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Should smart investors buy funds with high past returns?*

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Abstract. We fully characterize the equilibria in a game between a fund manager of unknown ability who controls the riskiness of his portfolio and investors who only observe realized returns. We derive two types of equilibria. The first one is such that (i) investors invest in the fund if the realized return falls within some interval, i.e., is neither too low nor too high, (ii) a good manager picks a portfolio of minimal riskiness and (iii) a bad manager picks a portfolio with higher risk, "gambling" on a lucky outcome. The second type of equilibrium is more traditional: (i) investors invest in the fund if the observed return is larger than some threshold, and (ii) good and bad managers choose the same risk level.

JEL Classification: G11 and G23

1. Introduction

Newspapers and weekly magazines catering to the investing crowd often produce tables showing "rat races" of mutual funds. They rank funds according to the returns generated in the previous year or over a period of several years. Aside from satisfying sheer curiosity, these numbers are probably also the basis on which investors pick a fund to invest in. Empirical work shows that flows in and out of funds are indeed positively correlated with past performances (see Ippolito (1992), Sirri and Tufano (1998), Chevalier and Ellison (1997) and Lettau (1997)). However, Zheng (1999) shows that the allocation of money *cannot* be fully explained by investors chasing past performance.

Under what conditions should investors buy funds with high past returns? To answer this question, we shall build on the premise that mutual fund managers differ in their ability to generate high returns, and that these abilities are persistent at least in the short run so that returns in year t can be indicative of performances in year $t+1$ (See Grinblatt and Titman (1992), Hendrick, Patel

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and Zeckhauser (1993), Brown and Goetzmann (1995) and Carhart (1997) and Zheng (1999)[1]). In such a situation, mutual fund managers with asset-based compensation schemes[2] are surely aware of the signaling function of their past performance, and will thus choose their portfolio strategies accordingly. For the sake of the argument, suppose that investors always pick the fund which generated the highest return in the past. Knowing this, a fund manager with an inferior ability may be tempted to gamble, i.e. to invest in risky portfolios, hoping to generate the highest return in the crowd. But if that is indeed the case, high past returns are not indicative of high ability: rather, they indicate high risk and a lucky outcome. Under such circumstances, smart investors should thus avoid funds that have realized the highest returns.

The contribution of this paper is not only to make this intuition precise, but to fully characterize the equilibria of the game played by investors and an active fund manager who is either good or bad, and knows his quality while investors do not. The mutual fund manager controls the riskiness of his portfolio. The investors observe a realization of the fund return, and invest in the fund, if it is sufficiently likely that the mutual fund manager is good. Hence, the manager will choose the riskiness of his to-be-observed portfolio return in order to maximize the chance that the investor will invest with him.

We derive conditions under which (i) investors invest in the fund if the realized return falls within some interval, i.e., is neither too low nor too high; (ii) a good fund manager picks a portfolio of minimal riskiness; and (iii) a bad fund manager picks a portfolio with higher risk, gambling on a lucky outcome. Thus smart investors may not want to invest in funds with the highest past returns.

The second type of equilibrium we derive is more "traditional". It is such that (i) investors invest in the active fund if the observed return is larger than some threshold, and (ii) good and bad fund managers choose the same risk level. This equilibrium holds for sets of parameters such that, ex-ante, investors are better off investing in the active fund and the difference in expected performance between a good and a bad fund is small.

Our results suggest two possible interpretations for the positive correlation between inflows and past performance empirically observed. The first one is that mutual fund investors are boundedly rational and do not pick mutual funds optimally given fund managers' investment strategy. The second

[1] Evidence of persistent under-performance seem to be stronger than that of over-performance.

[2] Khorana (1996) mentions that "The investment advisor's compensation is directly linked to the fund's size; the advisor receives a management fee based on a percentage of average net assets held during the year".

In a sample of 5, 198 funds, Deli (2002) finds that 4, 833 funds had advisory contracts based solely on a percent of assets. Elton, Gruber and Blake (2003) mention that "incentives fees are not widely used in the mutual fund industry. In 1999, only 108 out of total of 6,716 bond and stock mutual funds used incentive fees."

interpretation is that investors are fully rational, gambling incentives are small, hence can be neglected when picking mutual funds.

