

A Welfare Analysis of Regulation in Relationship Banking Markets

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Abstract. The increasing dependence of individuals on debt financing raises several welfare considerations that we analyze in this paper. We develop a dynamic, competitive model of relationship banking to determine how regulation influences borrowing and lending behavior, and analyze how it affects welfare in the market. We characterize the lending regimes that arise based on public policy, and evaluate the optimal choice by the government to induce particular lending practices to arise. Finally, we consider the effect that a credit reporting agency has on the market. In the paper, we highlight the new empirical implications that the model generates.

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1. Introduction

The increasing dependence of individuals on debt financing is staggering. Over the period from 1990–2004, the average debt burden in the United States rose from 86.2% to 105.1%, and between 26.2–28.2% of families in the bottom quintile of household income devoted more than 40% of their earnings to debt service (Jickling 2005)¹. This dependence, coupled with low (or negative) savings rates and rising rates of personal bankruptcy², has increased the importance of investigating the effect that government policies have on retail banking markets.

In this paper, we develop a dynamic, competitive model of relationship banking to address several important questions: How does regulation influence borrowing and lending behavior in retail banking markets? How should a government regulate markets to maximize social welfare? When does credit rationing improve welfare? Alternatively, when is it optimal to have a liberal lending policy with potentially

¹ The average debt burden is calculated as the average debt as a percentage of disposable income.

² Over the period from 1990–2004, the total number of personal bankruptcies doubled (1.6 million in 2004). According to White (2007), “. . . By 2004, more Americans were filing for bankruptcy each year than were graduating from college, getting divorced, or being diagnosed with cancer.” Quarterly savings rates after 2004 were typically less than 1% and were sometimes negative (U.S. Bureau of Economic Analysis).

high interest rates to low credit quality borrowers? What role does a credit reporting agency play in these decisions?

To address these questions, we consider a model that is based on three stylized facts in credit markets. First, default in credit markets is a strategic decision. That is, if borrowers are solvent, they face a tradeoff when deciding whether to honor their obligation to their lender. Strategic default has become quite common in credit markets (Fay, Hurst, and White 2002), and has been previously modeled by Parlour and Rajan (2001). Second, borrowers in the market are heterogeneous with respect to their ability to repay loans, and often have an information advantage over lenders. Therefore, in addition to the moral hazard problems associated with strategic default, this added information asymmetry causes adverse selection problems in the market as well. Third, the banking industry is a search market with significant switching costs (e.g., Baye and Morgan 2001). When borrowers switch lending institutions, they incur costs in the form of time, out-of-pocket expenses, or familiarity with the lender. For lenders, the cost is in the form of lost information, since a lender typically knows more about the credit history of a repeat borrower than one who is new to the bank.

With these stylized facts in mind, we proceed as follows. In each period of the model, a continuum of borrowers face a decision whether to keep patronizing the bank with which they have a relationship, or whether to start searching for a new bank. Borrowers are of two types, which is private information: either they are *insolvent*, in which case repayment of a loan is impossible, or they are *solvent* and decide optimally whether to pay back their loan or strategically default. Since the interaction between borrowers and lenders extends over an infinite horizon, solvent borrowers make this decision by weighing the benefit of a relationship with the bank, which generates future value through access to low-priced capital, versus the benefit of keeping the capital (reneging on their obligations) and having to search for a new institution.

Lenders are a continuum of homogeneous banks, which post interest rates based on the credit status (high or low quality) of the borrowers they attract. The banks not only set interest rates, but also set a rejection probability for each class. Because of imperfect inference regarding a borrower's true type, within each credit class there is a pool of solvent and insolvent types. The mix in each class depends on the interest rates and credit rationing imposed by the banks.

In the non-autarkic equilibrium that we characterize, two different regimes may arise, based on the market conditions present in the market. In Regime 1, there is no credit rationing, but the interest rate charged to low quality borrowers is high. In this case, the banks induce repayment by solvent borrowers by punishing them with high interest rates if they default and defect from the bank. In Regime 2, there is credit rationing for low quality borrowers, but the interest rates charged to these borrowers is not as high. In this case, banks induce repayment by limiting the access of low credit quality borrowers to debt.

The presence of either regime depends on the banks' profitability when they lend to low quality borrowers. If lending to low credit quality borrowers leads to non-negative profits, then there is no credit rationing, but higher interest rates for low quality borrowers to maintain proper incentives. If lending to all low quality borrowers leads to losses for the banks, then they will ration credit to break even.

Whether profits to low quality borrowers are positive or negative depends on three actions that the government takes. If the government raises the cost of funds (through the reference rate set by the central bank), taxes loans when they are originated, or places an interest rate ceiling, it makes it more likely that the banking market will be in the regime with credit rationing because these policies lower the profitability of lending to low quality borrowers. In contrast, if the government lowers the cost of funds, subsidizes loans, or refrains from placing an interest rate ceiling, it is more likely for the banks to set a liberal policy and lend to all borrowers. In this regime, however, banks will charge high interest rates to low quality borrowers.

The government, therefore, faces a tradeoff. In Regime 1, all borrowers gain access to funds, but low quality borrowers receive less capital since interest rates are higher. In Regime 2, some borrowers are excluded, but the ones who obtain financing get more capital at lower rates. Not surprisingly, the nature of the borrowers' production function affects the government's decision. If the production function achieves a sufficiently large return to scale for low levels of capital, then credit rationing may increase value. Absent this, it is more likely that lending to all borrowers at smaller amounts (and higher interest rates) is optimal. As we discuss in the paper, this has important cross-sectional and time-series empirical implications, and also may affect policy choice. As we note, a particular venue where this may have importance is in the U.S. small business sector where approximately 47% of businesses use personal credit cards as a source of non-collateralized borrowing (Mach and Wolken 2006). Based on the mix of businesses present (firms with different production functions), different regimes may be more or less attractive, which might guide choices about regulation.³

It must be mentioned that, in most of the paper, we make an assumption of "inside information" for the banks, in which only the bank who suffered a default from a particular borrower knows that this borrower did so. We extend our analysis to include the presence of a credit reporting agency, whose role is to identify borrower type (solvent versus insolvent) with some degree of precision. Interestingly, when an agency is present, the interest rates do not change from the base model. However, the degree of credit rationing increases when the market is in Regime 2. The reason is that relationships between the solvent borrowers and their banks are more

³ As noted in the paper, small businesses constitute half of the output in the private sector and are responsible for 60–80% of new jobs (Mach and Wolken 2006).

fragile because there is a positive probability that a solvent borrower gets correctly identified when they are in the pool of low quality borrowers. Therefore, to maintain proper incentives for solvent types to repay their loans the banks have to punish low quality borrowers more severely (through credit rationing). As precision rises, the degree of credit rationing necessary to maintain incentives becomes more severe. For a fixed set of parameters, there is a welfare loss in Regime 2 when a credit reporting agency is added. However, we show that the potential gain to a regime change (inducing a change to Regime 1) is higher with a credit reporting agency, which implies that the scope for tighter banking regulation is higher in the presence of credit agencies.

Our paper is related to several papers on multi-institutional banking (e.g., Bizer and DeMarzo 1992; Parlour and Rajan 2001), but is distinct in many respects. Bizer and Demarzo study the moral hazard that arises when a borrower takes on sequential loans from several banks. Parlour and Rajan (2001) analyze the ability to strategically default that arises from multi-bank lending. In contrast, in our paper, borrowers do not submit multiple loan applications or hold debt from more than one bank at any one time. The ability to strategically default arises from the fact that borrowers of different types pool in various credit classes, which is the primary motivation for posting different interest rates and for rationing credit. Further, the primary contribution of our paper is to consider the effect of regulation on the banking market, which is not explored in detail in either Bizer and DeMarzo (1992) or Parlour and Rajan (2001).

Our paper is also related to Diamond (1989) who focuses on the acquisition of reputation in dynamic debt markets and its effect on interest rates. Like Diamond, we show that reputation in the credit market is an important driver of interest rates. Whereas reputation guides project choice (risk-taking) and subsequently interest rates in his model, reputation provides incentives to repay in ours. Our analysis contrasts with Diamond (1989), however, in that we focus on the policy implications in our paper, and also consider credit rationing as a way in which lenders may induce solvent borrowers to honor their debt.

Our paper contributes to the extensive literature on credit rationing (e.g. Stiglitz and Weiss 1981)⁴. In many of these models, credit rationing arises when banks face an adverse selection problem in which borrowers have access to heterogeneous projects with different risks. In contrast, credit rationing arises in our paper because banks use credit rationing to induce solvent borrowers to repay their debt. Raising interest rates does not worsen adverse selection; rather, once solvent borrowers are induced to repay their loans, the expected fraction of high risk borrowers in each category is constant.

⁴ See also Hodgman 1960; Freimer and Gordon 1965; Jaffee 1969; Jaffee and Modigliani 1969; Jaffee and Russell 1976; Allen 1983; Wette 1983; Bester 1985; Besanko and Thakor 1987; Williamson 1987.

Finally, our analysis adds to the literature on information sharing in credit markets. Pagano and Jappelli (1993) show that information sharing reduces adverse selection in the market, but may have an ambiguous impact on aggregate lending, since more lending to safe borrowers may fail to offset the reduction on lending to risky borrowers. Vercammen (1995) and Padilla and Pagano (2000) highlight that information sharing can also improve borrowers' incentives to repay loans, as long as it makes past defaults publicly known: default is a signal of bad quality for outside banks and carries the penalty of higher interest rates, or no future access to credit. But if banks also share their information about customers' types, this disciplinary effect disappears as default is no longer a stigma. This result is akin to our finding that information sharing reduces incentives for solvent borrowers, which in our setting prompts more severe rationing. The latter prediction may appear in contrast to the empirical finding that the diffusion of information sharing arrangements correlates with more abundant and cheaper lending (Djankov, McLiesh, and Shleifer, 2007; Jappelli and Pagano, 2002; Brown, Jappelli, and Pagano, 2008; Love and Mylenko, 2003). However, this contrast is more apparent than real, considering that in our model information sharing raises the gains from switching from the credit rationing regime to the regime in which all applicants are served, and thus calls for public policies intended to increase lending. This highlights another channel through which information sharing may lead to a broader credit market.

