pricing models. ❖

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Appendix

Table A1: Augmented Dickey Fuller Test Statistics Examining the Stationarity of Risk-Free Interest Rate

This table shows Augmented Dickey-Fuller (ADF) test statistics examining the stationarity of risk-free interest rate series for the period spanning June 30, 1997 to December 18, 2006. Critical *t*-values are: 10% = –3.3588, 5% = –3.8288 and 1% = –3.48, with a constant included.

Variable	Dickey-Fuller Test Statistics	Durbin Watson Statistics
Risk-Free Interest Rate	–3.637930	2.056559

Table A2: Estimation of Parameters of Cox, Ingersoll and Ross Model

This table reports estimated parameters of interest rate model, using likelihood maximum approach. The sample contains monthly data for risk-free interest rate for the period 1997-2006.

Parameters	Estimation	*t*-statistics
κ	0.03536	4.312
θ	0.05124	4.031
σ	0.00728	54.245

Table A3: Estimation of Firm's Parameters

This table reports estimated parameters of the drift and volatility of the firm's asset value, for the period spanning June 30, 1997 to December 18, 2006.

Parameters	*ATL*	*CIL*	*TSL*	*GL*
σ_v	0.063467	0.071142	0.032067	0.023038
d	0.162400	0.085600	0.095500	0.150200
μ	0.058456	0.062202	0.046728	0.049481

(Cont.)

Appendix

Table A4: The Impact of Time to Maturity on the Default Spreads

This table reports the impact of time to maturity. Different scenarios of default, $\beta = -1$, $\beta = 0$ and $\beta = 1$, are used to calculate model default spreads with the following parameters: $\kappa = 0.03536$, $\theta = 0.051245$ and $\sigma = 0.0728$. The recovery values are consistent with the empirical findings in Carey (1998)—for bonds rated A (BBB) the recovery is set to be 0.76 (0.67).

Average of Default Spreads		
Default Scenarios	**<3 Years**	**+3 Years**
$\beta = -1$	1.1884	1.2245
$\beta = 0$	1.1909	1.2277
$\beta = 1$	1.2255	1.2363

Table A5: The Impact of the Volatility of Firm's Assets Values on the Default Spreads σ_v

This table reports the impact of the volatility of the corporate asset values for Tunisian leasing firms namely, CIL, ATL, TSL and GL. Different scenarios of default, $\beta = -1$, $\beta = 0$ and $\beta = 1$, are used to calculate model default spreads with the following parameters: $\kappa = 0.03536$, $\theta = 0.051245$ and $\sigma = 0.0728$. The recovery values are consistent with the empirical findings in Carey (1998)—for bonds rated A (BBB), the recovery is set to be 0.76 (0.67).

Average of Default Spreads				
Default Scenarios	σ_v *CIL*	σ_v *ATL*	σ_v *TSL*	σ_v *GL*
$\beta = -1$	1.2334	1.2188	1.2185	1.2010
$\beta = 0$	1.2375	1.2227	1.2209	1.2036
$\beta = 1$	1.2413	1.2264	1.2231	1.2061

Table A6: Decomposition of Credit Spreads

This table reports the decomposition of Tunisian corporate spreads observed on secondary market for the period April 23, 2004 to December 26, 2008. Different scenarios of default, $\beta = -1$, $\beta = 0$ and $\beta = 1$, are used to calculate model default spreads with the following parameters: $\kappa = 0.03536$, $\theta = 0.051245$ and $\sigma = 0.0728$.

Notation	**SD % SC**			**SR % SC**		
	$\beta = -1$	$\beta = 0$	$\beta = 1$	$\beta = -1$	$\beta = 0$	$\beta = 1$
<3 ANS	0.4944	0.4954	0.5054	0.5056	0.5046	0.4946
+3 ANS	0.4835	0.4848	0.4883	0.5165	0.5152	0.5117

(Cont.)

Appendix

Table A7: Descriptive Statistics

This table reports descriptive statistics of interest for all the variables used in this study. Our sample includes Tunisian corporate bonds observed in secondary market, spanning the period April 23, 2004 to December 26, 2008.

Variables	Mean	Median	Std. Dev.	Minimum	Maximum
$RS - \beta = -1$	1.288542	1.309600	0.121597	0.969800	1.458000
$RS - \beta = 0$	1.285513	1.307100	0.121717	0.966800	1.453500
$RS - \beta = 1$	1.270108	1.303700	0.172806	0.201500	1.449000
Independent Variables Bond Liquidity Variables					
Age	2.437591	1.882200	1.594792	0.200000	5.532600
Size	2.631221	2.565000	0.401700	2.302600	3.401200

Table A8: Regression Results for the Impact of Liquidity Variables on Residual Spreads: $\beta = -1$

This table reports the regression results of residual spreads on liquidity variables. Our sample is based on corporate bonds observed in secondary market, spanning the period April 23, 2004 to December 26, 2008. The following scenario of default is taken: $\beta = -1$. The dependent variable is the corporate residual spreads. The independent variables are defined below. t-statistics are reported in parentheses. Standard deviation and the p-value are also reported. The t-values in bold are statistically significant at 5% level.

Variables	Estimate	Std. Error	Prob.
Intercept	1.086833 **(10.12657)**	0.107325	0.0000
Age	–0.025683 **(–2.947959)**	0.008712	0.0041
Size	0.100453 **(2.904259)**	0.034588	0.0046

(Cont.)

Appendix

Table A9: Regression Results for the Impact of Liquidity Variables on Residual Spreads: $\beta = 0$

This table reports the regression results of residual spreads on liquidity variables. Our sample is based on corporate bonds observed in secondary market, spanning the period April 23, 2004 to December 26, 2008. The following scenario of default is taken: $\beta = 0$. The dependent variable is the corporate residual spreads. The independent variables are defined below. t-statistics are reported in parentheses. Standard deviation and the p-value are also reported. The t-values in bold are statistically significant at 5% level.

Variables	Estimate	Std. Error	Prob.
Intercept	1.088330 **(10.09469)**	0.107812	0.0000
Age	–0.025832 **(–2.951692)**	0.008752	0.0040
Size	0.098871 **(2.845603)**	0.034745	0.0055

Table A10: Regression Results for the Impact of Liquidity Variables on Residual Spreads: $\beta = 1$

This table reports the regression results of residual spreads on liquidity variables. Our sample is based on corporate bonds observed in secondary market, spanning the period April 23, 2004 to December 26, 2008. The following scenario of default is taken: $\beta = 1$. The dependent variable is the corporate residual spreads. The independent variables are defined below. t-statistics are reported in parentheses. Standard deviation and the p-value are also reported. The t-values in bold are statistically significant at 5% level.

Variables	Estimate	Std. Error	Prob.
Intercept	1.191802 **(7.114649)**	0.167561	0.0000
Age	–0.040970 **(–3.012082)**	0.013602	0.0034
Size	0.067715 (1.253970)	0.054001	0.2130

Reference # 37J-2009-06-03-01