Review of Finance (2007) 11: 527–560
doi: 10.1093/rof/rfm011
Advance Access publication: 6 July 2007

# Learning, Cascades, and Transaction Costs*

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**Abstract.** The paper analyzes the effect of transaction costs on social learning in an asset market with asymmetric information, sequential trading, and a competitive price mechanism. Both fixed and proportional transaction costs reduce the information content of trading orders and lead to informational cascades. If transaction costs are very high, an informational cascade may occur not only when beliefs converge on a specific asset value but also when there is extreme uncertainty about the asset's fundamental value. Finally, if the value in the bad state is sufficiently low, proportional transaction costs lead to an informational cascade only when prices are very high.

*JEL Classification*: D82, D83, G10

Much of the literature on market microstructure holds that when agents are asymmetrically informed and trades occur sequentially, the market should gradually learn the fundamental information possessed by market participants, so that prices should eventually converge to fundamental values.[1] In the long run, therefore, asset markets are viewed as strong-form informationally efficient.[2] However, many interesting regularities observed in capital markets are not easily addressed by standard models based on the efficient-markets hypothesis.

* I am grateful to Jean-Paul Decamps for his early input on this work. I would like to thank Abhijit Banerjee, Marcello D'Amato, Amil Dasgupta, Riccardo Martina, Marco Pagnozzi, Salvatore Piccolo, seminar participants at CSEF, University of Salerno, and an anonymous referee for comments and suggestions. The paper was improved by the helpful discussions with Alberto Bennardo and Marco Pagano.

[1] See Glosten and Milgrom (1985), Kyle (1985).

[2] Prices are strong-form informationally efficient if they fully reflect all publicly available as well as all private information (Brunnermeier, 2001).

Many financial economists suggest that herd behavior could be the source of several asset price inefficiencies.[3] For example, the empirical evidence on episodes of soaring prices and subsequent collapse, such as the Dutch Tulip Mania in the seventeenth century, the stock market crashes of 1929 and 1987, and, more recently, the international financial crisis in southeast Asia in 1997–1998 and the overpricing of U.S. technology stocks in the late 1990s, would be consistent with short-run mispricing caused by imitative behavior. Also, the recurrent high volatility of financial market prices even in the absence of news may be the consequence of information aggregation failure due to herding.

The present paper argues that transaction costs in financial markets can lead all informed investors to disregard their private information in making their decisions and act alike. This makes the markets (strong-form) informationally inefficient—because they fail to incorporate private information fully into prices—and prone to bubbles and crashes, despite the rationality of market participants.[4]

Originally, imitative behavior was not considered rational: herds were described as people blindly following others.[5] More recently, however, the literature on social learning has reconciled herd behavior with rationality in many economic environments.[6] The models of informational cascades–that is, situations where imitative behavior leads market participants to disregard completely their own private information–belong to this strand of literature on "rational herding".[7]

Standard models of informational cascades apply to environments where prices are exogenous. So they hardly fit the case of asset markets, where prices adjust continuously to reflect the information revealed by trades. Indeed, in

[3] Extensive reviews of recent theoretical and empirical literature on herd behavior in capital markets are in Bikhchandani and Sharma (2001), Hirshleifer and Teoh (2003), and Devenow and Welch (1996).

[4] Two recent articles, in seeking to explain large movements in asset prices, have developed models in which market imperfections drive incomplete information aggregation by affecting agents' trading strategies. Romer (1993) shows that if immediacy in trading is costly, then investors possessing relevant information may delay their trades. So, prices may move long after investors receive their information. Cao et al. (2002) analyze how setup costs of trading cause some investors with superior information to be sidelined. This leads price changes to exhibit path-dependent skewness.

[5] For references to irrational herding see Devenow and Welch (1996).

[6] See Chamley (2004) for a thorough analysis of the causes of rational herds.

[7] The term "informational cascade"was introduced by Bikhchandani et al. (1992) to describe a situation in which, in a sequential trade framework, every agent, based on the observation of previous agents, makes the same choice independent of his private information. Banerjee (1992) also wrote another paper on rational imitative behavior due to informational externalities.

financial markets, the choice of each investor affects the payoff structure of his successors in two ways. It not only induces an informational externality as in Bikhchandani et al. (1992) but also produces a payoff externality of an opposite sign owing to the change in price (Brunnermeier, 2001). In many situations the latter effect offsets the former, eliminating any incentive to herd.[8] However, some recent theoretical models have shown that herding–and in some cases informational cascades–may arise even in asset markets. The situations in which this may occur include: (i) multidimensional uncertainty (Avery and Zemsky, 1998; Gervais, 1996), (ii) reputational concerns of asset managers (Scharfstein and Stein, 1990; Dasgupta and Prat, 2004), (iii) heterogeneous preferences of market participants (Decamps and Lovo, 2002; Cipriani and Guarino, 2001), and (iv) brokerage fees (Lee, 1998).

This paper contributes to the literature on rational herding in financial markets, by providing an analysis of the existence of informational cascades arising from transaction costs in stock markets with asymmetric information, sequential trading, and competitive price formation. Furthermore, the paper investigates the empirical implications of the model.

First we analyze whether trading frictions in price formation may induce informational cascades in quote-driven stock markets with competitive dealers. Lee (1998) shows that stock markets may fail to aggregate information, and informational cascades may occur when there are fixed brokerage fees which deter informed market participants to place orders. However, Lee assumes that the market maker is not fully rational.[9] Our model shows that Lee's insight on informational cascades is robust to the introduction of competitive and profit maximizing market makers. Moreover, the full modeling of price formation paves the way to a more realistic analysis of an important element of market behavior, the bid–ask spread, and provides opportunity for interesting predictions on trades patterns and trading volume paths.

Second, this paper attempts to explore the correlation between the public uncertainty on the asset's fundamental value and the occurrence of informational cascades. While previous literature holds that cascades develop when public information becomes more accurate, we show that, in a financial market with costly market-making and competitive price mechanism, cascades may occur also when the public information is very noisy.

[8] Avery and Zemsky (1998) prove that, in the standard Glosten–Milgrom framework, the competitive price mechanism prevents informational cascades.

[9] In Lee's framework, the market maker is assumed to set the price equal to the expected asset value conditional on the history of past trades, rather than setting bid and ask prices so as to compete for incoming orders. This assumption results in the market maker earning negative expected profit.

Finally, we investigate whether in such a model herding is more often associated with crashes than with frenzies. Although models of herd behavior aim to explain the empirical evidence of financial bubbles and crashes, they exhibit no bias for crashes: herd behavior can just as easily generate frenzies, in contrast with the empirical evidence that frenzies are much less common than crashes. We show that this is no longer the case under proportional transaction costs.

The paper addresses these issues in a stock market economy *à* la Glosten and Milgrom, where sequential trades for a single risky asset are channeled through risk-neutral market makers who set competitive prices. Some traders have better information about the asset's fundamental value and trade so as to exploit this edge. The other traders are uninformed and trade for exogenous reasons. A key assumption is that market makers bear an exogenous cost for processing orders.[10]

We show that, when market makers bear a fixed cost per transaction, the market is not informationally efficient, that is, the competitive price mechanism cannot prevent the occurrence of informational cascades. Further, in tune with the finding of Lee (1998), during an informational cascade no informed trader chooses to trade.[11]

Equilibrium bid and ask prices can be decomposed into two components: the expected value of the asset, given all relevant public information, and the order-processing cost. The informational advantage of traders is the difference between the expected value conditional on their private signal and the expected asset value for market makers. As the price becomes a more precise estimate of the asset value, this difference shrinks. Before the uncertainty about the asset value is completely resolved, informed traders stop trading because their informational advantage is smaller than the order-processing cost. From then on, no new information reaches the market, so prices stay constant and the bid–ask spread narrows, because the adverse selection component of the spread disappears. Nevertheless, cascades are fragile and the release of a small amount of public information can break up a long-lasting cascade, leading to a sudden price change.

The analysis also shows that, in a setting where the asset's fundamental value can be low or high, contrarian signals are more valuable than confirming signals. When the expected asset value for market makers is low, also the

[10] Recent empirical works studying the determinants of price formation in asset markets suggest that order-processing costs are a predominant component of the bid–ask spread [e.g., George et al. (1991); Lin et al. (1995); Saporta et al. (1995)].

[11] In a recent and independently developed paper, Cipriani and Guarino (2006) also found, that in a setting *à la* Glosten and Milgrom, informational cascades with no trading eventually occur if traders have to pay a positive fee to transact.

bid and ask prices are low. As a consequence, informed buyers—that is traders observing a favorable signal on the fundamental value—have a larger expected profit than informed sellers—that is traders observing an adverse signal. Similarly, when the public belief is high, also the bid and ask prices are high. So, informed sellers have a larger expected profit than informed buyers.

An interesting implication of this result is the asymmetric price effect between sell and buy orders in bull and in depressed markets. Indeed, following a significant price run-up, in a bull market there will be a point where the informational advantage of traders observing a bullish signal is lower than the fixed transaction cost, while that of traders observing a bearish signal is not. At that point, informed traders with favorable signals stop trading and buy orders become uninformative. So, after an upward price trend, until a cascade develops the information component of transaction costs is lower on the ask side of the market, where the adverse selection problem is less severe, and sell orders have a larger price impact than buy orders. The reverse is true in a depressed market.[12]

Our results suggest that, if liquidity orders are endogenous and depend on the cost of selling and buying the asset,[13] the asymmetric price impact of trades may affect both the liquidity trading volume and the composition of the order flow.[14] In particular, if liquidity traders prefer to trade when the transaction costs are low, during an informational cascade the liquidity trading volume should reach its maximum level both in bull and in bear markets, since the only source of the spread is the order-processing cost. But, before a cascade develops, differences in transaction costs associated to purchases and sales should lead to a different behavior of liquidity traders on the two sides of the market.

In bear markets, on average, sales are associated with a lower transaction cost than buys. Hence liquidity-driven sell orders should be larger than liquidity-driven buy orders. Conversely, in bull markets, the narrowing of the transaction cost on the ask side of the market should lead to a larger asset demand from liquidity traders.

[12] This argument relies on the assumption that all market participants know that an information event has occurred and a fraction of traders possess superior information. In a market where there is also event uncertainty, one would expect, on the contrary, that confirming signals were worth more than contrarian ones, as shown by Cao et al. (2002).

[13] Admati and Pfleiderer (1988) analyze the volume and price variability in a model where discretionary liquidity traders time their trades to minimize transaction costs.

[14] For simplicity, in the model we develop, uninformed traders are assumed to buy, sell, or refrain from trading with equal probability, independently of the prices. However, all the results are robust to the introduction of discretionary liquidity traders.

The model also predicts that, if order-processing costs are high enough, an informational cascade may occur even when the market is extremely uncertain about the fundamental value. High transaction costs imply that traders observing a bad signal sell only when the bid price is high enough, and traders observing a favorable signal buy only at a sufficiently low ask price. Equilibrium prices are low if the the expected asset value for market makers is low, and high in the opposite case. When market makers attach equal probability to all the states of nature, prices are in the middle of their distribution, and in equilibrium no informed trader chooses to trade. This suggests that, in stock markets with competitive market makers, herd behavior due to informational cascades is not always related to extreme values of quoted prices. This result runs counter to the previous literature, which predicts that informational cascades occur as public beliefs concentrate at the tails of the asset value distribution and prices are either very high or extremely low [Dasgupta and Prat (2004); Lee (1998)].[15]

Finally, we extend the analysis to include the case of proportional –rather than fixed–exogenous transaction costs. In this setting, we show that if the asset value in the bad state of nature is sufficiently low, the probability of an informational cascade in a depressed market approaches zero.

