Persistence Characteristics of European Stock Indices

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This paper measures the degrees of persistence of the daily returns of eight European stock market indices, after their lack of ergodicity and stationarity has been established. The proper identification of the nature of the persistence of financial time series forms a crucial step in deciding what kind of diffusion modeling of such series might provide invariant results. The results indicate that ergodicity and stationarity are very difficult to establish with only daily observations of market indices and thus, various price diffusion models cannot be successfully identified. However, the measured degrees of persistence point to the existence of long-term dependencies or Long Memory (LM), most likely of a non-linear nature. Global Hurst exponents, computed from wavelet multi-resolution analysis using a fractal Brownian motion model, measure the degree of persistence of the data series. The FTSE turns out to be anti-persistent, i.e., an ultra-efficient market with abnormally fast mean-reversion, faster than that of a geometric Brownian motion. The various measurement methodologies reported in the financial literature produce non-unique empirical results. Thus, it is very difficult to obtain definite conclusions regarding the presence or absence of long-term dependence phenomena based on the global Hurst exponents. Most stock markets in Europe appear to be slightly anti-persistent, but more powerful methods, such as the computation of the multifractal spectra of financial time series from intra-day pricing data, may be required to establish this as a definite scientific conclusion. Still, we demonstrate that the visualization of the wavelet resonance coefficients and their power spectra, in the form of localized scalograms and averaged scalegrams, forcefully assist the detection and measurement of several types of persistent market price diffusion.

Introduction

For a long time financial researchers have struggled with the identification of properly specified econometric and time series models that can capture the dynamic dependencies in financial time series. The most popular models, accounting for lagged observations, are the family of ARIMA models and GARCH models. Some models from these families have become very popular among technical analysts, due to their ability to capture short-term dependence. But these, often linear, models are criticized for not being able to model long-term dependence (also called persistence or Long Memory, or LM) and for requiring (and often presuming) Gaussian distribution characteristics for the innovation residuals.

Loretan and Phillips (1994) recognize that distribution characteristics of time series often vary over time. Cochrane (1988) already pointed out several weaknesses of the ARIMA models and suggested a measure for long-term dependence, like unit root integration I(1), because of the presence of approximate common factors in the AR and MA polynomials.

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A very common and easily recognizable weakness of the ARIMA and GARCH models is the requirement for the modeled residual series to be stationary. Although one can achieve apparent wide-sense stationarity of the financial series after several adjusted differencing transformations, it has become clear that such a linear operation cannot remove the time dependence in the series between far-distant observations. The remaining Auto-Covariance Functions (ACFs) of the squared errors just do not die out, which proves that persistence or LM is a non-linear phenomenon. This LM feature has also been observed in certain hydrological studies of long-range rivers and is also called the Hurst effect (Mandelbrot and Van Ness, 1968). A short discussion on such persistence or LM is provided, for example, in Mills (1999). In the past decade more studies have attempted to empirically measure such persistence phenomena in financial data series applying newer measurement technologies as used in signal processing.

Interestingly, Mandelbrot (1969, 1972) introduced the concept of persistence originally in a study of time series of economic and financial prices. With Fama he researched the resulting non-Gaussian distributions of financial prices. Once the concept of persistence in prices was accepted in the late 1970s, financial researchers searched for models that could properly identify such non-linear LM behavior. Hosking (1981) and Granger and Joyeux (1980) built on the prevalence of the well-known ARIMA models and proposed fractionally integrated ARMA models to measure LM. These models are more recently discussed in greater detail in Beran (1992), Baillie (1996) and Robinson (1994).

Empirical econometric studies of LM often rely on the study of Geweke and Porter-Hudak (1983), who proposed a method for the calculation of Hoskings fractional differencing parameter d, which is related to the Hurst exponent $H = d + 0.5$.[1]

The finding of persistence or LM in financial data might be in contradiction with the Efficient Markets Hypothesis of Fama (1970)—which is based on the assumption of martingale behavior of financial market prices—but only when it is modeled in a monofractal form. When it is modeled as a multifractal price diffusion process, there appears to be no contradiction. The martingale theory requires an invariant stationarity and independence of the innovations of the historical price information sets, but it is difficult to show that this requirement is met either in weak form or, even less so, in strong form. Peters'work (1994) on the Fractional Market Hypothesis is an application of a persistence concept that is broader and appears to encompass Famas theory of market efficiency.

The objective of this paper is to identify the dynamic diffusion models of several European equity indices, which were expected to show persistence or LM. This is done primarily to demonstrate that, even though in the econometric modeling literature one can find various models that fit such financial data apparently well, one cannot always rely on the conclusions about the properness of the identification of these models. For example, conventional econometric and time series modeling emphasizes only the

[1] The Porter-Hudak method is scientifically insufficient or incomplete, since, it relies on one unidirectional orthogonal or LS projection, while it should execute at least two orthogonal projections in a bivariate correlation situation, the 'ordinary regression' and its 'reverse regression' to assess the complete accuracy range of the computed differencing parameter (Los, 1999a).

measurement of the first two moments of the residuals, but tends to ignore the measurement of higher-order distributional moments. The reason is that, it presumes 'linearity in the model coefficients' and 'Gausianness of the modeling residuals'. Therefore, caution with respect to these 'statistically estimated', 'linearity-in-the-coefficients-plus Gausianness' models is recommended. It is more often than not presumed that the data meet the assumptions of the theoretical models, even though the data can be shown to show significant discrepancies from those basic assumptions (Los, 2001 and, in particular, 2003; also Kyaw *et al.,* 2006).

To shed some light on such empirical modeling inconsistencies, we will try to answer several questions about the dynamic character of stock market index prices for several European countries. Answering the following questions is crucial for performing further econometric and time series analysis of the daily price series, which are used for the valuation of and hedging by derivatives and, thus, for serious portfolio risk management. Our research questions are:

1. Are the pricing series or their innovations ergodic?
2. Are the pricing series or their innovations stationary? And if so, are they strict or wide-sense stationary?
3. Do the pricing series, after proper Taylor expansion type differencing, exhibit independence, short-term dependence, or long-term dependence (LM)?
4. If the pricing series exhibit long-term dependence, are they persistent or anti-persistent?
5. What are the theoretical benchmark models for the analyzed pricing series?
6. How far do the empirically identified dynamic price diffusion models deviate from theoretical benchmark models?

In the context of this paper, a more detailed discussion of the approach to question four might be worthy somewhat more elaboration, because of its unfamiliarity among financial analysts and econometric researchers. In this paper, the degree of global persistence is measured by computing the Hurst exponent from the results of wavelet multiresolution analysis (MRA) developed by signal processing engineers, such as Mallat (1989).

MRA is a powerful analysis that allows one to simultaneously analyze time series in both time and frequency domains. This feature is a consistent way to identify the time/frequency characteristics of any time series data. It is more powerful than the conventional Fourier frequency analysis, since, it also allows for the measurement and visualization of non-linear dependencies, and thus of dependencies other than the usual collinearities between integer lags, in the form of Fourier frequencies, which only measure simple linear dependencies.

A wavelet scalogram is a color-coded visualization of the field of measured wavelet resonance coefficients, i.e., of the squared bivariate correlation coefficients (= coefficients of determination) between the time series and the wavelet tiling bases chosen for the analysis. It allows one to detect and identify discontinuities or dirac delta like shocks in financial markets and to measure price diffusion models and their localized frequency power or risk before and after such shocks. An example of such advanced stock market crash analysis is Los and Yalamova (2006).

The average of wavelet scalograms over time can be graphically represented by the logarithm of the average power spectrum of the financial time series, i.e., a so-called scalegram. In combination with the scalogram, such an averaged scalegram allows one to investigate also the autocorrelation function of the financial time series in the conventional Fourier-type frequency dimension and thus to identify possible periodicities but also cyclicities (= uncertain or aperiodic 'periodicities') in the stock indices. Such non-linear, time-varying phenomena cannot be easily viewed when the linear statistical methodologies of classical time series analysis are used.

Persistence (LM, or Long-Term Dependence)

One of the first finance researchers who formally recognized persistence (LM) in financial economic data was Mandelbrot (1969, 1972). By doing so, he launched a search for the proper model identification to account for this non-linear phenomenon. Granger and Joyeux (1980), and Hosking (1981) developed a method of determining LM with fractionally integrated ARMA or ARFIMA models.

Geweke and Porter-Hudak (1983) proposed calculating the fractionally differencing parameter *d* that allows one to determine the degree of LM. Beran (1992), Baillie (1996) and Robinson (1994) review such LM models and their applications. Ding *et al.* (1993) focus on the detection of the LM process in the second moments, which are of importance to financial risk analysis and management. Baillie *et al.* (1996) capture LM with their newly introduced class of Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic (FIGARCH) processes by applying it to daily Deutschmark-US dollar exchange rates.

Baillie *et al.* (1996) use the FIGARCH process to model financial market volatility, in particular in the foreign exchange markets, and assert, not completely convincingly, that a mean-reverting fractionally integrated process is superior in characterizing the volatility than any other financial market model. In a slightly different approach, Crato and de Lima (1994) find LM or persistent stochastic volatility in high-frequency stock market data.

All these LM identification results appear to depend very much on the particular analytic methodology used and therefore call into question how and if these analytic methods can really identify the dynamics of stock markets in a scientifically unique fashion.

For example, Green and Fielitz (1977) and Aydogan and Booth (1988) apply in their studies the R/S (range-over-scale) metric of Hurst (1951) to test the degree or LM in common stock returns. But, then Lo (1991) uses the modified rescaled range statistic for value and equal weighted CRSP index returns and finds that even though the Hurst re-scaled range statistic of Mandelbrot detects the existence of the long-memory in the data, his modified Hurst statistic rejects LM. Moreover, Lo also cannot find LM in annual returns for a long period from 1872 until 1986. But then his results appear to be very exceptional in the recently burgeoning financial market literature on LM. The reason may be that Los particular modification incorporates only the linear research technology of collinearity analysis, which, per definition, cannot detect the non-linear phenomenon of LM.

