Measuring Financial Cash Flow and Term Structure Dynamics in Turbulent Global Markets

Cornelis A Los*

Financial turbulence is a phenomenon usually occurs in anti-persistent markets. In contrast, financial crises tend to occur in persistent markets. A relationship can be established between these two extreme long memory phenomena and the older financial concept of (il-)liquidity. The measurement of the degree of market persistence and the measurement of the degree of market liquidity are related. To accomplish the scientific research objectives of measurement, analysis and simulation of different degrees of financial liquidity in the financial markets, this paper proposes to reformulate and reinterpret the classical laws of fluid mechanics into financial cash flow mechanics. The end results of these reformulations and reinterpretations are useful, and often already familiar, quantifiable financial quantities, which will assist in the measurement, analysis, and proper characterization of modern dynamic financial markets in ways that classical comparative static financial-economic analyses simply do not allow. The motivation for this study is directly derived from the currently increased turmoil in the globally linked financial markets, caused by the valuation adjustments emerging from the sharply adjusting residential housing markets, which are transmitted with increased speed via the swap markets, in particular the Credit Default Swap (CDS) markets.

Introduction

In the past decade, the accurate measurement of financial illiquidity risk has dramatically gained in importance, almost geometrically in the past few years. This is obvious when we look at the early example of the taxpayer-financed bail out of the Russian-default-traumatized Long Term Capital Management (LTCM) hedge fund in 1998 and compare that with the recent March 2008 example of the Federal Reserve—JP Morgan engineered electroshock of an already flat-lined mortgage portfolios-processing, Bear Sterns, or the more diversified Lehman Brothers. The LTCM debacle was considered an isolated incident. But recent history shows that it was just a predecessor of the current series of collapses of major financial houses. Therefore, this study first looks at LTCM in detail. It applied a trading strategy known as 'convergence arbitrage', which is based on the idea that when two securities have the same theoretical price, because they have the same return-risk profile, their market prices should eventually be the same. But this convergence strategy ignores the observation that financial risk is a time-dependent or long memory phenomenon and not a time-independent or no memory phenomenon to which the usual central limit theory based on i.i.d. assumptions applies. Indeed, in the summer of 1998, LTCM made a huge loss of $4 bn. This was triggered off when Russia

* Professor of Finance, University of Lethbridge, Lethbridge, Canada. E-mail: cornelis.los@uleth.ca

suddenly and unexpectedly defaulted on its debt, which caused a flight to quality in the German bond market.[1]

LTCM itself did not have a large exposure to Russian debt, but it tended to be long illiquid German (off-the-run) bonds and the short corresponding liquid German (on-the-run) bonds. The spreads between the prices of the illiquid bonds and the corresponding liquid bonds widened sharply and immediately after the Russian default and it suddenly imposed absolute illiquidity of the Russian debt. Credit default spreads also increased and LTCM was highly leveraged, with a debt/equity ratio of close to 300, so that the loss of profits on its spread hedges immediately translated into a very rapid vanishing of its market valued equity and threatened bankruptcy. When it was unable to make its projected 'risk-free' arbitrage profits, it experienced huge losses and there were margin calls on its positions which it was unable to meet.

LTCM's position was made more difficult by the fact that over a period of three years many other hedge funds had also learned this game of "gigantically vacuuming pennies all over the world" and implemented similar convergence arbitrage strategies.[2] When LTCM tried to liquidate part of its portfolio to meet its margin calls, by selling its illiquid off-the-run bonds and by buying its liquid on-the-run bonds, other hedge funds faced with similar problems did similar trades at the same time. Consequently, the price of the on-the-run bonds rose sharply relative to the price of the off-the-run bonds. This caused the illiquidity spreads to widen even further and to reinforce the original flight to quality. Thus, the illiquidity problem was exacerbated and not alleviated by the crowding effect. This was a clear example of a long-term illiquidity risk dependence accumulation that was not foreseen by the time-independence assumptions of geometric Brownian motion arbitrage models used by LTCM's managers and partners, which assumes perfectly competitive, generally liquid markets.[3]

The more recent examples of this year, such as Bear Sterns, Merrill Lynch, and even, when carefully examined, Bank of America, which took over Merrill Lynch, and most spectacularly perhaps, the vanished Lehman Brothers, show that the clash between the highly persistent real estate—in particular residential housing—markets, and the extremely anti-persistent Credit Default Swap (CDS) markets lead to unpredictable and very sharp valuation adjustments in not only the original real estate markets, or the

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[1] The first example is borrowed from Hull (2001, pp. 428-429). *Cf.* also Jorion (1999) and Dunbar (2000). Other examples of financial market discontinuities abound. They are now named 'Thanksgiving Turkey Surprises' when they are negative, or, more generally, 'black Swans', when they have an unexpected large impact, because they are based on avoidable (lack of business intelligence) ignorance (Taleb, 2007).

[2] The quote is from the 1999 NOVA/BBC movie *A Trillion Dollar Bet*.

[3] Some of LTCM's partners were the Nobel prize winners, who had proposed the option theory based on log-normal or geometric Brownian motion price diffusion in 1973. At least, they had courageously put their money where there mouths were. But the empirical experiment of 1998 drastically refuted the basic non-empirical assumption of ergodicity of their theory. *A Trillion Dollar Bet* is an excellent pedagogic video documentary of the rise and fall of LTCM, of the crucial role the Nobel prize winning Black-Scholes solution model in this financial market affair, and of the regulatory aftermath of LTCM's bailout by the Fed. Thanks to the Fed's guidance of a consortium of LTCM's investors and a generous bridge loan, the complex portfolio of LTCM was deconstructed within six weeks, and the liquidity constraint was dissolved without much of a ripple for the whole global financial system.

directly related mortgage and bond markets, but also the related stock and bond markets, because of the extreme leveraging executed by a mostly unregulated mortgage cash flows processing, repackaging and mortgage-backed securities-issuing firm like Bear Sterns (its set/equity leverage ratio was 34 times at its peak) or Lehman Brothers (with similarly stratospheric gauging ratios), and even the money markets (where some companies 'broke the buck'), which provided the rolled-over short-term cash financing of such operations, replacing the traditional deposit-taking of the well-regulated commercial banks.[4] Regulation increases persistence, so slightly better regulated cash markets should be brought in line with the newly deregulated real estate markets. It is clear from this year's events, that financial analysts, traders, regulators, and academics still have no firm grip on how to match the financial markets' respective, and currently, still widely varying persistence levels.

Despite its obvious importance, as suggested by the above mentioned examples, the proper empirical measurement and analysis of persistence or liquidity and of persistence or illiquidity risk is still in its infancy. We do not know much about the financial market behavior leading up to such illiquidity eddies, which can develop into very destructive Katrina- or Ike-like hurricane forces. For example, there is still no agreement on how changes in levels of financial market liquidity should be measured, because liquidity has traditionally been defined by price stability—a static concept—and not by the rapidity of the changes in price variability—a very dynamic concept, which we now measure by the level of market 'persistence'. More precisely stated: there does not yet exist a measurement standard for the various 'degrees of financial persistence and its relationship with changing levels of liquidity'.

Which transitions of financial illiquidity are prone to generate financial catastrophes? Which degrees of financial persistence allow regular, liquid trading activity? and which levels of ultra-financial persistence are prone to generate financial turbulence? Now these fundamental questions have been brought into sharper focus by the accumulating series of current sharp financial market adjustments.

Together with researchers such as Dacorogna *et al.* (2001) and Di Matteo *et al.* (2006), an attempt is made to stimulate more research in this area Los (2005), Kyaw *et al.* (2006), Jamdee and Los (2006 and 2007), Lipka and Los (2007), Los and Yu (2008) and to adopt at least a standard for measuring the degree of persistence in financial markets, as originally suggested by Mandelbrot (1982 and 1997). This study attempts to provide further guidance to where this sophisticated empirical financial market research should lead. Here at the University of Lethbridge we are now in the process of devoting laboratory-type controlled research in a new financial trading room to these transition issues of globally changing financial market structures, which, because of their nonlinear nature and complexity, can only be properly studied using high-frequency data and ingenious visualization software.

[4] For some interesting and timely commentary, see "Old-School Banking is Emerging Atop a Changing World of Finance", *The Asian Wall Street Journal*, Wednesday, September 17, 2008, pp. 14-15).

Peters (1989 and 1994) of PanAgora Asset Management already suggested that to understand financial turbulence the dynamics of cash flows between the various market participants, within and between different asset markets, should be analyzed and measured more carefully. Although there exists not yet a complete theory of physical turbulence, let alone a theory of financial turbulence, many parallels between the two phenomena have been noted by, for example, Mandelbrot (1982 and 1997) and Mandelbrot and Hudson (2004).

Therefore, in this study, we discuss the empirical measurement of the dynamics of the transitions in market illiquidity, in particular of the illiquidity of cash flows, which is directly related to the dynamics of the term structure of rates of return on cash investments and to the concept of (bond and equity) duration. For a fairly recent, but incomplete, survey of current theoretical work on term structure dynamics in the financial literature, see the recent study by Hu (2001). For a thorough discussion of a few recent but fairly standard approaches to the empirical model identification of term structure dynamics, in particular of its principal component analysis, see the companion article by Chapman and Pearson (2001).[5]

To accomplish the two research objectives of measurement and simulation of various degrees of financial liquidity (illiquidity), this study reformulates and reinterprets the classical physics laws of fluid mechanics into financial cash flow mechanics, in particular the laws of conservation of investment capital ('mass'), of the financial cash flow rate ('momentum') and of the rate of financial cash return (2 $\times$ 'energy'). These financial cash flow laws are modeled on the basic conservation laws of hydraulics and aerodynamics.[6] For example, we will study a "scale-by-scale cash flow rate of return budget equation", which allows us to interpret the transfer of cash returns among different scales of investment cash flow (*cf.* Los, 1999). For more detailed background on the formulation of these laws in the mechanics of incompressible fluids, see Batchelor (1970), Landau and Lifshitz (1987) and Tritton (1988).[7]

[5] Already in 1989 we thoroughly criticized and rejected principal component analysis, since it was non-scientifically subjective (Los, 1989) and it is based on a fundamental ergodicity assumption of wide-sense stationarity. This assumption has in the past decade been thoroughly refuted by empirical financial market research.