The reason is that when the asset price is low, the cost of trading is arbitrarily close to zero. Therefore, if a trader receives a favorable private signal about the true asset value, and if the signal is sufficiently informative, his potential gain from buying the asset, albeit small, is always greater than the negligible trading cost. So, the trader places a buy order and no cascade develops. Moreover, since all private information gets reflected in the asset price, there is no special tendency for shocks to induce a large jump in price. By contrast, when the asset price is high, the transaction cost is also high. A cascade can thus occur although the expected profit from selling the asset is nonnegligible. What is more, since during a cascade the market fails to aggregate hidden information, the equilibrium is fragile and there is a substantial chance that an informational shock triggers a significant downward move of the asset price.

The interesting by-product of these results is that cascades are asymmetrical: they rarely emerge in depressed markets, more often in bull markets. Hence, cascades are more likely to result in a crash—that is, a sharp drop in asset prices—, than in an anti-crash—that is, a significant increase in asset prices.

[15] This result is consistent with the fact that, in the real world, thinly traded stocks exist in which often there are no trades on a given day.

The article proceeds as follows. The basic model with fixed order-processing costs is presented in Section 1. Section 2 defines and derives the market equilibrium. Section 3 demonstrates the occurrence of informational cascades, and section 4 analyzes the market equilibrium before and during a cascade. The case of proportional exogenous transaction costs is examined in section 5. Finally, section 6 explores some extensions and concludes. Proofs of propositions, corollaries and lemmas are given in the Appendix.

## 1. Trading Mechanism and Information Structure

We take a sequential trade model similar to Glosten and Milgrom (1985), with a single risky asset whose value $\widetilde{V}$ depends on the state of nature. If the state is good, the value is $\overline{V}$; if bad, $\underline{V}$, with $\overline{V} > \underline{V} \geq 0$. We assume that the initial prior probability $\pi_0 = Pr(\overline{V})$ of the high value is nondegenerate, that is, $\pi_0 \in (0, 1)$. There are traders and market makers. Trades are sequential, and at any given point in the time, only one trader is allowed to transact. Before a trader arrives, market makers simultaneously announce their bid and ask prices. We assume that market makers are risk-neutral and act competitively. We further suppose that they bear a cost $c$ to process the order.[16] The trader arriving in the market observes the prices and has the option of selling or buying one unit of the asset at the best prices or refraining from trading. Once he has had the opportunity to trade, the trader leaves the market. He may trade further, but only after returning to the pool of traders and being selected again.[17]

A fraction $1 - \mu$ of traders are informed, while $\mu$ are uninformed liquidity traders.[18]

Informed traders are risk-neutral, price-taking agents[19] who privately observe a signal $\theta$ correlated with the value of the asset. They trade to maximize their expected profit. We denote as $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$ the set

[16] The cost $c$ represents all transaction costs unrelated to the underlying value of the stock, and includes clearing and settlement fees.

[17] As usual in models *à la* Glosten and Milgrom (1985), we make the simplifying assumption of fixed trade size equal to one unit of asset. The analysis would be totally unaffected by the normalization if the order-processing cost was proportional to the quantity traded (not its value). Moreover, it can be shown that the results on informational cascades are robust with respect to the hypothesis of different order sizes.

[18] Liquidity traders are needed to guarantee that trading occurs. In the absence of traders who trade for reasons other than speculation, the no-trade theorem of Milgrom and Stokey (1982) applies and the market breaks down.

[19] This assumption rules out their strategic behavior. It is reasonable given that there is an infinite number of informed traders in the pool and for each of them the probability of trading again is zero.

of private signals, and assume that these are conditionally independent and satisfy the monotone likelihood property:

$$0 < \lambda_1 < \lambda_2 < \cdots < 1 < \cdots < \lambda_{N-1} < \lambda_N < \infty,$$

where $\lambda_n = Pr(\theta_n | \widetilde{V} = \underline{V}) / Pr(\theta_n | \widetilde{V} = \overline{V})$.

The signal $\theta_n$ is "good" if the probability of observing $\theta_n$ when the true asset value is $\overline{V}$ exceeds that probability conditional on $\underline{V}$, that is $\lambda_n < 1$. In the opposite case it is "bad".

It is easy to see that a good signal is more informative when $\lambda$ is lower, a bad signal when $\lambda$ is higher. The information content of a good and a bad signal, $\theta_i$ and $\theta_j$, are equal if $\lambda_j = 1/\lambda_i$. For simplicity, we assume that good and bad signals are symmetrical, that is $\lambda_1 = 1/\lambda_N$, $\lambda_2 = 1/\lambda_{N-1}$, and so on.

Liquidity traders trade for reasons exogenous to the model. To simplify the analysis, we assume that they choose to sell, buy, or refrain from trading with equal probability.[20] However, we will also discuss the empirical predictions of the model under the more realistic hypothesis that the probability of a noninformation-driven order is endogenous, viz. it is a decreasing function of the bid–ask spread.[21]

Both market makers and traders are Bayesian agents who know the structure of the market. We denote by $\pi_t$ the probability of $\overline{V}$ conditional on the publicly observable history of trades up to time $t$. Thus, the public belief about the asset value at $t$ is:

$$E_t[\widetilde{V}] = \pi_t \cdot \overline{V} + (1 - \pi_t) \cdot \underline{V} = \underline{V} + \pi_t \cdot (\overline{V} - \underline{V}).$$

[20] In Glosten and Milgrom (1985), liquidity traders trade as long as the reservation value dictated by their liquidity needs exceeds the ask (bid) price for the buy (sell) order. As a consequence, if there are too many informed traders, then the market maker may have to set a spread so large as to preclude any trading at all. In order to simplify the analysis and avoid market collapse, we assume that the probability that liquidity traders buy and sell in each period is sufficiently high and is stationary. However, this assumption is not crucial and can be relaxed without changing the main results. It is straightforward to see that all the findings are robust to the introduction of an adequate fraction of liquidity traders who trade only if the market is thick enough.

[21] This analysis would not be possible in Lee's model where no endogenous transaction cost comes out. Indeed, in his framework, at the beginning of each trading round the market maker posts a single price at which he is willing to buy and sell whatever quantity, and no bid–ask spread is charged. The transaction cost that he considers is an exogenous fixed fee incurred by any agent only for his first trade.

## 2. Equilibrium

Before each trading round market makers set their prices. Bertrand competition and risk neutrality lead market makers to earn zero expected profit for every possible trade.

After prices are set, a trader is randomly selected to trade. He can place a sell order ($s$) at the highest bid price, $B_t$, place a buy order ($b$) at the lowest ask price, $A_t$, or refrain from trading ($r$). We denote by $\mathcal{A} \equiv \{s,\ b,\ r\}$ the traders' action space.

With probability $\mu$ the trader selected will be uninformed; with probability $1 - \mu$, informed. Agents do not observe the type of trader.

Uninformed traders trade for exogenous reasons and submit orders in the probabilistic way specified above. Informed traders observe the prices quoted and choose the strategy that maximizes the expected profit. We denote by $\sigma \equiv \{\sigma_\theta\}_{\theta\in\Theta}$ the informed traders' strategies, where $\sigma_\theta$ is the mixed strategy of the informed trader if he observes $\theta$. Clearly:

$$\sigma_\theta \equiv (\sigma_{\theta,s},\ \sigma_{\theta,b},\ \sigma_{\theta,r})$$

where $\sigma_{\theta,i}$ is the probability of action $i$ if the trader observes $\theta$, $\Sigma_{i\in\mathcal{A}}\, \sigma_{\theta,i} = 1$, and $\sigma_{\theta,i} \geq 0$ for all $i$ in $\mathcal{A}$.

The expected profit of a market maker is equal to $E_t[\widetilde{V}|s\ at\ B] - B - c$, if a sell order arrives at time $t$ and the bid price is $B$, and it is equal to $A - E_t[\widetilde{V}|b\ at\ A] - c$, if a buy order arrives at time $t$ and the ask price is $A$. The expected profit of a trader getting signal $\theta$, when the price schedule is $P = \{B,\ A\}$ and he plays the strategy $\sigma$, is $E_t[\Pi_\theta(\sigma|P)] = \sigma_{\theta,s}(B - E_t[\widetilde{V}|\theta]) + \sigma_{\theta,b}(E_t[\widetilde{V}|\theta] - A)$.

Assuming that market makers act competitively and that for each informed trader the probability of trading again is zero rules out all intertemporal strategic behavior. Therefore, each trading round can be viewed as a single two-stage game. In the first stage market makers set their prices, and in the second the trader observes the quotes and plays his strategy. What we are concerned with here is the Nash equilibrium.

**Definition 1.** *At each trading round* t, *the equilibrium in the asset market consists of a trading strategy correspondence $\sigma_\theta^*(P|t)$ for each informed trader $\theta \in \Theta$, and a price schedule $P_t^* = \{B_t^*,\ A_t^*\}$ such that:*

1. $\sigma_\theta^*(P|t) = \arg\max_\sigma E_t[\Pi_\theta(\sigma|P)]$, *for all price schedule $P \in R_+^2$ and $\theta \in \Theta$*
2. $B_t^* \in E_t[\widetilde{V}|s\ at\ B_t^*,\ \sigma^*(P_t^*|t)] - c$

3. $A_t^* \in E_t[\widetilde{V}|b \; at \; A_t^*, \; \sigma^*(P_t^*|t)] + c$
4. $B_t^* \leq A_t^*$

*where* $\sigma^*(P_t^*|t) = \{\sigma_\theta^*(P_t^*|t)\}_{\theta \in \Theta}$.

The first equilibrium condition states that informed traders maximize their expected profit, given the price schedule, trade by trade. Conditions (2) and (3) make market makers determine the price schedule so that they expect zero profits from each trade. Evidently, in the absence of order-processing costs, equilibrium bid and ask prices are equal to the expectation of $\widetilde{V}$ conditional, respectively, on a sell and on a buy order. The last condition says that at the equilibrium the market makers' buy price must be lower than (or equal to) the sell price.

We now offer a useful characterization of the equilibrium in terms of the informed traders' optimal strategy, and then prove the existence and the uniqueness of zero-profit equilibrium prices.

Informed traders are profit-maximizing agents. Hence, for any price schedule $P = \{B, \; A\}$ such that $B \leq A$, an informed trader places a sell order when his expected asset value is lower than the bid price, and places a buy order when it is higher. When the bid and ask prices straddle his valuation, he will not trade. Finally, if his expected value is equal to the bid or the ask price, he is indifferent between all mixed strategies with $\sigma_{\theta,\,b} = 0$ in the first case or $\sigma_{\theta,\,s} = 0$ in the second.