The proper detection and model identification of LM processes has crucial implications for the measurement of the efficiency, levels of risk and liquidity of the financial markets. If persistence or LM is confirmed in asset prices, then one can question the existence of the weakest form of efficiency, not to mention of other forms of financial market efficiency. Los (2000) and by Sadique and Silvapulle (2001) brought this particular issue into sharp focus. Los (1999b) used first non-parametric efficiency tests of markets of Hong Kong, Indonesia, Malaysia, Singapore, Taiwan, and Thailand and rejected their efficiency on the basis of proven lack of stationarity and independence of the time series innovations.

Sadique and Silvapulle (2001) looked specifically for long memory process in the stock market returns of Japan, Korea, New Zealand, Malaysia, Singapore, the USA and Australia, with the help of classical and modified rescaled range tests, the semi-parametric test proposed by Geweke and Porter-Hudak (1983), the frequency domain score test proposed by Robinson (1995a) and its time-domain counterpart derived by Silvapulle. Their methodologically far-ranging study finds LM in stock market returns in Korea, Malaysia, Singapore and New Zealand.

These early empirical results of Los (2000) and of Sadique and Silvapulle (2001) are in contradiction with the results of Cheung (1995), who did not find a persuasive support for the stock returns of 18 countries of Asia, Europe, and North America using the classical linear correlation and unidirectional projection techniques. Again, the problem of Cheungs study is that it uses a linear analytic method attempting to detect and identify the non-linear LM phenomenon. This is trying to measure a round object with a square box. Perhaps, the use of triangular measurement boxes would have shown progress. After all, famous geometers, called ‘pundits’, have used triangulation to measure uneven geographic objects such as the high-altitude mountain ranges on the border between India, Kashmir, Nepal and Tibet.

An analytic method used must be in tune with the non-linear phenomenon studied and wavelets have shown to be accomplishing such non-linear measurements and identification in the time-frequency domain with great ease and accuracy.

Thus, given the lack of agreement on the existence of long-term memory process in stock returns using the classical econometric correlation and projection techniques, it is necessary to study this non-linear LM phenomenon further using more powerful methodologies and technologies, as proposed and discussed by Gençay *et al.* (2002) and Los (2003).

In particular, Los (2003), reviews, in great detail, and in language understandable to financial and economic researchers, the latest wavelet MRA time-frequency signal processing methodologies to detect and measure financial market persistence, including the measurement not only of homogeneous or global Hurst exponents, but also of the multifractal spectra of the (more general) Lipschitz α metrics identifiable from financial market price series.

Indeed, most recently a very diverse group of mathematical financial researchers have come to the same conclusion as Los (2003), that financial time-series are actually multifractal, or more precisely, must be modeled by multifractal price diffusion models. Elliott and Van der Hoek (2003) came theoretically to that conclusion, and Calvet and Fisher (2002) and Mandelbrot and Hudson (2004) empirically. These authors agree that, in first instance, the existence persistence or LM can be properly identified by measuring the value of the monofractal Hurst exponent of a simple fractional Brownian motion *(cf.* Los, 2003; Kyaw *et al.,* 2006), although the process should be more accurately modeled by a multifractal price diffusion process to be theoretically arbitrage-neutral.

Data

The data used in this paper are daily deviations on eight European stock market indices and their various simple transformations. Detailed information concerning these indices is presented in Tables 1 and 2. The time period for the series varies from index to index due to limited data availability.

Table 1: The Stock Market Price Index Series Analyzed in this Paper

Country	Index	Index Yahoo Symbol	Symbol Used in the Project	Daily Data Range	Number of Observations
Austria	ATX-Index (Vienna)	ATX	ATX	11 Nov 92–23 Oct 00	2235
Denmark	KFX-Index (Copenhagen)	KFX	KFX	26 Jan 93–23 Oct 00	2194
France	CAC 40 Index (Paris)	FCHI	FCHI or CAC or CAC 40	1 Mar 90–23 Oct 00	2918
Germany	XETRA DAX Index	GDAXI	GDAXI or DAX	26 Nov 90–23 Oct 00	2737
Norway	Oslo Total Index	NTOT	NTOT or TOTX	1 July 97–23 Oct 00	1065
Spain	IBEX 35 Index (Barcelona)	IBEX	IBEX	9 Sept 97–23 Oct 00	435
Spain	Madrid GEN Index	SMSI	SMSI	29 Apr 99–23 Oct 00	550
United Kingdom	FTSE 100 Index (London)	FTSE	FTSE	2 Apr 84–23 Oct 00	4437

Table 2: Description of the Stock Market Price Indices Analyzed	
Country	**Details**
ATX (Austria)	
Long Name	Austrian Traded Index
Owner/publisher/sponsor	Wiener Börse AG (Vienna Stock Exchange)
Constituents	22 Austrian companies continuously traded on the Vienna Stock Exchange
Construction principle	Capitalization-weighted value ratio
Base date	January 2, 1991
Base value	1,000.00
Interval of calculation	Real time
KFX (Denmark)	
Long Name	Københavns Fondsbørs Index (Copenhagen Stock Exchange Index)
Owner/publisher/sponsor	Københavns Fondsbørs AS (Copenhagen Stock Exchange)
Constituents	21 Danish companies
Construction principle	Capitalization-weighted value ratio
Base date	July 3, 1989
Base value	100.00
Interval of calculation	one minute
CAC-40 (France)	
Long Name	Compagnie des Agents de Change 40 Index
Owner/publisher/sponsor	Société des Bourses Françaises (SBF)-Bourse de Paris (Association of French Stock Exchanges-Paris Stock Exchange)
Constituents	40 French companies listed on the Paris Stock Exchange that are also traded on the options market
Construction principle	Capitalization-weighted value ratio
Base date	December 31, 1987
Base value	1,000.00
Interval of calculation	30 seconds
DAX (Germany)	
Long Name	Deutscher Aktienindex DAX
Owner/publisher/sponsor	Deutsche Börse Group (German Stock Exchange)
Constituents	30 German companies
Construction principle	Capitalization-weighted total return Laspeyres index
Base date	December 30, 1987
Base value	1,000.00
Interval of calculation	one minute
Total Index (Norway)	
Long Name	Oslo Bors Total Index
Owner/publisher/sponsor	Oslo Bors
Number of constituents	All stocks registered on the Main List of the Oslo Stock Exchange
Construction principle	Capitalization-weighted total return value ratio
Base date	January 1, 1983
Base value	100.00
Interval of calculation	one minute

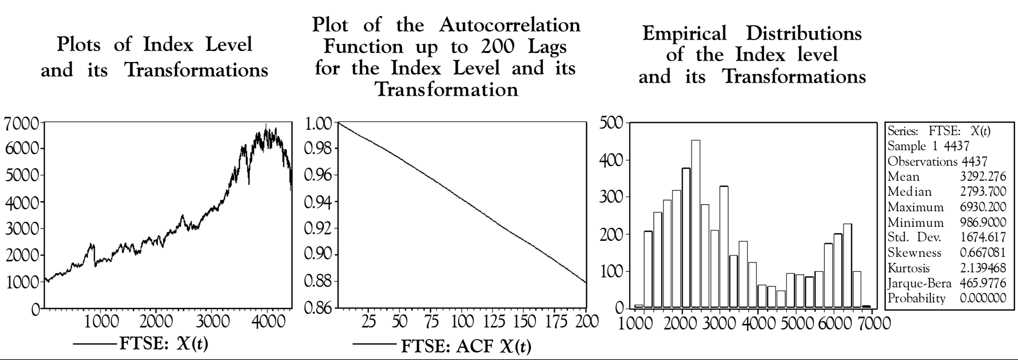
(Contd...)

Table 2: Description of the Stock Market Price Indices Analyzed *(...contd)*	
Country	**Details**
IBEX 35 (Spain)	
Long Name	IBEX 35
Owner/publisher/sponsor	Association of Stock Exchanges (Sociedad de Bolsas SA)
Constituents	35 Spanish companies
Construction principle	Capitalization-weighted value ratio
Base date	December 29, 1989
Base value	3000.00
Interval of calculation	Real time
FTSE 100 (UK)	
Long Name	Financial Times Stock Exchange 100 Index
Owner/publisher/sponsor	FTSE International Limited
Constituents	Shares of the top 100 UK companies ranked by market capitalization
Construction principle	Capitalization-weighted value ratio
Base date	December 31, 1983
Base value	1,000.00
Interval of calculation	one minute

Source: http://www.finix.at/-

We analyze various transformations of the stock market indices: the index levels, $X(t)$, logarithms of index levels, In $X(t)$, differenced index levels $D[X(t)]$, differenced logarithm of index levels, $x(t) = D[ln\{X(t)\}]$, which are the stock market returns, and differenced returns, $D[x;(t)]$. These transformations of index levels and their returns are then analyzed to find out whether the applied transformations allow one to find desirable properties of the data, such as stationarity and ergodicity. Also graphs are provided supporting the conclusions of our study.[2] For the sake of brevity of this paper, we focus most of the discussion on the FTSE (Figure 1).

Figure 1: Plots of the FTSE Stock Market Price Index Level and its Transformation, Autocorrelations up to 200 Lags and Empirical Distributions



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[2] The graphs included in this paper are only for the FTSE series, due to space limitations. The graphs for all other stock market series studied in this paper are available upon request from the non-corresponding author Lipka.