[6] A flow dynamics approach to the cash flows in financial markets is not new in economics. Compare, for example, the physical income flow dynamics modeling of an economy in the 1950s at the London School of Economics and Political Science (LSE) by the engineer and economist A W Phillips, familiar from the (expectations-augmented) Phillip's Curve in undergraduate textbooks on macroeconomics. Unfortunately, the financial economic research of markets has focused on the static 'equilibrium theory' of markets of Nobel Memorial prize winners Gerard Debreu and Kenneth Arrow, while a much more important (for fund managers) dynamic 'transport theory' has mostly been neglected. Consequently, the dynamics of cash flows has not been further developed since Phillips' heuristic macroeconomics research. But fluid dynamics has a long mathematical history going back to Dutch mathematician and engineer Simon Stevin's *Hydrostatics* (1605), Italian Evangelista Torricellis law of efflux (1644), and French Blaise Pascals uniform pressure law of hydrostatics (1653). Sir Isaac Newton devoted over one quarter of his famous *Principia* book to the analysis of fluids (Book II, 1713) and added some original ideas, like his hypothesis of viscosity. This was followed by the mathematical explanations of the mechanics of ideal, frictionless fluids mechanics by Bernoulli (1738) and Euler (1765). In 1827, Claude Navier derived the equations of viscous flow, which were published by Sir George Gabriel Stokes in 1845, the celebrated Navier-Stokes equations. Work by Reynolds (1883) on laminar (streamlined) flow and turbulent (erratic) flow extended these Navier-Stokes equations to turbulent flow, by including the Reynolds stresses and measurements.

[7] Van Dyke's (1982) album provides the author of this paper with inspiring visualizations of physical fluid dynamics and turbulence.

The results of these heuristic, and at the same time rather pragmatic, reformulations and reinterpretations are various useful quantifiable financial quantities, which will assist us with the measurement, analysis and proper dynamic characterization of the modern dynamic financial markets. Only then we can engineer and regulate their interlinked microstructures so that they do not cause major financial disruptions. This approach will do that in ways that cannot be achieved by the currently globally prevalent concepts of finance and economics, which were developed in Marshallian comparative static equilibrium models and not in the currently fashionable linearized dynamic partial differentiation equations, which are used in stochastic dynamic asset valuation by the current financial analysts and traders. The current underdeveloped status of the analysis and modeling of the transition phases of the liquidity dynamics of the financial markets forces one to explore and break new grounds, even when that may seem at the expense of 'financial' or 'economic' theoretical purity. The objective is to make slow but steady progress in this difficult area of risk measurement and analysis of what determines and drives market liquidity, so that we can implement some preventive engineering.[8]

The fundamental cash growth rate accounting framework has already been developed and its empirical applicability has been established by Karnosky and Singer (1994) and by Los (1998 and 1999) who extended it to Markowitz' classical mean-variance portfolio optimization and risk attribution framework (Los, 2002). It is easy to extend this multivariate theory further beyond mean-variance analysis taking account of the latest developments in stable distribution and extreme value theories as shown in Los (2003).

However, this paper focuses on the term structure of so-called 'laminar' and 'turbulent' financial cash flows. By way of several examples, this study demonstrates the empirical applicability of these ideas of physics and the computation of a coefficient of illiquidity ('kinetic viscosity of cash flows') for the term structure. This is a direct follow-up of our earlier, more philosophical remarks about the importance of time-frequency analysis of financial time series for financial management (Los, 2000a).

One important point we should emphasize for the benefit of keeping the attention of global fund managers is the following. financial turbulence is not a 'negative' or 'bad' phenomenon, in contrast to financial catastrophes, crises or bear markets. Turbulence is actually an 'efficiency-enhancing' dynamic phenomenon that we have to understand better for a full appreciation of the ongoing proper functioning of the financial markets. Financial turbulence is a limited phenomenon that can be observed under particular conditions of well-functioning financial markets such as in the foreign exchange and international swap markets of the Deutschemark, the Euro, or the Japanese Yen, when these markets are coping with 'differentials in liquidity or persistence', like they do currently.

Moreover, the occurrence of financial 'crises' is actually a fairly regular and a widespread phenomenon, that shows a continuing existence from the time of emergence

8 This is not unlike the increase in the understanding of hydrodynamics, which made possible the centuries of control of hydroforces in the Netherlands.

of financial markets based on the accounting system structures as we know them, i.e., from the early 16th century on, when the first system of interacting financial markets appeared under the 'globalizing' impact of the 80-year war between the Republic of the Netherlands and Monarchy of Spain.

In other words, financial 'turbulence', which can only occur in anti-persistent financial markets, must be sharply differentiated from the spectacular financial crises, which are unpredictable discontinuities and truly catastrophic crisis phenomena occurring in highly persistent financial markets, as emphasized by Taleb (2007). For example, the deflation of the residential housing bubble in the USA takes place in a persistent real estate market and leads to abrupt value adjustments. But the shock waves from the real estate value adjustments transmitted by the various swap lines around the world are dissipated in the very efficient and anti-persistent anchor-currency currency, money and derivatives markets.

Such financial crises we either need to prevent by changing the institutional structure of the existing real estate markets to make them less persistent and more liquid by deregulation and the elimination of restrictive asset allocation requirements, or against which we need to insure and hedge by trading financial catastrophe bonds or derivatives, to prevent their truly catastrophic adjustments, as has been suggested earlier by Chichilnisky and Heal (1992) and Chichilnisky (1996).

Thus, the question if financial crises emerging in persistent financial markets, such as real estate, stock, bond, and commercial loan markets, can generate financial turbulence in interlinked anti-persistent financial markets, like the foreign exchange and cash markets, must be answered affirmatively, they can and they do, like cataracts can interrupt smooth-flowing laminar rivers causing observable eddies or rotating temporary flow structures, increasing or diminishing in size after the cataracts as we have empirically observed and measured in several Asian financial markets in 1997-98. The reason for this interlinking is the existence of swaps, and also correspondence banking.

Nowadays, the risk premium spread of a bond can be split into genuine asset return swap spread and a credit default swap spread. The widening of this credit default swap spread immediately signals a deterioration of the quality and value of either the outstanding bonds or stocks (shares) or both, due to a deterioration of the revenue-generating capacity of the assets which those bonds and shares finance. The technical research questions are: which geography of financial market microstructures (rules and regulations) makes some of these generated eddies increase in size and which make them decrease in size? and how can we change that geography so that the energy of the value adjustments can dissipate quickly these eddies as fast as possible?[9]

[9] It would be an elusive goal, in my experience, to make prevent these eddies from emerging at all, since the capitalist system needs to create entrepreneurial innovations, like new ways of financing and of portfolio management for its continued productive existence. One cannot boil water well, without causing some bubbles. At higher oxygen pressure levels, water boils faster and is hotter. At lower oxygen levels it boils slower and cooler. One needs oxygen for life.

This paper is organized as follows. Under the subhead "Dynamic Financial Cash Flow Theory" this study proposes a simple measurement theory to develop the proposed fundamental dynamic financial cash flow theory without reinventing the wheel. We discuss the theoretical 'straw man' of perfectly efficient dynamic financial markets, and how financial pressure and cash flow power can be quantified and measured. Under the subhead "Illiquidity and Financial Turbulence" this paper makes a link between liquidity and financial turbulence. It shows that market illiquidity can be understood and measured as 'cash flow viscosity'. This allows us to distinguish between laminar or regular cash flows and turbulent or irregular cash flows. In particular, we propose a financial wavelet Reynolds number to categorize various kinds of financial turbulence and to measure the chaotic phenomenon of financial market intermittence, i.e., periods of relatively regular market trading interspersed by burst of irregular and frantic trading. Although this is intended as a conceptual paper, both subheads contain illustrative empirical measurement examples. In the Conclusion, we summarize our proposed dynamic cash flow theory and provide some motivating connection wits to the current turmoil in the financial markets.

Dynamic Financial Cash Flow Theory

When financial cash is in motion, its properties are described at each point in time by the properties of its cash flow rate as follows.

Definition 1: The financial 'cash flow rate', or 'cash volume flux', $\Delta X(t)$, is the product of the cash invested (cash position) $X(t-1)$ at the beginning of time period t and its cash rate of return (cash velocity) $x(t)$:

$$\Delta X(t) = x(t)X(t-1) \qquad \text{...(1)}$$

since the value of an asset grows as

$$X(t) = X(t-1).e^{x(t)} \qquad \text{...(2)}$$

where the cash rate of return is continuously computed as

$$x(t) = ln\frac{X(t)}{X(t-1)} \qquad \text{...(3)}$$

Example: An international investor needs to look at the size of his investment and at various rates of return in his or her investments. According to the exact cash accounting framework, at time t an investor has three possible investment instruments: (1) investment in an asset k in country i with rate of return $r_{ik}(t)$; (2) a cash swap with rate of return $c_j(t) - c_i(t)$, with $c_j(t)$ being the risk-free cash rate in country j into which the nominal is swapped, and $c_i(t)$ being the risk free cash rate in country i out of which the nominal is swapped; and (3) the foreign currency appreciation rate $f_j(t)$ of country j.[10] Thus, one particular bilateral investment strategy at time t is represented by the following strategic rate of return:

[10] In such cash accounting frameworks, one usually adopts the US dollar as the base currency, or numeraire.

$$x_{ijk}(t) = r_{ik}(t) + [c_j(t) - c_i(t) + f_j(t) \qquad ...(4)$$

where $i, j \in Z_1^2$ being the indices for bilateral cash flows between various countries and $k \in Z_2$ being the index for the various assets (stocks, bonds, real estate, commodities, etc.). Notice that such an international investment strategy is, according to the Capital Asset Pricing Model (CAPM), equivalent to the sum of the asset risk premium in local market $i[r_{ik}(t) - c_i(t)]$ and the cash return on currency $j, [c_j(t) + f_j(t)]$ (Los, 1998 and 1999; Leow, 1999).