The rest of the paper is organized as follows. In Section 2, we set up our benchmark model. In Section 3, we analyze the strategic choices of borrowers and lenders and characterize the incentives necessary for solvent borrowers to repay their debt. In Section 4, we derive and characterize the equilibrium of the game. Section 5 analyzes the welfare implications of the model. In Section 6, we extend our base model to include a credit reporting agency. Section 7 concludes. The appendix contains all of the proofs.

2. Market for Loans

Consider a dynamic lending market in which a continuum of identical banks offer loans to a continuum of potential borrowers in each period $t = 1, 2, \dots$ over an infinite horizon. All of the banks have the same access to depositors (capital) and no bank has an *a priori* advantage compared to the others. In each time period, the borrowers approach the banks and apply for loans. Based on a borrower's credit status, the interest rate may be different, and so may be the probability that the loan gets approved. If a loan is approved, borrowers invest the money that they borrow into a project. Then, based on the tendency for financial distress in each period and the strategic actions of solvent debtors, borrowers either default on the loan or repay their debt. This sequence is repeated in each period.

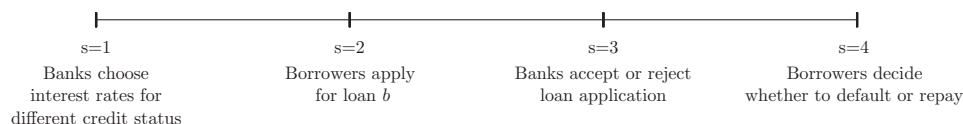


Figure 1. Relationship banking period game. At $s = 1$, banks post a menu of interest rates, given the credit status of each borrower. At $s = 2$, each borrower decides whether and where to apply for a loan, picking the dollar amount b . At $s = 3$, banks either accept or reject the loan application. At $s = 4$, if the bank made a loan to a solvent borrower, the borrower may either repay the loan at $b(1 + r)$, or they may default.

More precisely, the borrowing-lending situation in each period is modeled as a four-stage game. The stages are indexed by $s = 1, 2, 3, 4$ and a schematic summary of the overall game for each period is shown in Figure 1. In the first stage, $s = 1$, the banks post menus of interest rates that depend on each borrower's status or "label". That is, the interest rate offered to each borrower depends on whether he is known to be a returning customer who either repaid his loan or defaulted, or a new customer who has no previous relationship with the bank. Knowing this dependence, a borrower may either return to the same bank he already borrowed from or approach a different bank. The former class of borrowers are called "repeat customers", whereas the latter are called "search customers". We assume that each search customer picks a bank randomly.

When a repeat or search customer arrives at a particular bank at $s = 2$, he observes the posted interest rates, but cannot observe this information before arriving. He may then apply for a loan at that bank. If a borrower decides to apply for a loan, he chooses the amount of the loan b to request. If a borrower decides not to apply for a loan, he may continue his search by visiting another bank at a cost $\sigma > 0$, which is the same for all borrowers, and decide whether to apply for a loan at the new bank⁵. A borrower may sequentially approach as many banks as he wishes, but, in the end, he may only apply (if at all) for a loan at a single bank⁶.

At $s = 3$, after each borrower has picked the bank where to apply for a loan, the banks decide whether to approve or reject the loan applications. This process is time consuming, so if a loan is rejected, the applicant cannot apply for a loan at another bank during the same period. As such, a rejected loan application generates a zero payoff to the bank and to the applicant. Once the banks make their approval/rejection decisions, a borrower whose loan was approved invests the

⁵ We assume that the first visit to a bank is costless, and that σ is strictly positive and constant for subsequent visits. Based on the standard result by Diamond (1971), this will lead to monopoly pricing in equilibrium, independent of the magnitude of σ .

⁶ We exclude the possibility of multi-institutional lending. Though realistic in some contexts, if this were allowed, the banks would face an additional source of moral hazard: borrowers might apply for more loans and lower the probability of repaying earlier loans. We eliminate this additional source of complication to keep the analysis tractable and to focus on the implications for regulation. See Bizer and Demarzo (1992) or Parlour and Rajan (2001) for models of multi-institutional banking.

amount in a value-increasing project, which we specify shortly. Finally at $s = 4$, borrowers who invested the amount of the loan decide whether to repay the loan or default. If they repay, the payment consists of principal and interest. If they default, they pay nothing⁷.

We now formalize the strategies of the borrowers and lenders, as well as the information structure of the market.

Borrowers. Each borrower has access to a production technology that converts b dollars into $g(b)$ dollars. The production function $g(\cdot)$ is strictly increasing, continuously differentiable, and strictly concave. We assume that $1 < g'(0) < \infty$ so that the amount of the loan demanded is positive for sufficiently small interest rates and is zero for sufficiently high interest rates. Based on $g(\cdot)$, each borrower solves

$$\text{Max}_{b \geq 0} \{g(b) - (1 + r)b\}. \quad (1)$$

The maximizer of this program is the amount borrowed b^* at interest rate r , which we denote by $b^* = D(r)$. The maximized value of (1) for any interest rate r is given by $S(r)$. By the restrictions imposed on $g(\cdot)$, $D(r)$ is single-valued and continuously decreasing in r whenever it is positive. We define the choke rate $\hat{r} \equiv g'(0) - 1$ such that for all $r \geq \hat{r}$, $D(r) = 0$ and $S(r) = 0$. That is, for any interest rate higher than $\hat{r}$, the demand for funds $D(r)$ is zero.

Borrowers may be of two types: either they are financially constrained, insolvent types, or they are unconstrained, solvent types. Type is private information to each borrower. Insolvent types are those who are currently experiencing financial distress due to an exogenously determined liquidity shock (e.g., an unplanned hospitalization). Given such financial responsibilities, they are unable to pay the bank principal plus interest $(1 + r)D(r)$. That is, for any loan amount $D(r)$ they receive from the bank, even though they generate $g(D(r))$, they are unable to repay the bank and default.

Solvent types, on the other hand, face a choice. They may either repay the loan and receive the surplus $S(r)$, or they may strategically default and keep $g(D(r))$. We denote this repayment decision by α , where $\alpha = 0$ means that the borrower defaults and collects a period payoff of $g(D(r))$, whereas $\alpha = 1$ means the borrower pays back. For any $\alpha \in (0, 1)$, the solvent borrower randomizes between paying the loan and defaulting.⁸ As discussed below, solvent borrowers optimize by comparing

⁷ The assumption of paying nothing is for technical clarity. In reality, borrowers may not default in full, but rather hide capital in exempt assets under the rules governed by bankruptcy law (e.g. Chapter 7 and Chapter 13 in the U.S. system). The strategic use of such rules by solvent borrowers is what we have in mind here, though we have left the exact amount of exempt assets unmodeled. See Parlour and Rajan (2001) for a good institutional discussion of strategic default.

⁸ As will be seen, there may not exist equilibria in pure strategies, which is why we introduce mixing. However, since there is a continuum of borrowers, mixed-strategy equilibria may well be interpreted

their payoff to repaying or defaulting this period, as well as the effect that these repayment decisions have on future payoffs.

A particular borrower's type changes from period to period according to a Markov process as follows. If a borrower is a solvent type at time t , he becomes insolvent at time $t + 1$ with probability $\beta \in (0, 1)$. Otherwise, with probability $1 - \beta$, he remains solvent. On the other hand, if a borrower is currently insolvent, he recovers and becomes solvent with probability $\gamma \in (0, 1)$. We assume that types are sufficiently persistent, namely, that $\beta + \gamma < 1$.

Banks. Each bank has equal access to funds and pays the interest rate c that is linked to a reference rate set by the central bank. We suppose that all banks have equal footing in the interbank market and with the central bank, and thus, face the same cost of funds c . We finally assume that all banks also face a fixed cost $f > 0$ per loan that they issue to a borrower, which could be considered the cost of processing the loan. This cost f is the same across all banks.

Banks set interest rates, r , at the beginning of each period, which depends on the information that they have about borrowers. Then, once loan applications are submitted, banks decide whether to grant a loan to each borrower that applies for one. This decision is denoted by λ , where $\lambda = 0$ means that the loan is rejected and $\lambda = 1$ means that the loan is approved. For any $\lambda \in (0, 1)$, the bank randomizes between lending to a borrower and rejecting their application. For any $\lambda < 1$, the bank is said to “ration credit”, that is, to restrict the number of loans it makes despite the fact that it has unconstrained access to funds.

Information. As indicated above, banks are able to condition the interest rates that they charge and their loan approval decision on whatever information they have about borrowers. We make the informational assumption that only the bank to which a repeat customer returns knows whether this customer repaid their loan or defaulted. Therefore, lenders do not know anything about the credit history of their search customers. In what follows, we sometimes refer to this assumption as “inside information” since banks have information about the borrowers they dealt with, but not about borrowers with whom they have yet to develop a relationship^{9,10}.

as pure strategy equilibria in which some borrowers repay while others default. The same comment applies to the continuum of banks, namely some loans are approved while others are rejected.

⁹ As discussed in the Introduction, we relax this assumption in Section 6 when we consider the role of a credit reporting agency which makes this information publicly available to the banks. There, we show that adding a credit agency does not affect the interest rates in equilibrium, but may increase the severity of credit rationing.

¹⁰ We do not include other mechanisms for screening borrowers such as more detailed loan applications or collateral requirements. While we acknowledge that such mechanisms will decrease information asymmetry, it is unlikely that they will eliminate it altogether. Since our goal here is to model interest rates and loan approvals in the presence of adverse selection and moral hazard, we keep the analysis tractable and do not include such mechanisms in the model.

As will become clear, the assumption of inside information implies that borrowers who default on their loans will seek a new bank to do business with next period, that is, they will become search customers. In addition, we assume that borrowers who have their applications rejected will also seek a new bank next period. As such, information about potential borrowers reduces to whether a borrower has one of two labels (or status): a “high” label H for repeat, performing customers, and a “low” label L for search customers. The interest rates that are quoted to these two categories are denoted as r_H and r_L , and the loan approval probabilities are denoted as λ_H and λ_L . Likewise, a solvent borrower’s repayment decision may depend on their status, which is denoted as α_H and α_L .

It follows from this description that each borrower is in one of four states in each period, denoted as (I, H) , (I, L) , (S, H) , and (S, L) . The first component of a state refers to the borrowers’ types, “ I ” for insolvent and “ S ” for solvent. The second component refers to the borrowers’ labels, namely repeat and repaying customers, “ H ”, or search customers, “ L ”. Note that an (I, H) customer is sure to default, but nonetheless carries a misleading H label. The reason for that is that this borrower was a solvent type last period and repaid his loan, earning the H label. However, between last period and this period he experienced a financial shock converting him into an insolvent type. Therefore, borrowers have a one-period informational edge over lenders, as it takes banks one period to catch up with changes in their customers’ type. Analogous interpretations apply to the other three states¹¹.