Figure 1: Plots of the FTSE Stock Market Price Index Level and its Transformation, Autocorrelations up to 200 Lags and Empirical Distributions

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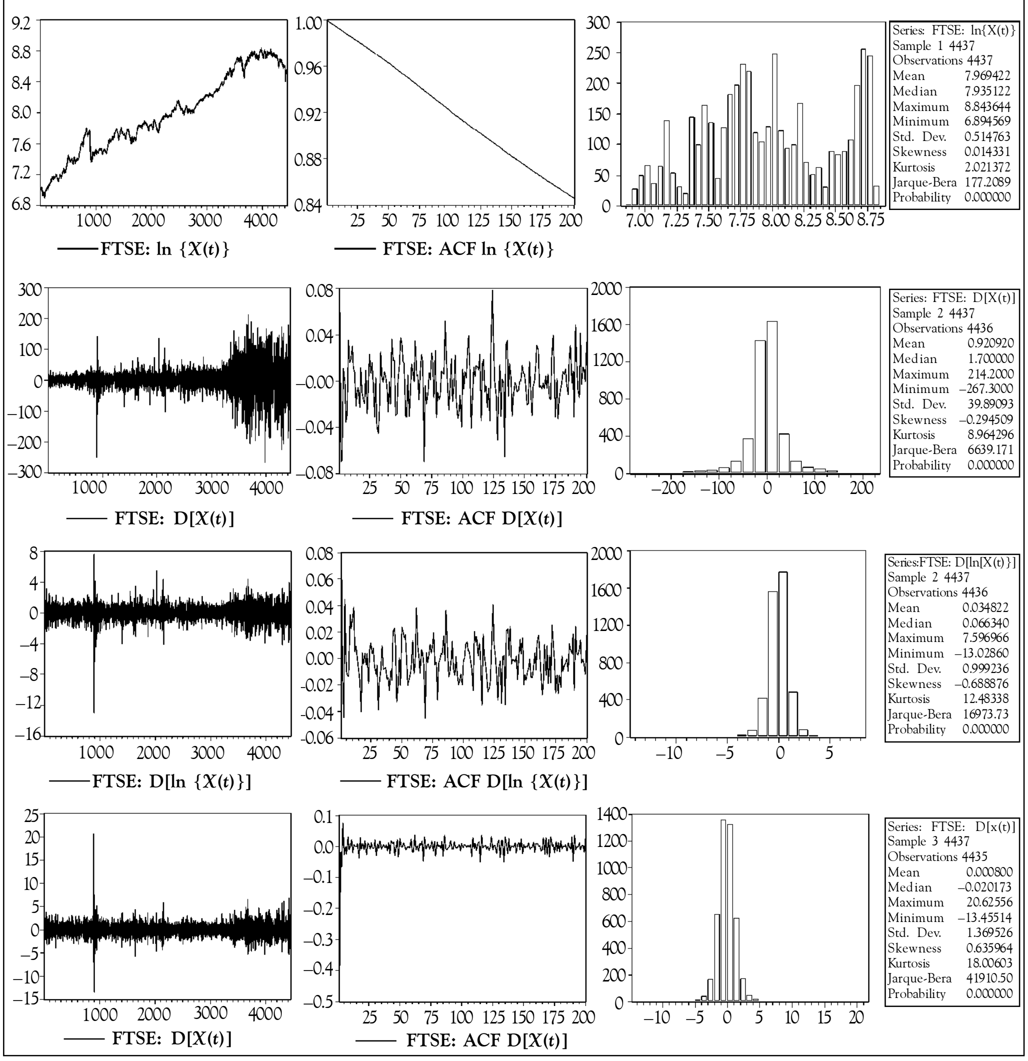

Methodology and Empirical Identification

Ergodicity, Stationarity and Independence

Definition 1: *Ergodicity is defined by Mills (1999, p. 9) as "... the process is ergodic, which roughly means that the sample moments for finite stretches of the realization approach their population counterparts as the length of the realization becomes infinite".*

Mills erroneously remarks that it is impossible to test for the ergodicity of time series using only one realization and thus he assumes that all time series have this property.[3]

[3] Our emphasis!

Obviously, one cannot have more than one historical realization of any time series. However, one can use time-ordered sample drawings of data from the available historical time series as substitutes for various length increasing realizations of a given population or infinite sample. Thus, moments for the five time series for each index using time windows of increasing size are computed and then these computed moments are plotted against their window length. One can then visually inspect whether the plots gradually converge to a flat line, which would suggest ergodicity of the time series.

In neither of our data series such gradual convergence to a flat time line is observed. Several sharp discontinuities and shifts occur, which is an indication of fractality in the time series, and no convergence points appear to exist. Thus, at least visually, it is reasonable to conclude that these series are not ergodic. Of course, it is an empirical scientific question, what realization is long enough to decide that the estimated moments can be relied on to make conclusions about ergodic property of a time series. Therefore, several time window sizes were tried. None provides results that even remotely could suggest ergodicity of these stock market time series. As an example of the increasing-window methodology, the first four moments of the analyzed FTSE in are plotted in Figure 2. This lack of visual ergodicity suggests that the usual procedure of using time moments as substitutes for ensemble moments is visually falsified.

Figure 2: Increasing Window Moments for FTSE Index Level

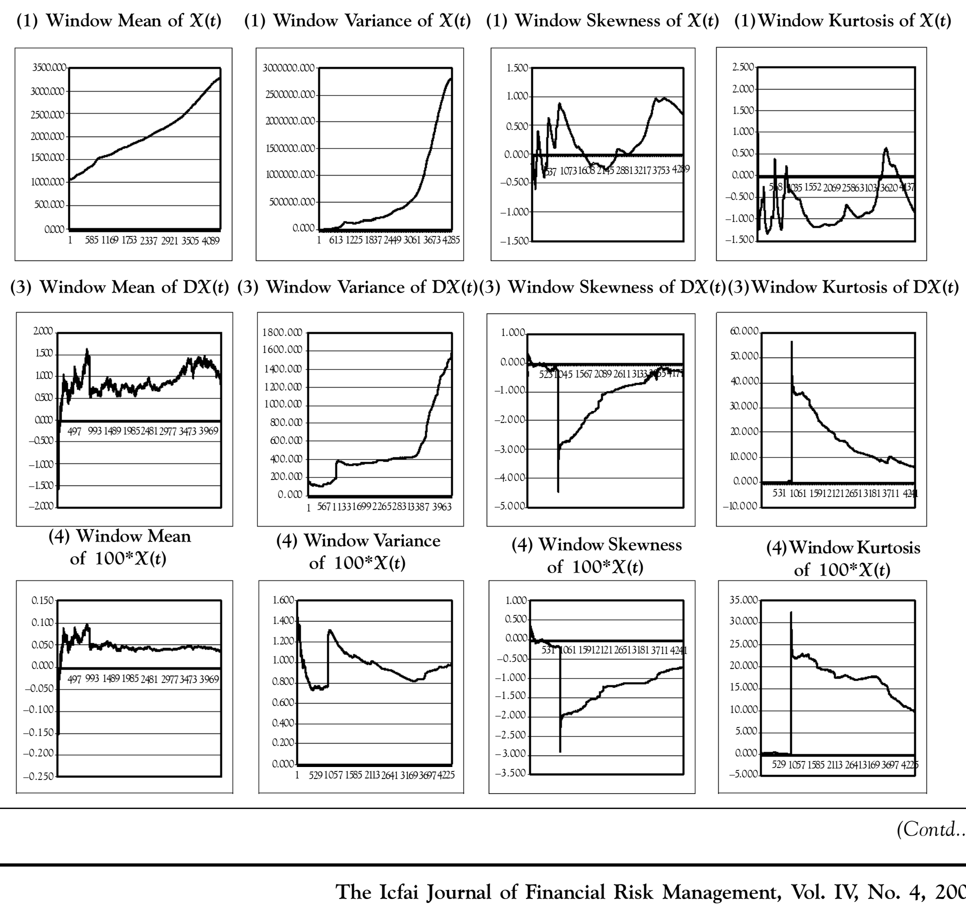

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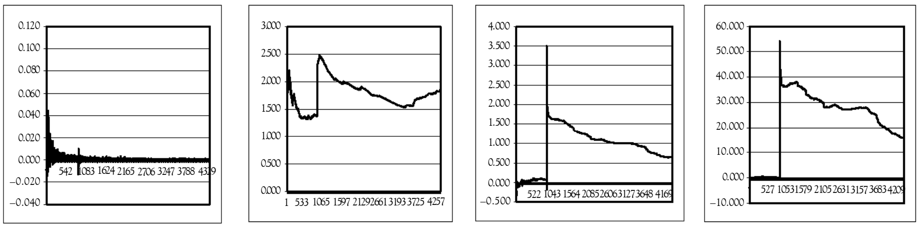
Figure 2: Increasing Window Moments for FTSE Index Level

(...contd)

(5) Window Mean of 100*D$X(t)$	(5) Window Variance of 100*D$X(t)$	(5) Window Skewness of 100*D$X(t)$	(5) Window Kurtosis of 100*D$X(t)$



Definition 2: *A time series is said to be strictly stationary if the joint distribution of any set of n observations* $X_{t_1}, X_{t_2}, ..., X_{t_n}$ *is the same as the joint distribution of* $X_{t_{1+k}}, X_{t_{2+k}}, ..., X_{t_{n+k}}$ *for all n and k.*

Strict stationarity is difficult to observe in financial time series data, because we would have to compute an infinite set of moments, since for an unknown distribution it is unknown how many moments exist. Thus, the strict stationarity assumption is often relaxed to weak stationarity.