Remark: Since we first proposed this cash accounting framework, these asset return swap spreads have been further split in genuine risk-free cash-flow asset return swap spreads and Credit Default Swap (CDS) spreads, which measure the relative riskiness of the assets underlying those spreads. This simple conceptualization leads directly to the following distinguishing definitions of continuous and steady cash flows.

Definition 2: 'Continuous' financial cash flow occurs in a particular cash flow channel, when its cash flow rate $\Delta X(t)$ is constant:

$$\Delta X(t) = Constant \text{ (independent of time)} \qquad ...(5)$$

Remark: This equation is called the 'cash continuity equation'.

Definition 3: 'Steady' financial cash flow occurs in a particular investment channel, when its cash flow return rate $x(t)$ = *constant* (independent of time).

Thus, under conditions of continuous cash flow, the cash flow return rate $x(t)$ is high when the cash flow channel is constricted, i.e., where the investment position $X(t-1)$ is small and the cash flow return rate $x(t)$ is low where the channel is wide, i.e., where the investment position $X(t-1)$ is large. In other words, under the conditions of continuous cash flows, a small investment with high cash returns generates as much cash as a large investment with low cash returns.

Remark: Day traders, who have very short investment horizons, trade very quickly with small investment positions in the order of a \$1,000 and reap highly fluctuating returns $x(t)$ with very large amplitudes $|x(t)|$. In contrast, institutional investors, such as pension funds, insurance funds, or large mutual funds, who usually have long-term investment horizons, trade much slower with very large investment positions in the order of \$100 mn, and who invest for steady rates of return $x(t)$ with moderate amplitudes $|x(t)|$. The important research question, therefore, is: Can such different types of traders, with investment positions of different size and investment horizons of different lengths, coexist in the same global markets without financial turbulence, or is that a necessary and unavoidable consequence? This study is of the opinion, since day traders add liquidity and reduce the persistence of a financial market, it is likely that they reduce the possible occurrence of financial crises, which is a phenomenon occurring in illiquid, persistent financial markets, when suddenly large investment positions are set up or are being unwound.

Perfectly Efficient Dynamic Financial Markets

The motion of empirical cash flows in globally interconnected financial markets is very complex and still not fully understood, because the flows start and stop at various times and go back and forth between millions of bilateral trading positions. Until very recently, financial economists have proceeded by making several heroic, simplifying assumptions for stationary, and often static, markets. Following the classical definitions of ideal, perfectly efficient physical flows, the study suggests the following for a new theoretical definition of perfectly efficient dynamic financial markets, which can provide a new standard for the comparative measurement of dynamic cash flows in non-stationary markets.

Definition 4: 'Perfectly efficient dynamic financial' markets have cash flows, which are:

- Perfectly Liquid (or non-viscous): There is no internal (institutional) friction in the markets and all assets of different maturities in all markets can be instantaneously purchased and liquidated;
- Steady: The rate of return $x(t)$ of each cash flow remains constant;
- Incompressible: The density of the cash transactions remains constant in time, i.e., the volume of market buy and sell transactions or 'open interest' remains uniform over time.
- Irrotational: There is no angular momentum of the cash flows, i.e., all cash flows in a particular investment channel at a time t are in the same direction. All decision makers with different time horizons for investment receive the same information at the same time and interpret that information correctly and simultaneously (the market's rational 'herd instinct').

Thus, in the ideal situation of a perfectly efficient dynamic financial market, the cash flows in the various investment channels are all streamlined in the same direction (there are no crossover trades) and move at the same constant flow rates $x(t)$. There are only an even number of 'bulls' (positively inspired investors) and 'bears' (negatively inspired investors) trading at the same maturity levels. This means, for example, that the term structure of interest rates can only move parallel to itself and is not allowed to rotate. This is, of course, an abstraction from empirical financial market reality, where rotations in the term structures of interest rates are a (difficult to model) empirical phenomenon (*cf*. Chapman and Pearson, 2001).

Financial Pressure and Cash Flow Power

Under such abstract, perfectly efficient dynamic financial market conditions, one can formulate a Bernoulli equation for cash flows which quantifies the concept of financial pressure.[11]

[11] Daniel Bernoulli's study, *Hydrodynamics* (1738), is both a theoretical and practical study of equilibrium, pressure and velocity of fluids. He demonstrated that as the velocity of fluid flow increases, its pressure decreases.

Definition 5: (Bernoulli equation for cash flows) In perfectly efficient dynamic financial markets:

$$FP(t) + \frac{1}{2}\rho x^2(t) = constant \qquad \text{...(6)}$$

where $FP(t)$ is the financial pressure at time t and $\rho = \frac{1}{s^2}$ is the uniform cash density.

Remark: The Bernouilli cash flow equation is also called the 'cash energy equation'. This Bernoulli equation for cash flows states that the sum of financial pressure *FP(t)* and the kinetic energy of the cash flow per dollar, $\frac{1}{2}\rho x^2(t)$, is constant along a cash flow investment channel. This Bernoulli equation for cash flows describes a 'venturi cash flow channel' to measure the difference in financial pressure between two different places along that investment channel, as follows. This equation implies that

$$FP_i(t) + \frac{1}{2}\rho x_i^2(t) = FP_j(t) + \frac{1}{2}\rho x_j^2(t), \quad \text{where } (i, j) \in \mathbb{Z}^2 \qquad \text{...(7)}$$

so that the difference in financial pressure $FP_i(t) - FP_j(t)$, measured simultaneously at two different points i and j in a particular investment cash flow channel at time t is the difference between the kinetic energies of the uniform cash flow at these two points:

$$\begin{aligned}\Delta_{ij}FP(t) &= FP_i(t) - FP_j(t) \\ &= \frac{1}{2}\rho\,[x_j^2(t) - x_i^2(t)] \\ &= -\frac{1}{2}\rho\,\Delta_{ij}x^2(t)\end{aligned} \qquad \text{...(8)}$$

When the financial pressure at an 'upstream' point i is higher than at a 'downstream' point j, the cash flow velocity at the 'downstream' point j is higher than at the 'upstream' point i, and vice versa. Financial pressure is relatively high where investment cash flow moves slowly, and is relatively low where it moves fast.

This point about the cash flow pressure differential becomes clearer, when we take account of the size of the investment positions. When the cash flow is continuous, as we thus far have assumed, thus, when

$$x_i(t)X_i(t-1) = x_j(t)X_j(t-1) = constant \qquad \text{...(9)}$$

so that

$$x_j(t)=\frac{X_i(t-1)}{X_j(t-1)}x_i(t) \qquad \text{...(10)}$$

then

$$\begin{aligned}
\Delta_{ij}FP(t)&=-\frac{1}{2}\rho\,\Delta_{ij}x^2(t)\\
&=\frac{1}{2}\rho[x_j^2(t)-x_i^2(t)]\\
&=\frac{1}{2}\rho\left[\left(\frac{X_i(t-1)}{X_j(t-1)}x_i(t)\right)^2-x_i^2(t)\right]\\
&=\frac{1}{2}\rho\left[\left\{\left(\frac{X_i(t-1)}{X_j(t-1)}\right)^2-1\right\}x_i^2(t)\right]\\
&=\frac{1}{2}\rho\left[\left\{\frac{X_i^2(t-1)-X_j^2(t-1)}{X_j^2(t-1)}\right\}x_i^2(t)\right]
\end{aligned} \qquad \text{...(11)}$$

Note that $\Delta_{ij}FP(t)>0$ if and only if $X_i^2(t-1)>X_j^2(t-1)$, i.e., if the amount of cash invested at point *i* is larger than what is invested at point *j*. Thus, a financial market cash flow system operates as follows. Consider a particular very crowded asset investment market with many market participants where a lot of cash is squeezed together. As soon as an extra investment channel is opened and cash begins to exit that particular asset market, the squeezing, or pressure *FP*, is least near this small exit where the motion (cash flow rate of return $x(t)$) is greatest. Thus, under the assumption of perfectly efficient financial markets the lowest financial pressure occurs where the investment channel is the smallest and the cash return rate is the highest, e.g., in emerging financial markets where the marginal productivity and the risk of capital investment is the highest. The highest financial pressure occurs where the investment channel is largest and the cash return rate is lowest, e.g., in the developed financial markets,where the marginal productivity and the risk of capital investment is the lowest.

Remark: One should be on the alert for financial turbulence when the difference in financial pressure between the upstream developed and downstream emerging financial markets, $\Delta_{ij}FP(t)$, is large, because the difference in the cash investments in each of the two connected markets $X_i^2(t-1)>X_j^2(t-1)$ is very large so that the kinetic energy

$$X_j^2\left(\frac{X_i(t-1)}{X_j(t-1)}x_i(t)\right)^2$$ in the underdeveloped downstream market is very large since the uniform kinetic energy term $\frac{1}{2}\rho x^2(t)$ figures prominently in the financial Reynolds number. A high Reynolds number empirically measures the onset of financial turbulence.

Multiplying the kinetic energy of the uniform cash flow by its flow rate provides the rate at which the energy is transferred, i.e., it provides its power.