The paper proceeds as follows. Section 2 discusses related literature. Section 3 provides a simple two-quality model, and provides the key results. Section 4 provides analyzes a more general framework. Section 5 discusses some assumptions of the model and Section 6 concludes.

2. Related literature

The most related models are those of Huddart (1999), Berk and Green (2004) and Dangl, Wu and Zechner (2006). Huddart studies a two-period model in which, at the end of the first period, risk-averse investors reallocate their wealth between two funds after having observed the performance of each fund. It is shown that in the first period, both an informed and an uninformed fund choose overly risky investment strategies; and, in the second period, investors should always invest in the fund that has realized the higher return in the first period. There are several differences between Huddart's model and ours. First, Huddart assumes that portfolios are observable while we do not. Huddart's assumption implies that he studies a standard signaling model in which some additional information is given by realized returns. If portfolios are not observable, as in our model, managers' skills can only be derived from statistical inference. Furthermore, we do not find the assumption of observable portfolios very appealing since disclosure is not frequent[3] and managers window-dress their portfolio around disclosure dates in practice (see Lakonishok, Thaler, Shleifer and Vishny (1991), Musto (1999) and Carhart, Kaniel, Musto and Reed (2001)). Second, Huddart assumes that returns are binomially distributed. Therefore, it is very "easy" for an uninformed manager to obtain exactly the same return as an informed manager. Hence, there is not much information an investor can gather from observed returns.

Berk and Green (2004) and Dangl, Wu and Zechner (2006) also study fund flows as a response to performance. Both models show that the absence of performance persistence can be reconciled with rational investment in best performing funds if one assumes decreasing returns to scale. There is one main difference between these models and ours: they assume that there is no asymmetric information between managers and investors as far as managers' quality is concerned. Both investors and managers learn managers' quality from past performances. As a consequence, managers have no incentive to act strategically when choosing their investment strategy.

Other models have studied the game played by mutual fund managers (see, for example, Chen and Pennacchi (2005), Taylor (2003) and Palomino (2005)). However all these models take investors' fund picking rule as exogenous and

[3] In the United States, mutual fund portfolios have to be disclosed semiannually. However, other countries such as the Netherlands only require disclosure once a year.

study the consequences of such a rule on the game played by fund managers. Conversely, in our model, investors' picking rule is endogenous.

Other papers such as Heinkel and Stoughton (1994) and Das and Sundaram (2002) have studied the fee structure. Heinkel and Stoughton focus on the extraction of the right effort from the manager by conditioning a continued relationship on the observed return. Das and Sundaram (2002) show that the fee structure can be used as a way to signal skills.

Finally, our model is a special case of problem studied by Crawford and Sobel (1982) in which a sender (the fund manager in our case) sends a *costless* (noisy) signal to a receiver (the investor) who makes inference about the sender's type and then chooses an action.

3. The Model

We consider a situation in the spirit of Ippolito (1992). The economy contains two funds, an actively managed fund which charges a high management fee and an index fund which charges a low management fee. We denote μ_o the expected return (net of fees) of the index fund.

The manager of the active fund is of good quality with some probability $\psi > 0$ and of bad quality with probability $1 - \psi$.

There are two periods: 0 and 1. At the beginning of period 0, the manager of the active fund picks a portfolio which generates a net-of-fee return

$$R = \mu + \sigma\epsilon,\ \epsilon \sim \mathcal{N}(0; 1) \tag{1}$$

If the manager is of good quality, then $\mu = \mu_g$ and if the manager is of bad quality, then $\mu = \mu_b < \mu_g$. In picking the portfolio, the fund manager only controls the riskiness $\sigma \geq \underline{\sigma} > 0$ of the portfolio, i.e., the fund manager cannot reduce the risk in his portfolio below some positive lower bound, but he can add as much risk as he wants. In order to make the problem interesting, we assume that $\mu_b < \mu_o < \mu_g$. That is, if the manager is good (bad), the net expected return derived from investing in the active fund is larger (smaller) than the expected return derived from investing in the index fund.

There is a continuum of identical risk-neutral investors who, on aggregate, have one unit of capital to invest at the beginning of period 1. Investors do not know the quality of the active fund manager (i.e., whether he is good or bad). At the end of period 0, investors only observe the return realized by the active fund in this period, but not the portfolio (hence the risk level) he has selected.