Definition 3: A *time series is said to be weakly stationary if its first moment or mean is constant and its second moment or ACF depends only on the time lags.*

In order to test for wide sense stationarity with an expansion to third and fourth moments, rolling windows are computed for the first four moments of all stock market indices and their transformations. As a representative example of the plots for these four rolling-window moments, the moments for FTSE are plotted in Figure 3. In neither case constant moments are observed. Again sharp shifts occur in the rolling-window moments, an indication of the possible fractality of the time series. Thus, we conclude that our series are neither strict-sense nor wide-sense stationary.

Figure 3: Plots of Moving Moments (50 Observations Window) for FTSE Index Level and its Transformations

(1) Moving Mean of $X(t)$	(1) Moving Variance of $X(t)$	(1) Moving Skewness of $X(t)$	(1) Moving Kurtosis of $X(t)$

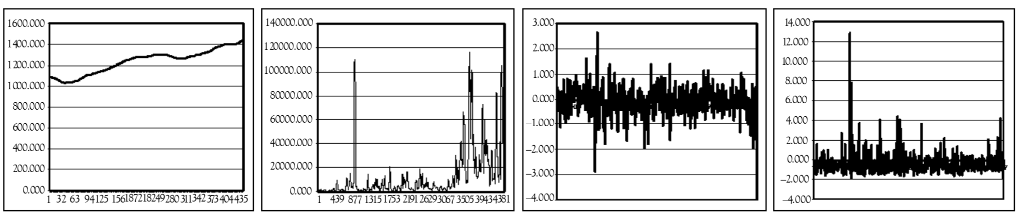


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Figure 3: Plots of Moving Moments (50 Observations Window) for FTSE Index Level and its Transformations

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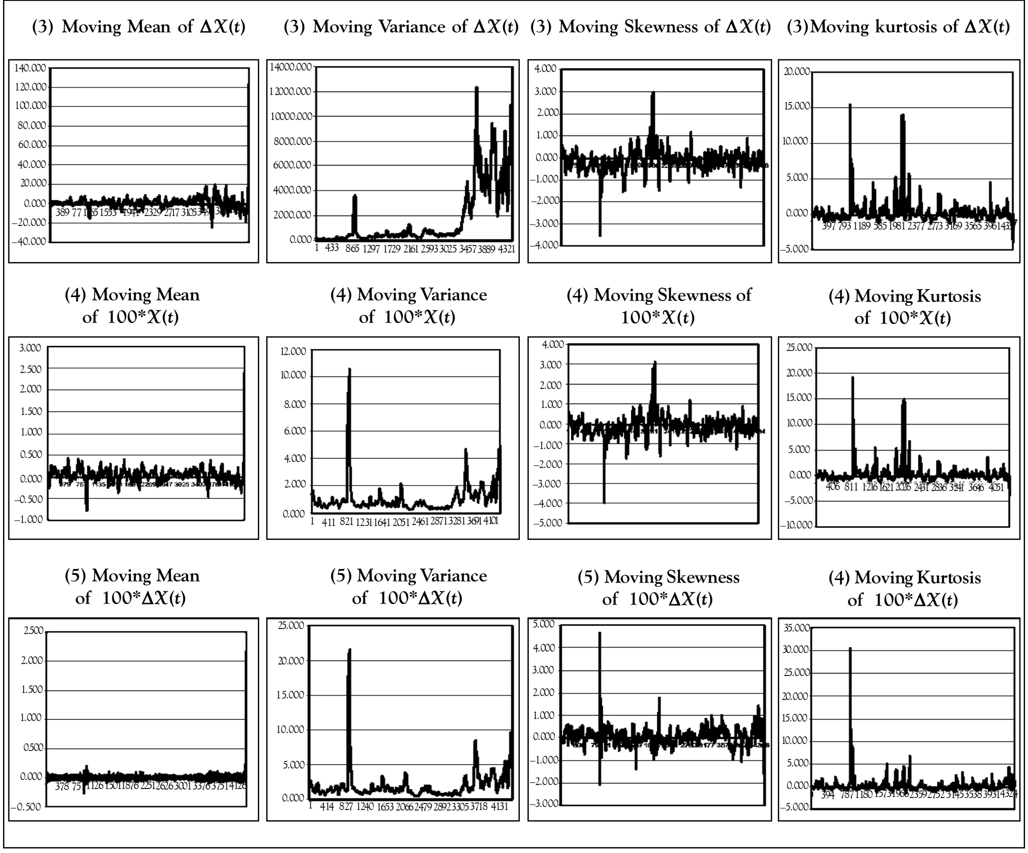


Definition 4: *The elements of a time series are independent if the autocorrelation function of this series equals one for the lag equal to zero and zero for any lag different from zero.*

Definition 5: *A time series is persistent or shows Long Memory (LM) if the autocorrelation function for the series decays at some hyperbolic rate.*

The decay at a hyperbolic rate is much slower than the decay at the geometric rate. To investigate the nature of the dependence of the time series, one can thus visually inspect the autocorrelation function of each of the five series. The autocorrelation functions, or ACFs of the five stock market indices, and their transformations were computed and inspected up to 200 lags.

Table 3 shows that the behavior of the autocorrelation function varies for each of the five series. For prices, logarithms of prices, and differences of returns, one can easily detect short-term dependence, because the autocorrelation function takes significant values for initial lags. For example, for one lag the absolute values of the ACFs vary from 0.441

up to 0.999 for these series. ACFs for the differences of prices and for returns are much smaller than for other series and their absolute values vary between 0.036 and 0.176. For the price levels and for the logarithmic transformations of prices, the autocorrelation function has a clear pattern, and dies off very slowly, thereby suggesting, but not definitely confirming the existence of LM.

Table 3: Autocorrelation Function Values for One Lag and for Two Hundred Lags

	$X(t)$	$\ln\{X(t)\}$	$D[X(t)]$	$D[\ln\{X(t)\}]$	$D[x(t)]$
ACF for 1 Lag					
ATX	0.996	0.996	0.076	0.084	–0.441
KFX	0.999	0.999	–0.176	–0.157	–0.590
CAC	0.999	0.999	0.039	0.040	–0.466
DAX	0.999	0.999	0.036	0.040	–0.471
TOTX	0.993	0.993	0.049	0.047	–0.464
IBEX	0.999	0.973	0.079	0.097	–0.477
SMSI	0.984	0.984	–0.079	–0.086	–0.442
FTSE	0.999	0.999	0.060	0.060	–0.462
ACF for 200 Lags					
	$X(t)$	$\ln\{X(t)\}$	$D[X(t)]$	$D[\ln\{X(t)\}]$	$D[x(t)]$
ATX	0.328	0.290	0.010	0.004	0.024
KFX	0.711	0.735	–0.006	0.001	0.005
CAC	0.796	0.818	0.020	0.014	0.014
DAX	0.804	0.826	–0.004	0.006	–0.010
TOTX	0.002	–0.008	–0.036	–0.032	–0.048
IBEX	0.879	–0.125	–0.067	–0.067	–0.053
SMSI	–0.187	–0.166	0.003	0.002	–0.014
FTSE	0.879	0.846	0.033	0.018	0.034

However, an empirical question is for how many lags the autocorrelation function should be different from zero in order to undoubtedly admit the existence of LM. Assuming that 200 days is a long enough period to determine the long-term dependence for our daily data, large values for ACFs for 200 lags for prices and logarithms of prices suggest that one can detect LM in these series. For other transformations of prices, like for the differences of prices, returns, and differences of returns, there are no clear patterns in the autocorrelation functions. The plots oscillate around zero without visible decline in the amplitudes of the ACF.

The nature of the ACF functions for the studied series is indicated in Table 4, which reports maximum, minimum, and the max-min ranges for the various ACFs.