Definition 6: The power of a cash flow at time t is defined by:

$$\begin{aligned} Power(t) &= \left[\frac{1}{2}\rho x^2(t)\right]\Delta X(t) \\ &= \left[\frac{1}{2}\rho x^2(t)\right]\,[x(t)X(t-1)] \end{aligned} \qquad \text{...(12)}$$

$$= \frac{1}{2}\rho x^3(t)X(t-1) \qquad \text{...(13)}$$

Therefore, the 'available cash flow power per dollar invested' is measured by:

$$\frac{Power(t)}{X(t-1)} = \frac{1}{2}\rho x^3(t) \qquad \text{...(14)}$$

and the 'average available cash flow power per dollar invested' is:

$$\frac{1}{2}\rho E\left\{x^3(t)\right\} \qquad \text{...(15)}$$

Notice that this last expression involves the third moment (or skewness) $m_3 = E\{x^3(t)\}$ of the frequency distribution of the cash flow rates of return in the term structure of spot rates. For (symmetric) Gaussian investment rates returns, the average available cash flow power per dollar invested $m_3 = 0$. For asymmetric distributions of investment rates of return, the average available cash flow power per dollar invested $m_3 \neq 0$, which can be positive (long tail to the right), or negative (distributional long tail to the left). Thus, the average available cash flow power per dollar invested is nil when the distribution of the rates of return is symmetric around zero, e.g., when it is as likely to make money as it is to lose it. The average cash flow power per dollar invested is positive when the distribution of the rates of return is positively skewed, and negative when their distribution is negatively skewed. Most of the time, rates of asset return are positively skewed and market prices move into a steady upwards direction, driven by positive productivity increases and thus profits. But occasionally, this process is interrupted, often by emerging bad performance conditions. On the margin, investments start not to perform as initially expected.

Illiquidity and Financial Turbulence

In the imperfect empirical international financial markets, in which asset investors with different cash investment horizons—for example, day traders, bank treasurers, and pension fund managers—trade, friction can occur between the various investment cash flows, which show a great diversity of changing strategic cash flow rates of return $X_{ijk}(t)$. Such friction can cause either financial turbulence or financial crises depending on the degree of persistence of the financial markets as measured by the persistence of these time series of the strategic rates of return. Now that we have defined continuous and steady cash flows, let us, initially rather informally, define turbulent cash flow.

Definition 7: Turbulent financial cash flow means irregular cash flow characterized by small whirlpool-like regions, vortices, or 'eddies'.

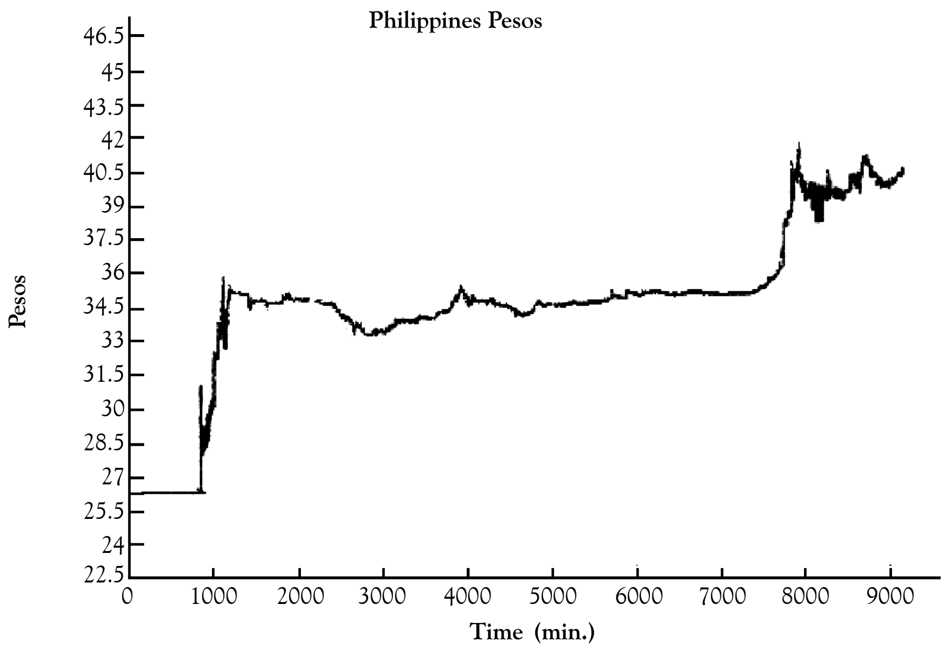
$$\Delta X(t) = x(t)X(t-1) = \textit{irregular, with possible cash vortices} \qquad \ldots(16)$$

Remark: This effectively means that a financial cash flow is irregular when the 'rates of return' series $x(t)$ is irregular. We have already characterized and measured various degrees of irregularity by computing the fractal spectrum of financial cash flow rates based on the Lipschitz-α_L and the corresponding fractal dimensions (Karuppiah and Los, 2000). But how can we imagine a cash flow vortex or 'eddy' to be? We suggest the following definition.

Definition 8: A financial cash flow 'vortex' occurs, when the fractal differences $\Delta^d x(t)$ first rapidly increase in density (become quickly more rapid) and then rapidly decreases in density (become quickly less rapid). In other words, a cash vortex occurs, when the fractional differentiation (in particular the second order differentiation, or convexity) of $x(t)$ is non-homogeneous and there exists a fractal spectrum of irregularities.

Thus cash flow vortices are regions of non-homogeneous fractional differentiation of the spot rates of return *x(t)*, which can lead to period-doubling and intermittent behavior. Period-doubling and intermittence can lead to chaos, which is actually an efficiency-enhancing, although also for many a people disturbing, phenomenon. It shows up in the records of financial time series as 'clustering' of the increments of the Fractional Brownian Motion (FBM). For an example, notice the cash vortex in the Philippine Pesos after the Thai baht crisis on July 2, 2001 at the right side in Figure 1.

Figure 1: Philippine Pesos After the Thai Baht Crisis



Note: More than 9,300 minute-by-minute quotations on the Philippine Pesos collected in real time for the month of July 1997. Immediately after the Thai baht crisis break on July 2, 2001, there is adjustement turbulence in the Pesos/US$ exchange rate, when it adjusts to its new, more stable, depreciated level. Later in July, there is another sharp discontinuity, immediately followed by financial adjustment turbulence, which includes a financial vortex at about the 8,000th observation.

Illiquidity = Cash Flow Viscosity

Why would such financial turbulence consisting of a series of financial vortices occur? The physical theory *of* 'flow dynamics' may provide us with some clues, or, at least, with some quantifiable entities which can measure when turbulence occurs even though no completely acceptable model or theory of financial turbulence exists. The term 'viscosity' is used in flow dynamics to characterize the degree of internal friction, or illiquidity, in a fluid. Equivalently, we can define in finance:[12]

Definition 9

Financial Cash Flow Viscosity = Financial Cash Flow Illiquidity
= Degree of Illiquidity of Adjacent Financial Cash Flows ...(17)

This internal cash flow friction is associated with the resistance of two adjacent layers of cash flows to move relative to each other due to 'Newton's Second Law' for flows.

Proposition 1 *(Newton's Second Law)***:** *Any force applied to a flow results in an equal but opposite reaction which in turn causes a rate of change of momentum in the flow.*

[12] Similarly, Koch and Lazarov (2002) define two measures of bunching in the DAX equity index derivatives market based on the volume of transactions in adjacent maturity 'buckets' of options. The first measure takes the ratio of transaction volumes and the second takes the ratio of average transaction volumes.

Because of the illiquidity of investment cash flows, part of their steady kinetic energy represented by a finite, integrated amount of energy $\int |x(t)|^2 dt < \infty$ is converted to thermal ('random') energy, represented by an infinite integrated amount of energy $\int |x(t)|^2 dt \to \infty$, which can cause cash vortices ('eddies') on the edges of the adjacent investment cash flow channels where investors go in and out of adjacent investment positions unsure of themselves.

Again, analogously to definitions in physical flow dynamics, we can define a 'coefficient of cash flow illiquidity' which would allow us to measure the degree of illiquidity in the financial markets as follows.

Definition 10: The *ex-post* term structure $x_\tau(t)$ of any asset at time t is defined by the expression:

$$\begin{aligned} x_\tau(t) &= \left[\frac{X(t)}{X(t-\tau)}\right]^{\frac{1}{\tau}} - 1 \\ &= \frac{ln\, X(t) - ln\, X(t-\tau)}{\tau} \end{aligned} \qquad \ldots(18)$$

This is easy to see, since

$$[1 + x_\tau(t)]^\tau = \frac{X(t)}{X(t-\tau)} \qquad \ldots(19)$$

so that by approximation,

$$\begin{aligned} \tau\, ln\, [1 + x_\tau(t)] &= \tau\, x_\tau(t) \\ &= ln\, X(t) - ln\, X(t-\tau) \end{aligned} \qquad \ldots(20)$$

It is important to understand that the *ex-post* term structure $x_\tau(t)$ of an asset is non-linearly related to the corresponding cash flows $X(t)$ and $X(t-\tau)$ defining the asset and its value. This becomes even clearer in the following examples of the fundamental and derivative instruments used in the financial markets and the way they are accounted for by the double-entry bookkeeping system of market values.