In the sequel, we focus on anonymous steady-state equilibria in which the same λ_H is chosen across banks and over time, and likewise for λ_L . Similarly, all borrowers choose the same (α_H, α_L) . Given these endogenous choices, we calculate the steady-state measures of borrowers in each of the four states. The measure of insolvent types in the market is $\frac{\beta}{\beta + \gamma}$ and the measure of solvent types is $\frac{\gamma}{\beta + \gamma}$ at any time t . Defining x as the measure of (I, H) borrowers, and y as the measure of (S, H) borrowers, the two-dimensional classification of (type, label)-pairs of borrowers (i.e. borrowers’ states) is given in Table I.

Continuing to take borrowers’ and banks’ strategies as given, the intertemporal equations corresponding to Table I are

$$\begin{aligned} x' &= \beta \left[\alpha_H \lambda_H y + \alpha_L \lambda_L \left(\frac{\gamma}{\beta + \gamma} - y \right) \right], \\ y' &= (1 - \beta) \left[\alpha_H \lambda_H y + \lambda_L \alpha_L \left(\frac{\gamma}{\beta + \gamma} - y \right) \right]. \end{aligned}$$

¹¹ An equivalent model could be posed in which customers do not know their type until they make their repayment decision. Such a model would still involve moral hazard, and the banks would still have to take the tendency for borrowers to switch types and the incentives of solvent borrowers into account when they set their policies.

Table 1. The joint distribution over (type, label)-pairs (or “states”) of borrowers.

	H	L
I	x	$\frac{\beta}{\beta+\gamma} - x$
S	y	$\frac{\gamma}{\beta+\gamma} - y$

In steady state, $x' = x$ and $y' = y$, so that we may solve for the steady state distribution:

$$x = \frac{\alpha_L \lambda_L \beta \gamma}{(\beta + \gamma) [1 - (\alpha_H \lambda_H - \alpha_L \lambda_L)(1 - \beta)]}, \quad (2)$$

$$y = \frac{\alpha_L \lambda_L (1 - \beta) \gamma}{(\beta + \gamma) [1 - (\alpha_H \lambda_H - \alpha_L \lambda_L)(1 - \beta)]}. \quad (3)$$

Given that banks can only observe labels (H or L), we compute the composition of borrower types within labels. We define p as the fraction of solvent types among H -labelled borrowers, and q as the fraction of solvent types among L -labelled borrowers. Given (2) and (3), we can calculate these fractions as

$$p \equiv \frac{y}{x + y} = 1 - \beta, \quad (4)$$

$$q \equiv \frac{\frac{\gamma}{\beta+\gamma} - y}{\frac{\gamma}{\beta+\gamma} - y + \frac{\beta}{\beta+\gamma} - x} = \frac{\gamma [1 - \alpha_H \lambda_H (1 - \beta)]}{(\beta + \gamma) [1 - (\alpha_H \lambda_H - \alpha_L \lambda_L)(1 - \beta)] - \gamma \alpha_L \lambda_L}. \quad (5)$$

We observe that q depends continuously on the endogenous choices of the banks (λ_H, λ_L) and borrowers (α_H, α_L), and denote this dependence as $q = Q(\lambda_H, \lambda_L, \alpha_H, \alpha_L)$. In contrast, p is always equal to $1 - \beta$, and is thus independent of the endogenous strategic actions of the market participants. We also observe that $Q(\lambda_H, \lambda_L, \alpha_H, \alpha_L) \leq p$ for any $(\lambda_H, \lambda_L, \alpha_H, \alpha_L)$. Labels are therefore indicative of type (albeit with residual noise), so an H -labeled borrower represents a better credit-risk than an L -labeled borrower. Accordingly, we sometimes refer to an H (L)-labeled borrower as a high (low) credit-quality borrower.

3. Borrowing and Lending

We now study the maximization programs of the borrowers and lenders. We start by considering the incentives for solvent borrowers to repay their debt. As we will see, the higher the difference in credit rationing for the two labels $\Delta\lambda \equiv \lambda_H - \lambda_L$ and the higher the difference between the two interest rates $\Delta r \equiv r_L - r_H$, the more likely a solvent borrower is to repay their debt. These are the two channels through which the market may impose penalties for defaulting: if the disadvantage

from becoming L -labeled is sufficiently high, then borrowers will repay their debt whenever they are solvent.

3.1 BORROWER BEHAVIOR

At $s = 4$, borrowers whose loan applications were approved decide whether to repay their loans. This decision is determined by an intertemporal tradeoff between the present cost of repaying and the future benefit of developing a good relationship with a bank (being an H -labeled customer). We formulate this tradeoff as a dynamic programming problem (see Equations 6–9 below).

At $s = 3$, lenders decide whether to approve loan applications given the borrowers' expected repayment decision, as just discussed, and given the amount of the loan b^* that they applied for at $s = 2$. As such, at $s = 2$, borrowers may use their loan request b^* as a mechanism to signal their type since banks may use this information to infer a borrower's true type. Therefore, the interaction between the banks and borrowers in stages 2 and 3 constitutes a signalling (sub-) game. Since we assume that loan applications are costless, insolvent borrowers can freely imitate solvent borrowers and the game has no separating equilibrium¹². We focus then on the efficient pooling equilibrium of this game in which both types (insolvent and solvent) apply for the same loan amount. In this case, borrowers in state (S, H) and (I, H) apply for amount $b_H \equiv D(r_H)$ and borrowers in state (S, L) and (I, L) apply for amount $b_L \equiv D(r_L)$.

Folding back stages 2–4, and assuming that the parameters r_H , r_L , λ_H , and λ_L are constant over time, we can write the following reduced-form programs for each of the borrowers, which will help determine their intertemporal incentives to repay their loans.

$$v_{SH} = \text{Max}_{\alpha \in [0,1]} \{ \lambda_H [\alpha s_H + (1 - \alpha)g_H] + \delta \{ \lambda_H \alpha [(1 - \beta)v_{SH} + \beta v_{IH}] + (1 - \lambda_H \alpha) [(1 - \beta)v_{SL} + \beta v_{IL}] \} \}, \quad (6)$$

$$v_{SL} = \text{Max}_{\alpha \in [0,1]} \{ \lambda_L [\alpha s_L + (1 - \alpha)g_L] + \delta \{ \lambda_L \alpha [(1 - \beta)v_{SH} + \beta v_{IH}] + (1 - \lambda_L \alpha) [(1 - \beta)v_{SL} + \beta v_{IL}] \} \}, \quad (7)$$

$$v_{IH} = \lambda_H g_H + \delta [(1 - \gamma)v_{IL} + \gamma v_{SL}], \quad (8)$$

$$v_{IL} = \lambda_L g_L + \delta [(1 - \gamma)v_{IL} + \gamma v_{SL}], \quad (9)$$

¹² Of course, the ability for insolvent customers to pool with solvent types depends in part on the absence of alternative screening mechanisms such as more detailed loan applications or collateral requirements. If such screening were perfect, the problem at hand would become uninteresting as the banks would simply reject all applications from insolvent customers (fully separating equilibrium). With imperfect screening, we may still get pooling or partial pooling, but the model would be less tractable.

where $s_H \equiv S(r_H)$, $s_L \equiv S(r_L)$, $g_H \equiv g(b_H)$, and $g_L \equiv g(b_L)$. The expressions inside the $Max\{\}$ operators in (5) and (6) are the sum of the value from the short-term repayment decision plus the present value of the future borrowing-lending relationship given the decision to repay in the current period.¹³ Since insolvent types do not have discretion over whether to repay, their value functions in (8) and (9) are merely the sum of the amount they can steal in the current period plus their future payoffs, which depend on whether they become solvent (with probability γ).

The following proposition defines conditions under which solvent types have proper incentives to repay their debt.

Proposition 1. (*Conditions for Repayment of Loan*) For each label $k \in \{H, L\}$, a solvent borrower repays his loan (chooses $\alpha_k = 1$) iff

$$\delta \left[\frac{\beta(\lambda_H s_H - \lambda_L s_L) + (1 - \beta)(\lambda_H g_H - \lambda_L g_L)}{1 - \delta(1 - \beta)(\lambda_H - \lambda_L)} \right] \geq (1 + r_k)D(r_k). \quad (10)$$

The condition in (10) is easier to satisfy for a production function $g(\cdot)$ with higher payoffs and for a higher value of δ . For a fixed value of r_H (or r_L), (10) is easier to satisfy as Δr increases. Likewise, for a fixed value of λ_H (or λ_L), (9) is easier to satisfy as $\Delta \lambda$ increases. Finally, (10) is harder to satisfy for H-labeled solvent types if, and only if, $(1 + r_H)D(r_H) > (1 + r_L)D(r_L)$.

According to Proposition 1, borrowers who are otherwise solvent will only repay their loans in the current period if the future benefits to doing so are high enough relative to the cost $(1 + r_k)D(r_k)$ or, equivalently, if the costs of being labeled a low-credit quality are sufficiently severe. The function $g(\cdot)$ measures the benefit that a borrower receives when he puts his loan to good use and δ measures the present value of a dollar earned one-period into the future. For a higher level of productivity per unit input (i.e. $g(\cdot)$ with higher payoffs *ceteris paribus*) and a higher level of δ , the benefit from receiving more money in the future to be put to better use, increases the value of repaying the current debt. If we consider that the function $g(\cdot)$ is tied to aggregate economic growth, then Proposition 1 implies that empirically we should observe strategic default to be less rampant during economic booms.

Proposition 1 also characterizes how the banks' strategies affect the incentives for solvent borrowers to repay their debt. If access to loans is relatively poor for low quality borrowers (high $\Delta \lambda$), then solvent borrowers will repay their debt to

¹³ For notational clarity, all of the value functions in (6)–(9) are computed gross of switching costs. The corresponding value functions net of these costs could be obtained by subtracting the discounted value of switching costs over a borrower's lifetime. Since our analysis hinges on differencing value functions and since the equilibria we find are such that the discounted value of switching costs are the same in all states, it is unaffected by working with gross value functions.

minimize the chances that they will be denied future credit because they get labeled as having low credit quality. Likewise, if the interest rates for L -labeled borrowers is relatively high compared to more favorable H -labeled borrowers, that is, if Δr is large, then solvent borrowers will be more likely to repay their debt to avoid the interest rate penalty incurred by being L -labeled.