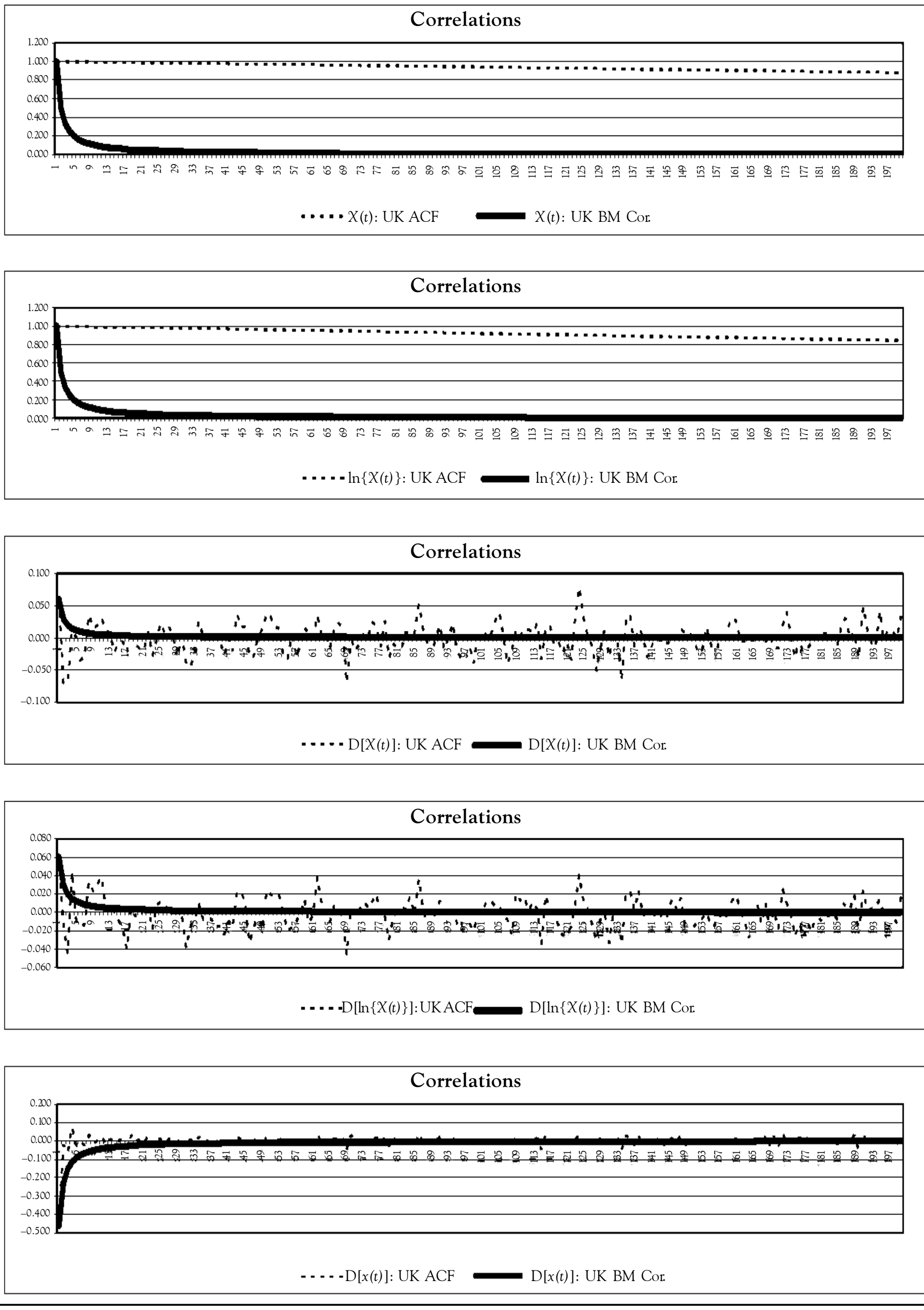
The minimum and maximum values constitute a 'bandwidth' for the ACF. Because the autocorrelation function is a decreasing function for prices and for logarithms of prices for the first 200 lags, small values of the difference between the maximum and minimum for ACFs suggest a stronger degree of persistence, and large values for the difference between maximum and minimum for ACFs suggest a weaker degree of persistence.

One can thus, see that the strongest persistence in prices occurs for the FTSE and the weakest persistence for prices and for logarithm of prices occurs for the SMSI. The differences between maximum and minimum for ACFs for the differences of prices, returns, and differences of returns are very small. However, they remain different from zero. Because the plot for these series is rather rough, with visible positive and negative spikes occurring at different lags, this suggests that these series retain weak LM. Examples of plots of the autocorrelation functions are provided in Figures 4 and 5.

Table 4: Maximum, Minimum and Maximum-Minimum Range for ACFs for the Analyzed Stock Market Price Index Series

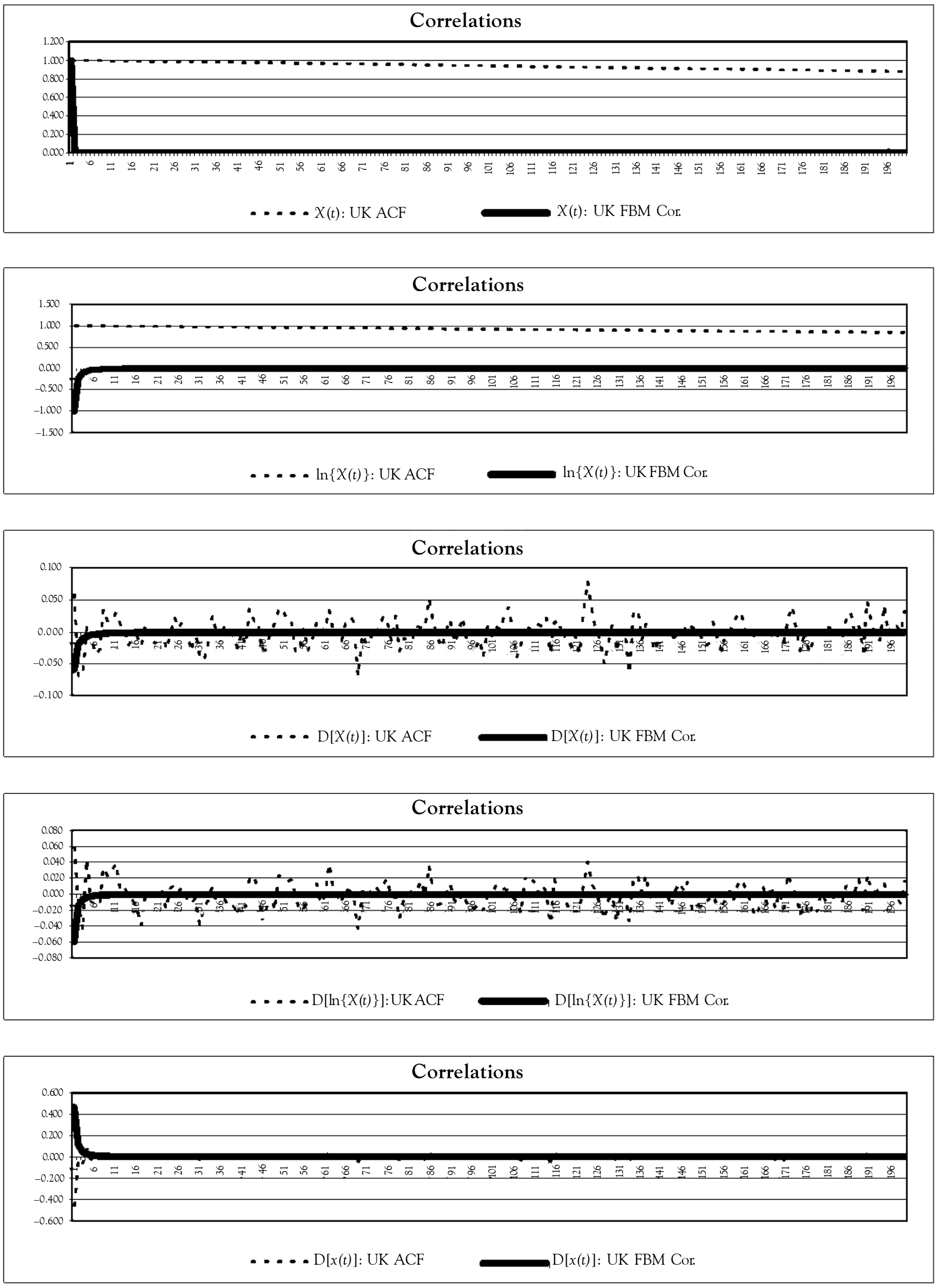
	$X(t)$ Max.	$X(t)$ Min.	$X(t)$ Max.–Min.	$ln\{X(t)\}$ Max.	$ln\{X(t)\}$ Min.	$ln\{X(t)\}$ Max.–Min.	$D[X(t)]$ Max.	$D[X(t)]$ Min.	$D[X(t)]$ Max.–Min.	$D[ln\{X(t)\}]$ Max.	$D[ln\{X(t)\}]$ Min.	$D[ln\{X(t)\}]$ Max.–Min.	$D[X(t)]$ Max.	$D[X(t)]$ Min.	$D[X(t)]$ Max.–Min.
ATX	0.996	0.328	0.668	0.996	0.290	0.706	0.097	–0.067	0.164	0.087	–0.059	0.146	0.090	–0.441	0.531
KFX	0.999	0.711	0.288	0.999	0.735	0.264	0.082	–0.176	0.258	0.084	–0.157	0.241	0.109	–0.590	0.699
CAC	0.999	0.796	0.203	0.999	0.818	0.181	0.076	–0.080	0.156	0.057	–0.050	0.107	0.066	–0.466	0.532
DAX	0.999	0.804	0.195	0.999	0.826	0.173	0.093	–0.068	0.161	0.059	–0.054	0.113	0.070	–0.471	0.541
TOTX	0.993	0.002	0.991	0.993	–0.008	1.001	0.083	–0.075	0.158	0.085	–0.085	0.170	0.090	–0.464	0.554
IBEX	0.999	0.879	0.120	0.973	–0.125	1.098	0.088	–0.277	0.365	0.097	–0.237	0.335	0.120	–0.477	0.597
SMSI	0.984	–0.187	1.171	0.984	–0.166	1.150	0.105	–0.079	0.184	0.089	–0.086	0.174	0.111	–0.442	0.553
FTSE	0.999	0.879	0.120	0.999	0.846	0.153	0.078	–0.070	0.148	0.060	–0.045	0.105	0.075	–0.462	0.537

Figure 4: ACF Functions for Five Series, $X(t)$, $\ln\{X(t)\}$, $D[X(t)]$, $D[\ln\{X(t)\}]$ and $D[x(t)]$ (Where D = Differentiation Operator) for FTSE Index and ACF Functions for Geometric Brownian Motion



Note: In the legends provided under the figures ACF stands for empirical correlation and FBM stands for autocorrelation that would exist if the data would be generated by geometric Brownian motions.

Figure 5: ACF Functions for Five Series, $X(t)$, $\ln\{X(t)\}$, $D[X(t)]$, $D[\ln\{X(t)\}]$ and $D[x(t)]$, for FTSE Index and ACF Functions for Fractal Brownian Motion



Note: In the legends provided under the figures ACF stands for empirical correlation and FBM stands for autocorrelation that would exist if the data would be fractal Brownian motion.

Persistence or LM

Based on the visual inspection of the ACF function, the series appear to exhibit LM, and therefore can be better modeled by a Fractal Brownian Motion (FBM) then by the conventional geometric Brownian motion. For the model of fractal Brownian motion, the Hurst exponents, H, are computed for all the series, in order to determine the degree if their persistence. The three manifestations of LM are anti-persistence, when $0 < H < 0.5$, white (independent) noise when $H = 0.5$, and persistence when $0.5 < H < 1$.