There are several reasons for which the riskiness of the investment strategy may neither be observable nor estimated. First, as mentioned in Section 2, mutual fund regulation does not require funds to disclose their portfolio very often and managers window dress around disclosure dates. Therefore, for young funds with a short track record, estimating the return volatility may not be possible. Window dressing and a low frequency of portfolio disclosure also imply that investment decisions may have to taken without knowing the return volatility if the manager's quality is only persistent in the short run. This happens, for example, if a good quality manager is an informed one, a bad quality manager is an uninformed one, and if a manager is informed (uninformed) in period t, he is informed (uninformed) in period $t+1$ with probability $\theta \in (1/2, 1)$. In such a case, the risk level chosen by the fund manager is time dependent. For any ex-ante probability of the manager being informed in period t, only the risk level chosen in this period (σ_t) is useful for investors' decision at the beginning of period $t+1$. The observation of σ_t would require the observation of the portfolio selected in period t by the fund manager. However, the low frequency of portfolio disclosure will force investors to choose between the active and the index funds at time $t+1$ without knowing σ_t.

A second reason for which the non-observability of return volatility may seem reasonable if we consider private equity funds.[4] These are funds for which a return volatility may be difficult to estimate since their net asset value is not very often observed. For example, the European Private Equity and Venture Capital Association (EVCA) advises funds to release net asset values on a quarterly basis (see EVCA Reporting Guidelines (2000)). Furthermore, these reported values are often based on funds' self valuation of their portfolio companies (see EVCA Valuation Guidelines (2004)).

3.1 INVESTORS' PROBLEM

Given that all investors have the same information, they make the same investment decision. Investors are assumed to be fully rational. Hence, they invest in the actively managed fund in period 1 if they expect to earn a higher net return than if investing in the index fund. I.e., upon observing a return R realized by the active fund in period 0, they invest in this fund in period 1 if

$$P(good|R)\mu_g + P(bad|R)\mu_b > \mu_o. \tag{2}$$

Hence, investors invest in the active fund if

$$P(good|R) = P(\mu_g|R) > \frac{\mu_o - \mu_b}{\mu_g - \mu_b}. \tag{3}$$

Otherwise, they invest in the index fund in period 1.

Let

$$\tau = \frac{\mu_o - \mu_b}{\mu_g - \mu_b}. \tag{4}$$

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[4] We thank Josef Zechner, the Editor, for this suggestion.

Investors choose to invest in the active fund if the probability of the manager being of good quality exceeds some threshold $\tau \in (0, 1)$.

3.2 THE FUND MANAGER'S PROBLEM

We assume that the manager of the active fund is risk neutral and aims at attracting investors' money in period 1. Hence, the objective of the fund manager is to maximize the probability that investors will choose to invest in the active fund rather than in the index fund.

We are interested in the sequential equilibria of the game played by the investors and the fund manager.

Before proceeding, three remarks are in order. First, we have assumed that the market price of risk is zero, i.e., changing the riskiness of the portfolio does not affect its mean return. This is not restrictive in principle: if there is a market price for risk, one can rewrite everything in risk-adjusted terms, i.e., do a change of measure so that assets can be compared by comparing their expected returns only, using the new measure. Of course, while this is a standard and elegant procedure in theory, it may makes comparisons to the data tricky.

Second, the rather mechanical decision by the investors shortcuts a much lengthier derivation of the portfolio choice based on first principles in some multiperiod, multifund model. Such a derivation would likely complicate the analysis in an unnecessary way.

Last, we assume that funds only use returns to signal their quality, while a good fund could use an appropriate performance-based fee scheme to signal its quality.

3.3 THE EQUILIBRIUM

We analyze the game backwards, searching for the portfolio risks σ_g^* and σ_b^* to be chosen by the good manager and the bad manager, respectively, and the fund picking rule for the investors.