Therefore, there are two channels through which solvent borrowers are induced to repay their debt whenever possible. Which channel (or which combination thereof) is used will depend on the constellation of economic conditions and on the actions of the central bank (e.g. through setting c). In what follows, the pair of incentive compatibility constraints in (10), as well as fundamentals, will guide the banks when they set r_H , r_L , λ_H , and λ_L , which is what we consider next.

3.2 LENDER BEHAVIOR

At $s = 1$, each bank maximizes its profits given its expectations of how borrowers and the other banks behave. Of particular importance to banks are the probabilities, z_H and z_L , that H - and L -labeled borrowers repay their loan. Given any pair (z_H, z_L) , the expected profit for each bank may be written as

$$\Pi(r_k, \lambda_k; z_k) \equiv \lambda_k [z_k(1 + r_k)D(r_k) - (1 + c)D(r_k) - f]. \quad (11)$$

As such, for any loan that a bank grants, it receives $(1 + r_k)D(r_k)$ from a borrower with probability z_k and incurs the costs of servicing the loan $(1 + c)D(r_k) + f$ with probability one. Note that r_k and λ_k are a particular bank's decision, whereas z_k is a parameter induced by the decisions of all of the banks together.

For H -labeled borrowers, the probability z_H may be calculated as $z_H = \alpha_H(1 - \beta)$. For L -labeled borrowers, $z_L = \alpha_L q$ or, by (5),

$$z_L = \alpha_L \frac{\gamma [1 - \alpha_H \lambda_H (1 - \beta)]}{(\beta + \gamma)[1 - (\alpha_H \lambda_H - \alpha_L \lambda_L)(1 - \beta)] - \gamma \alpha_L \lambda_L}. \quad (12)$$

By differentiation, $\frac{\partial z_L}{\partial \lambda_L} < 0$, so that when more loans are approved for low-quality customers, the probability that the bank receives repayment decreases.

Before characterizing the lending equilibria of the game, it is important to note some technical assumptions and notation that we use in what follows. First, we assume that the function $\Pi(r, 1, z)$ is strictly quasi-concave in r for every z , and that the revenue function $(1 + r)D(r)$ is also strictly quasi-concave.¹⁴ Second, we define $R(z)$ to be the unique r -maximizer of $\Pi(r, 1, z)$. By our foregoing assumptions,

¹⁴ A sufficient condition for $\Pi(r, 1, z)$ to be strictly quasi-concave is for $D(r)$ to be weakly concave, and in particular for $D(r)$ to be linear. Since $D(r)$ is derived from (1), the usual results from the Theorem of the Maximum ensure that $D(r)$ is continuous as long as $g(\cdot)$ is concave. To our knowledge, however, there are no underlying restrictions on $g(\cdot)$ that guarantee $D(r)$ to be concave.

$R(z) \in [0, \hat{r}]$. Lemma A.1 in the appendix characterizes some of the properties of $\Pi(r, 1, z)$ and $R(z)$ that are important in characterizing the equilibria of the game.¹⁵ It is shown, in particular, that $R(z)$ is continuous and decreasing in z . As such, if we confine our attention to $z \in [\gamma, \frac{\gamma}{\beta+\gamma}]$, R achieves its maximum at $\bar{r}_L \equiv R(\gamma)$ and its minimum at $\underline{r}_L \equiv R(\frac{\gamma}{\beta+\gamma})$. Finally, there exists a unique $z_0 \in (0, 1)$ such that $\Pi(R(z_0), 1, z_0) = 0$. That is, there exists a particular repayment probability in which banks earn zero expected profits when approving all loan applications. We denote the interest rate in this case $r_0 \equiv R(z_0)$.

Now, we are ready to define and characterize the equilibria of the game.

4. Equilibrium

The object to be determined in equilibrium is the vector $(r_H, r_L, \lambda_H, \lambda_L, \alpha_H, \alpha_L)$ corresponding to the banks' and borrowers' behavior. Implicit in this notation is the fact that we focus on anonymous steady-state equilibria, in which all of the parameters are constant over time. Each bank's choices are optimal given the actions of other banks and the optimal behavior of borrowers in the market. As such, the banks' behavior only depends on their borrowers' labels, and not on the time period. Likewise, borrowers' behavior is optimal given the behavior of the banks, and hence only depends on their own type and label.¹⁶

We define a steady-state equilibrium as follows.

Definition 1. *A vector $(r_H^e, r_L^e, \lambda_H^e, \lambda_L^e, \alpha_H^e, \alpha_L^e)$ is a steady-state equilibrium if*

- i. *For $k \in \{H, L\}$, (r_k^e, λ_k^e) maximizes (11) under the z_k that $(r_H^e, r_L^e, \lambda_H^e, \lambda_L^e, \alpha_H^e, \alpha_L^e)$ induces and subject to the incentive constraints in (10).*
- ii. *α_H^e and α_L^e maximize the right-hand side of (6) and (7).*

Proving that an equilibrium exists in this game is trivial. For example, $\lambda_H^e = \lambda_L^e = \alpha_H^e = \alpha_L^e = 0$ is associated with a degenerate equilibrium (autarky) in which the entire market breaks down and no loans are extended. Such an equilibrium is, in fact, the unique equilibrium of a static, one-period version of our banking game, where relationship banking would play no role in the market. Since we are naturally interested in more efficient (non-autarkic) equilibria, where relationship banking enables positive lending and borrowing, we consider now the conditions under which such equilibria exist and how to characterize them.

¹⁵ To be clear, in what follows we use lower-case r to indicate particular interest rates and upper-case R to designate the function that is the unique r -maximizer of $\Pi(r, 1, z)$.

¹⁶ The equilibria are identified with fixed points of a best-response mapping for all market participants, and are therefore Nash equilibria.

To simplify the analysis, we consider equilibria in which banks always lend to H -labeled borrowers ($\lambda_H = 1$), and solvent H -labeled borrowers always repay their debt ($\alpha_H = 1$). There are two reasons for doing this. First, if it is not profitable for the banks to lend to borrowers with high credit quality, that is, if $\Pi(r_H, \lambda_H, z_H) \leq 0$, then they will also not lend to borrowers with low credit quality and, thus, the only equilibrium is the degenerate one. Second, we are most interested in deriving expressions for Δr and $\Delta \lambda$. So, anchoring the banks' behavior for borrowers with high credit quality and then deriving their optimal choices for λ_L and r_L serves this purpose.

Recall that $R(z_k)$ is the interest rate for each type that maximizes the profit function $\Pi(r_k, 1, z_k)$ for any given probability z_k . As such, since $z_H = \alpha_H(1 - \beta)$ and $\alpha_H = 1$, the interest rate for high-credit quality borrowers is pinned down at $\bar{r}_H \equiv R(1 - \beta)$. In contrast, both r_L^e and λ_L^e will vary based on the parameters of the model.

The following proposition characterizes a class of steady state, non-degenerate equilibrium of the game, namely those for which $\lambda_H = \alpha_H = 1$.

Proposition 2. (*Existence and Characterization*) Suppose that (10) is satisfied for $(r_H, r_L, \lambda_H, \lambda_L) = (\bar{r}_H, \underline{r}_L, 1, 1)$, and that

$$\Pi\left(\underline{r}_L, 1, \frac{\gamma}{\beta + \gamma}\right) \geq 0. \quad (13)$$

Then,

- i. If $\Pi(\bar{r}_L, 1, \gamma) \geq 0$ (Regime 1), then there exists an equilibrium in which $\alpha_L^e = \lambda_L^e = 1$. The interest rate charged to L -labeled borrowers is $\bar{r}_L$, which is higher than the interest rate charged to H -labeled borrowers, $\bar{r}_H$.
- ii. If $\Pi(\bar{r}_L, 1, \gamma) < 0$ (Regime 2), then there exists an equilibrium in which $\lambda_L^e < 1$ and $\alpha_L^e = 1$. The interest rate charged to L -labeled borrowers is r_0 , which is again higher than the interest rate charged to H -labeled borrowers. It always holds that $r_0 < \bar{r}_L$, so that r_L^e is lower in Regime 2 than in Regime 1. Finally, banks make lower (zero) profits when lending to L -labeled borrowers in Regime 2 than in Regime 1.

According to Proposition 2, there are two types of equilibria: one with credit rationing ($\lambda_L^e < 1$, Regime 2) and one without ($\lambda_L^e = 1$, Regime 1). If the profit on L -labeled borrowers is non-negative (when all loans are approved), then the equilibrium features no credit rationing (i.e. $\lambda_L^e = 1$). In this case, loans to L -labeled are relatively risky, and as such, the interest rate charged r_L^e is relatively high. On the other hand, if the profit on L -labeled borrowers is negative (again, when all loans are approved), then the equilibrium features credit rationing ($\lambda_L^e < 1$). Intuitively, credit rationing lowers the riskiness of loans by keeping more solvent borrowers

in the pool of L -labeled borrowers, and as such serves to raise the profitability of loans to this group (to zero). In this case, loans to L -labeled borrowers are less risky (as compared to regime 1), and the interest rate charged is accordingly lower.

Which equilibrium regime materializes depends how profitable it is to lend to L -labeled borrowers when all loan applications are approved. This in turn depends on the primitives of the model. In particular, for higher γ , lower β and f , and steeper $g(\cdot)$, it is more profitable to lend to all borrowers, which makes it more likely that the regime 1 equilibrium arises, and vice versa.

We note that the profits on H -labeled borrowers are higher than the profits on L -labeled borrowers because H -labeled borrowers represent better risk. All in all, therefore, the banks' profits are positive in Regime 1, and exceed their profits in Regime 2. We also note that solvent borrowers' incentive to repay stems from a large interest rate differential in Regime 1, and from a large approval probability differential in Regime 2.

We finally observe that credit rationing by one bank raises the profitability of lending to L -labeled borrowers to *all* banks. This externality motivates the welfare analysis of public policies aimed at affecting credit markets. An added force in the model that has a bearing on this welfare analysis is the distortion that arises from consumers' search costs and the banks' consequent monopoly interest rates. As Proposition 2 indicates, this monopoly distortion operates either through cutting off some borrowers from loans altogether, or by limiting the amounts of the loans they receive (by raising interest rates). In the next section, we analyze the interplay between these forces. Specifically, we study the effects that regulation has on interest rates, credit rationing, and welfare.