We computed the Hurst exponent H by seven different methods and summarized the results in Tables 5 and 6: (1) R/S Analysis Method (R/S), (2) Power-Spectral Analysis Method (P-S), (3) Roughness-Length Relationship Method (R-L), and (4) Variogram Method (V), (5) the method proposed by developers of the IDL Wavelet Toolkit software, (6) the method developed by Veitch and Arby of the University of Melbourne, and (7) the method proposed by developers of FracLab software.

Table 5: Hurst Exponents Identified for the Analyzed Stock Market Price Index Series

	IDL Wavelet Toolkit	D Veitch and P Abry Procedure	Frac-Lab Software	R/S (Benoit Software)	P-S (Benoit Software)	R-L (Benoit Software)	V (Benoit Software)
Index							
Austria: ATX	0.42	0.48	0.47	0.55	0.50	0.55	0.47
Denmark: KFX	0.55	0.28	0.40	0.50	0.51	0.41	0.52
France: CAC 40	0.46	0.41	0.46	0.51	0.50	0.44	0.56
Germany: DAX	0.47	0.43	0.43	0.51	0.54	0.44	0.52
Norway: TOTX	0.45	0.49	0.50	0.53	0.52	0.49	0.53
Spain: IBEX	0.46	0.46	0.39	0.46	0.51	0.51	0.40
Spain: SMSI	0.48	0.46	0.23	0.41	0.47	0.36	0.46
UK: FTSE	0.33	0.41	0.44	0.51	0.53	0.45	0.49

Table 6: Long-Term Dependence of the Series

	IDL Wavelet Toolkit	D Veitch and P Abry Procedure	Frac-Lab Software	R/S (Benoit Software)	P-S (Benoit Software)	R-L (Benoit Software)	V (Benoit Software)
Index							
Austria: ATX	AP	AP	AP	P	WN	P	AP
Denmark: KFX	P	AP	AP	WN	P	AP	P
France: CAC 40	AP	AP	AP	P	WN	AP	P
Germany: DAX	AP	AP	AP	P	P	AP	P
Norway: TOTX	AP	AP	WN	P	P	AP	P
Spain: IBEX	AP	AP	AP	AP	P	P	AP
Spain: SMSI	AP	AP	AP	AP	AP	AP	AP
UK: FTSE	AP	AP	AP	P	P	AP	AP

Note: P = persistence; AP = anti-persistence, WN = white noise.

Table 7 reports even more extensive results obtained from a procedure developed by Veitch and Arby.

Table 7: Homogeneous Hurst Exponents Identified Using Veitch and Arby

	Goodness of Fit (Probability of Data Assuming Linear Regression)	Scaling Parameter Alpha (LRD) (slope of log-log plot)	Scaling Parameter H	Scaling Parameter D (Fractal Dimension, if Alpha in (1,3)
Austria: ATX	0.00314	1.950	0.475	1.525
CI's:		[1.880,2.019]	[0.440,0.510]	[1.490,1.560]
Denmark: KFX	0.01618	1.567	0.283	1.717
CI's:		[1.496,1.637]	[0.248,0.319]	[1.681,1.752]
France: CAC 40	0.99767	1.814	0.407	1.593
CI's:		[1.755,1.874]	[0.377,0.437]	[1.563,1.623]
Germany :DAX	0.25144	1.853	0.427	1.573
CI's:		[1.791, 1.915]	[0.396, 0.457]	[1.543, 1.604]
Norway: TOTX	0.12103	1.985	0.493	1.507
CI's:		[1.877,2.094]	[0.438, 0.547]	[1.453, 1.562]
Spain: IBEX	0.52905	1.92	0.46	1.54
CI's:		[1.715,2.124]	[0.358, 0.562]	[1.438, 1.642]
Spain: SMSI	0.04523	1.927	0.464	1.536
CI's:		[1.758,2.097]	[0.379, 0.548]	[1.452, 1.621]
U1UK: FTSE	0.00567	1.821	0.411	1.589
CI's:		[1.774, 1.868]	[0.387, 0.434]	[1.566, 1.613]

Table 7 presents the identified homogeneous Hurst exponents of the stock indices. The parameters were obtained with the LDestimate function developed by Veitch and Abry of The University of Melbourne. The LDestimate function estimates the parameter of the long-range dependence process (LRD), the scalegrams spectrum slope parameter α, from which the Hurst exponent and the fractal dimension D can be computed, using the wavelet-scalegram-based LS estimation of Abry and Veitch. The original method of Abry and Veitch uses a (downward biased) uni-directional orthogonal projection (= LS regression). However the biases are very small as we found when we also did also the reverse unidirectional orthogonal projections (= the 'reverse regressions'). For α we took the mid-point of the resulting scaling parameter α ranges, given by the CI ranges of the α (= 'confidence' or, more precisely, computed accuracy intervals). The lower figure of the CI range is the computed downward biased α, the upper figure is the computed upward biased α. For a further explanation, *cf.* Los, 2001, Chapter 4, pages 57-73. The relationship between the slope of the power spectrum α and the Hurst exponent H is as follows: $\alpha = (2H + 1)$, from which we find $H = \alpha - 1/2$. $H = 0.50$ indicates that market prices follow a geometric Brownian motion (log-normal price diffusion), while $0.5 < H < 1$

indicates that the market prices are persistent and $0 < H < 0.5$ indicates that the market prices are anti-persistent. The fractal dimension $D = 2-\alpha$. The mathematical relationships between the Hurst exponent H, the fractal dimension D and the power spectrum slope a are explained in Los, 2003, Table 4.3, p. 124.[4]

Based on the calculated Hurst exponents, we find that stock market index series might be either persistent, P, or anti-persistent, AP, or white noise WN. The degree of persistence clearly depends on the particular stock market. In order to draw any general conclusion about the data, one might decide that the series has a given property, if the majority of the proposed methods of analysis identify the given property. Thus, based on the results in Table 6, the ATX, CAC 40, DAX, IBEX, SMSI, and FTSE prices appear persistent.

In case of KFX and TOTX, however, the results remain inconclusive. If one requires that all methods allow for the same unique conclusion about the nature of long-term dependence, for stock market in general, then the empirical results remain inconclusive. It would clearly be of no added value to require some sort of 'significance' criterion, since each of the methods has different residual noise characteristics, because of the different projections involved, although now an accuracy criterion has been found for the Hurst exponent H after this paper was originally researched (*Cf.* Kyaw et al., 2006).

Our conclusion remains that the degree of the measured persistence depends on the particular stock market. Some stock markets are anti-persistent and are thus ultra-efficient. Some stock markets show independent innovations, and thus are efficient in the traditional Samuelson definition of efficiency.

But some stock markets are persistent and thus inefficient and could be even dangerous: long periods of calmness in pricing may be disrupted by sudden and large discontinuities and draw-downs. Such differences in the degrees of persistence between the various financial markets are probably caused by the differences in their institutional organization. More research is now needed between the measured degrees of persistence of trading in financial markets and their respective degrees of freedom of trading as determined by their institutional arrangements.

Because in modeling of financial series the idea has always been accepted to test whether well-established models can fit the data, this paper also examines whether the European indices can be identified by some theoretical models available in the theoretical financial literature. The theoretical models that can be used to compare with the empirical time series are the random walk model, the geometric Brownian motion model and the fractional Brownian motion model. These models are defined in the following way (Los, 2003):

Definition 6: *A random walk model is a particular wide sense Markov or unit root process of the original variables with independent innovations:*

$X(t)-X(t-1) = (1-L)X(t) = e(t)$,

where $e(t) \sim i.i.d.(0,\sigma_e^2)$ *and L is the one-period lag operator.*

[4] The slope parameter α of the scalegram spectrum of Abry and Veitch is not to be confused with either the Lipschitz α_L or the stability α_Z in Los (2003), which made a valiant attempt to clarify the mathematical relationships between all these parameters, symbolically captured by an 'alpha'.

Definition 7: *A geometric Brownian motion is a random walk of the natural logarithm of the original process. Thus,* $lnX(t) - lnX(t-1) = x(t)$ *are the rates of return and for Brownian motion:* $\Delta x(t) = x(t) - x(t-1) = (1 - L)\ x(t) = e(t)$, *where* $e(t) \sim i.i.d.(0, \sigma_e^2)$.

Definition 8: *Fractional Brownian Motion (FBM) is defined by the fractionally differenced time series* $(1 - L)dx(t) = e(t)$, $d \in (-0.5, 0.5)$ *with* $e(t) \sim i.i.d.(0, \sigma_e^2)$.

Based on the rolling window test of the first four moments and based on the ACF function, the first differences of prices, rates of return (= first differences of logarithms of prices), and the first differences of rates of return are not identically and independently distributed. Thus, prices, logarithms of prices and returns are clearly not processes integrated from or driven by white noise and the Random Walk model is therefore immediately falsified.

To compare our series with the geometric Brownian motion, the ACF function for the analyzed series is compared with the correlations of geometric Brownian motion. (ACF comparison for FTSE series is provided in Figure 4). In almost all cases, the correlations of geometric Brownian motion substantially differ from the calculated ACFs for the original series. Thus, we also reject the geometric Brownian motion as a good model to fit the analyzed series.

To compare the empirical series with the theoretical FBM, one can compare the autocorrelation function of the series with the FBM based autocorrelation function. Because we obtain different Hurst exponents with different estimation methods, one needs to compute different ACFs functions and then to compare the obtained ACFs functions with ACFs of the original series. We suspected that FBM based ACFs are the closest to the original ACFs. Thus we computed ACFs for the theoretical FBMs for all series, but even these ACFs still do not approximate well the original ACFs. (The ACF comparison for FTSE series is provided in Figure 5).