The valuation of the asset for informed traders depends on their information set, which includes both the whole history of trades and the private signal. From the maximum likelihood ratio property of signals, the expected value for traders observing signal $\theta_i$ is higher than that for those observing $\theta_j$ for any $j < i$. So, if it is profitable for traders with signal $\theta_n$ to sell when the price schedule is $P$, then also all traders observing signals $\theta_{n+i}$, with $i \in \{1, 2, \ldots, N-n\}$, optimally prefer to sell. Similarly, if the traders getting signal $\theta_n$ prefer to buy, then so do all traders getting signals $\theta_{n-j}$, with $j \in \{1, 2, \ldots, n-1\}$. Proposition 1 resumes this result.

**Proposition 1.** *If traders observing signal $\theta_n$ optimally sell the asset with positive probability, then traders observing any signal $\theta_{n+i}$, with $i \in \{1, 2, \ldots, N-n\}$, optimally sell the asset with probability* 1. *Similarly, if traders observing signal $\theta_n$ optimally buy the asset with positive probability, then traders observing any signal $\theta_{n-j}$, with $j \in \{1, 2, \ldots, n-1\}$, optimally buy the asset with probability* 1.

Proposition 1 implies that a sell order generally indicates $V$ because the probability of its occurrence is greater in the bad state of nature, while a buy order generally indicates $\overline{V}$. To see this, note that the proposition implies that the likelihood ratio of a sell order at $t$, given the price schedule $P = \{B, A\}$, is:

$$\lambda_t^s(B) = \frac{Pr_t(s\ at\ B|\underline{V})}{Pr_t(s\ at\ B|\overline{V})} = \frac{\frac{\mu}{3} + (1-\mu)\sum_{i=n_t^s(B)}^{N} Pr(\theta_i|\underline{V})}{\frac{\mu}{3} + (1-\mu)\sum_{i=n_t^s(B)}^{N} Pr(\theta_i|\overline{V})},$$

and the likelihood ratio of a buy order is:

$$\lambda_t^b(A) = \frac{Pr_t(b\ at\ A|\underline{V})}{Pr_t(b\ at\ A|\overline{V})} = \frac{\frac{\mu}{3} + (1-\mu)\sum_{i=1}^{n_t^b(A)} Pr(\theta_i|\underline{V})}{\frac{\mu}{3} + (1-\mu)\sum_{i=1}^{n_t^b(A)} Pr(\theta_i|\overline{V})},$$

where $\frac{\mu}{3}$ is the probability that the order comes from a liquidity trader, and $\theta_{n_t^s(B)}$ and $\theta_{n_t^b(A)}$ are respectively the signals of marginal selling and buying traders.[22] Since $\sum_{i=n}^{N} Pr_t(\theta_i|\underline{V}) \geq \sum_{i=n}^{N} Pr_t(\theta_i|\overline{V})$ for all $n = 1, 2, \ldots, N$, the probability of a sell order is higher in the bad state of nature and, since $\sum_{i=1}^{n} Pr_t(\theta_i|\underline{V}) \leq \sum_{i=1}^{n} Pr_t(\theta_i|\overline{V})$ for all $n = 1, 2, \ldots, N$, the probability of a buy order is higher in the good state.

Another important implication of Proposition 1 is that if equilibrium bid and ask prices exist, then they straddle the unconditional expected asset value, which is the price that would prevail in the absence of adverse selection and order-processing costs. Indeed, since $\lambda_t^s(B) \geq 1$ and $\lambda_t^b(A) \leq 1$ for all $t$ and $P$, the expected value conditional on a sell order never exceeds the unconditional expectation, while the expected value conditional on a buy order is never less than that expectation.

The next proposition states the existence and the uniqueness of equilibrium bid and ask prices.

**Proposition 2.** *In each period $t$ there exists a unique equilibrium price schedule $P_t^* = \{B_t^*, A_t^*\}$.*

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[22] Given the price schedule $P = \{B, A\}$, the marginal selling and buying traders at $t$, are those getting, respectively, signals $\theta_{n_t^s(B)}$ and $\theta_{n_t^b(A)}$ such that:

$$E_t[\widetilde{V}|\theta_{n_t^s(B)}] \leq B \quad \text{and} \quad E_t[\widetilde{V}|\theta_{(n_t^s(B)-1)}] > B$$

and

$$E_t[\widetilde{V}|\theta_{n_t^b(A)}] \geq A \quad \text{and} \quad E_t[\widetilde{V}|\theta_{n_t^b(A)+1)}] < A.$$

For simplicity, here we suppose that the marginal traders choose to place an order also if they are indifferent between trade and do not.

## 3. Informational Cascades with Costly Market-Making

In this section we study the occurrence of informational cascades in the market equilibrium just described.

If all informed traders make the same choice regardless of their private signal, no new information reaches the market. Hence, during an informational cascade, the market equilibrium is such that: $\sigma^*_\theta(P^*_t|t) = \sigma^*(P^*_t|t)$ for all $\theta$ in $\Theta$, and $B^*_t = E_t[\widetilde{V}] - c$ and $A^*_t = E_t[\widetilde{V}] + c$ since orders are uninformative about the asset value, that is: $\lambda^s_t(B^*_t) = \lambda^b_t(A^*_t) = 1$.

It is straightforward to see that in equilibrium traders observing a bad signal never buy and those getting a good signal never sell. Indeed, the valuation of traders with bad news is always lower than the unconditional expected value, while that of traders with good news is always higher. As we have seen, by Proposition 1 the unconditional expected value is the upper bound of the equilibrium bid price and the lower bound of the equilibrium ask price. Therefore, traders with a bad signal will never find it worthwhile to buy, while those with a good signal will never place a sell order. This implies that in the market equilibrium an informational cascade in which all informed traders place the same type of order will never occur.

Nevertheless, an informational cascade can still occur if all informed traders abstain from trading, since also in this case they behave identically. This may occur if market-making has a cost. The intuition is the following. Order-processing costs induce wider bid–ask spreads, but the expectation of market makers and that of informed traders converge as the number of transactions increases.[23] Hence, there would be a moment at which the informational advantage is so small relative to the order-processing cost that it is no longer profitable for any informed trader to place an order.

A no-trade informational cascade starts if $B^*_t < E_t[\widetilde{V}|\theta] < A^*_t$ for any signal $\theta$. Clearly, if an informational cascade occurs, the orders have no information content and then $B^*_t = E_t[\widetilde{V}] - c$ and $A^*_t = E_t[\widetilde{V}] + c$.

The next proposition specifies a necessary and sufficient condition for the occurrence of informational cascades in market equilibrium.

**Proposition 3.** *The equilibrium bid price* $B^*_t$ *is equal to* $E_t[\widetilde{V}] - c$ *if, and only if:*

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c,$$

[23] See Glosten and Milgrom (1985).

*and the equilibrium ask price $A_t^*$ is equal to $E_t[\widetilde{V}] + c$ if, and only if:*

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c.$$

Proposition 3 states that the equilibrium bid price is equal to the public belief about the fundamental value less the order-processing cost $c$ if and only if the informational advantage of the traders with the most informative bad signal $\theta_N$ is smaller than the fixed transaction cost. Symmetrically, the equilibrium ask price is equal to the public belief plus $c$ if and only if the informational advantage of traders endowed with the most informative good signal $\theta_1$ is smaller than the fixed transaction cost.

The belief about the asset value of traders getting bad signals less informative than $\theta_N$ exceeds that of traders observing $\theta_N$. This means that $E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c$ implies that $E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta] \leq c$ for all bad signals $\theta$. Likewise, the belief of traders with good signals less informative than $\theta_1$ is lower than that of traders who observe it. This means that $E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c$ implies that $E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}] \leq c$ for all good signals. If the informational advantage of all traders is lower than the order-processing cost, no informed trader will place a order. Since all traders observing a private signal act alike, no new information reaches the market and an informational cascade is triggered.

It is straightforward that until the uncertainty is resolved, the informational advantage of informed traders is strictly positive. As a consequence, in the absence of exogenous transaction costs, an informational cascade would never occur. On the other hand, if for any possible history of trades the transaction cost is greater than the advantage of informed traders, then orders will never be information-based. This last property is stated in the following lemma.

**Lemma 1.** *There exists a threshold cost $\overline{c} = \frac{|1-\sqrt{\lambda_1}|}{1+\sqrt{\lambda_1}} \cdot (\overline{V} - \underline{V})$ such that, if the transaction cost $c$ is larger than $\overline{c}$, in equilibrium all informed traders refrain from trading regardless of trading history.*

Since the threshold cost $\overline{c}$ is increasing in the information content of the best private signals, one can conclude that in markets where private information is less sharp, informed traders will be more likely to refrain from trading, so price changes will not reflect information about the fundamental.

The next proposition establishes that when order-processing costs are lower than the threshold $\overline{c}$, an informational cascade occurs as the public belief approaches the extremes of the distribution of the true asset value.

**Proposition 4.** *If $c$ is lower than $\overline{c}$, there exist unique $\pi^l_{\theta_1}$ and $\pi^u_{\theta_N}$, with $\pi^l_{\theta_1} < \pi^u_{\theta_N}$, such that when $\pi$ is lower than $\pi^l_{\theta_1}$ or larger than $\pi^u_{\theta_N}$ all informed traders refrain from trading in equilibrium.*

Proposition 4 states that when the market's expected asset value is lower than $\underline{V} + \pi^l_{\theta_1} \cdot (\overline{V} - \underline{V})$ or higher than $\underline{V} + \pi^u_{\theta_N} \cdot (\overline{V} - \underline{V})$ no informed trader prefers to trade because the transaction cost exceeds the trader's informational advantage for all signals in $\Theta$.

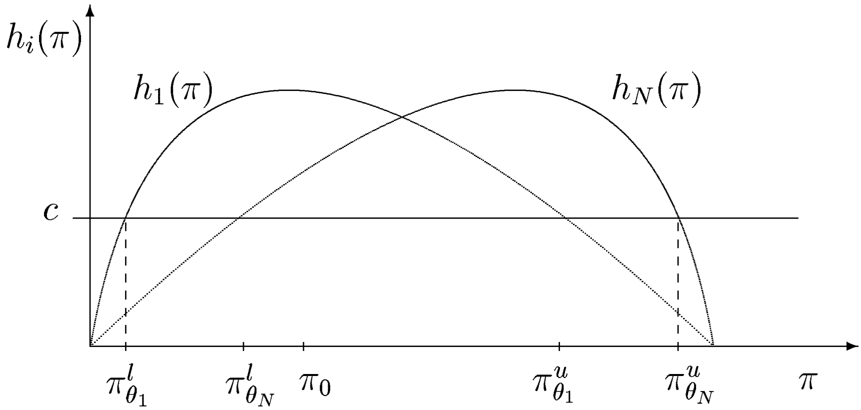
**Corollary 1.** *There exist unique $\pi^u_{\theta_1}$ and $\pi^l_{\theta_N}$, with $\pi^u_{\theta_N} > \pi^u_{\theta_1} > \pi^l_{\theta_1}$ and $\pi^l_{\theta_1} < \pi^l_{\theta_N} < \pi^u_{\theta_N}$, such that (1) when $\pi$ is larger than $\pi^u_{\theta_1}$ no informed trader buys the asset, (2) when $\pi$ is lower than $\pi^l_{\theta_N}$ no informed trader sells the asset.*

When the market assigns high probability to the bad state, the equilibrium bid and ask prices are low. Hence, the expected profit from buying of traders getting a favorable signal is greater than the expected profit from selling of traders getting an equally informative negative signal. In the opposite case, equilibrium prices are high and then the expected profit from buying is smaller than the expected profit from selling. This means that traders whose signal is consistent with the public belief have a smaller potential gain from trading than traders observing a contrarian signal. So, there exists a "sufficiently optimistic"public belief $\pi^u_{\theta_1}$, lower than $\pi^u_{\theta_N}$, such that, when $\pi$ is larger than $\pi^u_{\theta_1}$, the transaction cost exceeds the informational advantage of traders getting a good signal and buy orders do not convey any information about the asset value. Similarly, there exists a "sufficiently pessimistic"public belief $\pi^l_{\theta_N}$, larger than $\pi^l_{\theta_1}$, such that, when $\pi$ is lower than $\pi^l_{\theta_N}$, the transaction cost exceeds the informational advantage of traders observing a bad signal and sell orders are uninformative.