5. Welfare Analysis and Regulation

According to our model, there are three potential ways in which public policy may affect the market for loans. First, by inspection of the profit function in (11), when the central bank changes its reference rate, it not only changes interest rates, but also the likelihood that the banks are in a particular regime. Raising c makes Regime 2 more likely because it reduces the banks' profits, whereas lowering c makes Regime 1 more likely. Second, the government can alter how much they subsidize or tax loans by changing the fixed costs of initiating a loan (by changing f), which may induce a similar regime change. Higher (lower) f makes it less (more) attractive for the banks to lend to everyone. Finally, the government can pass usury laws that cap the interest rate that may be charged to low credit-quality borrowers (L -labeled). In our model, restricting r_L lowers the banks' profits and makes it more difficult (or impossible) for the constraints in (10) to be satisfied. Thus, capping r_L may cause the banks to institute credit rationing in order to induce solvent borrowers to

repay their debt. Therefore, profit maximizing behavior, in the face of usury laws, may involve a transition from Regime 1 to Regime 2.

All three of these policies may cause either a change in interest rates alone (when the policy does not affect the equilibrium regime), or a change in both interest rates and credit rationing (when the policy induces a regime change). In the first instance, the policy is welfare-improving if it reduces interest rates¹⁷. In the second instance, and as commented on before, the tradeoff is between lending to all L -labeled borrowers, but granting them relatively small loans versus denying credit to some borrowers, but granting the ones who gain access to capital relatively larger loans. Our aim here is to elicit a condition under which the government will optimally choose to induce a particular regime switch.

Let $\Delta W = W_1 - W_2$ be the difference between the social welfare derived from the two types of regimes. By construction, when $\Delta W > 0$, it is optimal for the government to establish policies that encourage banks to lend to all types of borrowers, even if this means that low-credit quality borrowers pay high interest rates. As such, ΔW may be calculated as

$$\Delta W = H_1 g_{H,1} + L_1 g_{L,1} - H_2 g_{H,2} - \lambda_L L_2 g_{L,2}, \quad (14)$$

where H_i (L_i) is the proportion of H -labeled (L -labeled) borrowers in each regime $i \in \{1, 2\}$ and $g_{k,i}$ is the value of the production function evaluated at the interest rate for each label $k \in \{H, L\}$ in each regime¹⁸. Recalling (2) from Section 2 and Proposition 2, H_i and L_i will depend on the primitives of the model as well as the equilibrium behavior of the different parties to the transaction (specifically, λ_L^e).

To illustrate how regulatory policy affects the market, let us focus on policies that affect the fixed cost of making loans f . Inspection of (11) reveals that varying f has no impact on the interest rate $\bar{r}_H$ charged to H -labeled borrowers in either regime, nor does it have an impact on the interest rate $\bar{r}_L$ charged to L -labeled borrowers in Regime 1. The only impact of varying f is on z_0 : increasing f causes z_0 to rise and λ_L^e to drop, and vice versa. As a result, $g_{H,1}$, $g_{H,2}$, and $g_{L,1}$ are constants (with respect to policies that affect f only) and $g_{H,1} = g_{H,2}$.¹⁹

Now, Proposition 2 tells us that $\alpha_L^e = 1$ in either regime. So, setting $\alpha_L^e = 1$ and using (2), the proportion of H -labeled and L -labeled borrowers in regime $i \in \{1, 2\}$

¹⁷ This assumes, as usual, that the policy does not generate distortions elsewhere (e.g. when the policy requires higher taxes to be collected to offset any subsidies offered by the policy).

¹⁸ Note that we assume that changes in f do not by themselves cause other welfare changes (e.g., a deadweight loss).

¹⁹ By contrast, if government policy affects c , then $\bar{r}_H$ and $\bar{r}_L$ are affected as well, which implies that $g_{H,1}$, $g_{H,2}$, and $g_{L,1}$ are not constants. Similar, albeit somewhat more involved, analysis applies in that case.

is given by²⁰

$$H_1 = \frac{\gamma}{\beta + \gamma} \quad H_2 = \frac{\lambda_L^e \gamma}{(\beta + \gamma)[1 - (1 - \lambda_L^e)(1 - \beta)]} \quad (15)$$

and

$$L_1 = \frac{\beta}{\beta + \gamma} L_2 = \frac{\beta[1 - (1 - \lambda_L^e)(1 - \beta - \gamma)]}{(\beta + \gamma)[1 - (1 - \lambda_L^e)(1 - \beta)]}. \quad (16)$$

To evaluate ΔW , let us first consider the social welfare derived from H -labeled borrowers, which we denote by $\Delta W_H = g_H(H_1 - H_2)$, where g_H is the common value of $g_{H,1}$ and $g_{H,2}$. Using (15), we calculate

$$\Delta W_H = \frac{\gamma}{\beta + \gamma} \left[1 - \frac{\lambda_L^e}{1 - (1 - \lambda_L^e)(1 - \beta)} \right] g_H. \quad (17)$$

By inspection and differentiation, ΔW_H is strictly positive and decreasing in λ_L^e ($\frac{\partial \Delta W_H}{\partial \lambda_L^e} < 0$). The intuition for this result is straightforward. Since all H -labeled borrowers are offered the same interest rate in both regimes, $\bar{r}_H$, the only difference in welfare between the regimes is the proportion of H -labeled borrowers that are present. The credit rationing in Regime 2 prevents solvent borrowers who are L -labeled from getting the chance to prove themselves, thereby lowering the steady state proportion of H -types (i.e. $H_1 > H_2$). This leads to a welfare loss because these borrowers are prevented from putting capital to good use. Therefore, from the standpoint of analyzing H -labeled borrowers, Regime 1 is always superior to Regime 2, *ceteris paribus*.

Now, consider the social welfare derived from L -labeled borrowers, which we denote by $\Delta W_L = g_{L,1}L_1 - \lambda_{L,2}^e g_{L,2}L_2$. Upon inspection, it is unclear whether ΔW_L is positive or negative. Further, since ΔW_L is non-monotonic in λ_L^e , it is not possible at this level of generality to determine the impact of varying λ_L on ΔW_L , and hence on ΔW . Nonetheless, we are able to pin down a condition under which one regime or the other is optimal.

Proposition 3. *Regime 1 is welfare-superior to Regime 2 ($\Delta W > 0$) if, and only if,*

$$\frac{g_H}{g_{L,1}} \frac{\gamma}{\beta} M + 1 > \frac{g_{L,2}}{g_{L,1}} \lambda_{L,2}^e N, \quad (18)$$

where

$$M \equiv 1 - \frac{\lambda_L^e}{1 - (1 - \lambda_L^e)(1 - \beta)}$$

²⁰ With a small abuse of notation, we use λ_L^e to refer to the credit rationing in Regime 2, since we know that $\lambda_L = 1$ in Regime 1.

and

$$N \equiv \frac{1 - (1 - \lambda_L^e)(1 - \beta - \gamma)}{1 - (1 - \lambda_L^e)(1 - \beta)}.$$

The condition in (18) is a necessary and sufficient condition that determines which regime is optimal. As is evident, the optimality of either regime depends on the model parameters, namely β , γ , and $g(\cdot)$, as well as the endogenous data λ_L^e and r_L^e . The interaction of these parameters admittedly makes definitive comparative statics challenging, though it is clear that the nature of the production function is a key input. Indeed, for a credit rationing regime to be attractive, $\frac{g_{L,2}}{g_{L,1}}$ needs to be sufficiently large, where a little more capital goes a long way.²¹ This has a direct implication regarding the scalability of production: if returns to scale are sufficiently large at low levels of capital, then it may be more attractive for regulators to induce credit rationing.

This analysis has empirical import in the U.S. small business sector, where non-collateralized loans play a large role in financing businesses activities. As of 2003, 47% of small businesses used the owners' personal credit card for financing. Given that this sector represents 50% of the output from the private sector and accounts for 60–80% of new jobs in the economy, regulation has a significant impact on welfare in the market. Different industries within this sector have varied ability to achieve returns to scale and the mix of businesses changes over time.²² This not only affects the contribution that each industry has on the aggregate production function of the sector (which drives regulation according to our model), but also changes how efficient regulation is given each industry that is represented within the mix. Taking the theory to the data then might involve constructing an aggregate production function from the panel of data from this sector, and correlating a proxy for $\frac{g_{L,2}}{g_{L,1}}$ with the type of regulation that is present. To our knowledge this has not been done to date, but is the subject of future research.

6. Credit Reporting Agencies

So far, we have made the assumption that each bank privately observes the behavior of its own borrowers, that is, whether they repaid or defaulted on their loans (inside information). Now, we relax that assumption and analyze the model when a credit

²¹ Note that the condition that $\frac{g_{L,2}}{g_{L,1}}$ be large is necessary but not sufficient to guarantee that credit rationing is optimal. Of course, $\frac{g_H}{g_{L,1}} > \frac{g_{L,2}}{g_{L,1}}$, but each are multiplied by different parameters. Indeed, $\frac{g_H}{g_{L,1}}$ is multiplied by $\frac{\gamma}{\beta}$ and $\frac{g_{L,2}}{g_{L,1}}$ is multiplied by λ_L^e . Also, by inspection, $N > M$. Therefore, large $\frac{g_{L,2}}{g_{L,1}}$ is necessary, but not sufficient to make Regime 2 more attractive.

²² For example, see Table 2 in Mach and Wolken (2006).

reporting agency is present to assist banks in resolving asymmetric information in the market. The purpose of this section is two-fold. First, we explore how a credit reporting agency affects the optimal strategies of both the banks and borrowers, which in turn affects the feasibility of a non-autarkic equilibrium and the amount of credit rationing in the market. Second, we determine the effects that a credit reporting agency has on welfare, and how it may change public policy.

Given the model setup in Section 2, consider now a situation in which a credit reporting agency is present that assists banks in identifying borrowers' types. Suppose that the credit reporting agency is successful in identifying a borrower's true type with probability ϕ . So, we consider ϕ to measure the precision with which the credit reporting agency assists the banks. When $\phi = 1$, all borrowers are identified accurately, whereas when $\phi = 0$ we are back to the situation modeled in Section 2. Therefore, when a solvent type of borrower is identified and has either an H -label or an L -label, they receive a loan from the bank at an interest rate r_H . When an insolvent borrower is identified with either label, however, their loan application is rejected. When a borrower (solvent or insolvent) is not identified, the interest rate and credit-rationing probability are determined, as before, by their label. So, with probability ϕ the banks can sort each customer, and with probability $(1 - \phi)$ lending terms are determined by labels as previously modeled.²³

We solve this version of the model, as we did in Section 3. With regard to the borrowers' programs, we fold back stages 2–4 of the game, and assume that the parameters r_H , r_L , λ_H , and λ_L are constant over time. We then write reduced-form programs for each of the borrowers, and derive their intertemporal incentives to repay their loans.