Example 1: The fundamental security of a simple (Treasury) 'bond' at time t can be decomposed into a series of zero coupon bonds. A zero coupon bond, or pure discount bond, with maturity τ and principal payment $B_\tau(t)$, has zero coupons so that its (discounted) present value is:

$$PB_0(t) = \frac{B_\tau(t)}{[1 + x_\tau(t)]^\tau} \qquad \ldots(21)$$

Thus, we have, *ex-post,*

$$[1+x_\tau(t)]^\tau = \frac{B_\tau(t)}{PB_0(t)}$$
$$= \frac{X(t)}{X(t-\tau)} \qquad ...(22)$$

A τ-year 'spot interest rate' at time t is the interest rate of a zero bond:

$$x_\tau(t) = \left[\frac{B_\tau(t)}{PB_0(t)}\right]^{\frac{1}{\tau}} - 1 \qquad ...(23)$$

This term structure of interest rates is the nonlinear relationship between the spot interest rate $x_\tau(t)$ and the maturity τ for a particular grade of credit quality of obligations (e.g., bills, notes, bonds) (Vasicek, 1977; Cox *et al.*, 1985; Heath *et al.*, 1992; and Los, 2001, p. 52).

Example 2: For the second fundamental security of a 'stock', we have the following valuation. For (not necessarily constant) dividend payments $D_\tau(t)$ and a cost of capital $x_\tau(t)$ at time t, the Dividend Discount Model (DDM) of stock valuation for an ongoing concern (a firm with an infinite life time) computes the stocks present value at time t to be equivalent to an infinite series of 'zero coupon bonds' with maturities τ = 1, 2,..., ∞

$$PS_0(t) = \sum_{\tau=1}^{\infty} \frac{D_\tau(t)}{[1+x_\tau(t)]^\tau} \qquad ...(24)$$

Thus, for each of the dividend payments we have the cash flow relationship:

$$[1+x_\tau(t)]^\tau = \frac{D_\tau(t)}{PS_0(t)}$$
$$= \frac{X(t)}{X(t-\tau)} \qquad ...(25)$$

so that the τ-year 'spot dividend yield' is

$$x_\tau = \left[\frac{D_\tau(t)}{PS_0(t)}\right]^{\frac{1}{\tau}} - 1 \qquad ...(26)$$

Example 3: The rational law of 'pricing by arbitrage' implies that all other financial instruments like derivatives can be derived from these two fundamental asset valuations by simple linear transformations. Call and put options can be synthesized from a linear portfolio of bonds and stocks according to the 'call-put balance equation' (*cf.* Los, 2001, pp. 164-166). Forwards and futures can be represented as time-shifted bonds or stocks (*cf.* Los, 2001, pp. 225-227). Swaps can be represented by a combination of a long and a short bond, or, equivalently, by a series of zero coupon bonds, or a series of long and short forwards (*cf.* Los, 2001, pp. 239-243). Of course, this arbitrage assumes that we deal with free-entry financial markets with minimal transaction costs, narrow bid-ask spreads, etc., thus high liquidity.

Example 4: The double-entry bookkeeping model of accounting shows that the value of any concern can be represented as a linear portfolio of fundamental and derivative financial instruments (*cf.* Los, 2001, p. 24). The Accounting Identity of the balance sheet, or 'accounting balance equation' with current market values is:

$$\begin{aligned}&Assets(t) = Liabilities(t) + Equity(t)\\&\text{or } A(t) = L(t) + E(t)\\&\text{or, rewritten, } E(t) = A(t) - L(t)\end{aligned} \quad \text{...(27)}$$

This accounting balance equation is the basic model for exact financial modeling already since the 14th century since financial instruments can be viewed as combinations of long and short positions of the fundamental securities. Changes in the net equity position $\Delta_\tau E(t)$ over the accounting period τ, which is usually one quarter of a year or a year are produced by the corresponding changes in the assets and liabilities as reflected in the derived 'income statement'[13]

$$\begin{aligned}&\Delta E_\tau(t) = \Delta A_\tau(t) - \Delta L_\tau(t)\\&\text{or } x_\tau^E(t)E(t-\tau) = x_\tau^A(t)A(t-\tau) - x_\tau^L(t)L(t-\tau)\\&\text{or Net } Income_\tau(t) = Revenues_\tau(t) - Expenses_\tau(t)\end{aligned} \quad \text{...(28)}$$

on market value basis where $x_\tau^A(t)$ is the market valued Rate of Return on Assets (ROA) over the period $\tau, x_\tau^L(t)$ is the market valued rate of debt liability expense (rate of interest on all debt) over the period τ, and $x_\tau^E(t)$ is the market valued Rate of Return on Equity (ROE) over the period τ. Thus, the accounting balance equation combines the bundles of discounted cash flows in a linear fashion and these discounted cash flows are non-linearly related to the rates of return.

In physics, 'shear stresses' are the internal friction forces opposing flowing of one part of a substance past the adjacent parts. In finance, in recent years, we have become more concerned about the shear stresses of investment cash flows with different degrees of illiquidity adjacent in the complete term structure flowing past each other within and between the various financial term markets and their maturity 'buckets'.

[13] It is easy to view liabilities as "negative assets".

Example 5: One can form a (not necessarily square) matrix of bilateral cash flow channels between two countries so that every cash flow investment channel associated with a particular term rate of return $x_{\tau_i}(t)$ is adjacent to every other cash flow investment channel resulting in a net rate of return for both cash channels together $x_{\tau_i,\tau_j}(t) = x_{\tau_i}(t) - x_{\tau_j}(t)$:

$$x(t) = \begin{Bmatrix} x_{11}(t) & x_{12}(t) & \dots & x_{1,\tau_2}(t) \\ x_{21}(t) & x_{22}(t) & \dots & \dots \\ \dots & \dots & \dots & \dots \\ x_{\tau_1,1}(t) & \dots & \dots & x_{\tau_1,\tau_2}(t) \end{Bmatrix} \qquad \dots(29)$$

Each side of the matrix can represent, say, the term structure of assets in a country whereby one country may have more investment channels and different asset term structures than another (as is empirically the case) so that this matrix is not necessarily symmetric, or square. For example, the USA has a very fine term structure with T_1 = 30-year bonds while other countries have only T_2 = 10-year bonds. Using tensor algebra, Los (1998 and 1999) provides empirical examples of such bilateral cash rates of return matrices based on the cash rates and stock market return rates in ten Asian countries plus Germany.

But how does cash flow shear stress and strain occur? What do these terms mean in financial terms? and How can they be measured?

Definition 11: Cash flow shear stress is the change in the *ex-post* asset term structure relative to the original asset prices, $\Delta x_\tau(t)/X(t-\tau)$. It is the quantity that is proportional to the cash flow supply of earnings on an investment made τ periods ago causing deformation of the term structure of the asset return rates.

Definition 12: Cash flow shear strain is the change in an asset price relative to its maturity $\Delta X_\tau(t)/\tau$. It is the quantity that is proportional to the cash flow demand.

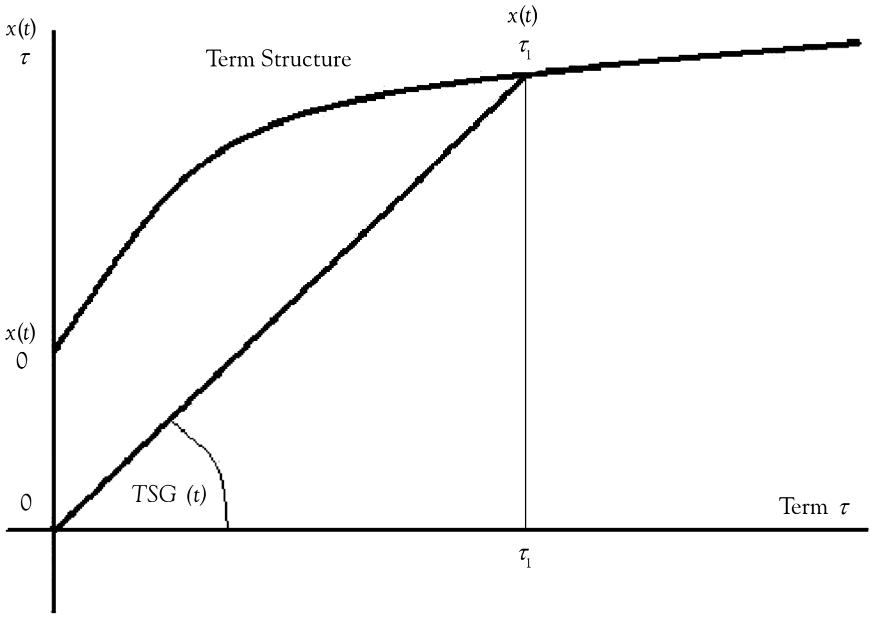
When we express the cash strain per unit of time $\Delta t = 1$, we have the cash rate of return (velocity), or term structure gradient.

Definition 13: The Term Structure Gradient (TSG) is given by:

$$\begin{aligned} TSG_\tau(t) &= \frac{\textit{Cash flow shear strain}}{\Delta t} \\ &= \frac{\Delta X_\tau(t)/\tau}{\Delta t} \\ &= \frac{X_\tau(t)}{\tau} \end{aligned} \qquad \dots(30)$$

This term structure gradient in Figure 2 is the cash flow rate of return of an asset with maturity τ relative to its term as a measure of the 'degree of term structure deformation' caused by cash flow shearing, i.e., caused by the differences between the cash flow rates of return with different time horizons for investment. This term structure gradient or TSG is conceptually similar to the laminar velocity profile of the hydraulic flow in pipes.

Figure 2: The term structure gradient, *TSG(t)*, is the cashflow rate of return $x_t(t)$ of an asset with investment horizon or maturity *t* relative to its term *t*, as a measure of the degree of term structure deformation caused by cashflow shearing. The *TSG(t)* measures the steepness of the term structure of interest rates from its origin, at time *t*



The $TSG_\tau(t)$ is largest when the short-term rates $x_\tau(t)$ (for small τ) are large, e.g., because of an inverted term structure, or because of high inflation expectations. However, the size of the $TSG_\tau(t) > 0$ is less important than the speed by which the $TSG_\tau(t)$ changes for a given term τ. The $TSG_\tau(t)$ for a particular maturity term τ changes most when the term rate $x_\tau(t)$ changes a lot, *i.e.*, when $|\Delta x_\tau(t)|$ or $|\Delta x_\tau(t)\,\tau|$ is large. Interestingly, the financial term structure of interest rates can be deformed inversely in contrast to physical gradients.