These results are a consequence of the convergence of beliefs. To see this, consider the following example. Suppose that at $t = 0$ the unconditional expected asset value is such that at least the traders getting signals $\theta_1$ and $\theta_N$ choose to trade (see Figure 1). Suppose also that at $t = 1, 2, \ldots, T$ a sequence of buy orders arrives. Because of the new information that reaches the market, the public belief about the asset value moves toward $\overline{V}$. Hence the bid and ask prices rise, and so does the valuation of informed traders.

If the public belief rises enough ($\pi$ exceeds $\pi^u_{\theta_1}$), the informational advantage of traders with signal $\theta_1$ becomes so small that it is no longer worthwhile for them to buy.

Moreover, at the beginning of the arrival process the expected profit of traders getting signal $\theta_N$ increases because the bid price rises. Nevertheless, if the buy orders are very numerous, the expected profit from selling begins to




$$h_i(\pi) = |E[\widetilde{V}|\, \pi, \theta_i] - E[\widetilde{V}|\, \pi]|, \quad i = 1, N$$

*Figure 1.* Informational cascades in a market with fixed order-processing costs. *The curve $h_1(\pi)$ represents the informational advantage of a trader who receives the best good signal, as a function of the probability of $\overline{V}$, $\pi$; while the curve $h_N(\pi)$ represents the informational advantage of a trader who receives the best bad signal, as a function of $\pi$. As $\pi$ becomes larger than $\pi^u_{\theta_N}$ or smaller than $\pi^l_{\theta_1}$ the fixed transaction cost c overcomes the informational advantage of traders, resulting in a cascade.*

fall off and eventually turns negative, because the traders' evaluation depends both on the history of trades and on their private signal. When the public belief converges to 1 or to 0, the weight of the information inferred from the history of trades increases and that of private information decreases. Then, if enough buy orders are placed and the unconditional expected value goes above $\underline{V} + \pi^u_{\theta_N} \cdot (\overline{V} - \underline{V})$, no informed trader will sell even though the bid price is very high. At that point, an informational cascade occurs.

An analogous argument demonstrates the occurrence of an informational cascade when the public belief tends to $\underline{V}$.

The previous example emphasizes that an informational cascade develops because the informed traders' expectation depends not only on their private information but also on the market history. But the weight of market history in the updating of informed traders' expectations also depends on the sharpness of their private signal. When in the market some traders observe a perfect signal, an informational cascade never occurs, as the public belief get concentrated on the high or on the low asset value. Indeed, if signals $\theta_N$ and $\theta_1$ are perfectly informative (i.e., $Pr(\theta_N|\underline{V}) = Pr(\theta_1|\overline{V}) = 0$) the expected asset value conditional on those signals does not depend on the history of trades. As a consequence, the profit upon sale of a trader observing $\theta_N$ increases when the public belief tends to $\overline{V}$ because of the price effect. Similarly, the expected profit from buying of traders getting $\theta_1$ increases when the public belief tends to $\underline{V}$.

In the following we show that if order-processing costs are high enough, an informational cascade may start even when the market is extremely uncertain over the true value, that is, when $\pi$ is close to 1/2. Notice that, if signals are symmetrical, when $\pi = 1/2$ the informational advantage of the traders getting signal $\theta_1$ is equal to that of the traders observing $\theta_N$. Let be $\underline{c} = E[\widetilde{V} \mid \pi = 1/2, \theta_1] - (\overline{V} + \underline{V})/2$. Proposition 5 establishes that if the order-processing cost exceeds $\underline{c}$, there exists a neighborhood of $(\overline{V} + \underline{V})/2$ such that an informational cascade occurs also if the unconditional expected asset value is within this neighborhood.

**Proposition 5.** *If $\underline{c} < c < \overline{c}$, then $\pi^u_{\theta_1} < 1/2 < \pi^l_{\theta_N}$, and all informed traders refrain from trading in equilibrium when $\pi$ is larger than $\pi^u_{\theta_1}$ and lower than $\pi^l_{\theta_N}$.*

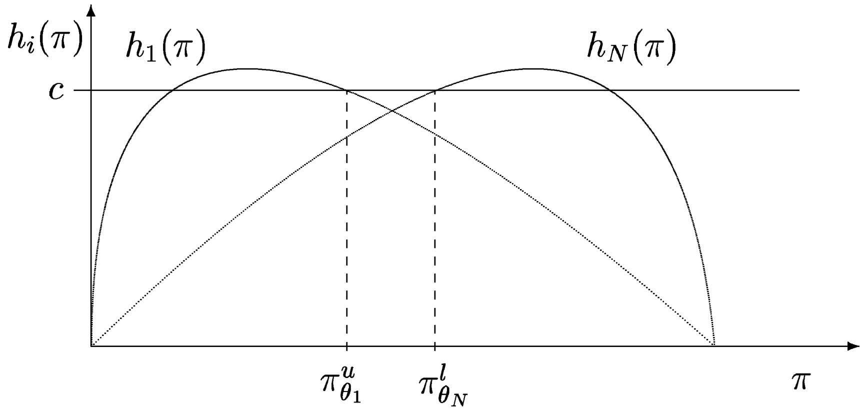
To understand intuitively why cascades can occur under the conditions described in Proposition 5, consider the following argument.

High order-processing costs require large informational advantages to induce informed traders to enter the market.

If $\pi$ is below 1/2, both the signal $\theta_N$ and the public belief indicate that the true value is $\underline{V}$. Hence, the informational advantage of traders observing the confirming signal $\theta_N$, is lower than the informational advantage of traders observing the good signal $\theta_1$, which contradicts public belief. By the same argument, when $\pi$ exceeds 1/2, the informational advantage of traders observing $\theta_1$, is lower than the informational advantage of traders observing $\theta_N$. Moreover, if $\pi$ is 1/2, both the informational advantage of traders observing $\theta_N$ and the informational advantage of traders observing $\theta_1$ equal $\underline{c}$.

This implies that if the order-processing cost is higher than $\underline{c}$, no trader with a bad signal chooses to sell when $E_t[\widetilde{V}]$ is below $(\overline{V} + \underline{V})/2$, and no trader observing a good signal chooses to buy when $E_t[\widetilde{V}]$ exceeds $(\overline{V} + \underline{V})/2$ (see Figure 2). As a consequence, sell orders are uninformative about the true value when prices are low and buy orders are uninformative when prices are high. And, if $E_t[\widetilde{V}]$ is larger than $\underline{V} + \pi^u_{\theta_1} \cdot (\overline{V} - \underline{V})$ and lower than $\underline{V} + \pi^l_{\theta_N} \cdot (\overline{V} - \underline{V})$, no informed trader places an order and there is an informational cascade.

Therefore, if order-processing costs are high enough, an informational cascade may develop even when the public belief is not concentrated on $\overline{V}$ or




$$h_i(\pi) = |E[\widetilde{V} \mid \pi,\, \theta_i] - E[\widetilde{V} \mid \pi]|, \quad i = 1,\, N$$

*Figure 2.* Informational cascades in a market with high fixed order-processing costs. *The informational advantage of a trader receiving the best good signal (i.e. $h_1(\pi)$) and the informational advantage of a trader receiving the best bad signal (i.e., $h_N(\pi)$) are lower than c also when the probability of $\overline{V}$, $\pi$, is in the interval $(\pi^u_{\theta_1}, \pi^l_{\theta_N})$.*

$\underline{V}$. This result contrasts with the typical finding of the literature, namely, that cascades occur when there is convergence of beliefs.[24]

Finally, a further point of interest is that if order-processing costs are very large, an informational cascade may start when $\pi$ is close to 1/2 also in the case where some traders are perfectly informed. Indeed, if the true asset value is $\underline{V}$, the profit of a perfectly informed trader is high when the bid price is high and shrinks as the bid price decreases—that is, $\pi$ tends to 0. Similarly, if the true asset value is $\overline{V}$, the profit of a perfectly informed trader is high when the ask price is low and decreases as the ask price increases—that is, $\pi$ tends to 1.

As a result, if the transaction cost is low [i.e., $c < \frac{\mu}{3-\mu}(\overline{V} - \underline{V})$], the probability of an information-based order is always a positive. On the contrary, for large values of $c$, a perfectly informed seller transacts only when the price is large enough, while a perfectly informed buyer transacts only at a sufficiently low price. Then, no information-based order is placed if the bid and the ask prices are in the middle of their distribution, that is, if $\pi$ is near to 1/2.

[24] An informational cascade may occur when the market attaches equal probability to any asset value because (1) signals are monotone and (2) contrarian signals are more valuable than confirming signals. In the model I develop, I consider a binary space of the states. However, the conclusions of Proposition 5 would be robust with respect to different assumptions on the asset value space if cascades arise in the standard setting where the adoption cost is fixed, and if conditions (i) and (ii) on the distribution of signals are satisfied.

This has the interesting consequence that in asset markets where the exogenous component of transaction costs is extremely high and speculators are very well informed:

- price volatility is low when prices are in the middle of their distribution and increases in both depressed and bull markets;
- the bid-ask spread has a U-shaped relationship with prices.

## 4. Information Content of Orders, Bid–Ask Spread and Fixed Order-Processing Costs

In this section we analyze the effect of order-processing costs on the adverse selection component of the bid–ask spread. First, we prove that in the absence of exogenous transaction costs the competitive price mechanism produces equilibrium prices that maximize the information content of orders. Then we show that order-processing costs reduce the amount of information that the market can infer from both buy and sell orders. Finally, we discuss some empirical predictions of the model.

An order's information content is related to its likelihood ratio. An order is totally uninformative about the true value of the asset if and only if the likelihood ratio is equal to 1; the more the ratio differs from 1, the more informative the order. More specifically, a sell order is more informative when the likelihood ratio is higher, a buy order when it is lower. So we can define the information content of a sell order as its likelihood ratio and that of a buy order as the reciprocal of its likelihood ratio.