To avoid expositional repetition, the value functions for borrowers in each state and conditions under which solvent borrowers repay their loans are given in the appendix (i.e., Proposition A.1). There, we show that as ϕ rises, it is more difficult to induce solvent types to repay their loans. This occurs because the credit reporting agency decreases the penalty for the solvent type to having an L -label. That is, if a solvent type gets a loan as an H -labeled borrower and defaults, when they search for a new bank (as an L -labeled borrower), they get identified as a solvent borrower by the credit reporting agency with probability ϕ and as such are not penalized by having an L -label. This decreases the penalty that they suffer and, in turn, makes the incentive constraints in (11) harder to satisfy.

²³ Admittedly, this approach to modeling information sharing is not general, as there are many mechanisms available in which banks might pool information. The goal here, though, is to relax our assumption of inside information and compare the results with those of the base model. As an alternative, one might consider a model in which credit reporting agencies report without error the entire credit history of borrowers, rather than their types. That approach would be less tractable and less comparable to our base model since one would have to handle three borrower labels: those who repaid their loans, those who defaulted, and those whose loans were denied.

In Proposition A.2 in the appendix, we consider the effect that the credit reporting agency has on interest rates, credit rationing, and the existence of non-autarkic equilibria. As such, Proposition A.2 characterizes the existence of equilibria when a credit reporting agency is present. The proof follows in the same way as the proof of Proposition 2 so we provide an outline of the proof there.

According to Proposition A.2, when Regime 1 arises with a credit reporting agency, the interest rates are unchanged when compared to the case without one. Therefore, in this case, the same values of the endogenous variables equilibrate the market. In contrast, when Regime 2 arises with a credit reporting agency present, the interest rates remain the same, but the degree of credit rationing for L -labeled customers is more severe. That is, λ_L^e is lower with a credit reporting agency than without one. The intuition behind this result, again, is that it is easier for solvent borrowers to avoid having an L -label because the credit reporting agency may indeed identify them as being solvent, thereby giving them access to good credit. Given this, solvent borrowers have weaker incentives to repay their loan *ceteris paribus*, so λ_L^e is reduced in equilibrium to induce solvent borrowers to nonetheless repay their loans.

Next, we turn to the question regarding the welfare effects of a credit reporting agency. As in Proposition 3, all of the model primitives determine whether it is optimal for a social planner to choose one regime over another. Given this, we compare the welfare that arises in each regime, with and without a credit reporting agency. To that end, define W_i^j to be the welfare that forms in Regime $i \in \{1, 2\}$ in each lending environment $j \in \{a, n\}$, where a signifies the presence of an credit reporting agency and n signifies that an agency is not present. Let us also define $\Delta W^j = W_1^j - W_2^j$.

Proposition 4. *Fix values for f , c , ϕ , and δ such that an equilibrium exists with a credit reporting agency. Then,*

- i. $W_1^n = W_1^a$.
- ii. $W_2^n > W_2^a$.
- iii. $\Delta W^a > \Delta W^n$.

According to Proposition 4, the welfare that is generated is (weakly) higher without a credit reporting agency than with one. Indeed, since interest rates are the same in either case, the welfare that forms in Regime 1 is the same, whether or not a credit reporting agency is present. In contrast, the welfare that forms in Regime 2 with a credit reporting agency is strictly lower. This result arises because credit rationing is more severe with a credit reporting agency, which lowers the fraction of H -labeled customers who could employ capital to produce g_H .

The fact that $\Delta W^a > \Delta W^n$ for a particular set of parameters implies that there is more to be gained when a credit reporting agency is present if the government

induces a regime switch in which all borrowers receive loans. Since the government may reduce f to induce this change, they will be more likely to do so in the presence of a credit reporting agency. Therefore, the model predicts that the scope for tighter regulation is higher when a credit reporting agency is present.

The analysis here adds to the literature on information sharing in credit markets (e.g. Pagano and Jappelli 1993; Jappelli and Pagano 2001). Pagano and Jappelli (1993) analyze how information sharing arises endogenously and show that information is more likely to be shared when borrowers are heterogeneous, when the credit market is large, when borrowers are mobile, and when the cost of exchanging information is low. Sharing information leads to freer lending because it decreases adverse selection in the market.

Our analysis adds to their work in two ways. First, it provides another empirical motivation for the observation that the volume of lending increases when information sharing occurs: since there is more to be gained from decreasing credit rationing, the government will effect a switch from Regime 2 to Regime 1. Empirically, this predicts that with information sharing, the cost of funds should be lower and/or the fixed cost of initiating loans should be lower. Second, our analysis predicts that information sharing by itself (i.e. without regulatory action) may not increase welfare. This difference with Pagano and Jappelli (1993) comes about because we consider that solvent borrowers have the ability to default. Testing this hypothesis with Pagano and Jappelli (1993) in mind might involve controlling for the laxity of personal bankruptcy laws by locale. This work is the subject of future research.

7. Conclusions

Given the increasing dependence of U.S. households on debt financing, we develop a dynamic model of relationship banking to study the effects of regulation on borrower and lending behavior, and to analyze the welfare implications that such policies have on the market.

The model that we analyze in the paper incorporates three stylized facts about banking markets. First, solvent borrowers may hide assets from lenders and strategically default on their obligations. Second, borrowers are heterogeneous with respect to their ability to repay loans, which is private information. Third, finding a new bank is a costly activity for both borrowers and lenders.

In the model, two regimes may arise in equilibrium. Regime 1 involves all borrowers obtaining access to capital, though low quality borrowers are charged high interest on their loans. In Regime 2, some of the low quality borrowers do not have access to capital (via credit rationing), but the interest rate they are charged is not as high as in Regime 1. We analyze the effect that regulation has on which

regime arises and characterize when it is optimal for the government to intervene. Finally, we complete our analysis by considering the effect that a credit reporting agency has on the dynamics in the market.

Appendix

Proof of Proposition 1. We start by using (6) to derive an incentive compatibility condition under which a borrower in state (S, H) repays their loan. Comparing their value function when $\alpha = 0$ and when $\alpha = 1$, an (S, H) borrower prefers to repay the bank iff

$$\delta[(1 - \beta)(v_{SH} - v_{SL}) + \beta(v_{IH} - v_{IL})] \geq (1 + r_H)D(r_H). \quad (19)$$

Likewise, using (7) we can derive an incentive compatibility condition for an (S, L) borrower to repay their loan

$$\delta[(1 - \beta)(v_{SH} - v_{SL}) + \beta(v_{IH} - v_{IL})] \geq (1 + r_L)D(r_L). \quad (20)$$

Since the left-hand side of (19) and (20) are the same, the incentive constraint in (19) is more restrictive than that in (20) if, and only if, $(1 + r_H)D(r_H) > (1 + r_L)D(r_L)$.

From (8) and (9), we can write $v_{IH} - v_{IL} = \lambda_H g_H - \lambda_L g_L$. Further, from (6)–(9), we can calculate that

$$v_{SH} - v_{SL} = \frac{\lambda_H s_H - \lambda_L s_L + \beta\delta(\lambda_H - \lambda_L)(\lambda_H g_H - \lambda_L g_L)}{1 - \delta(1 - \beta)(\lambda_H - \lambda_L)}.$$

Substituting these expressions, we can re-write (19) as

$$\begin{aligned} \delta \left[(1 - \beta) \left(\frac{\lambda_H s_H - \lambda_L s_L + \beta\delta(\lambda_H - \lambda_L)(\lambda_H g_H - \lambda_L g_L)}{1 - \delta(1 - \beta)(\lambda_H - \lambda_L)} \right) \right. \\ \left. + \beta(\lambda_H g_H - \lambda_L g_L) \right] \geq (1 + r_H)D(r_H). \end{aligned} \quad (21)$$

Combining the summands under one common denominator, (21) is written as

$$\delta \left(\frac{\beta(\lambda_H s_H - \lambda_L s_L) + (1 - \beta)(\lambda_H g_H - \lambda_L g_L)}{1 - \delta(1 - \beta)(\lambda_H - \lambda_L)} \right) \geq (1 + r_H)D(r_H). \quad (22)$$

Likewise, we can re-write (20) as

$$\delta \left(\frac{\beta(\lambda_H s_H - \lambda_L s_L) + (1 - \beta)(\lambda_H g_H - \lambda_L g_L)}{1 - \delta(1 - \beta)(\lambda_H - \lambda_L)} \right) \geq (1 + r_L)D(r_L). \quad (23)$$

Thus, (19) and (20) can be written simply as (10).

The comparative statics on $g(\cdot)$, δ , Δr , and $\Delta \lambda$ are generated by simple differentiation. ■

Proof of Proposition 2. We begin by proving the following lemma that summarizes some important properties of $\Pi(r, 1, z)$ and $R(z)$, which will be important in characterizing the equilibria of the game.

Lemma A.1. *Suppose that $c < \bar{r}$ and that f is sufficiently small. Then,*

- i. $\Pi(R(z), 1, z)$ is continuous and increasing in z , with $\Pi(R(0), 1, 0) < 0 < \Pi(R(1), 1, 1)$. Therefore, there exists a unique $z_0 \in (0, 1)$ such that $\Pi(R(z_0), 1, z_0) = 0$.
- ii. $R(z)$ occurs on the downward-sloping portion of $(1 + r)D(r)$, for all $z \in [0, 1]$.
- iii. $R(z)$ is continuous and decreasing in z .
- iii. If $z' < z''$, $r' = R(z')$, and $r'' = R(z'')$, then $(1 + r')D(r') < (1 + r'')D(r'')$.