The models that most likely can be used to identify abnormal stock market returns are those models which represent the persistence of the time series, that is models for which $0.5 < H < 1.0$. The series that appear to be anti-persistent with $0 < H < 0.5$ are abnormally fast mean-reverting and will not generate abnormally high returns, since those markets are ultra-efficient.

Persistence and wavelet MRA

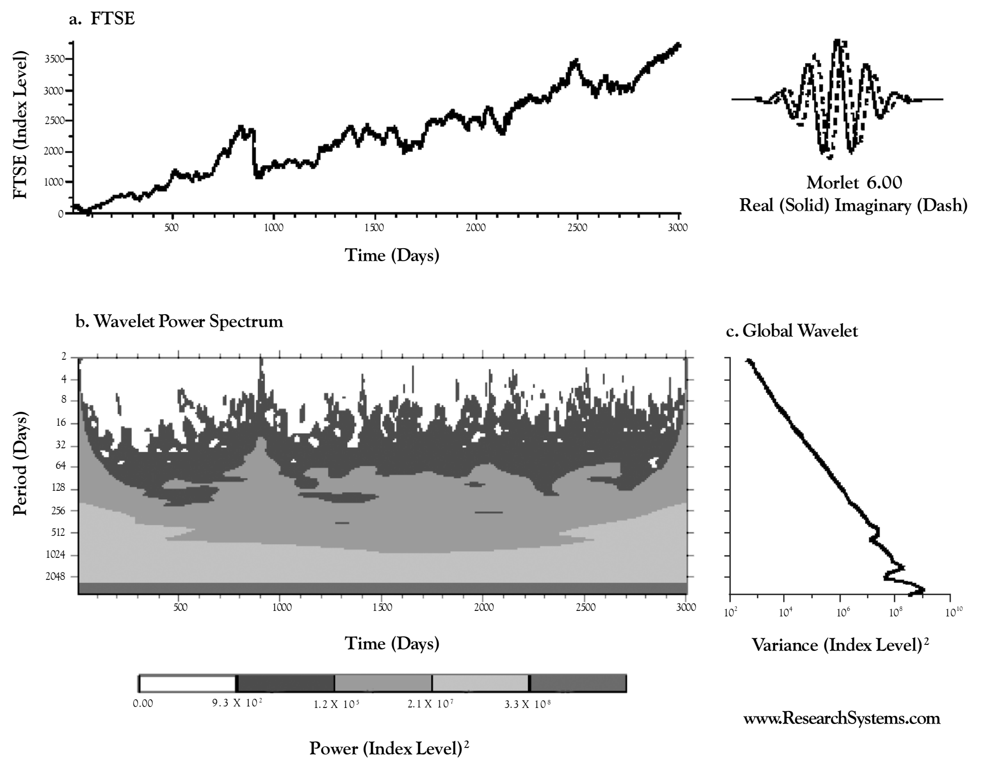
Examples of the scalogram and scalegram results of the wavelet MRA are plotted in Figure 6. A scalogram measures all power spectra localized in time and frequency (= 1/scale) domains at various scales and for various times. The wavelet resonance coeffi cients are computed by Mallats (1989) wavelet MRA with the use of the quite general Morlet-6 wavelet (which measures distributions with up to six non-zero moments). Such a scalogram, which is a visualization of the colorized wavelet resonance coefficients, allows the identification of the precise timing and power of the innovations or shocks occurring in the markets.

In contrast, scalegrams are averaged based on scalograms based on linear wavelet bases, and thus comparable to the Fourier spectra based on trigonometric bases. Together with the fields of the scalograms, help to detect the institutional periodicities or, more precisely, the aperiodic cyclicities (= uncertain 'periodicities') of the financial markets, which cannot be easily identified by the static ergodicity-based methodologies. Scalegrams also assist with the identification of the global or homogeneous Hurst exponent for each time series and can determine if the residuals are, indeed, white noise.[5]

There are three parts in each plot in Figure 6. Part (a) is the plot of original time series and the type of wavelet used to analyze the time series, c.q., the Morlet-6 wavelet, often used for the analysis of meteorological and environmental time series, such as the El Nino effect, or the level of CO_2. Part (b) is the scalogram, which is the color-coded plot of the absolute magnitude of the wavelet resonance coefficients. Finally, part (c) is the scalegram, which is the logarithm of the power spectrum or Fourier transform of the series' autocorrelation function (ACF).

Figure 6: Scalogram and Scalegram from Wavelet Multiresolution Analysis (MRA) of the FTSE Stock Price Index. This Time-Frequency (or time-scale) Analysis Uses a Morlet-6 Wavelet-Base.

I. FTSE Index Level (Observations for April 2, 1984—February 12, 1996)

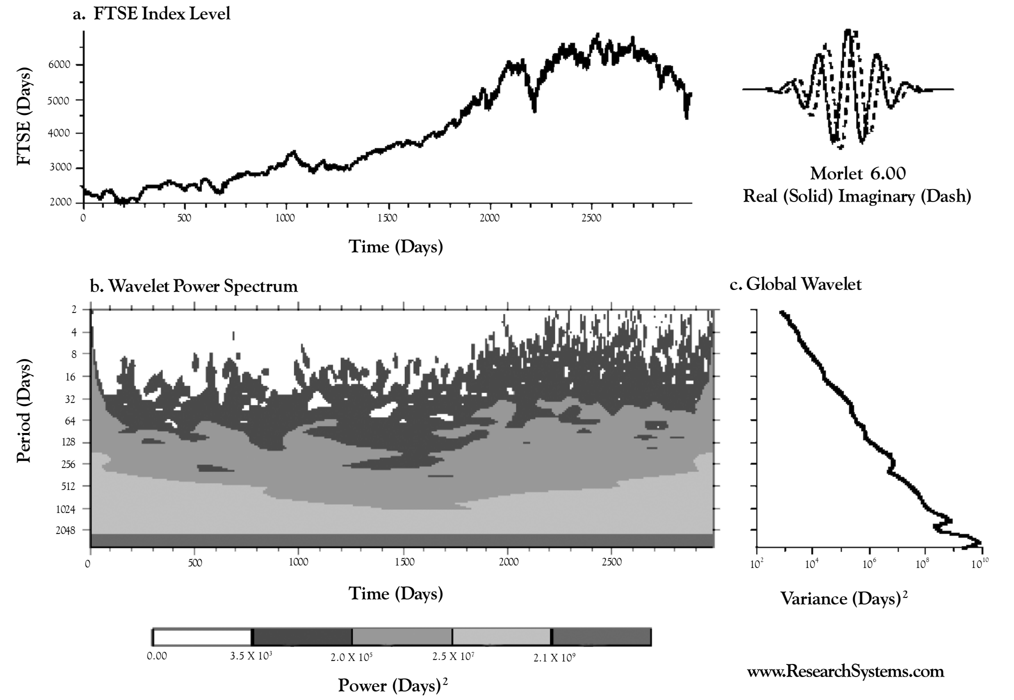


(Contd...)

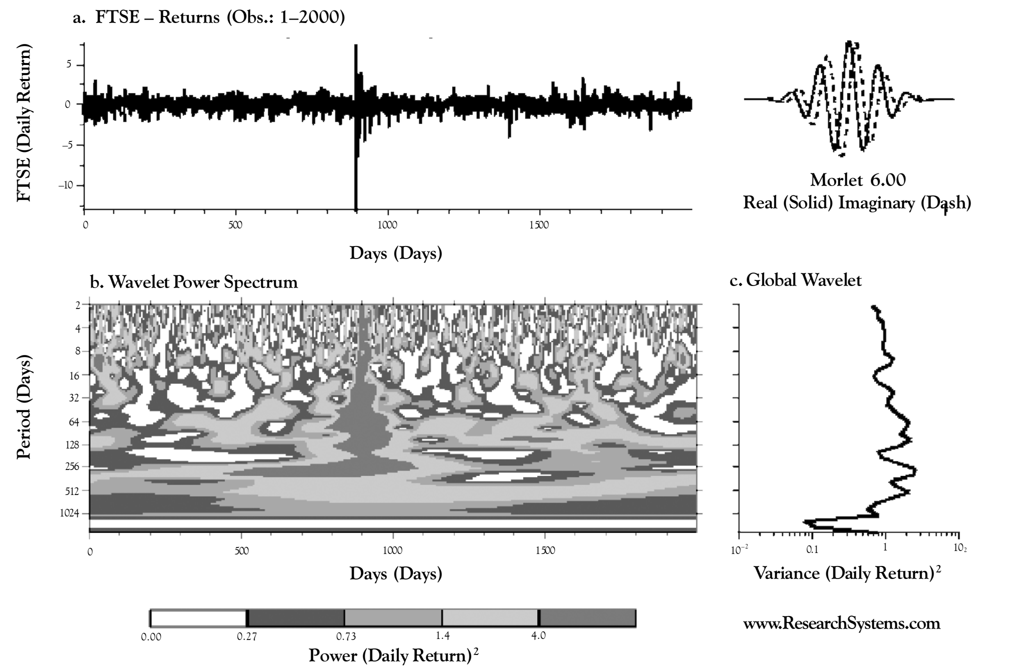
5 The discussed scalograms and scalegrams in this paper are computed with the help of software available on the following website: http://ion.researchsystems.com/IONScript/wavelet

(...contd.)