This leads us to the coefficient of dynamic cash flow illiquidity. For small cash flow stresses, stress is proportional to strain and we can define a coefficient of cash flow illiquidity, or kinetic cash viscosity, based on this simple linear stress-deformation relationship.

Definition 14: The coefficient of cash flow illiquidity or kinetic cash flow viscosity at time t is:

$$\eta_{\tau}(t) = \left| \frac{\textit{cash flow shear stress}}{\textit{cash flow shear strain}} \right|$$

$$= \left| \frac{\Delta x_{\tau}(t) / X(t-\tau)}{x_{\tau}(t) / \tau} \right|$$

$$= \left| \frac{\Delta x_{\tau}(t)\tau}{x_{\tau}(t) X(t-\tau)} \right| \qquad \text{...(31)}$$

This relative illiquidity coefficient measures the change in the term structure $\Delta x_{\tau}(t)$ multiplied by its term (maturity) τ relative to the cash flow rate $x_{\tau}(t)X(t-\tau)$. When the investment cash flow is continuous, i.e., when the investment cash flow rate $x_{\tau}(t)X(t-\tau)$ is constant, all illiquidity is measured by the empirical changes in the term structure $\Delta x_{\tau}(t)$ for each term τ.

When $\Delta x_{\tau}(t)$ varies inversely with the term τ, the illiquidity coefficient is constant, $\eta_{\tau}(t) = \eta_{\tau}$. Another way of expressing this is to state:

$$\Delta x_{\tau}(t)\tau = \eta_{\tau} x_{\tau}(t) X(t-\tau) \qquad \text{...(32)}$$

Thus, for 'laminar cash flow', the 'force of illiquidity resistance' is

$$\Delta x_{\tau}(t)\tau \propto x_{\tau}(t) X(t-\tau) \qquad \text{...(33)}$$

i.e., proportional to the rate of cash return $x_{\tau}(t)$ for maturity τ and the size of the original cash investment $X(t-\tau)$. Still a different way of interpreting this relative illiquidity coefficient is to state that it measures the absolute relative change in the interest rates of a particular maturity $\frac{\Delta x_{\tau}(t)}{x_{\tau}(t)}$, relative to the size of the average invested cash flow $\frac{X(t-\tau)}{\tau}$ in particular maturity market:

$$\eta_{\tau}(t) = \left| \frac{\dfrac{\Delta x_{\tau}(t)}{x_{\tau}(t)}}{\dfrac{X(t-\tau)}{\tau}} \right| \qquad \text{...(34)}$$

After these introductory definitions which can all be calculated from the basic data, we are now in a position to define perfect cash flow liquidity.

Definition 15: 'Perfect cash flow liquidity' exists when $\eta_\tau(t) = 0$. That occurs either: (1) when $\Delta x_\tau(t) = 0$ for finite invested cash flows $X(t-\tau)$ and finite investment horizon terms τ, implying that the finite term rates of return structure remains unchanged under financial stress, thus $x_\tau(t) = constant$; or (2) when unrealistically the term rates of return are infinite $x_\tau(t) = \infty$ for all finite cash flows and finite terms; or when the cash flow for a finite term τ and a finite term structure $x_\tau(t)$ is infinite $X(t-\tau) = \infty$; or when the term equals zero, $\tau = 0$, for a finite investment cash flow $X(t-\tau)$ and a finite spot rate of return $x_\tau(t)$.

Remark 1: Most real world cash flows are non-perfectly liquid with $\eta_\tau(t) > 0$ because most real world cash flows are term dependent. There is not much instantaneous cash in comparison. Also, infinite term rates of return and invested cash flows empirically do not exist, nor do zero investment horizons. Thus, the only realistic outcome is that perfect cash flow liquidity exists when $\Delta x_\tau(t) = 0$ for finite invested cash flows $X(t-\tau)$ and finite investment horizon terms τ.

This particular expression for the liquidity coefficient is only really valid when the term structure $x_\tau(t)$ varies linearly with the term τ, i.e., when the rate of cash return gradient $x_\tau(t)/\tau$ is uniformly constant. When this is the case, one may speak of 'Newtonian cash flows', in which stress, illiquidity, and rate of strain are linearly related. This is, of course, seldom the case with the 'dynamic term structures' in the international financial markets. Empirically, the term structures of the cash investment markets vary non-linearly in such a way that the short-term cash rates of return vary relatively more (have larger amplitudes), and vary more frequently than the long-term cash rates of return.[14]

Remark 2: Term structure modelers have attempted to capture this nonlinear dynamics by, first, inter-correlating the various terms $x_\tau(t)$ for a limited number of terms $1 < \tau < T$, and, then, by analyzing the resulting covariance matrix using a static principal component, or factor analysis (*cf.* the study by Chapman and Pearson, 2001). The objective of such spectral analysis is to capture most of the variation of the term structure by retaining a small number, say three, principal component factor loadings, like Factor 1 = Level, Factor 2 = Slope, and Factor 3 = Curvature of the term structure. But such inexact identification scheme of a covariance matrix is inherently subjective because we do not know which eigenvalues are to be considered significant and to be retained and which should be considered representing noise. (For a detailed discussion of such 'prejudices' of principal components, *cf.* Los, 1989).

When the rate of return gradient is not uniform, as is usually the case, we must express the illiquidity coefficient in the general marginal form:

[14] Professor Craig Holden of Indiana University has made a simple historical demonstration of this nonlinearly varying term structure: *Craig Holdens Excel-Based Interactive* movie of Term Structure Dynamics (1996), which can be downloaded from his web page.

$$\eta_\tau(t) = \left|\frac{\Delta x_\tau(t)/X(t-\tau)}{\partial x_\tau(t)/\partial\tau}\right|$$

$$= \left|\frac{\Delta x_\tau(t)\partial\tau}{\partial x_\tau(t)X(t-\tau)}\right| \qquad \ldots(35)$$

Thus, we compare the empirical change in the required cash flow over an infinitesimal small change in the term, $\Delta x_\tau(t)\partial\tau$, with the infinitesimal change in the available cash flow rate $\partial x_\tau(t)X(t-\tau)$. That is a comparison between the marginal investment cash flow shear stress and the marginal investment cash flow shear strain.[15]

Laminar and Turbulent Financial Cash Flows

We will now define the important concepts of laminar and turbulent cash flows.

Definition 16: If the adjacent layers of illiquid term cash flow smoothly alongside each other in a financial market, and the term cash flow rates $x_\tau(t)$ are constant, the stable streamline flow of investment cash through a particular financial market (of a particular term and risk profile), is called laminar.

Definition 17: If the streamlined term cash flows in a financial market at sufficiently high term rates of return $x_\tau(t)$, change from a laminar cash flow to a highly irregular cash flow with highly irregular cash flow (velocity) rates $x_\tau(t)$, it is called turbulent.

Both laminar and turbulent flows are easily illustrated by the hot cigarette smoke which initially is laminar, but then turns turbulent due to the nonlinear constraints imposed by the interactive heat exchange with the cooler environment (Figure 16 in Schroeder, 1991, p. 26). One can measure such cash flow velocity rate changes by a spectrogram or a scalogram.

My current conjecture is that the nonlinear expectation constraints inherent in the logical term structure $x_\tau(t)$ cause such chaos in the cash flows in financial markets to emerge when particular parametric thresholds in the interlocking term structures are surpassed. In other words, particular shapes of swap-interlinked national term structures at sufficiently high term rates can cause various irregularities to emerge, mostly because of information gaps or lagging information flows.

Remark: Small, perfectly liquid, laminar cash flow motions are called 'acoustic' cash flow motions and (linear) Fourier Transform analysis can be used to decompose such steady and continuous global cash flow motions in subcomponent flows. However, when any of these conditions do not apply, and we deal with large-scale, so-called 'non-acoustical' cash flow motions, like cash flow turbulence, we need (nonlinear) Wavelet Transform analysis to analyze and decompose them.

[15] We use the partial derivatives ∂ to indicate that we sre interested in variation on the term structure $x_\tau(t)$ caused by a marginal change in the term τ at a fixed time t.

At this moment, analogously to physical turbulence measurements, this study also conjectures that the onset of cash flow turbulence can be identified by a dimensionless parameter called the Reynolds number for uniform cash flow. In hydrodynamics and aerodynamics, the 'Reynolds number' is a dimensionless ratio related to the velocity at which smooth flow shifts to turbulent flow.[16] There is no fundamental reason why we should not be able to measure a similar Reynolds numbers for financial cash flows.

Definition 18: The financial Reynolds number $Re_\tau(t)$ for a uniform cash flow of maturity term τ is given by:

$$\begin{aligned} Re_\tau(t) &= \left| \frac{a\rho x_\tau(t)}{\eta_\tau(t)} \right| \\ &= \left| \frac{\rho x_\tau(t) X(t-\tau)}{\frac{\Delta x_\tau(t)\tau}{x_\tau(t)X(t-\tau)}} \right| \\ &= \left| \frac{\rho x_\tau^2(t) X^2(t-\tau)}{\Delta x_\tau(t)\tau} \right| \end{aligned} \qquad \dots(36)$$

where $\rho = \frac{1}{s^2}$ is again the uniform cash flow density which renders the Reynolds number dimensionless, and $a = X(t-\tau)$ is the scale of the investment cash flow channel.