Proof of Lemma A.1

- i. The monotonicity of $\Pi(R(z), 1, z)$ follows from the fact that $\Pi(r, 1, z)$ increases in z . Continuity follows from the Theorem of the Maximum. Note also that when $z = 0$, $\Pi(R(z), 1, z) < 0$. And, when $z = 1$, then $\Pi(R(z), 1, z) > 0$ as long as $c < \bar{r}$ and f is sufficiently small. Since $\Pi(R(z), 1, z)$ is continuous and strictly increasing in z , there exists a unique $z_0 \in (0, 1)$ such that $\Pi(R(z_0), 1, z_0) = 0$.
- ii. Suppose that $R(z)$ does not occur on the downward portion of $(1 + r)D(r)$. Then, there would exist an r' such that $r' > r = R(z)$ and such that $(1 + r')D(r') > (1 + r)D(r)$. Since, $D(r)$ is strictly decreasing in r , it follows that $(1 + c)D(r') < (1 + c)D(r)$. Taking these two inequalities together implies that r' yields higher profits than r , contradicting the optimality of r .
- iii. Consider two z -values, $0 \leq z' < z'' \leq 1$, and let $r' = R(z')$ and $r'' = R(z'')$. Then, since r'' maximizes under r'' , while r' need not, we have

$$z''(1 + r'')D(r'') - (1 + c)D(r'') \geq z''(1 + r')D(r') - (1 + c)D(r'). \quad (24)$$

By the same token,

$$z'(1 + r')D(r') - (1 + c)D(r') \geq z'(1 + r'')D(r'') - (1 + c)D(r''). \quad (25)$$

Subtracting the right-hand side of (25) from the left-hand side of (24), and subtracting the left-hand side of (25) from the right-hand side of (24) yields

$$(z'' - z')(1 + r'')D(r'') \geq (z'' - z')(1 + r')D(r').$$

Since $z'' > z'$, this implies

$$(1 + r'')D(r'') \geq (1 + r')D(r'),$$

which proves (iv). Also, since, r'' and r' occur on the downward sloping portion of $(1 + r)D(r)$ (by (ii)), it must be that $r'' < r'$. Therefore, $R(z)$ is decreasing in z . The continuity of $R(z)$ follows from the Theorem of the Maximum. ■

Now we proceed to prove Proposition 2.

Regime 1: $\Pi(\bar{r}_L, 1, \gamma) \geq 0$.

We have to show that $\lambda_H^e = \lambda_L^e = \alpha_H^e = \alpha_L^e = 1$, $r_H^e = \bar{r}_H$, and $r_L^e = \bar{r}_L$ is an equilibrium. To do so, we first observe that the borrowers' incentive constraints in (10) are satisfied. For H -labeled borrowers this follows from $r_L^e > \underline{r}_L$ (since $\gamma < \frac{\gamma}{\beta+\gamma}$ and R is decreasing), from the fact that (10) is easier to satisfy as Δr increases (see Proposition 1), and from the assumption that (10) is satisfied for $\bar{r}_H, \underline{r}_L$, and $\lambda_H = \lambda_L = 1$. For L -labeled borrowers, this follows from the fact that $(1+r)D(r)$ is decreasing at the optimum (see Lemma A.1) and from what we have just proven, namely that (10) is satisfied for H -labeled borrowers. This establishes that $\alpha_H^e = \alpha_L^e = 1$ represents maximizing behavior for the borrowers.

Turning to the lenders, $\lambda_H^e = \lambda_L^e = \alpha_H^e = \alpha_L^e = 1$ implies that $z_H = 1 - \beta$ and $z_L = \gamma$ (see (4) and (5)). But, then $r_H^e = \bar{r}_H$, $r_L^e = \bar{r}_L$, and $\Pi(\bar{r}_L, 1, \gamma) \geq 0$, which implies that $\lambda_H^e, \lambda_L^e, r_H^e$, and r_L^e represents maximizing behavior for the banks.

Regime 2: $\Pi(\bar{r}_L, 1, \gamma) < 0$.

In this case, Lemma A.1 along with (13) shows that there is a unique $z_0 \in (\gamma, \frac{\gamma}{\beta+\gamma}]$, such that $\Pi(r_0, 1, z_0) = 0$.

Now, we set $r_H^e = \bar{r}_H$ and $r_L^e = r_0$, and let λ_L^e be the unique value in $[0, 1)$ for which $Q(1, \lambda_L, 1, 1) = z_0$. The reason such a λ_L exists is that $Q(1, \lambda, 1, 1)$ is strictly decreasing in λ_L , which can be verified from (5), and that $Q(1, 0, 1, 1) = \frac{\gamma}{\beta+\gamma}$ and $Q(1, 1, 1, 1) = \gamma$.

Next, we verify that this configuration, along with $\lambda_H^e = \alpha_H^e = \alpha_L^e = 1$, constitutes an equilibrium. Indeed, (10) is satisfied for H -labeled borrowers because $r_L^e = r_0 \geq \underline{r}_L$ (since $z_0 \leq \frac{\gamma}{\beta+\gamma}$), and $\lambda_L^e < 1$. So, the monotonicity of the condition in (10) in Δr and $\Delta \lambda$ establishes the claim.

Further, (10) is satisfied for L -labeled borrowers because $(1+r)D(r)$ is decreasing at the optimum and $r_L^e > r_H^e$.

Now, with this $(\alpha_L^e, \lambda_L^e)$ -pair, $z_L = z_0$. Therefore, the banks' choices for L -labeled borrowers, λ_L^e and $r_L^e = r_0$ are profit maximizing (with zero profit). And, as already stated, the banks choices for H -labeled borrowers ($\lambda_H^e = 1$ and $r_H^e = \bar{r}_H$) are profit maximizing as well. ■

Proof of Proposition 3. According to (14)–(16), $\Delta W > 0$ if

$$g_H \frac{\gamma}{\beta + \gamma} \left[1 - \frac{\lambda_L^e}{1 - (1 - \lambda_L^e)(1 - \beta)} \right] + \frac{\beta}{\beta + \gamma} g_{L,1} - g_{L,2} \lambda_L^e \frac{\beta[1 - (1 - \lambda_L^e)(1 - \beta - \gamma)]}{(\beta + \gamma)[1 - (1 - \lambda_L^e)(1 - \beta)]} > 0,$$

which may be expressed as

$$\frac{g_H}{g_{L,1}} \frac{\gamma}{\beta} \left[1 - \frac{\lambda_L^e}{1 - (1 - \lambda_L^e)(1 - \beta)} \right] + 1 > \frac{g_{L,2}}{g_{L,1}} \lambda_L^e \frac{[1 - (1 - \lambda_L^e)(1 - \beta - \gamma)]}{[1 - (1 - \lambda_L^e)(1 - \beta)]}$$

■

Propositions with a credit reporting agency

The reduced-form programs for each of the borrowers are expressed as

$$v_{SH}^c = \text{Max}_{\alpha \in [0,1]} \{ \lambda_H [\alpha s_H + (1 - \alpha) g_H] + \delta \{ \lambda_H \alpha [(1 - \beta) v_{SH}^c + \beta v_{IH}^c] + (1 - \lambda_H \alpha) [(1 - \beta) v_{SL}^c + \beta v_{IL}^c] \} \}, \quad (26)$$

$$v_{SL}^c = \text{Max}_{\alpha \in [0,1]} \{ \phi v_{SH}^c + (1 - \phi) [\lambda_L [\alpha s_L + (1 - \alpha) g_L] + \delta \{ \lambda_L \alpha [(1 - \beta) v_{SH}^c + \beta v_{IH}^c] + (1 - \lambda_L \alpha) [(1 - \beta) v_{SL}^c + \beta v_{IL}^c] \} \}, \quad (27)$$

$$v_{IH}^c = (1 - \phi) \lambda_H g_H + \delta [(1 - \gamma) v_{IL}^c + \gamma v_{SL}^c], \quad (28)$$

$$v_{IL}^c = (1 - \phi) \lambda_L g_L + \delta [(1 - \gamma) v_{IL}^c + \gamma v_{SL}^c]. \quad (29)$$

As before, the expressions inside the $\text{Max}\{\}$ operators in (26) and (27) are the sum of the value from the short-term repayment decision plus the present value of the future borrowing-lending relationship, given the decision to repay in the current period.

The following proposition defines conditions under which solvent types have the proper incentives to repay their debt.

Proposition A.1. *(Conditions for Repayment of Loan with Credit Reporting Agency) For each label $k \in \{H, L\}$, a solvent borrower repays their loan (chooses $\alpha_k = 1$) iff*

$$\Psi(r_H, r_L, \lambda_H, \lambda_L, \phi) \geq \frac{(1 + r_H)D(r_H)}{1 - \phi} \quad (30)$$

and

$$\Psi(r_H, r_L, \lambda_H, \lambda_L, \phi) \geq (1 + r_L)D(r_L) \quad (31)$$

where

$$\Psi \equiv \delta \left(\frac{(1 - \beta)(\lambda_H s_H - \lambda_L s_L) + \beta(\lambda_H g_H - \lambda_L g_L)}{1 - \delta(1 - \beta)(1 - \phi)(\lambda_H - \lambda_L)} \right). \quad (32)$$

The conditions in (30) and (31) are easier to satisfy for a production function $g(\cdot)$ with higher payoffs and for a higher value of δ . For a fixed value of r_H (or r_L), both conditions are easier to satisfy as Δr increases. Likewise, for a fixed value of λ_H (or λ_L), it is easier to satisfy both conditions as $\Delta \lambda$ increases.

The condition in (30) is harder to satisfy than the condition in (31) if, and only if, $\frac{(1+r_H)D(r_H)}{1-\phi} > (1+r_L)D(r_L)$. For increasing ϕ , both conditions become harder to satisfy.

Proof of Proposition A.1. We start by using (26) to derive an incentive compatibility condition under which an (S, H) borrower repays their loan. Comparing this value function when $\alpha = 0$ and when $\alpha = 1$, an (S, H) borrower prefers to repay the bank iff

$$\delta[(1-\beta)(v_{SH}^c - v_{SL}^c) + \beta(v_{IH}^c - v_{IL}^c)] \geq (1+r_H)D(r_H). \quad (33)$$

Likewise, using (27) we can derive an incentive compatibility condition for an $\{S, L\}$ -type to repay their loan

$$\delta[(1-\beta)(v_{SH}^c - v_{SL}^c) + \beta(v_{IH}^c - v_{IL}^c)] \geq (1-\phi)(1+r_L)D(r_L). \quad (34)$$

Since the left-hand side of (33) and (34) are the same, the incentive constraint in (33) is more restrictive than that in (34) if, and only if, $(1+r_H)D(r_H) > (1-\phi)(1+r_L)D(r_L)$.

