Investor Confidence in an Underdeveloped Stock Market

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This paper studies the behavior of the stock price in a market characterized by the presence of one large trader and a fringe of marginal 'noise' traders. The study shows that price volatility and sensitivity excessively depend on the large trader's behavior. The paper also comments on the implications of these properties for the derivatives market.

Introduction

Globally, investor confidence in the capital market is considered important for the economic development of a country. Deep bear market, corporate scandals, insider trading, and inaccuracy of financial statements are cited as reasons for lack of investor confidence in many countries. Revival of such confidence is necessary to improve the efficiency of the securities market in converting savings to investment.

In the Indian context, it is indicated that regulation for investors' protection in the new issues market is vital. The number of small investors in this market is massive. Unfortunately, market studies show that in the Indian stock market higher risk is not priced (Madhusoodanan, 1997), and Indian IPOs are overpriced in the long run (Kakati, 1999). As a result, the equities markets suffer serious inadequacies as a mechanism for raising capital (Gokarn, 1996). A SEBI-NCAER report (2000) indicates that despite the expansion of the securities market, only 1.4% of household savings is channeled into it. 80% of these households are first generation investors, so, retaining their confidence is important. Overall, the secondary market is made safer with reforms but primary market is perceived riskier than before.

From the above discussion, we find that the Indian market for financial securities may be considered to be in its developing stage, and that it is still characterized by a few large traders and a largely marginal and passive fringe of small retail investors. These can be thought of as 'noise traders' (given that their participation in the primary market is minimal) who operate on the expectation of the market converging to a corrected trend. So, their trading gives rise to a movement that can be modeled by a mean corrected noise process. The large traders also rely on the mean correction, switching from bullish to bearish trend, depending on their individual expectations of under or over valuation. Thus, we may assume that their trading is described by a process with a mean correcting trend. In this paper, we aim to explore the consequences of these features for the stock market and the derivative market.

In particular, we study stock market volatility and market sensitivity with respect to the behavior of these large traders, i.e., whether individual traders can influence the price

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movement in a significant way. A related but equally interesting question is that of the underdevelopment of the derivatives market. Whether the features of the stock market mentioned above can explain this is the other issue that we address here. The analysis proceeds in the usual tradition of modelling the stock price by a suitable stochastic differential equation involving a Wiener process (see Oksendal (2003) for details). Here, we formally establish the volatility and sensitivity relationships. In the conclusion, we settle the issue of underdevelopment in the derivatives market.

The Analysis

For the sake of simplicity we assume that there is one large buyer and a fringe of small passive traders for a stock (or index) in the market. The demand equations are as follows: For the large trader, it is given by:

$$dB_t = (\theta - S_t)\, dt + \sigma dW_t \qquad \text{...(1)}$$

The monopolistic power of the large trader is incorporated in Equation (1) through the mean correction term. In a competitive market, the traders will be 'price takers'. The fringe trading is modelled as:

$$dF_t = f(\theta - S_t)\, dW_t \qquad \text{...(2)}$$

Therefore, instantaneous change in total demand in the market can be written as:

$$dU_t = dB_t + dF_t = (\theta - S_t)\, dt + [\sigma + f(\theta - S_t)]\, dW_t \qquad \text{...(3)}$$

The supply for the stock (or index) is governed by the relationship

$$dV_t = v dt + \alpha dS_t \qquad \text{...(4)}$$

One can interpret θ as the secular growth rate in demand and v as the secular growth rate in supply. We assume that the market is in a dynamic equilibrium at each instant of time. So, we have $dU_t = dV_t\ \forall t$. This implies:

$$dS_t = \frac{1}{\alpha}[(\theta - v) - S_t]\, dt + \frac{1}{\alpha}[(\sigma + f\theta] - fS_t] dW_t \qquad \text{...(5)}$$

This is a mean correction type Stochastic Differential Equation (SDE) which may be rewritten as:

$$dS_t = (a - bS_t)\, dt + (c - gS_t)\, dW_t, \qquad \text{...(6)}$$

where $a = (\theta - v)/\alpha$, $b = 1/\alpha$, $c = (\sigma + f\theta)/\alpha$ and $g = f/\alpha$. This is similar to the well-known 'Ornstein-Uhlenbeck' process.