The beauty of the Reynolds number is that it measures dynamic similarity: flows with the same Reynolds number, look the same, whereas flows with different Reynolds numbers look quite different. It is a measure of the amount of cash flow present at a particular time in a particular channel. When the cash flow Reynolds number is constant, $Re_\tau(t) = Re_\tau$, for turbulent investment cash flows show

$$\Delta x_\tau(t)\tau = \frac{\rho x_\tau^2(t) X^2(t-\tau)}{Re_\tau} \qquad \dots(37)$$

Thus, for turbulent cash flow, the 'force of illiquidity resistance' is a quadratic term. It is proportional to the squares of the rate of cash return and the size of the cash investment,

$$\Delta x_\tau(t)\tau \propto x_\tau^2(t) X^2(t-\tau) \qquad \dots(38)$$

[16] Osborne Reynolds (1842-1912) was an English scientist, whose studies on fluid dynamics and turbulence remain basic to turbine operation, lubrication, fluid flow and streamlining, cavitation, and the effects of tides.

This quadratic expression has a much higher value than the 'force of illiquidity resistance' or viscosity for laminar cash flow which we know to be linearly proportional.

$$\Delta x_\tau(t)\tau \propto x_\tau(t)X(t-\tau) \qquad \text{...(39)}$$

Remark 1: The cash flow Reynolds number is proportional to the uniform cash flow energy $\frac{1}{2}\rho x_\tau^2(t)$ at the ('downstream') measurement point. When the financial pressure 'upstream' is high, this 'downstream' energy increases rapidly, producing a large Reynolds number. Turbulent cash flows have violent and erratic fluctuations in velocity and pressure which are not associated with any corresponding fluctuations in the exogenous forces driving the flows.

Physical turbulence is generally considered to be a manifestation of the nonlinear nature of the underlying fundamental equations. The "force of illiquidity resistance" is a quadratic, therefore, highly nonlinear, term. We know from chaos theory that this quadratic term is the essential constraint 'causing' deterministic chaos or turbulence to occur at particular values of the growth rate parameters of the underlying simple dynamic processes. Feedback loops of the simple dynamic investment cash flow processes quickly increases the complexity of the aggregated dynamic investment cash flow process of several interconnected financial markets.

Remark 2: The physical flows in round pipes are laminar for $Re_\tau(t)<2000$, but physical turbulence begins to occur for Reynolds numbers, $Re_\tau(t)>3{,}000$. On the other hand, in physical turbulent flow over flat surfaces $Re_\tau>500{,}000$. We do not know yet what the critical values are for the investment cash flow Reynolds numbers.

Since the scale of the investment cash flow channel is measured by a uniform dollar a = \$1, we have $a\rho\frac{1}{\$}$, so that the Reynolds number for uniform cash flows can be interpreted as a term-based 'return/liquidity risk ratio':

$$Re_\tau(t) = \left|\frac{x_\tau(t)}{\eta_\tau(t)}\right| \qquad \text{...(40)}$$

The higher the cash rate of return on an investment asset with maturity τ, and the lower the cash illiquidity (the higher the cash liquidity) for that same maturity, the higher the Reynolds number and, thus, the opportunity for financial turbulence.[17]

[17] The value of an asset is computed as the sum of the discounted (future) cash flows associated with that asset. Each asset has a particular maturity, even though it is sometimes assumed to have an infinite maturity, as in the case of, for example, the assets of a firm, based on the accounting principle of an "ongoing concern." However, in reality, any firm has finite assets, i.e., projects with finite maturities.

In other words, the financial Reynolds number measures the cash rate of return on an investment relative to its illiquidity (or viscosity) risk which is a quite different, and, perhaps, a much more important measure than the usual market volatility risk $\sigma_\tau(t)$.

It is a matter of current empirical financial research to determine at which magnitudes of this cash flow Reynolds number financial turbulence begins to occur and if it is a real empirical problem. But this expression shows that the Reynolds number for finite term rates of return $x_\tau(t)$ only becomes very large when the illiquidity risk approaches zero, $\eta_\tau(t) \to 0$. Thus, financial turbulence is a phenomenon of very liquid financial markets and not of illiquid markets, as is often erroneously asserted in the popular media. In fact, the efficiency-enhancing financial turbulence contrasts sharply with the high risk discontinuity phenomena of the very illiquid and efficient markets (which show very low Reynolds numbers).

Remark 3: Indeed, the empirical evidence this study collected in Singapore during the Asian Financial Crisis in 1997 shows that turbulence is likely to occur in the anti-persistent anchor currency markets of the (then D-Mark, now) Euro, Swiss Franc, and the Yen, relative to the US dollar.

Financial Wavelet Reynolds Numbers and Financial Intermittence

How can we translate this financial Reynolds number in terms of the wavelet Multiresolution Analysis (MRA). Analogously to Farge *et al.* (1996), we have the following definition:

Definition 19: The local financial wavelet spectrum, $S_W(\tau, a)$ *of* $x(t)$, *is* defined as its scale-standardized scalogram:

$$\begin{aligned} S_W(\tau,a) &= \frac{P_W(\tau,a)}{a} \\ &= \frac{|W(\tau,a)|^2}{a} \end{aligned} \qquad \text{...(41)}$$

Remark: With the suggested financial uniform scale a = \$1 that standardization is very simple and

$$S_W(\tau,a) = |W(\tau,a)|^2 / \$ \qquad \text{...(42)}$$

A characterization of the local 'activity' of $x(t)$ is given by its 'wavelet intermittence', which measures local deviations from the mean spectrum of $x(t)$ at every time lag τ and scale a. It is a local measure *of* financial risk, *i.e.*, a measure of risk localized in the scale (frequency)-time domain. Thus, we no longer need the conventional, idealized statistical concept of ergodicity, a time averaging measure, to measure financial risk since we can now 'locally' measure financial risk. While we already provided here a way of quantitatively measuring intermittence, the concept of intermittence has been explained in greater detail in Los (2000b) when we discuss the various phases of degeneration into chaos of nonlinear financial pricing processes.

Definition 20: The 'financial wavelet intermittence' of $x(t)$ is defined as:

$$I_W(\tau, a) = \frac{|W(\tau, a)|^2}{\int_R |W(\tau, a)|^2 d\tau} \quad \text{...(43)}$$

Remark: One advantage of this wavelet intermittence measure is that it is dimension-free.

Next, we implement the suggestion of Farge *et al.* (1996, p. 651) to express the Reynolds number in terms of wavelets as follows.

Definition 21: The 'financial wavelet Reynolds number' is defined as:

$$\begin{aligned} Re_W(\tau, a) &= \left| \frac{a\rho|W(\tau, a)|}{\eta_\tau(t)} \right| \\ &= \left| \frac{a|W(\tau, a)|}{\eta_\tau(t) V_\psi^{0.5}} \right| \end{aligned} \quad \text{...(44)}$$

so that $\rho = 1/V_\psi^{0.5}$ where the constant

$$V_\psi = \int_{R^2} |\psi_{\tau,a}(t)|^2 d\tau \, da \quad \text{...(45)}$$

is the uniform variance of the analyzing wavelet $\psi_{\tau,a}(t)$ of the scalogram $W(\tau, a)$ over all time horizons τ and scales a.

Remark: Our expectation is that at large (inverted frequency) scales a, the wavelet financial Reynolds number coincides with the usual large-scale Reynolds number $Re_\tau(t)$. In the smallest scales (say $a = \sigma_\varepsilon$, where σ_ε is the Kolmogorov scale of turbulent cash flow, as in Chap. 11), one expects this wavelet Reynolds number to be close to unity when averaged over time.

Analogously to Farge *et al.* (1996), the current empirical research objective regarding the analysis of financial turbulence is the following:

Are there time lags τ where the financial Reynolds number at some very small scale is much larger than at others, and how do such time periods correlate with time periods of small-scale activity within the cash flow?

If so, then $Re_W(\tau, a)$ could give an unambiguous answer to the activity at small scales (or at any desired scale a). Such time periods of high $Re_W(\tau, a)$ can then be interpreted as periods of 'strong nonlinearity', i.e., periods of financial turbulence. In other words, if the interpretation of Farge *et al.* (1996) is correct, we can just look at 'scalograms of financial

Reynolds numbers' to detect periods of strong nonlinearity and thus of financial turbulence.[18]

Conclusion

The turmoil in the current global financial markets was initiated by the collapse of the residential housing real estate values in certain immigration areas in the United States of America, in particular in Southern California and Southern Florida. The relaxation of the mortgage lending standards, legislated by the Community Reinvestment Act (CRA) of 1977 put a heavy emphasis on the 'affordability' of housing in the inner cities and other immigration areas. The politically motivated US Congress which chartered the two main semi-government mortgage lenders—Fannie Mae and Freddie Mac—leaned on those two financial institutions to reduce the quality standards for measurement of creditworthiness of the mortgagees to make it easier for immigrants to obtain a mortgage for their first home.

Such relaxed creditworthiness standards, combined with a relatively easy money policy of the Federal Reserve (central banking system in the USA) led ultimately to a housing 'bubble' on top of regular residential housing growth induced by continuing immigrant flows into the USA. Rising speculative residential values contaminated various inner city markets, first within the USA (e.g., Chicago) and then overseas, e.g., Shanghai and Moscow where creditworthiness measurement standards were either nonexistent or not enforced. When ultimately the global housing prices became stratospheric and the truly marginal mortgage loans became non-performing (with increasingly deficient monthly mortgage payments of interest and capital), the valuation of the mortgages backing the securities sold into the global markets to raise the capital for such mortgage-lending in the first place, started to drop, putting the valuation of the mortgage-portfolios-backed securities under pressure.