The likelihood ratio of an order depends on the set of informed traders who prefer to place the order. From Proposition 1 we know that, given the price schedule $P = \{B, A\}$, the set of informed sellers is composed of traders observing $\theta_{n_t^s(B)}$ or any bad signal more informative than $\theta_{n_t^s(B)}$, and the set of informed buyers is composed of traders observing $\theta_{n_t^b(A)}$ or any good signal more informative than $\theta_{n_t^b(A)}$, with $\theta_{n_t^s(B)}$ and $\theta_{n_t^b(A)}$ being the signals of the marginal sellers and buyers. These sets are denoted by $\Theta_t^s(B)$ and $\Theta_t^b(A)$, respectively.

If the set of buying (selling) traders is very large but also includes many traders with very noisy signals—that is, $\lambda_{n_t^b(A)}$ ($\lambda_{n_t^s(B)}$) near to 1—the information content of a buy (sell) order is low. However, the information content of an order may be low even if the set of active informed traders includes only a few traders observing very accurate signals.

Lemma 2 establishes that there exists a set of informed sellers and a set of informed buyers that maximize the information content of sell and buy orders, and that these sets do not depend on the public belief.

**Lemma 2.** *There exist a bad signal $\theta_{n^s}$ and a good signal $\theta_{n^b}$ such that, for all trading histories:*

- *if $\theta_{n^s}$ is the signal of the marginal selling trader, then the information content of a sell order is maximum,*
- *if $\theta_{n^b}$ is the signal of the marginal buying trader, then the information content of a buy order is maximum.*

Let us denote by $\Theta_{n^s}$ and $\Theta_{n^b}$ the sets of all traders getting any signal $\theta_n$ in $\Theta$ with, respectively, $n \geq n^s$ and $n \leq n^b$. A price schedule $P = \{B, A\}$ maximizes the information content of orders at time $t$ if it prompts all traders in these two sets, and only those traders to place orders, that is, $\Theta_t^s(B) = \Theta_{n^s}$ and $\Theta_t^b(A) = \Theta_{n^b}$. Proposition 6 states that in the absence of order-processing costs, perfect competition among market makers leads to equilibrium bid and ask prices which maximize the information content of orders for all trading histories.

**Proposition 6.** *If $c = 0$, the equilibrium bid and ask prices, $B_t^*$ and $A_t^*$, are such that the information content of both sell and buy orders is maximum for all trading histories, that is: $\Theta_t^s(B_t^*) = \Theta_{n^s}$ and $\Theta_t^b(A_t^*) = \Theta_{n^b}$ for all $t$.*

To gain an intuitive understanding of this result, denote by $E_t[\widetilde{V}|\Theta_{n^b}]$ the expected asset value conditional on a buy order, when the set of informed buyers is $\Theta_{n^b}$.

In the absence of order-processing costs, the market makers' expected profit from selling is given by the difference between the ask price and their conditional expectation. Bertrand competition between market makers implies that, in equilibrium, this difference is equal to zero.

Traders observing $\theta_{n^b}$ refrain from trading when the ask price is greater than $E_t[\widetilde{V}|\theta_{n^b}]$. But, if $A > E_t[\widetilde{V}|\theta_{n^b}]$, the market makers' expected profit from selling is positive because $E_t[\widetilde{V}|b \text{ at } A] \leq E_t[\widetilde{V}|\Theta_{n^b}] \leq E_t[\widetilde{V}|\theta_{n^b}]$ for all $A$, by the definition of $\Theta_{n^b}$. Hence, the equilibrium ask price cannot exceed $E_t[\widetilde{V}|\theta_{n^b}]$ and the set of informed buyers includes $\Theta_{n^b}$.

On the other hand, if traders observing a good signal which is less informative than $\theta_{n^b}$ buy the asset—that is, $A < E_t[\widetilde{V}|\theta_{(n^b+i)}]$—the information that market makers can infer from a buy order is more accurate than the signal $\theta_{(n^b+i)}$. This implies that $E_t[\widetilde{V}|b \text{ at } A]$ is higher than the valuation of marginal traders, hence market makers' expected profit from selling is negative. As a consequence, the equilibrium ask price belongs to the interval $(E_t[\widetilde{V}|\theta_{(n^b+1)}],\ E_t[\widetilde{V}|\theta_{n^b}])$, and the set of informed buyers is $\Theta_{n^b}$.

The result of Proposition 6 points out that in an asset market with a competitive price mechanism, if adverse selection is the only source of the bid–ask spread, then in equilibrium the information content of both sell and buy orders is always at its maximum. This means that the price schedule that maximizes the information content of orders is such that the bid and ask prices equal the expectation of the asset value conditional on a sell and on a buy, respectively.

Under positive order-processing costs, the equilibrium bid price is lower than the conditional expectation of the asset value given a sell order, and the equilibrium ask price exceeds the conditional expectation, given a buy order. This implies that, in each side of the market, the set of active informed traders is smaller than in the case of zero transaction costs. Since the private information possessed by inactive traders does not get revealed to the market, costly market-making lowers the information content of both types of orders. Moreover, the difference between the equilibrium prices with and without order-processing costs is increasing in $c$. As a result, when fixed transaction costs are higher, the set of informed traders who enter to the market is smaller and orders convey less information.

This analysis yields the following empirical predictions:

- the adverse selection component of the bid–ask spread is smaller in asset markets where the order-processing costs are significant;
- price volatility decreases as $c$ increases.[25]

A further implication of Proposition 6 is that in the absence of order-processing costs, the set of informed traders who are active in the market does not depend on the trading history and is constant over time. As a result, the probability of an order, conditional on the true asset value, is the same at all $t$. If we consider the expected trading volume as the probability of an order, Proposition 6 suggests that, in equilibrium, expected trading volume is constant over time and does not depend on the public belief about the true value of the asset.

By contrast, in a market with positive transaction costs, the probability of an information-driven order is not constant over time, but decreases as the public belief approaches $\overline{V}$ or $\underline{V}$.[26]

During an informational cascade, no order comes from informed traders and the expected volume equals the probability of a liquidity trader's being selected to trade and placing an order. Furthermore, as the market tends

[25] These predictions are consistent with empirical evidence. For example Affleck-Graves et al. (1994) examine the components of the quoted bid–ask spread for a sample of stocks traded on different markets and find that the order-processing cost component of the spread is negatively correlated with the adverse selection component.

[26] If order-processing costs are high, the probability of an information driven order also decreases as the public belief approaches $(\overline{V} + \underline{V})/2$.

to a cascade, the expected volume from informed agents gradually decreases because traders getting less informative signals progressively refrain from trading. In particular, in a bull market, when the asset price is high and grows further as transactions go on, the model predicts that informed buyers leave the market ahead of informed sellers ($\pi^u_{\theta_1}$ is lower than $\pi^u_{\theta_N}$). Then the ask price becomes simply equal to the unconditional expected asset value plus the fixed order-processing cost before a cascade develops, and buy orders have no price impact. The opposite should arise in depressed markets when the price trend is decreasing (see Figure 1).

This analysis entails the following empirical predictions:

- the bid–ask spread is positively correlated with the speculative trading volume;
- in bull markets, sell orders have, on average, a larger price impact than buy orders;
- in depressed markets, buy orders have, on average, a larger price impact than sell orders.

So far, we assumed that orders from liquidity traders are exogenous to the model and constant over time. It would be interesting to extend the analysis to the more realistic case where the probability of a liquidity order on each side of the market is negatively correlated with the total transaction cost on that side of the market (i.e., the difference between the transaction price and the unconditional expected asset value).[27]

Under this assumption, the liquidity volume should rise even before a cascade starts, as the adverse selection component of the transaction cost shrinks. Moreover, in bull markets, an increasing price path should involve liquidity trading volume raising more quickly on the ask side of the market, where adverse selection problems are less severe. Symmetrically, in depressed markets, a decreasing price path should entail liquidity volume raising more quickly on the bid side of the market.

To conclude this section, note that the effect of order-processing costs on the speed of convergence to a cascade is ambiguous. On the one hand, an increase in $c$ implies higher $\pi^l_{\theta_1}$ and lower $\pi^u_{\theta_N}$, which should obviously speed up convergence. But, an increase in $c$ also involves lower information content of orders, which should slow down convergence. Thus, the sign of the final effect of an increase in order-processing costs on the speed of convergence to a cascade depends on which of these two effects prevails.

[27] See Dow (2004).

## 5. Proportional Transaction Costs

Let us now examine informational cascades when trading costs are proportional to the value of the transaction.[28] The only difference from the framework of the previous sections is that the expected profit of the market maker is now equal to $E_t[\widetilde{V}|s\ at\ B] - (1+c)B$ if the market maker buys at $B$, and to $(1-c)A - E_t[\widetilde{V}|b\ at\ A]$ if he sells at $A$.

Perfect competition gives market makers zero expected profit on either side of the market. Hence, the equilibrium price schedule, $P_t^* = \{B_t^*,\ A_t^*\}$, is such that:

$$B_t^* \in \frac{E_t[\widetilde{V}|s\ at\ B_t^*,\ \sigma^*(P_t^*|t)]}{1+c} \tag{1}$$

$$A_t^* \in \frac{E_t[\widetilde{V}|b\ at\ A_t^*,\ \sigma^*(P_t^*|t)]}{1-c}. \tag{2}$$

Since equilibrium bid and ask prices straddle the unconditional value expectation, in equilibrium traders with a bad signal will never buy, while those with a good signal will never sell. Then, as before, in equilibrium there will never be informational cascades in which all informed traders place the same type of order. However, the price schedule may be such that informed traders refrain from trading. In this case, the orders convey no information; hence $B_t^* = E_t[\widetilde{V}]/(1+c)$ and $A_t^* = E_t[\widetilde{V}]/(1-c)$.

The optimal correspondence strategies of informed traders do not change if exogenous transaction costs are proportional rather than fixed. Thus, no informed trader wishes to sell when the bid price is lower than the valuation of traders observing signal $\theta_N$—that is, $E_t[\widetilde{V}|\theta_N] > B_t$. Symmetrically, none wishes to buy when the ask price is higher than the valuation of traders observing signal $\theta_1$—that is, $E_t[\widetilde{V}|\theta_1] < A_t$. Proposition 7 states that an informational cascade will occur in equilibrium if and only if the informational advantage of traders observing the most informative good signal or the most informative bad signal is lower than the transaction cost multiplied by their expected asset value.

**Proposition 7.** *The equilibrium bid price $B_t^*$ is equal to $E_t[\widetilde{V}]/(1+c)$ if and only if:*

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c \cdot E_t[\widetilde{V}|\theta_N],$$

[28] Proportional trading costs also include many explicit costs such as stock market taxes and brokerage commissions which are usually expressed as a percentage of the value of the transaction.

*and the equilibrium ask price* $A_t^*$ *is equal to* $E_t[\widetilde{V}]/(1-c)$ *if and only if:*

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c \cdot E_t[\widetilde{V}|\theta_1].$$

To gain an insight into this proposition, consider the ask side of the market and denote by $C(A) = cA$ the cost to the market maker of selling the asset at the price $A$. In equilibrium, when the set of informed buyers is nonempty, $A^* \leq E[\widetilde{V}|\theta_1]$ and then $C(A^*) \leq c \cdot E[\widetilde{V}|\theta_1]$. This implies that $c \cdot E[\widetilde{V}|\theta_1]$ represents the upper bound of the cost to the market maker of selling the asset in equilibrium. So traders getting signal $\theta_1$ always choose to buy the asset when their informational advantage exceeds $c \cdot E[\widetilde{V}|\theta_1]$. On the other hand, if the public belief is such that $E[\widetilde{V}|\theta_1] - E[\widetilde{V}] \leq c \cdot E[\widetilde{V}|\theta_1]$, then in equilibrium informed traders prefer to refrain from trading because their expectations are below $E[\widetilde{V}]/(1-c)$, which is the lowest possible equilibrium ask price.