In the last stage of the game, having observed the realized return R and in knowledge of the equilibrium strategies (σ_g^*, σ_b^*) investors can calculate the likelihood ratio $L(R; \sigma_g^*, \sigma_b^*)$ that the manager is good:

$$\begin{aligned} L(R; \sigma_g^*, \sigma_b^*) &= \frac{P(R \mid \mu_b)}{P(R \mid \mu_g)} \\ &= \frac{\sigma_g^*}{\sigma_b^*} e^{(R-\mu_g)^2/(2\sigma_g^{*2}) - (R-\mu_b)^2/(2\sigma_b^{*2})} \end{aligned}$$

Using Bayes' rule, the probability of a good manager having realized the return R is then

$$P(\mu_g \mid R) = \frac{P(R \mid \mu_g)P(\mu_g)}{P(R \mid \mu_g)P(\mu_g) + P(R \mid \mu_b)P(\mu_b)}$$
$$= \frac{\psi}{\psi + (1-\psi)L(R; \sigma_g^*, \sigma_b^*)}$$

Given that investors choose the active fund if $P(\mu_g \mid R) \geq \tau$ (see Equations (3) and (4)) and given the equilibrium strategies (σ_g^*, σ_b^*), the fund manager will choose σ so as to maximize his chances of receiving investors' money in period 1:

$$\max_{\sigma} P\left(\frac{\psi}{\psi + (1-\psi)L(R; \sigma_g^*, \sigma_b^*)} \geq \tau\right), \tag{5}$$

where the return distribution of R depends both on σ and on the quality of the manager via the mean μ_g or μ_b. (See Equation (1).)

An equilibrium is then a pair (σ_g^*, σ_b^*) such that

$$\sigma_x^* = \text{argmax}_{\sigma} P\left(\frac{\psi}{\psi + (1-\psi)L(R; \sigma_g^*, \sigma_b^*)} \geq \tau\right) \text{ s.t. } R = \mu_x + \sigma\epsilon, \epsilon \sim \mathcal{N}(0;1)$$

for $x = g, b$.

We are now ready to state our main results.

Theorem 1. *There exists $\overline{\rho} < 1$ such that if the parameters $\mu_g, \mu_b, \mu_o, \underline{\sigma}$ satisfy*

$$0 > \log\left(\frac{\psi(1-\tau)}{(1-\psi)\tau}\right) > Max\left(-\frac{1}{2} - \log\left(\frac{\mu_g - \mu_b}{\underline{\sigma}}\right), \log(\overline{\rho})\right) \tag{6}$$

with $\tau = \frac{\mu_o - \mu_b}{\mu_g - \mu_b}$, then,

(i) there exists an equilibrium (σ_g^*, σ_b^*) with the following features:

1. *A good manager picks the minimal feasible risk level, $\sigma_g^* = \underline{\sigma}$.*
2. *A bad manager picks a risk level, which is strictly greater than the feasible minimum, $\sigma_b^* > \underline{\sigma}$.*
3. *Investors invest in the active fund, if the return R falls in the interval $[R_l, R_h]$ where R_l and R_h solve the quadratic equation*

$$\left(\frac{1}{2\sigma_g^{*2}} - \frac{1}{2\sigma_b^{*2}}\right)R^2 - \left(\frac{\mu_g}{\sigma_g^{*2}} - \frac{\mu_b}{\sigma_b^{*2}}\right)R$$

$$+\left(\frac{\mu_g^2}{2\sigma_g^{*2}}-\frac{\mu_b^2}{2\sigma_b^{*2}}\right)-\log\left(\frac{\sigma_b^*}{\sigma_g^*}\frac{\psi(1-\tau)}{(1-\psi)\tau}\right)=0$$

and satisfy

$$\mu_b < R_l < \mu_g < R_h < \infty$$

(ii) There is no equilibrium in which investors invest in the active fund if $R \geq \underline{R}$ for some $\underline{R}$.
Proof. See Appendix.

The following two theorems provide the investors' equilibrium investment decisions for other sets of parameters.

Theorem 2. *Suppose that the parameters $\mu_g, \mu_b, \mu_o, \underline{\sigma}$ satisfy*

$$\frac{(\mu_g-\mu_b)^2}{2\underline{\sigma}^2} > \log\left(\frac{\psi(1-\tau)}{(1-\psi)\tau}\right) > 0. \tag{7}$$

Then, part (i) of Theorem 1 holds.
Proof. See Appendix.