The system of Credit Default Swaps (CDS), refined and developed in the last decade created a much faster credit-problem signaling system than originally existed when commercial bank intermediaries did the mortgage lending and kept the portfolios on their books. Next, the distressed value adjustment problems began to appear in those mortgage-transfer and mortgage-backed securities and then swap markets where the illiquid and persistent real estate valuation finally met the ultra-liquid and anti-persistent money markets.

Circular 'eddies' began to appear: on page 26 of *The Asian Wall Street Journal* on Monday, September 15, 2008 was published a revealing box titled: "Playing the Market." The box item explains how traders bought CDS to drive up their spreads which then signals to others that the credit quality of the firms involved declines leading to the selling of their stock and putting even more stress on their bonds. However, some financial traders had already set up ahead of this game short-selling positions so that they then profit from the

[18] At the University of Lethbridge, we are currently creating such financial scalograms.

decline in the bond and share prices they themselves have engineered. This can only happen when the bond and stock prices reflect negative credit information with more delay than the CDS markets. The signaling of the default stress was quicker in the CDS markets than in the bond and stock markets. As this study suspected, the very low (even anti-) persistence of the CDS cash-based markets combined with the higher persistence of the less liquid bond and stock markets, or, worse, with the much higher persistence of the original real estate markets makes such circular game-playing of the markets not only possible but profitable with negative consequences for all these financial markets since consistent valuations are literally 'ripped apart'. The CDS market of $62 tn nominal value was no longer related to the originating bond markets: the CDS traded independently and was six times larger, nominally, than the originating bond market. There was no longer any correlation discernible between the CDS rates and the bond rates. Or in physics terms: the financial atoms were smashed to particles.

This study shows that if we really want to measure, analyze and model such advanced financial system phenomena, so that they can be tracked and monitored in real time, we need to adopt a much more sophisticated framework for measurement than is currently offered by the traditional financial and economics and financial engineering theories and empirical research. It also shows that it can be done and that we can speak in quantifiable terms of the persistence or viscosity or particular cash flows, or of the demand or supply pressure on the money markets, or of the stresses and shears of various financial markets.

However, one fundamental problem exists and can no longer be ignored. These financial market phenomena are not only dynamic, but they are also interlinking swap-accounting systems with a combined dimensionality considerably higher than the four dimensions of hydrological physical systems. They resemble more the very complex meteorological models by which some scientists (and some politicians!?) try to model the global warming system (which is also still not well understood!). ❖

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References

1. Batchelor G K (1970), *Introduction to Fluid Dynamics,* Cambridge University Press, Cambridge, UK.

2. Bernoulli D (1738), *Hydrodynamics*, Johann Reingold, Strassbourg.

3. Chapman and Pearson (2001), "Recent Advances in Estimating Term-Structure Models", *Financial Analysts Journal,* Vol. 57, No. 4, pp. 77-95.

4. Chichilnisky G (1996), "Markets With Endogenous Uncertainty: Theory and Policy", *Theory and Decision,* Vol. 41, No. 2, pp. 99-131.

5. Chichilnisky G and Heal G M (1992), “Xilobal Environmental Risks”, *Journal of Economic Perspectives,* Vol. 7, No. 4, pp. 65-86.

6. Cox J C, Ingersoll J E and Ross S A (1985), “A Theory of the Term Structure of Interest Rate”, *Econometrica,* Vol. 53**,** No. 2, pp. 385-407.

7. Dacorogna M M, Gençay R, Müller U, Olsen R B and Pictet O V (2001), *An Introduction to High-Frequency Finance*, Academic Press, San Diego.

8. Di Matteo T, Aste T and Dacorogna M M (2006), “Long Term Memories of Developed and Emerging Markets: Using the Scaling Analysis to Characterize Their Stage of Development”, *Journal of Banking & Finance*, Vol. 29, No. 4, pp. 827-851.

9. Dunbar N (2000), *Inventing Money: The Story of Long-Term Capital Management and the Legends Behind It,* John Wiley & Sons, Chichester, UK.

10. Euler L (1765), “Theory of the Motions of Rigid Bodies”.

11. Farge M, Kevlahan N, Perrier V and Goirand É (1996), “Wavelets and Turbulence”, *Proceedings of IEEE*, Vol. 84, No. 4, pp. 639-669.

12. Heath D, Jarrow R and Morton A (1992), “Bond Pricing and the Term Structure of Interest Rates: A New Methodology”, *Econometrica,* Vol. 60**,** No. 1, pp. 77-105.

13. Hong Y (2001), “Dynamic Models of the Term Structure”, *Financial Analysts Journal*, Vol. 57, No. 4, pp. 60-76.

14. Hu (2001), *Financial Analysts Journal,* Vol. 57, July 4/August.

15. Hull John C (2001), *Fundamentals of Futures and Options Markets,* 4th Edition, Prentice Hall, Upper Saddle River, NJ.

16. Jamdee S and Los C A (2006), “Dynamic Risk Profile of the U.S. Term Structure by Wavelet MRA”, *International Research Journal of Finance and Economics*, Vol. 1, No. 5, pp. 19-47.

17. Jamdee S and Los C A (2007), “Long Memory Options: LM Evidence and Simulations”, *Research in International Business and Finance* , Vol. 21, No. 2, pp. 260-280.

18. Jorion P (1999), “How Long-Term Lost Its Capital”, *RISK,* September.

19. Karnosky D S and Singer B D (1994), “Global Asset Management and Performance Attribution”, *The Research Foundation of the Institute of Chartered Financial Analysts*, Charlottesville.

20. Karuppiah Jeyanthi and Cornelis A Los (2000), “Wavelet Multiresolution Analysis of High-Frequency FX Rates”, Quantitative Methods in Finance & Bernoulli Society 2000 Conference (Program, Abstracts and Papers), University of Technology, Sydney, December 5-8, pp. 171-98, Australia.

21. Koch Alexander and Lazarov Zdravetz (2002), Working Paper, Bonn University.

22. Kyaw N A, Los C A and Zong S (2006), "Persistence Characteristics of Latin American Financial Markets", *Journal of Multinational Financial Management*, Vol. 16, No. 3, pp. 269-290.

23. Landau L D and Lifshitz E M (1987), *Fluid Mechanics,* 2nd Edition, Pergamon Press, Oxford, UK.

24. Leow Cheng Boon (1999), "Optimal Global Investment Strategy", M.Sc. Thesis (under extra-mural supervision by Dr. Los), University of Durham, Durham UK.

25. Lipka J M and Los C A (2007), "Persistence Characteristics of European Stock Indices", *The Icfai Journal of Financial Risk Management*, Vol. 4, No. 4, pp. 13-40.

26. Los Cornelis A (1989), "The Prejudices of Least Squares, Principal Components and Common Factors", *Computers & Mathematics With Applications,* Vol. 17, Nos. 8 and 9, pp. 1269-1283.

27. Los Cornelis A (1998), "Optimal Multi-Currency Investment Strategies With Exact Attribution in Three Asian Countries", *Journal of Multinational Financial Management,* Vol. 8, Nos. 2 and 3, pp. 169-198.

28. Los Cornelis A (1999), "Comment on 'Combining Attribution Effects Over Time'", *The Journal of Performance Measurement,* Vol. 4, No. 1, pp. 5-6.

29. Los Cornelis A (2000a), "Frequency and Time Dependence of Financial Risk", *The Journal of Performance Measurement,* Vol. 5, No. 1, pp. 72-73.

30. Los Cornelis A (2000b), "Visualization of Chaos for Finance Majors", 2000 Finance Educators Conference: Finance Education in the New Millennium, Proceedings of the 2000 Annual Conference, Deakin University, Burwood, Victoria, November 30, pp. 187-226, Australia.

31. Los Cornelis A (2001), *Computational Finance: A Scientific Perspective,* World Scientific Publishing Co., Singapore.

32. Los C A (2002), "Optimal Asian Multi-Currency Strategy Portfolios With Exact Risk Attribution", in Batten Jonathan A and Thomas A Fetherston (Eds.), *Financial Risk and Financial Risk Management,* Vol. 16, pp. 215-260, Research in International Business and Finance, JAI Press, Inc.- Elsevier Science, Amsterdam, The Netherlands.

33. Los Cornells A (2003), *Financial Market Risk: Measurement and Analysis,* Routledge/ Francis & Taylor Group, London.

34. Los C A (2005), "Measurement of Financial Risk Persistence," *The Icfai Journal of Financial Risk Management,* Vol. 2, No. 3, pp. 7-33.

35. Los C A and Yu B (2008), "Persistence Characteristics of the Chinese Stock Markets", *International Review of Financial Analysis*, Vol. 17, No. 1, pp. 64-82

36. Mandelbrot B B (1982), *The Fractal Geometry of Nature*, W H Freeman, New York.

37. Mandelbrot B B (1997), *Fractals and Scaling in Finance: Discontinuity, Concentration, Risk*, Springer-Verlag, New York.

38. Mandelbrot B and Hudson R L (2004), *The (mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward*, Basic Books, New York.

39. Peters E (1989), "Fractal Structure in the Capital Markets", *Financial Analysts Journal*, Vol. 45, No. 4, pp. 32-37.

40. Peters Edgar E (1994), *Fractal Market Analysis,* John Wiley & Sons, New York.

41. Reynolds O (1883), "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and the Law of Resistance in Parallel Channels", *Philosophical Transactions of the Royal Society of London*, Vol. 174, pp. 935-982.

42. Taleb N M (2007), *The Black Swan: The Impact of the Highly Improbable*, Random House, New York.

43. Tritton D J (1988), *Physical Fluid Dynamics,* 2nd Edition, Clarendon, Oxford, UK.

44. Van Dyke M (1982), *An Album of Fluid Motion,* The Parabolic Press, Stanford, CA.

45. Vasicek O A (1977), "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics,* Vol. 5, No. 2, pp. 177-188.

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