Let us redefine (6) by assuming $c - gS_t = M_t$; so, $dS_t = -(dM_t/g)$. This implies

$$dM_t = b(m - M_t)dt - gM_t dW_t \qquad \text{...(7)}$$

where $m = (bc - ag)/b = (\sigma + fv)/\alpha$.

Solving for M_t

To solve (7), we multiply both sides of the equation by the integrating factor e^{bt} and rearranging terms, we obtain:

$$d(e^{bt} M_t) = e^{bt} bm\,dt - e^{bt} g M_t\, dW_t$$

$$\Rightarrow e^{bt}(M_t - M_0) = m(e^{bt} - 1) - g\int_0^t M_s e^{bs}\, dw_s$$

$$\Rightarrow M_t = m + (M_0 - m)e^{-bt} - g\int_0^t M_s e^{b(s-t)}\, dW_s \qquad ...(8)$$

It is easy to compute the expectation for this process as $E(M_t) = m + (M_0 - m)e^{-bt}$. We, now compute its variance. Let $f(t) = V(M_t)$. Then,

$$f(t) = V(M_t) = E(M_t - E(M_t))^2$$

$$= E(-g\int_0^t M_s e^{b(s-t)}\, dW_s)^2$$

$$= g^2 E\int_0^t M_s^2 e^{2b(s-t)}\, ds$$

$$= g^2 \int_0^t E(M_s^2) e^{2b(s-t)}\, ds$$

$$= g^2 \int_0^t [V(M_s) + E^2(M_s)] e^{2b(s-t)}\, ds$$

Therefore,

$$f(t) = g^2 \int_0^t [f(s) + (m + (M_0 - m)e^{-bs})^2] e^{2b(s-t)} ds$$

Differentiating above with respect to t we get:

$$f'(t) = g^2 f(t) + g^2 (m + (M_0 - m)e^{-bt})^2$$

We again use integrating factor and rearrange terms to get:

$$d(e^{-g^2 t} f'(t)) = e^{-g^2 t} g^2 (m + (M_0 - m)e^{-bt})^2$$

So, finally, integrating, we get:

$$f(t) = e^{g^2 t} \int_0^t e^{-g^2 s} g^2 (m + (M_0 - m)e^{-bs})^2\, ds$$

or finally,

$$V(M_t)=f(t)$$

$$=e^{g^2 t}g^2\left[\frac{m^2}{g^2}(1-e^{g^2 t})+\frac{2m(M_0-m)}{b+g^2}(1-e^{(b+g^2)t})+\frac{(M_0-m)^2}{2b+g^2}(1-e^{(2b+g^2)t})\right] \quad \text{...(9)}$$

Thus, the variance of M_t is of the order of $e^{g^2 t}$ which grows exponentially with time.

Solving for B_t

We now solve for B_t (the large traders' trading process, as in (1)) in the same way as above and obtain the following:

$$B_t=\theta+(B_0-\theta)e^{-t}+\sigma\int_0^t e^{s-t}\,dW_s \quad \text{...(10)}$$

Routine calculations show $E(B_t)=\theta+(B_0-\theta)e^{-t}$, and

$$V(B_t)=\frac{\sigma^2}{2}(1-e^{-2t}). \quad \text{...(11)}$$

Correlation and Market Sensitivity

To study the sensitivity of the market price with respect to the large traders' behavior, we need to compute the correlation between S_t and B_t. To find this we first compute

$$Cov(B_t,M_t) = E[(B_t-E(B_t))(M_t-E(M_t))]$$

$$=-E\left[(\sigma\int_0^t e^{s-t}\,dW_s)(g\int_0^t M_s e^{b(s-t)}dW_s\right]$$

$$=-\sigma g\int_0^t E(M_s)e^{(b+1)(s-t)}\,ds$$

After simplification, this finally yields:

$$Cov(B_t,M_t)=-\sigma g\left[\frac{m}{b+1}(1-e^{-(b+1)t})+(M_0-m)(e^{-bt}-e^{-(b+1)t})\right] \quad \text{...(12)}$$