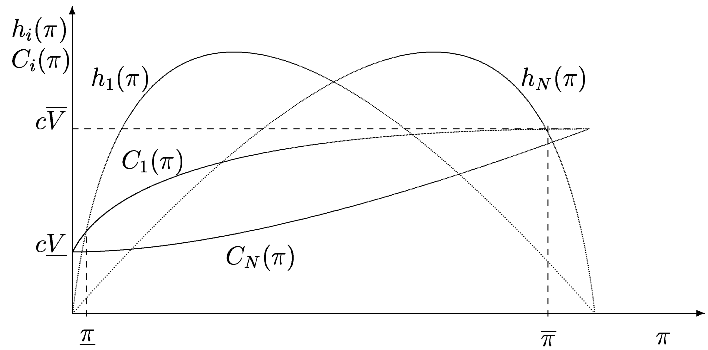
The total cost to the market maker of selling or buying now depends on the value of the transaction. Higher bid and ask prices produce higher transaction costs, so their effect on social learning should be greater when the probability that the market assigns to $\overline{V}$ is high, and should depend on the magnitude of $\underline{V}$ and $\overline{V}$. In particular, if the low asset value is zero, transaction costs vanish as $\pi$ converges to 0. Hence, the two cases, $\underline{V} > 0$ and $\underline{V} = 0$, should be considered separately. First consider $\underline{V} > 0$. As in the previous framework, excessively high exogenous transaction costs can inhibit informed traders from trading regardless of the public belief, so that in equilibrium orders are never information-based.

**Lemma 3.** *There exists a threshold cost* $\overline{c}_p = \frac{\overline{V}-\underline{V}}{(\sqrt{\overline{V}}+\sqrt{\lambda_N \underline{V}})^2} \cdot (\lambda_N - 1)$ *such that, if the proportional transaction cost c is larger than* $\overline{c}_p$*, in equilibrium all informed traders refrain from trading, regardless of the history of trades.*

Interestingly, the threshold cost $\overline{c}_p$ is increasing in $\lambda_N$. So, if the information content of signals is modest, even a small proportional cost can prevent private information to be revealed through trading.

Proposition 8 states that if the low asset value is strictly positive and the proportional transaction cost does not exceed $\overline{c}_p$, an informational cascade develops both when the public belief about the asset value tends to $\overline{V}$ and when it approaches $\underline{V}$. Also, if the exogenous transaction costs are high enough and if signals are sufficiently precise, an informational cascade may also occur when the probabilities that the market assigns to $\underline{V}$ and to $\overline{V}$ are not far apart.

**Proposition 8.** *If c is lower than* $\overline{c}_p$ *and if* $\underline{V}$ *is strictly positive, there exist unique* $\underline{\pi}$ *and* $\overline{\pi}$*, with* $\underline{\pi} < \overline{\pi}$*, such that when* $\pi$ *is lower than* $\underline{\pi}$ *or larger than* $\overline{\pi}$




$$h_i(\pi) = |E[\widetilde{V}|\ \pi,\ \theta_i] - E[\widetilde{V}|\ \pi]|;\ C_i(\pi) = c \cdot E[\widetilde{V}|\ \pi,\ \theta_i],\quad i = 1,\ N$$

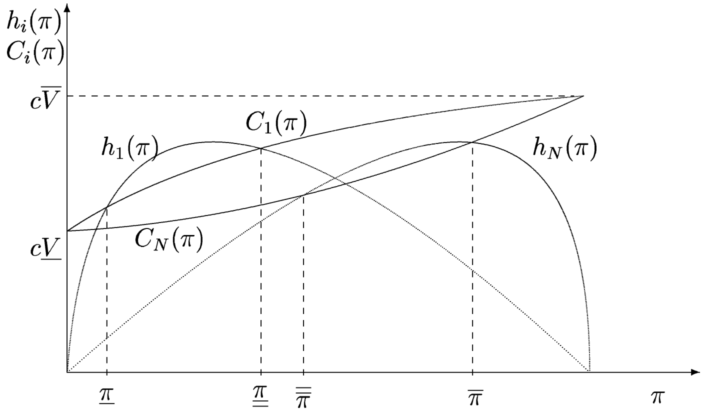
*Figure 3.* Informational cascades in a market with low proportional transaction costs. *The curve $C_1(\pi)$ represents the upper threshold cost on the ask side of the market, as a function of the probability of $\overline{V}$, $\pi$; while the curve $C_N(\pi)$ represents the upper threshold cost on the bid side of the market, as a function of $\pi$. As $\pi$ becomes larger than $\overline{\pi}$ or smaller than $\underline{\pi}$, the upper threshold cost on the ask side of the market overcomes the informational advantage of a trader who receives the best good signal [i.e., $h_1(\pi)$], and the upper threshold cost on the bid side of the market overcomes the informational advantage of a trader who receives the best bad signal [i.e., $h_N(\pi)$], resulting in a cascade.*

*all informed traders refrain from trading in equilibrium. Further, if $\underline{c}_p < c < \overline{c}_p$, with $\underline{c}_p = \frac{\overline{V}-\underline{V}}{\overline{V}+\underline{V}} \cdot \frac{\lambda_N - 1}{\lambda_N + 1}$, and if $\underline{V}/\overline{V}$ exceeds $\lambda_1$, there exist unique $\underline{\underline{\pi}}$ and $\overline{\overline{\pi}}$, with $\underline{\pi} < \underline{\underline{\pi}} < \underline{V}/(\overline{V} + \underline{V}) < \overline{\overline{\pi}} < \overline{\pi}$ such that an informational cascade occurs also when $\pi$ is larger than $\underline{\underline{\pi}}$ and lower than $\overline{\overline{\pi}}$.*

Informed traders find it profitable to trade as long as their informational advantage is greater than the transaction cost. The informational advantage decreases both when $\pi$ approaches 0 and when it tends to 1, and it is nil when $\pi$ is exactly equal to 0 or 1. The exogenous transaction costs are an increasing function of the public belief and are always strictly positive if $\underline{V}$ is greater than zero. As a result, the transaction cost exceeds the informational advantage of informed traders, so a cascade occurs, both in the case where the public belief about the asset value is close to $\underline{V}$ and when it is near to $\overline{V}$ (see Figure 3).

What is more, for large values of $c$, an informational cascade may develop also if the the public belief is far from the tails of the asset value distribution; that is, if $\pi$ is near to $\underline{V}/(\overline{V} + \underline{V})$.

If the cost to trade is high, traders observing good signals buy the asset only at a sufficiently low price, while traders observing bad signals sell the asset only if the price is large enough. If $\pi$ is close to $\underline{V}/(\overline{V} + \underline{V})$, the bid and ask




$$h_i(\pi) = |E[\widetilde{V}|\ \pi,\ \theta_i] - E[\widetilde{V}|\ \pi]|;\ C_i(\pi) = c \cdot E[\widetilde{V}|\ \pi,\ \theta_i],\quad i = 1,\ N$$

*Figure 4.* Informational cascades in a market with high proportional transaction costs. *The informational advantage of a trader receiving the best good signal [i.e., $h_1(\pi)$] and the informational advantage of a trader receiving the best bad signal [i.e., $h_N(\pi)$] are lower than transaction costs on both sides of the market [i.e., $C_1(\pi)$ and $C_N(\pi)$] also when the probability of $\overline{V}$, $\pi$, is in the interval $(\underline{\underline{\pi}}, \overline{\overline{\pi}})$.*

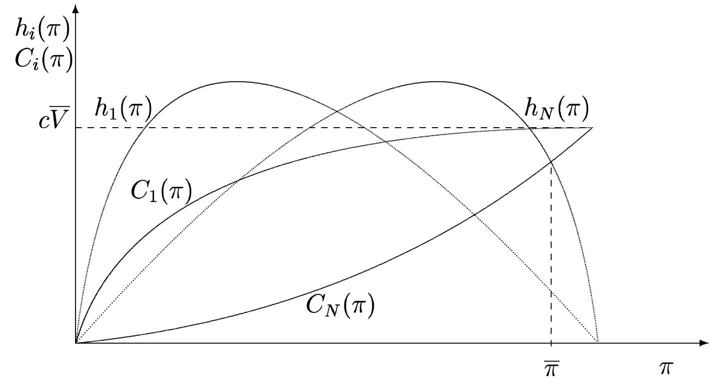
prices are in the middle of their distribution, then no informed trader enters to the market and an informational cascade takes place (see Figure 4).

However, differently from the case of fixed costs, under proportional costs a cascade may occur in the absence of convergence of beliefs only if $\underline{V}/\overline{V}$ exceeds $\lambda_1$.

To understand intuitively this result, first note that informed buyers pay, on average, a larger transaction cost than informed sellers. Indeed, the trading cost is proportional to the expected value of the transaction, which is larger for a buy than for a sell. As a result, proportional costs discourage traders observing a favorable signal to buy the asset, while encourage traders observing a bad signal to sell the asset. In particular, if the trading cost on the ask side of the market is too much larger than the trading cost on the bid side of the market—that is, $\underline{V}/\overline{V}$ is very low—, and/or if private signal $\theta_1$ is poorly informative—that is, $\lambda_1$ is near 1—, then, in equilibrium, traders observing a good signal trade very rarely and never find it profitable to enter to the market when traders observing a bad signal do not.

**Corollary 2.** *The likelihood of an informational cascade when $\pi$ is low decreases as $\underline{V}$ tends to zero.*

Under proportional transaction costs, the cost to trade decreases as $\pi$ tends to zero and reaches its minimum, $c\underline{V}$, when $\pi = 0$. If $\underline{V}$ is very small, it




$$h_i(\pi) = |E[\widetilde{V}|\ \pi,\ \theta_i] - E[\widetilde{V}|\ \pi]|;\ C_i(\pi) = c \cdot E[\widetilde{V}|\ \pi,\ \theta_i],\quad i = 1,\ N$$

*Figure 5.* Informational cascades in a market with proportional transaction costs and $\underline{V} = 0$.
*The informational advantage of a trader receiving the best good signal [i.e., $h_1(\pi)$] and the informational advantage of a trader receiving the best bad signal [i.e., $h_N(\pi)$] are lower than transaction costs on both sides of the market [i.e., $C_1(\pi)$ and $C_N(\pi)$] only when the probability of $\overline{V}$, $\pi$, is larger than $\overline{\pi}$.*

becomes negligible as the public belief about the asset value moves toward $\underline{V}$. Then, the likelihood of an informational cascade with low prices declines as the low asset value approaches 0. Moreover, in the extreme case of $\underline{V} = 0$, it falls to zero, as stated by Proposition 9.