Using algebraic manipulation as in the proof of Proposition 1, (33) and (34) can be written simply as (30) and (31).

The comparative statics on $g(\cdot)$, δ , Δr , $\Delta \lambda$, and ϕ are generated by simple differentiation. ■

Proposition A.2. (Existence and Characterization under Credit Reporting Agency) Suppose that

$$\Pi\left(\underline{r}_L, 1, \frac{\gamma}{\beta + \gamma}\right) \geq 0 \quad (35)$$

and that

$$\Psi(\bar{r}_H, \underline{r}_L, 1, 1, \phi) \geq \frac{(1+\bar{r}_H)D(\bar{r}_H)}{1-\phi}. \quad (36)$$

Then,

- i. If $\Pi(\bar{r}_L, 1, \gamma) \geq 0$ (Regime 1), then there exists an equilibrium in which $\alpha_L^e = \lambda_L^e = 1$. The interest rate charged to L-labeled borrowers is $r_L^e = R(\gamma)$, which is higher than the interest rate charged to H-labeled borrowers $r_H^e = R(1-\beta)$. Both interest rates are the same as when a credit reporting agency is not present, but the condition under which a non-autarkic equilibrium exists (i.e. the inequality in (36)) is more restrictive.

- ii. If $\Pi(\bar{r}_L, 1, \gamma) < 0$ (Regime 2), then there exists an equilibrium in which $\lambda_L^e < 1$ and $\alpha_L^e = 1$. This λ_L^e is smaller than the corresponding λ_L^e when a credit agency is not present (i.e. banks are more stringent about approving loans). The interest rate charged to L -labeled borrowers is $r_L^e = r_0$, which is higher than the interest rate charged to H -labeled borrowers. Both interest rates are the same as when a credit reporting agency is not present, but the condition under which such a non-autarkic equilibrium exists is more restrictive.

Outline of Proof of Proposition A.2. The proof of Proposition A follows the same logic as in the proof of Proposition 2, noting the modifications added by the presence of a credit reporting agency. The profit function for the banks is the same as before since it only depends on $D(\cdot)$, c , and f . Hence, z_0 is larger when f increases. On the other hand, as noted in Proposition A.1, the incentive constraints are more restrictive, implying that it is harder to achieve a non-autarkic equilibrium. In addition, we show in Lemma A.2 (which follows next) that the credit worthiness q of the L -labeled pool of borrowers after screening by the credit reporting agency is lower. That is, q is smaller for $\phi > 0$ *ceteris paribus*. It is furthermore shown that $Q(1, \lambda, 1, 1)$ is decreasing in λ . Therefore, the λ_L that satisfies $Q(1, \lambda, 1, 1) = z_0$ is smaller when $\phi > 0$, even if z_0 is the same and more so if z_0 increases. ■

Lemma A.2. Assume that $\alpha_H \lambda_H - \alpha_L \lambda_L > 0$. Then,

- i. The steady-state y under $\phi > 0$ is

$$y = \frac{(1 - \beta)\gamma[\phi\alpha_H\lambda_H + (1 - \phi)\alpha_L\lambda_L]}{(\beta + \gamma)[1 - (1 - \beta)(1 - \phi)(\alpha_H\lambda_H - \alpha_L\lambda_L)]}. \quad (37)$$

- ii. The steady-state y is larger when $\phi > 0$ than when $\phi = 0$.
 iii. The steady-state q is smaller when $\phi > 0$ than when $\phi = 0$.
 iv. Setting $\lambda_H = \alpha_H = \alpha_L = 1$, the steady-state y is increasing in λ_L and hence, the steady-state q is decreasing in λ_L .

Proof.

- i. The intertemporal equations corresponding to Table 1 under the presence of ϕ are

$$x' = \beta \left\{ \alpha_H \lambda_H \left[y + \phi \left(\frac{\gamma}{\beta + \gamma} - y \right) \right] + \alpha_L \lambda_L (1 - \phi) \left(\frac{\gamma}{\beta + \gamma} - y \right) \right\} \quad (38)$$

and

$$y' = (1 - \beta) \left\{ \alpha_H \lambda_H \left[y + \phi \left(\frac{\gamma}{\beta + \gamma} - y \right) \right] + \alpha_L \lambda_L (1 - \phi) \left(\frac{\gamma}{\beta + \gamma} - y \right) \right\}. \quad (39)$$

Therefore, the steady state y solves

$$y = (1 - \beta) \left[\alpha_H \lambda_H (1 - \phi)y - \alpha_L \lambda_L (1 - \phi)y + \alpha_L \lambda_L (1 - \phi) \frac{\gamma}{\beta + \gamma} + \alpha_H \lambda_H \phi \frac{\gamma}{\beta + \gamma} \right].$$

Solving for y , we obtain (37).

- ii. For given λ_H , λ_L , α_H , α_L , and using (37), the steady-state y is larger under $\phi > 0$ than under $\phi = 0$ if, and only if,

$$\frac{\phi \alpha_H \lambda_H + (1 - \phi) \alpha_L \lambda_L}{1 - (1 - \beta)(1 - \phi)(\alpha_H \lambda_H - \alpha_L \lambda_L)} > \frac{\alpha_L \lambda_L}{1 - (1 - \beta)(\alpha_H \lambda_H - \alpha_L \lambda_L)}.$$

Multiplying through, this is equivalent to

$$\begin{aligned} & [\phi \alpha_H \lambda_H + (1 - \phi) \alpha_L \lambda_L][1 - (1 - \beta)(\alpha_H \lambda_H - \alpha_L \lambda_L)] \\ & > \alpha_L \lambda_L [1 - (1 - \beta)(1 - \phi)(\alpha_H \lambda_H - \alpha_L \lambda_L)], \end{aligned}$$

which is equivalent to

$$\begin{aligned} & [\alpha_L \lambda_L + \phi(\alpha_H \lambda_H - \alpha_L \lambda_L)][1 - (1 - \beta)(\alpha_H \lambda_H - \alpha_L \lambda_L)] \\ & > \alpha_L \lambda_L \{ [1 - (1 - \beta)(\alpha_H \lambda_H - \alpha_L \lambda_L)] + \phi(1 - \beta)(\alpha_H \lambda_H - \alpha_L \lambda_L) \}. \end{aligned}$$

After algebraic manipulation and using that $\alpha_H \lambda_H - \alpha_L \lambda_L > 0$, this condition may be expressed as

$$1 > (1 - \beta)\alpha_H \lambda_H,$$

which is always true since $\beta > 0$, and $\alpha_H \lambda_H \leq 1$.

- iii. By definition,

$$q = \frac{\frac{\gamma}{\beta + \gamma} - y}{\frac{\gamma}{\beta + \gamma} - y + \frac{\beta}{\beta + \gamma} - x}. \quad (40)$$

Using (38) and (39), $x = \frac{\beta}{1 - \beta}y$. Substituting this into (40), we obtain

$$q = \frac{\frac{\gamma}{\beta + \gamma} - y}{1 - \frac{\gamma}{1 - \beta}}. \quad (41)$$

We will now prove that the right-hand side of (41) is decreasing in y , which by (ii) of this lemma is sufficient to establish the claim in (iii). Indeed, differentiating the right-hand side of (41) with respect to y and looking at the numerator, we have that the right-hand side is decreasing in y if, and only if,

$$\frac{1}{1 - \beta} \frac{\gamma}{\beta + \gamma} < 1,$$

which is equivalent to

$$\gamma < (1 - \beta)(\beta + \gamma) = \beta + \gamma - \beta(\beta + \gamma).$$

Therefore, if

$$\beta - \beta(\beta + \gamma) > 0,$$

the right-hand side of (41) is decreasing in y , which is always true since $\beta + \gamma < 1$ by assumption.

iv. Setting $\lambda_H = \alpha_H = \alpha_L = 1$ in (37), we have

$$y = \frac{\phi + (1 - \phi)\lambda_L}{1 - (1 - \beta)(1 - \phi)(1 - \lambda)},$$

which is equivalent to

$$y = \frac{\phi + (1 - \phi)\lambda_L}{\beta + \gamma - \beta\phi + (1 - \beta)(1 - \phi)\lambda}. \quad (42)$$

Differentiating (42) with respect to λ_L and inspecting the numerator, we have that $\frac{\partial y}{\partial \lambda_L} > 0$ if, and only if,

$$(1 - \phi)[\beta + \phi - \beta\phi + (1 - \beta)(1 - \phi)\lambda_L] - (1 - \beta)(1 - \phi)[\phi + (1 - \phi)\lambda_L] > 0.$$

which reduces to the condition

$$\beta(1 - \phi)^2 > 0,$$

which is always true.

From the proof of (iii.), we know that q decreases in y . Therefore, q decreases in λ_L . ■

Proof of Proposition 4. Fix values for f, c, ϕ , and δ such that an equilibrium exists with a credit reporting agency. We know that under the same parameter conditions that an equilibrium would exist without one. Indeed, according to Proposition A.2, the condition under which such a non-autarkic equilibrium exists with a credit reporting agency is more restrictive. Therefore, we can compare the welfare induced under each scenario.

By Proposition A.2, we know that interest rates in both regimes and both lending environments (with and without an agency) are the same. This implies that $H_1, L_1, g_H, g_{L,1}$, and $g_{L,2}$ are the same in both lending environments. The only differences will be in H_2, L_2 , and λ_L^e .

Consider that Regime 1 is present with and without a credit reporting agency. Then, the welfare that is generated in the market is the same. That is, $W_1^n = W_1^a$. Specifically,

$$H_1^n g_H + L_1^n g_{L,1} = H_1^a g_H + L_1^a g_{L,1}.$$

Now, consider that Regime 2 is present with and without a credit reporting agency. The welfare for each can be calculated as

$$W_2^n = H_2^n g_H + \lambda_L^n L_2^n g_{L,2}$$

and

$$W_2^a = H_2^a g_H + \lambda_L^a L_2^a g_{L,2}.$$

Since $\frac{\partial H_2}{\partial \lambda_L} > 0$, $\frac{\partial H_2}{\partial \lambda_L} = -\frac{\partial L_2}{\partial \lambda_L}$, $g_H > g_{L,2}$, and $\lambda_L^n > \lambda_L^a$, it follows that $W_2^n > W_2^a$. Finally, since $W_1^n = W_1^a$ and $W_1^n > W_1^a$, $\Delta W^a > \Delta W^n$. ■

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