Theorem 3. *Suppose that the parameters $\mu_g, \mu_b, \mu_o, \underline{\sigma}$ satisfy*

$$\log\left(\frac{\psi(1-\tau)}{(1-\psi)\tau}\right) > \frac{(\mu_g-\mu_b)^2}{2\underline{\sigma}^2} \tag{8}$$

Then there exists an equilibrium (σ_g^*, σ_b^*) with the following features:

1. *Both managers pick the minimal feasible risk level, $\sigma_g^* = \sigma_b^* = \underline{\sigma}$.*
2. *Investors invest in the active fund if the return R is larger than some threshold $\underline{R}$ with*

$$\underline{R} = \frac{\mu_g+\mu_b}{2} - \frac{\underline{\sigma}^2}{\mu_g-\mu_b}\log\left(\frac{\psi(1-\tau)}{(1-\psi)\tau}\right) \leq \mu_b \tag{9}$$

Proof. See Appendix.

Theorem 1 derives conditions under which the unique equilibrium is such that the realization of a high return signals that the manager is of bad quality and chose a large variance. As a consequence, a rational investor should not invest in the active fund in period 1.

The first inequality of (6) means that ex-ante (i.e., before observing the return) investors are better off investing in the index fund.[5] The interpretation of Theorem 1 is then the following. If, ex-ante, investors are better off investing in the index fund, a large return should always be interpreted as the performance of a bad manager gambling for a lucky outcome, rather than a sign of good quality. As a consequence, after having observed R and updated their beliefs about the quality of the fund, rational investors should keep investing in the index fund. Conversely, if the active fund has realized a good but not great return, then rational investors should invest in this fund.

The difference between Theorems 1 and 2 is the following. If, ex-ante, investors are better off investing in the active fund (the first Inequality of (6) does not hold) and the difference in expected returns between a good and a bad fund is large (so that Inequality (8) does not hold), there also exists an equilibrium such that investors always invest in the active fund no matter what return is observed. Such an equilibrium does not exist under the conditions of Theorem 1.

The equilibrium proposed by Theorem 3 is such that bad fund managers do not gamble, i.e., good and bad managers choose the same risk level. In such a case, investors' optimal fund picking rule is more "conventional": pick the active fund if it generates a return large enough.

This equilibrium holds if (i) ex-ante, investors are better off investing in the active fund and (ii) the difference in expected returns between a good and a bad fund is small.

From these three theorems, we can describe what a rational investor's strategy should be as a function of the observed return R and the active fund's ex-ante expected quality:

- If ex-ante, investors are better off investing in the index fund (the first inequality of (6) holds):
  - If the ex-ante expected return is very low (the second inequality of (6) does not hold), then whatever the observed return, a rational investor never invests in the active fund.
  - If the ex-ante expected return is not too low (the second inequality of (6) holds), a rational investor invests in the active fund at time 1, if the observed return R falls in the interval $[R_l, R_h]$.
- If ex-ante, investors are better off investing in the active fund (the first inequality of (6) does not hold):
  - If the difference $(\mu_g - \mu_b)$ is small (Inequality (8) holds), then a rational investor invests in the active fund at time 1 if the observed return is larger than some threshold $\underline{R}$.

[5] Given that investors are fully rational, these beliefs imply that the manager of the active fund has no investor money under management in period 0. We implicitly assume that, in such a situation, the manager uses one unit of his own money to produce a return in period 0.

– If the difference $(\mu_g - \mu_b)$ is large (Inequality (8) does not hold), then a rational investor invests in the active fund at time 1 whatever the observed return.

4. The General Case

The preceeding section concentrated on a highly restrictive case in which there are only two possible qualities for the manager. We shall now proceed to the general case of fairly arbitrary a priori expected returns. Our aim is now more modest: we want to state sufficient conditions under which we can *rule out* that, for any threshold, a smart investor should always invest in an active fund if its performance exceeds the threshold. In other words, we want to derive conditions under which the answer to the question in the title is negative.