**Proposition 9.** *If $c$ is lower than $\overline{c}_p$ and if $\underline{V} = 0$, there exists unique $\overline{\pi}$ such that all informed traders refrain from trading in equilibrium if and only if $\pi$ is larger than $\overline{\pi}$.*

If $\underline{V}$ is zero, the transaction cost vanishes when $\pi = 0$. The informational advantage of traders getting private signals also shrinks as $\pi$ tends to 0. However, it overcomes the transaction cost if private information is accurate, and $c$ is lower than $\overline{c}_p$.[29] Hence, an informational cascade never occurs when equilibrium prices are low. Conversely, in a bull market a cascade is possible since when $\pi$ raises, the transaction costs increase, whereas the informational advantage of traders decreases (see Figure 5).

These findings carry the interesting empirical implication that proportional transaction costs in financial markets produce asymmetry between propensity

[29] The slope of the transaction costs' curve depends on $c$. Lemma 3 states that informed traders enter to the market only if $c$ does not exceed the upper threshold $\overline{c}_p$, which depends on the precision of the most informative signal. This leads transaction costs approaching zero at least faster than the informational advantage of traders observing the most accurate signal.

for crashes and anticrashes. Indeed, both in a bull market and in a depressed market, during a cascade, the equilibrium is fragile. Hence, the release of a small amount of public information can overturn a long-lasting informational cascade and induce a sudden change in asset price. However, since cascades emerge more often in bull markets than in depressed markets, they are more likely to trigger a market crash than a buying surge.

## 6. Concluding Remarks and Extensions

We have presented a model of the effect of costly market-making on the informational efficiency of prices in an asset market characterized by asymmetric information and sequential trading, examining the impact of both fixed and proportional transaction costs on price discovery.

Standard microstructure models show that in the short run trading frictions may decouple market prices from fundamentals, but assume that prices ultimately converge to fundamental values [Glosten and Milgrom (1985), Kyle (1985)]. Models of informational cascades question this conclusion, by showing that the deviations can be persistent, because there are circumstances where markets may stop impounding fundamental information in prices. One of these circumstances is precisely the presence of trading frictions, as shown by Lee (1998), who considers fixed transaction costs in a framework of asymmetric information and sequential trading. The limitation of Lee's model, however, is that it does not allow for optimizing behavior by market makers as normally done in microstructure models, so that the question arises if informational cascades can still arise when this assumption is made.

This paper answers this question. When market makers are assumed to behave optimally and competitively in setting prices, informational cascades can still occur in equilibrium. Moreover, fixed order-processing costs may allow a cascade not only when the market assigns a high probability to the high or to the low value of the asset, so that prices are either very high or very low, but even when there is extreme uncertainty about the fundamental value and prices are in the middle of their distribution.

The paper also studies what happens if transaction costs are proportional to asset prices, rather than fixed. The main novel finding is that in this case the probability of an informational cascade is greater when prices are high. Specifically, if the fundamental value in the bad state approximates zero, an informational cascade will never occur when the price is low. This implies that when transaction costs depend on prices, cascades are asymmetrical: they are rare in depressed markets, and more likely to develop in bull markets.

Recent work on informational cascades in financial markets seeks to explain bubbles and crashes [Avery and Zemsky (1998), Dasgupta and Prat (2004), Lee

(1998)]. These empirical phenomena generally come with increasing trading volume. However, we find that as the market tends to an informational cascade, the expected volume of trading decreases because informed traders stay on the sidelines.

The model assumes that liquidity traders submit orders in a probabilistic way according to a stationary distribution. This is a very strong assumption. In reality, if liquidity traders have any discretion as to the timing of their orders, one should expect their trading to be negatively correlated with the bid–ask spread. Since the bid-ask spread tends to decrease before and during informational cascades, owing to the reduction of informed trading, we should find an increase in liquidity trading before and during cascades, when the adverse selection component of the spread disappears. In a such a setting, informational cascades should involve a change in the composition of total volume in favor of liquidity trading. And, if liquidity traders are more numerous, total expected volume could actually increase during a cascade. A natural extension of this paper would be to investigate the empirical implications of informational cascades in financial markets with discretionary liquidity traders.

## Appendix

**Lemma 4.** *The marginal selling trader increases the information content of a sell order if, and only if,* $\lambda_{\theta_{n_t^s(B)}} > \lambda_t^s(B)$.
*The marginal buying trader increases the information content of a buy order if, and only if,* $\lambda_{\theta_{n_t^b(A)}} < \lambda_t^b(A)$.

**Proof.** We prove the result for the ask side. The result for the bid side follows by symmetry.

Let $f^b(n) = \frac{\frac{\mu}{3}+(1-\mu)\sum_{i=1}^{n} Pr(\theta_i|\underline{V})}{\frac{\mu}{3}+(1-\mu)\sum_{i=1}^{n} Pr(\theta_i|\overline{V})}$. To prove the lemma for the ask side, we have to show that:

$$f^b(n) \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n).$$

By using algebraic calculus, it is easy to show that:

1. $f^b(n) \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n-1)$
2. $\lambda_n \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n)$.

By combining these results, we obtain:

$$f^b(n) \geq f^b(n-1) \Longleftrightarrow \lambda_n \geq f^b(n).$$

Since $f^b(n_t^b(A)) = \lambda_t^b(A)$, the lemma for the ask side is proved. ■

*Proof of Proposition* 2. We prove the proposition for the ask price. The proof for the bid price is by symmetrical argument.

By the monotone likelihood property of signals, the expected asset value conditional on a buy order is always greater than (or equal to) the unconditional expected value. This implies that if the ask price is lower than $E[y|H] + c$, the market maker's expected profit is negative. Moreover, $E[y|H, b \ \ at \ \ A]$ is upper-bounded by $\overline{V}$. Hence, by the zero-expected-profit condition, the equilibrium ask price cannot be greater than $\overline{V} + c$. Let us define the correspondence $F_t^b : [E_t[\widetilde{V}] + c, \ \overline{V} + c] \rightsquigarrow [E_t[\widetilde{V}] + c, \ \overline{V} + c]$ as:

$$F_t^b(A) \equiv E_t[\widetilde{V}|b \ at \ A] + c.$$

$F_t^b(A)$ is an upper semicontinuous convex-valued correspondence that maps the set $[E_t[\widetilde{V}] + c, \ \overline{V} + c]$ onto itself. By Kakutani's fixed point theorem, $F_t^b(A)$ has a fixed point $A_t^*$; that is, an equilibrium ask price always exists.

Uniqueness is proved using the results of Lemma 4, which states that the marginal buying (selling) trader increases the information content of a buy (sell) order. That is, the likelihood ratio of the buy (sell) order increases (decreases) when the marginal buyer (seller) refrains from trading, if the signal that he observes is more accurate than the information that the market maker infers from a buy (sell) order.

Suppose, by way of obtaining a contradiction, that there exist two equilibrium ask prices: $A_1$ and $A_2$, with $A_1 < A_2$. Denote $n_t^b(A_1)$ and $n_t^b(A_2)$ respectively the marginal buying trader given $A_1$ and the marginal buying trader given $A_2$. Clearly, since we suppose $A_1 < A_2$, it must be that $\lambda_{n_t^b(A_1)} > \lambda_{n_t^b(A_2)}$. First, note that at the equilibrium, the value assessment of the marginal buying trader is greater than (or at most equal to) the market maker's conditional expected value. Hence $\lambda_{n_t^b(A_1)} \leq \lambda_t^b(A_1)$. This implies that $\lambda_t^b(A_1) < \lambda_t^b(A_2)$ because, when the ask price rises to $A_2$, the traders getting the signal $\theta_{n_t^b(A_1)}$ refrain from trading, and then the likelihood ratio of a buy order increases by Lemma 4. As a consequence, if the ask price is equal to $A_2$, the market maker's expected profit is strictly positive. Hence, by the zero-expected-profit condition, $A_2$ cannot be an equilibrium ask price. ■

*Proof of Proposition* 3. We prove the proposition for the ask price; the proof for the bid price is symmetrical. First we assume that $A_t^* = E_t[\widetilde{V}] + c$ and we prove that this implies $E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c$. Suppose by way of obtaining a contradiction that $E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] > c$. Since at least the traders observing $\theta_1$ prefer to buy when the ask price is $E_t[\widetilde{V}] + c$, the expected asset value conditional on a buy order at $E_t[\widetilde{V}] + c$ is greater than the unconditional expected asset value. This implies that the market maker's expected profit is negative, which contradicts the fact that $E_t[\widetilde{V}] + c$ is the equilibrium ask price.

On the other hand, if $E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c$ then no informed trader will buy. Hence $\inf\{E_t[\widetilde{V}|b\ at\ E_t[\widetilde{V}] + c,\ \sigma^*(P_t^*|t)] + c\} = E_t[\widetilde{V}] + c = A_t^*$. ■

*Proof of Lemma* 1. Define the functions $h_\theta(\pi)$ as:

$$h_\theta(\pi) \equiv |E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}]| = \frac{\pi - \pi^2}{\pi + (1-\pi)\lambda_\theta} \cdot |1 - \lambda_\theta| \cdot (\overline{V} - \underline{V}).$$

$h_\theta(\pi)$ gives the informational advantage of traders observing $\theta$ for any $\pi \in [0,\ 1]$. Putting Proposition 3 and the assumption of symmetrical signals together, it follows that:

$$\overline{c} = \max h_{\theta_N}(\pi) = \max h_{\theta_1}(\pi) = \frac{|1 - \sqrt{\lambda_1}|}{1 + \sqrt{\lambda_1}} \cdot (\overline{V} - \underline{V}).$$ ■

*Proof of Proposition* 4. Consider the function $h_\theta$ defined in the proof of Lemma 1. Note that:

- $h_\theta(0) = h_\theta(1) = 0$,
- $\max h_\theta(\pi) = \frac{|1-\sqrt{\lambda}|}{1+\sqrt{\lambda}} \cdot (\overline{V} - \underline{V})$,
- $h''_\theta(\pi) = -\frac{2\lambda}{(\pi+(1-\pi)\lambda)^3} \cdot |1 - \lambda_i| \cdot (\overline{V} - \underline{V}) < 0$ for all $\pi \in [0,\ 1]$.