Thus, considering leading terms only, we get:

$$r(B_t,M_t)=Cov(B_t,M_t)/\sqrt{V(B_t)V(M_t)}\simeq-\frac{\sigma g\frac{m}{b+1}}{\frac{\sigma}{\sqrt{2}}e^{g^2 t/2}g\frac{m}{g}}=-\frac{\sqrt{2}g}{(b+1)e^{g^2 t/2}}$$

Therefore,

$$r(B_t,S_t)=-r(B_t,M_t)\simeq\frac{\sqrt{2}f}{(\alpha+1)e^{f^2 t/2\alpha^2}} \quad \text{...(13)}$$

For moderate values of t, this correlation is significant. To illustrate, consider $f = \alpha = 1$. Then we have the following values for $r\ (B_t,\ S_t)$ for sample of values of t.

t	0.1	0.5	1.0	1.5	2.0
$r(B_t, S_t) = \frac{1}{\sqrt{2}\, e^{t/2}}$	0.67	0.55	0.43	0.33	0.26

Using Girsanov's Theorem, it can be shown that:

$$dS_t = \gamma\, dt + \frac{1}{\sigma} dB_t \qquad \text{...(14)}$$

where $\gamma = \frac{1}{\sigma}\left[v + \frac{f}{\sigma}(\theta - S_t)^2\right]$. Thus, the instantaneous correlation between dS_t and dB_t is perfect.

From the above analysis, we note that the supply Equation 4 may be changed to one with time dependent secular growth rate (to make it potentially consistent with a market growing at an increasing rate) $dV_t = v(t)dt + \alpha dS_t$. At the cost of some algebraic tedium, it can be shown that our results are qualitatively unchanged under this modification.

Consider next the case when there are more than one (but a small finite number of) large traders in the market. Without loss of generality, assume there are two. Then the situation may be modeled as a two person stochastic differential game, assuming that these two players participate in a strategic interaction in terms of their demand for the stock. We can write:

$$dB_{it} = (\theta_i - S_t)dt + \sigma_i dW_t \qquad \text{...(15)}$$

for the two traders, $i = 1, 2$. Assume that the value function for each player is given by $J_i(t, S_t, B_1, B_2)$, $i = 1, 2$. A Nash equilibrium for such a game exists under general regularity conditions (Buckdahn *et al.*, 2004). Under the usual market conditions, the value functions, J_i's, will be inverted U-shaped in B_i, and hence, one can compute the Nash equilibrium for this game and verify that the results will remain qualitatively unchanged.

Conclusion

The analysis shows that under the assumptions of our model, the stock market will be characterized by excess sensitivity with respect to individual behavior of large traders (discussion following Equation 13) which is contradictory to a competitive and smooth functioning market. We also see that the volatility of the market price increases exponentially with time (see Equation 9) and hence, even though this eliminates the excess sensitivity in the long run, it causes instability in the market, damaging the growth prospects of the economy. Both these phenomena will result in lack of confidence in the stock market on the new and retail investors' part, and a low flow of investment.

The conclusions are qualitatively unchanged if we assume a time dependent secular growth rate in the supply of stock (reflecting a commensurate growth of the economy).

Turning now to the other issue, the above analysis implies bad news for the derivatives market too. Given that market volatility increases exponentially with time, let us consider the case of pricing a simple call and put option.

The pricing equation for a call option with strike price K and time to maturity T on S_t is given by (see Hull, 2003 for details):

$$c = S_0 N(d_1) - K e^{-rT} N(d_2) \qquad \text{...(16)}$$

where, $d_1 = \frac{\log(S_0/K) + (r + \sigma_s^2/2)T}{\sigma_s \sqrt{T}}$, $d_2 = d_1 - \sigma_s \sqrt{T}$, $\sigma_s = \sqrt{V(S_t)}$, and $N(d_2)$ is the standard normal cumulative distribution function. As T becomes large, $d_1 \rightarrow \infty$ and $d_2 \rightarrow -\infty$. Thus, $c \rightarrow S_0$. The same argument shows that the price of a put option, $p \rightarrow K e^{-rT}$. So, no new information is available from the options market for contracts of a longer duration. This will result in lack of interest from potential investors in the derivatives market, and hence, to absence of development in this market.

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