Assume that the quality of the manager is measured by the mean return μ he can achieve, which is drawn according to some prior probability measure $\pi(d\mu)$ with compact support $[\underline{\mu}, \overline{\mu}]$. The manager picks σ within a given interval $[\underline{\sigma}, \overline{\sigma}]$, $0 < \underline{\sigma} < \overline{\sigma}$ and realizes the return $R = \mu + \sigma\epsilon$, where $\epsilon \sim N(0, 1)$. The investor observes R and forms posterior beliefs π_R about the managerial quality μ. He will invest in the fund iff $E_{\pi_R}[\mu] \geq \mu_o$. Our general result is as follows.

Theorem 4. *Suppose that, a priori, $E_\pi[\mu] < \mu_o$. Then, for $\overline{\sigma}$ sufficiently large, there is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some $\underline{R}$.*
Proof. See Appendix.

The theorem generalizes the result of Theorem 1 part (ii): whenever investors are better off investing in the index fund a priori, then investors should always interpret very large returns as a sign of a bad fund gambling for a good performance rather than the sign of good ability.

5. Discussion

5.1 THE FEE STRUCTURE

In the previous sections, it was assumed that the fee structure is given, i.e., the active fund charges a fraction-of-fund type of fee. It can be argued that the fee structure can be used to signal quality (see, for example, Das and Sundaram (2002)). However, empirical evidence suggests that only a very small minority of funds use incentive fees. Elton, Gruber and Blake (2003) report that less that 2 percent of US equity and bond mutual funds use incentive fees. Furthermore, Elton, Gruber and Blake conclude that "while at this time funds with incentive fees seem to offer superior performance with respect to other actively managed funds, we don't know whether this is true because of the motivation supplied by incentive fee or because skilled managers adopt incentive fees to advertise their skills to the public".

Casual observation suggests that funds do not modify their fee structure very often. Furthermore, short run future performance is in part predictable from past returns (Gruber (1996)). We do not believe that all the funds charging asset-based fees have the same quality for two reasons. First, money is not only invested into funds with incentive fees, and second, inflows into funds charging asset-based fees are not allocated uniformly among funds. Empirical evidence shows that these inflows are performance dependent. Therefore, funds charging asset-based fees compete with each other for inflow. Hence, our model can be seen as one studying short run competition between funds that have chosen asset-based fee schemes.[6]

5.2 DECREASING RETURNS TO SCALE

Our model assumes that the fund's expected return is independent of the size of assets under management. Berk and Green (2004) and Dangl, Wu and Zechner (2006) consider situations in which there is difference of ability between managers but decreasing returns to scale in deploying these abilities. Chen, Hong, Huang and Kubik (2003) study specifically the impact of fund size on performance. They find an inverse relationship between size and performance which is mainly explained by the trading costs associated with liquidity and price impact. To take the possibility of diseconomies of scale into account, we modify the model of Section 3 as follows. We assume that the expected return of an investment in the active fund is a decreasing linear function of the size of assets under management. Denote s the amount of money invested in the active fund. If this fund is of good quality, then

$$\mu = \mu_g(s) = \overline{\mu}_g - (\overline{\mu}_g - \mu_o)s$$

with $\overline{\mu}_g > \mu_o$.

Similarly, if the active fund is of bad quality, then

$$\mu = \mu_b(s) = \mu_o - (\mu_o - \overline{\mu}_b)s$$

with $\overline{\mu}_b < \mu_o$.

We deduce that at time 0, the amount of fund invested in the active fund (s_o) is such that

$$\psi \mu_g(s_o) + (1 - \psi)\mu_b(s_o) = \mu_o$$

[6] To take into account the short run predictive power of past returns, we can modify our model as follows: If a fund is good (bad) in Period t, it is good (bad) in Period $t + 1$ with probability $\theta > 1/2$, and it is bad (good) with probability $(1 - \theta)$.

At time 1, i.e., after observation of the realized return R, the amount invested in the active fund is such that

$$Prob(good|R)\mu_g(s) + Prob(bad|R)\mu_b(s) = \mu_o.$$

This is equivalent to

$$s = \frac{1}{1 + \frac{1-\psi}{\psi}\frac{\mu_o - \overline{\mu}_b}{\overline{\mu}_g - \mu_o}\frac{f(R|\mu_b(s_o), \sigma_b)}{f(R|\mu_g(s_o), \sigma_g)}} = H(R|\sigma_g, \sigma_b),$$

where $f(.|\mu, \sigma)$ is the normal density with mean μ and variance σ^2.