Since by assumption $0 < c \leq \frac{|1-\sqrt{\lambda}|}{1+\sqrt{\lambda}} \cdot (\overline{V} - \underline{V})$, from the strict concavity of $h_\theta(\pi)$ it follows that there exist unique $\pi_\theta^l$ and $\pi_\theta^u$, with $\pi_\theta^l < \pi_\theta^u$, such that $h_\theta(\pi_\theta^l) = h_\theta(\pi_\theta^u) = c$ and $h_\theta(\pi) > c$ if, and only if, $\pi \in (\pi_\theta^l,\ \pi_\theta^u)$. Thus, when $\pi \in (0,\ \pi_\theta^l) \cup (\pi_\theta^u,\ 1)$, traders getting signal $\theta$ refrain from trading. Moreover,

$$\arg\max h_{\theta_1}(\pi) = \frac{\sqrt{\lambda_1}}{1 + \sqrt{\lambda_1}} < \frac{1}{2} < \arg\max h_{\theta_N}(\pi) = \frac{\sqrt{\lambda_N}}{1 + \sqrt{\lambda_N}}$$

and

$$h_{\theta_1}(\pi) \geq h_{\theta_N}(\pi) \quad for\ all\ \pi \leq \frac{1}{2}$$

As $\pi_{\theta_1}^l < \arg\max h_{\theta 1}(\pi)$ and $\pi_{\theta_N}^u > \arg\max h_{\theta_N}(\pi)$, then $\pi_{\theta_1}^l < \pi_{\theta_N}^l$ and $\pi_{\theta_N}^u > \pi_{\theta_1}^u$. Hence, when either $\pi < \pi_{\theta_1}^l$ or $\pi > \pi_{\theta_N}^u$, all informed traders will refrain from trading. ■

*Proof of Proposition* 5. From the proof of Proposition 4, we know that there exist $\pi_{\theta_1}^u > \pi_{\theta_1}^l$ and $\pi_{\theta_N}^l < \pi_{\theta_N}^u$ such that $h_{\theta_1}(\pi_{\theta_1}^u) = h_{\theta_N}(\pi_{\theta_N}^l) = c$. Since

$h'_{\theta_1}(\frac{1}{2})<0$ and $h'_{\theta_N}(\frac{1}{2})>0$, and since $\frac{|1-\lambda_1|}{2(1+\lambda_1)} \cdot (\overline{V} - \underline{V}) < c < \overline{c}$, then $\pi^u_{\theta_1} < \pi^l_{\theta_N}$ and both $c > h_{\theta_1}(\pi)$ and $c > h_{\theta_N}(\pi)$ for any $\pi \in [\pi^u_{\theta_1}, \pi^l_{\theta_N}]$. ■

*Proof of Lemma* 2. We prove the lemma for the ask side. The proof for the bid side is analogous.

A buy order indicates the good state of nature. Hence, the information content of a buy order is maximum when $f^b(n)$ (defined in the proof of Lemma 4) reaches its minimum value. If all informed traders prefer to buy the asset, then $f^b(N) = 1 < \lambda_N$. By Lemma 4, it follows that the marginal buying trader reduces the information content of the buy order; that is: $f^b(N-1) < f^b(N)$. If no informed trader buys, then $f^b(0) = 1 > \lambda_1$, and the marginal buying trader increases the information content of the buy order; that is: $f^b(1) > f^b(0)$. By the maximum likelihood property of signals, it follows that there exists a unique signal $\theta_{n^b}$ such that:

$$f^b(n^b+1) > f^b(n^b) \geq \lambda_{n^b} \ \ and \ \ f^b(n^b) \leq f^b(n^b-1).$$

If the equilibrium ask price $A_t^*$ is such that $n_t^b(A_t^*) = n^b$, then the equilibrium information content of a buy order is maximum. ■

*Proof of Proposition* 6. We prove the proposition for the ask side. The proof for the bid side can be obtained symmetrically.

Since $c = 0$, the competitive ask price has to be equal to the expected asset value conditional on a buy order, that is:

$$A_t^* \in E_t[\widetilde{V}|b \ at \ A_t^*, \ \sigma^*(P_t^*|t)].$$

Lemma 2 states that there exists a good signal $\theta_{n^b}$ such that $f^b(n^b) \leq f^b(n)$ for every $n \in \{1, 2, \ldots, N\}$, and $\lambda_{n^b} \leq f^b(n^b)$. Moreover, by combining Lemmas 4 and 2 it easy to see that $\lambda_{(n^b+1)} > f^b(n^b+1)$. As a consequence, for any history of trades:

$$E_t[\widetilde{V}|\theta_{n^b+1}] < \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t) f^b(n^b+1)} \cdot (\overline{V} - \underline{V}) \tag{A1}$$

and

$$E_t[\widetilde{V}|\theta_{n^b}] \geq \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t) f^b(n^b)} \cdot (\overline{V} - \underline{V}). \tag{A2}$$

Equations (A1) and (A2) imply that there exists an ask price $A_t^*$ that belongs to $(E_t[\widetilde{V}|\theta_{n^b+1}], \ E_t[\widetilde{V}|\theta_{n^b}]$ and that satisfies the market maker's zero-profit condition. By Proposition 2, we know that there exists a unique ask price that satisfies that condition. Thus, in equilibrium, $n_t^b(A_t^*) = n^b$. ■

*Proof of Proposition* 7. We prove the proposition for the ask price; the proof for the bid price is symmetrical. First assume that $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$ and prove that this implies $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c$. Suppose by way of obtaining a contradiction that $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} > c$. This means that $E_t[\widetilde{V}|\theta_1] > \frac{E_t[\widetilde{V}]}{1-c}$. Hence, at the least the traders observing $\theta_1$ prefer to buy when the ask price is $\frac{E_t[\widetilde{V}]}{1-c}$. As a consequence, the expected asset value conditional on a buy order at $\frac{E_t[\widetilde{V}]}{1-c}$ is greater than the unconditional expected asset value. This implies that the market maker's expected profit is negative, which contradicts the fact that $\frac{E_t[\widetilde{V}]}{1-c}$ is the equilibrium ask price.

On the other hand, if $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c$ then no informed trader buys.[30] Hence:

$$\inf\{\frac{E_t[\widetilde{V}|b\ at\ \frac{E_t[\widetilde{V}]}{1-c},\ \sigma^*(P_t^*|t)]}{1-c}\} = \frac{E_t[\widetilde{V}]}{1-c} = A_t^*. \qquad \blacksquare$$

*Proof of Lemma* 3. Define the functions $g_\theta(\pi)$ as:

$$g_\theta(\pi) \equiv \frac{|E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}]|}{E_t[\widetilde{V}|\theta]} = \frac{\pi - \pi^2}{\pi\overline{V} + (1-\pi)\lambda_\theta\underline{V}} \cdot |1-\lambda_\theta| \cdot (\overline{V} - \underline{V}).$$

$g_\theta(\pi)$ gives the relative informational advantage of traders observing $\theta$ for any $\pi \in [0,\ 1]$. The relative informational advantage of traders getting the signals $\theta_N$ and $\theta_1$ is always greater than that of traders observing a different signal, hence the threshold cost $c > \overline{c}_p$ is given by:

$$\overline{c}_p = \sup\{\max_\pi g_{\theta_1}(\pi),\ \max_\pi g_{\theta_N}(\pi)\} = \frac{\overline{V} - \underline{V}}{(\sqrt{\overline{V}} + \sqrt{\lambda_N\underline{V}})^2} \cdot (\lambda_N - 1). \qquad \blacksquare$$

*Proof of Proposition* 8. First we prove that there exist unique $\underline{\pi}$ and $\overline{\pi}$ such that when $\pi \in [0,\ \underline{\pi}) \bigcup (\overline{\pi},\ 1]$, all informed traders will refrain from trading.

Consider the function $g_\theta(\pi)$ defined in the proof of Lemma 3. Notice that:

- $g_\theta(0) = g_\theta(1) = 0$
- $\max g_\theta(\pi) = \frac{\overline{V}-\underline{V}}{(\sqrt{\overline{V}}+\sqrt{\lambda_\theta\underline{V}})^2} \cdot |\lambda_\theta - 1|$
- $g_\theta''(\pi) = -\frac{2\lambda_\theta\underline{V}\overline{V}}{(\pi\overline{V}+(1-\pi)\lambda_\theta\underline{V})^3} \cdot |1-\lambda_\theta| \cdot (\overline{V} - \underline{V}) < 0$ for all $\pi \in [0,\ 1]$.

[30] The relative informational advantage of informed traders increases with the signal's precision.

Since by assumption $0 < c \leq \overline{c}^p$, from the strict concavity of $g_\theta(\pi)$ it follows that there exist unique $\underline{\pi}_\theta$ and $\overline{\pi}_\theta$, with $\underline{\pi}_\theta < \overline{\pi}_\theta$, such that $g_\theta(\underline{\pi}_\theta) = g_\theta(\overline{\pi}_\theta) = c$ and $g_\theta(\pi) > c$ if, and only if, $\pi \in (\underline{\pi}_\theta,\ \overline{\pi}_\theta)$. Thus, $\underline{\pi} = \inf\{\underline{\pi}_\theta\}_{\theta\in\Theta}$ and $\overline{\pi} = \sup\{\overline{\pi}_\theta\}_{\theta\in\Theta}$.

In order to prove the second part of the theorem, note first that $g_{\theta_1}(\frac{\overline{V}}{\overline{V}+\underline{V}}) = g_{\theta_N}(\frac{\overline{V}}{\overline{V}+\underline{V}}) = \underline{c}_p$, and $g_{\theta_1}(\pi) < g_{\theta_N}(\pi)$ for all $\pi > \frac{\overline{V}}{\overline{V}+\underline{V}}$. Moreover, $\frac{\overline{V}}{\overline{V}+\underline{V}}$ is always lower than $\arg\max g_{\theta_N}(\pi)$, whereas it is larger than $\arg\max g_{\theta_1}(\pi)$ if and only if $\lambda_1 < \underline{V}/\overline{V}$. As a consequence, when $c$ is greater than $\underline{c}_p$, if $\lambda_1 > \underline{V}/\overline{V}$, then $\underline{\pi}_{\theta_N}$ is lower than $\underline{\pi}_{\theta_1}$ and the relative informational advantage of traders getting $\theta_N$ is strictly positive for all $\pi \in [\underline{\pi}_{\theta_1}, \overline{\pi}_{\theta_1}]$. In contrast, if $\lambda_1 < \underline{V}/\overline{V}$, then $\overline{\pi}_{\theta_1}$ is lower than $\underline{\pi}_{\theta_N}$ and both $g_{\theta_1}(\pi)$ and $g_{\theta_N}(\pi)$ are lower than $c$ for all $\pi \in (\overline{\pi}_{\theta_1}, \underline{\pi}_{\theta_N})$. Hence an informational cascade occurs also when $\pi \in (\overline{\pi}_{\theta_1}, \underline{\pi}_{\theta_N})$. ■

*Proof of Corollary* 2. In order to prove the result we need to show that $d\underline{\pi}/d\underline{V} > 0$. For the implicit function theorem, this derivative is given by:

$$\frac{d\underline{\pi}}{d\underline{V}} = -\frac{\partial g_\theta(\pi, \underline{V})/\partial \underline{V}}{\partial g_\theta(\pi, \underline{V})/\partial \pi}$$

The numerator is always negative. The denominator is positive for low values of $\pi$. Then, $d\underline{\pi}/d\underline{V}$ is greater than zero. ■

*Proof of Proposition* 9. If $\underline{V} = 0$, the relative informational advantage of traders getting signal $\theta$ is $g_\theta(\pi) = (1 - \pi) \cdot |\lambda_\theta - 1|$ and $\lim_{\pi\to 0} g_\theta(\pi) = |\lambda_\theta - 1|$. Since by assumption $0 < c \leq \overline{c}^p$, with $\overline{c}^p = \lambda_N - 1$ when $\underline{V} = 0$, it follows that $\lim_{\pi\to 0} g_{\theta_N}(\pi)$ is larger than $c$ and there exists a unique $\pi = \overline{\pi}$ such that $g_{\theta_N}(\overline{\pi}) = c$, and $g_{\theta_N}(\pi) > c$ if, and only if, $\pi < \overline{\pi}$. ■

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