II. FTSE Index Level (Observations for January 2, 1990 – October 23, 2001)



III. FTSE Index – Returns (Observations for April 2, 1984 – February 25, 1992)



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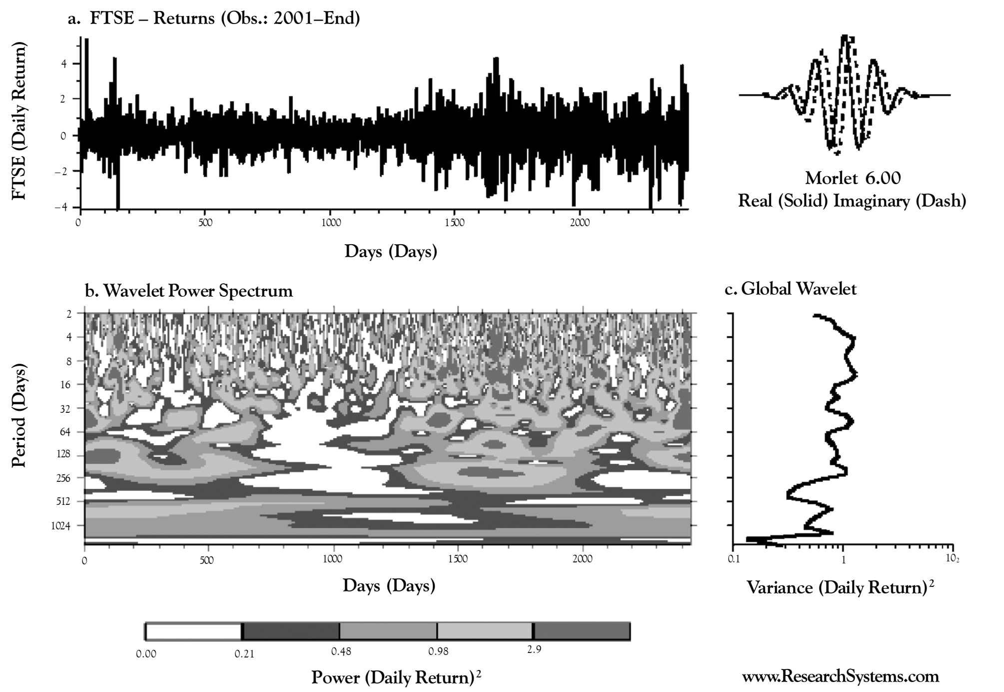
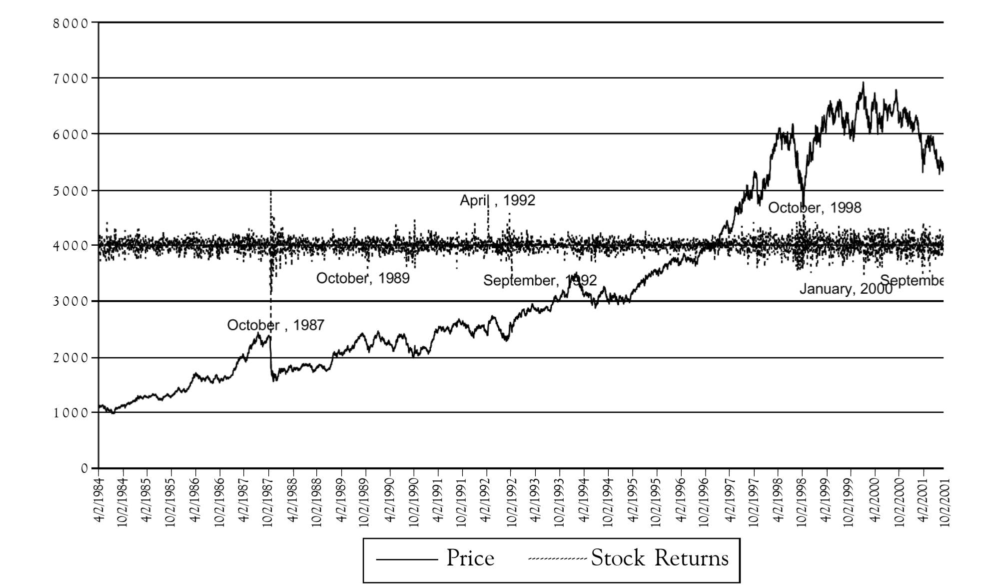
(...contd.)

IV. FTSE Index—Returns (Observations for February 26, 1992 – October 23, 2001)



On the basis of the price and return time series of FTSE index, one can see in Figure 7 that there are numerous spikes in the processes, which are consistent with sudden discontinuities in the stock market prices. Figure 7 shows that the most significant price

Figure 7: FTSE Stock Index (Close Price)



changes in the FTSE have occurred in October 1987, October 1989, April 1992, September 1992, October 1998, January 2000, and September 2001. For example, the sudden decline in the FTSE stock index in October, 1987 followed the crash in the US stock markets (black Monday), caused by rapidly rising of short term US interest rates, followed by rapidly rising long-term US interest rates, a weakening US dollar, deteriorating US current account deficit, unjustifiably high domestic price-earnings-ratios, very low dividend yields, and, most likely, too optimistic investor sentiment.

In terms of the wavelet analysis, stock market crashes can be easily detected by sudden spikes in power (dirac deltas or singularities), indicated in the scalogram by a steep upward migration of blue, green to red color. In October 1987, on the scalogram, one sees the burst of higher power through all frequencies for both stock market prices and returns, spreading from the high frequencies (at the top) to the low frequencies (at the bottom). The scalegram makes it easy to calculate the Hurst exponent from slope of the line fitted to the scalegram, which is $2H + 2$ for the price indices. The Hurst exponents calculated from the slope of the line fitted to the scalegram are reported in Table 5 under the title IDL Wavelet Toolkit. In case of the FTSE, the Hurst exponent is 0.33, indicating anti-persistence in the FTSE stock market returns data and definitely not consistent with a conventional LM or persistent process of $H > 0.5$. It indicates that the FTSE is an ultra-efficient market with abnormally fast mean-reversion, faster than identified by a geometric Brownian motion model (which has $H = 0.5$).

Conclusion

This paper identifies the ergodicity, stationarity, independence, and persistence (LM) of the eight European index prices and their transforms, or the lack thereof. We find that most of the data analyzed are far from being either ergodic, or stationary or independent. Thus, such series cannot be modeled with ARIMA or GARCH family models that assume stationarity of the final residual series.

The stock market prices and their returns and their various transformations are then compared with theoretical benchmark models, which are white (independent) noise (which integrates to Brown noise), geometric Brownian motion, and fractional Brownian motions. Even though some series appear to be fitted quite well by the white noise residual model (based on the computed global Hurst exponent), the estimated ACFs contradict often this finding. This demonstrates that the indiscriminate use of the global, homogeneous Hurst exponent computed from the average power spectrum (or Fourier transform of the ACF) is also not completely substantiated and that, indeed, we will have to start using the complex multifractal models of Elliott and van der Hoek, 2003, (originally presented in 2000 in Melbourne) in empirical research of financial markets. Although not easy, this is quite possible, as was already demonstrated in Los (2003).

The fractional Brownian motion model encompasses the geometric Brownian motion model. But also the fractional Brownian motion cannot capture all the empirically observed intricacies, such as 'cyclicities' or 'uncertain and time-varying periodicities' and the extremely valued power spikes (= infinite variance) observable in the power spectra of some stock market returns (Los and Yalamova, 2006), as was originally suggested by Mandelbrot (1969).

In addition, the question should be raised whether such models can be used to earn abnormal stock market returns, in particular when persistence is observed. The methods thus far suggested in the financial literature appear not to lead to unique scientific conclusions regarding stock market returns in general. For example, not all European stock markets are conventionally efficient, but some appear to be anti-persistent or ultra-efficient, such as the FTSE, but others are persistent and inefficient.

Therefore, the more important question for regulators and risk managers in Europe is to identify which of the European stock market indices are persistent and thus inefficient and illiquid and which can therefore produce abnormal returns? By strictly focusing on Long Memory, i.e., persistence, and by not allowing for the possibility of anti-persistence, many research analysts have been going into blind alleys, since we find that most of the European stock market indices appear to exhibit anti-persistent behavior and are thus ultra-efficient, i.e., more efficient than the standard lognormal price diffusion (= geometric Brownian motion) model suggests and also more than expected based on journalistic impressions.

However, we want to emphasize that the conventional linear methods currently suggested in the financial literature tend to lead to conficting, inconsistent, non-unique, and thus non-scientific, identification results. There still exists no commonly accepted stock market model for the European stock markets. But that the various European stock markets differ in their degrees of persistence and efficiency is now indisputable. The next step would be to discover what the mathematical relationships between such various degrees of persistence and the degrees of trading freedom in each of those European stock markets and, finally, what institutional arrangements are the constraining and defining factors for those degrees of trading freedom.

This paper does find that visualization of the time-frequency spectra by wavelet scalograms is a useful way to visualize the important localized characteristics of the high frequency financial time series produced by a stock market. That is a first step on the road to the identification of a unique empirical stock market pricing model for European stock markets that can properly measure the various degrees of persistence of those various stock markets.

Scalegrams and spectrograms are also based on computed averages, be it based on wavelets or on Fourier transforms in the scale, respectively frequency domains and thus on the ergodicity in the frequency domain. Accordingly they also tend to obscure the important and not easily modeled, localized risk and time-variant higher moment phenomena, which are clearly observable in the colorized, analytic wavelet fields of scalograms.

This strongly suggests that researchers must pay more attention to those rapid changes in the frequencies of stock market price or return time series over time, when they model stock markets. Those rapid changes can be detected and properly identified by wavelet multiresolution analysis (MRA) based on scalograms. As Benoit Mandelbrot already forewarned us in the 1960s, many risk managers have become aware in the past two decades that stock market crashes, 'turkey surprises' or 'black swans' do now more often occur in such financial markets than could normally be expected from historical evidence (Taleb, 2007).

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