First, we study investors' behavior. It is easy to show that if $\sigma_g < \sigma_b$, then $\partial H/\partial R > 0$, is equivalent to

$$R < \frac{\frac{\mu_g(s_o)}{\sigma_g} - \frac{\mu_b(s_o)}{\sigma_b}}{\frac{1}{\sigma_g} - \frac{1}{\sigma_b}} = R^*(\sigma_g^*, \sigma_b^*)$$

It is straightforward that $R^* > \mu_b(s_o)$. Therefore, if $\sigma_g^* < \sigma_b^*$, we have a result along the lines of Theorems 1 and 2 in our model: beyond some threshold $(R^*(\sigma_g^*, \sigma_b^*))$, the allocation of money into funds is decreasing in performances.

If $\sigma_g^* = \sigma_b^*$, then s is always increasing in R. Hence, this result is similar to Theorem 3 in our model. This equilibrium investment strategy is also similar to those obtained in Berk and Green (2004) and Dangl, Wu and Zechner (2006) in the sense that (i) investors' expected return is that of the index fund, and (ii) investment in the active fund is an increasing function of its realized return.

Now, we turn to the problem of the manager. As in Section 3, we assume that when picking a portfolio, the fund manager only controls the riskiness of the portfolio (σ). With respect to our basic model, the problem of the manager is modified in the sense that his objective is now to maximize $E(s)$, i.e., the expected size of assets under management. An equilibrium is then a pair (σ_b^*, σ_g^*) such that

$$\sigma_b^* \in \text{Argmax}_{\sigma_b} \int_{-\infty}^{+\infty} H(R|\sigma_b, \sigma_g^*) f(R|\mu_b(s_o), \sigma_b) dR$$

$$\sigma_g^* \in \text{Argmax}_{\sigma_g} \int_{-\infty}^{+\infty} H(R|\sigma_b^*, \sigma_g) f(R|\mu_g(s_o), \sigma_g) dR$$

We have not managed to show analytically that there exists an equilibrium with $\sigma_g^* = \underline{\sigma}$ and $\sigma_b^* > \underline{\sigma}$. Therefore, we rely on numerical simulations to show that, at least for some sets of parameters, equilibria with $\sigma_g^* = \underline{\sigma}$ and $\sigma_b^* > \underline{\sigma}$ exist when funds exhibit decreasing returns to scale. Tables I and II provide

Table I. Equilibrium value of σ_b for various values of ψ and $\underline{\sigma}$. Other exogenous parameters are: $\overline{\mu}_g = 1.2$, $\overline{\mu}_b = 1.05$, $\mu_o = 1.1$

	$\underline{\sigma}$			
ψ	0.03	0.05	0.07	0.09
0.3	6.15	6.92	7.83	9.64
0.4	5.66	6.45	7.73	9.43
0.5	5.41	6.27	7.70	9.38
0.6	5.33	6.27	7.68	9.37
0.7	5.37	6.40	7.81	9.52

Table II. Equilibrium value of σ_b for various values of ψ and $\underline{\sigma}$. Other exogenous parameters are: $\overline{\mu}_g = 1.1$, $\overline{\mu}_b = 1$, $\mu_o = 1.05$

	$\underline{\sigma}$			
ψ	0.03	0.05	0.07	0.09
0.3	4.92	5.88	7.42	9.26
0.4	4.74	5.64	7.27	9.17
0.5	4.68	5.54	7.22	9.15
0.6	4.74	5.63	7.27	9.17
0.7	4.91	5.88	7.41	9.27

the equilibrium risk level chosen by a fund of bad quality for various sets of parameters. In all cases, $\sigma_g^* = \underline{\sigma}$, in equilibrium. Hence, if funds exhibit decreasing returns to scale, there exist equilibria similar to that described in Theorem 1, at least for some sets of parameters.

6. Conclusion

This paper provides a simple, highly stylized theory of the game played between a mutual fund manager and a collection of investors. The mutual fund manager is either bad or good, and is thus able to realize either a low or a high return on investment on average. The mutual fund manager controls the riskiness of his portfolio; investors observe a realization of the fund return, and invest in the fund, if it is sufficiently likely that the mutual fund manager is good. In such a situation, the manager will choose the riskiness of his to-be-observed portfolio return in order to maximize the chance that investors invest with him.

We obtain different types of equilibria for different conditions on parameter values. The first type of equilibrium is such that (i) investors invest in the active fund if the realized return falls within some interval, i.e., is neither too low nor too high, (ii) a good mutual fund manager picks a portfolio of minimal riskiness, and (iii) a bad mutual fund manager will pick a portfolio with higher risk, "gambling" on a lucky outcome. Thus, smart investors may not want to buy the fund with the highest past returns.

The second type of equilibrium we derive is more "traditional". It is such that (i) investors invest in the active fund if the observed return is larger than some threshold, and (ii) good and bad fund managers choose the same risk level. This equilibrium holds for sets of parameters such that, ex-ante, investors are better off investing in the active fund and the difference in expected performance between a good and a bad fund is small.

Our results suggest two possible interpretations for the positive correlation between inflows and past performance empirically observed. The first one is that mutual fund investors are boundedly rational and do not pick mutual funds optimally given fund managers' investment strategy. The second interpretation is that investors are fully rational, gambling incentives are small, hence can be neglected when picking mutual funds.

7. Appendix

Proof of Theorem 1.

Proof of (i)

The proof proceeds in several steps. We concentrate on finding equilibria, satisfying $\sigma_g^* \leq \sigma_b^*$. We will make ample use of this inequality, which obviously needs to be verified in the end.

The Investment Decision of the Investors

Given (4), the criterion (3) can be written as

$$(R-\mu_g)^2/(2\sigma_g^{*2}) - (R-\mu_b)^2/(2\sigma_b^{*2}) \leq \log\left(\frac{\sigma_b^*}{\sigma_g^*}\frac{\psi(1-\tau)}{(1-\psi)\tau}\right)$$

or, equivalently,

$$q(R;\sigma_g^*,\sigma_b^*) \leq 0 \tag{10}$$

where $q(R;\sigma_g^*,\sigma_b^*)$ is a quadratic function in R, given by

$$\begin{aligned} q(R;\sigma_g^*,\sigma_b^*) = {} & \left(\frac{1}{2\sigma_g^{*2}} - \frac{1}{2\sigma_b^{*2}}\right)R^2 \\ & - \left(\frac{\mu_g}{\sigma_g^{*2}} - \frac{\mu_b}{\sigma_b^{*2}}\right)R + \left(\frac{\mu_g^2}{2\sigma_g^{*2}} - \frac{\mu_b^2}{2\sigma_b^{*2}}\right) \\ & - \log\left(\frac{\sigma_b^*}{\sigma_g^*}\frac{\psi(1-\tau)}{(1-\psi)\tau}\right) \end{aligned} \tag{11}$$

Inequality (10) is satisfied, iff

$$R \in [R_l(\sigma_g^*, \sigma_b^*), R_h(\sigma_g^*, \sigma_b^*)] \tag{12}$$

where $R_l(\sigma_g^*, \sigma_b^*) \leq R_h(\sigma_g^*, \sigma_b^*)$ are the two solutions to the quadratic equation

$$q(R; \sigma_g^*, \sigma_b^*) = 0 \tag{13}$$

Let $\rho = \frac{\psi(1-\tau)}{(1-\psi)\tau}$. A sufficient condition for Equation (13) to have two real solutions is that

$$\Delta = \left(\frac{\mu_g}{\sigma_g^{*2}} - \frac{\mu_b}{\sigma_b^{*2}}\right)^2 + \left(\frac{1}{\sigma_g^{*2}} - \frac{1}{\sigma_b^{*2}}\right) \times \left[\left(\frac{\mu_b^2}{\sigma_b^{*2}} - \frac{\mu_g^2}{\sigma_g^{*2}}\right) + 2\log\left(\frac{\sigma_b^*}{\sigma_g^*}\rho\right)\right] > 0 \tag{14}$$

This can be rewritten as

$$\Delta = \frac{(\mu_g - \mu_b)^2}{\sigma_g^* \sigma_b^*} + 2\left(\frac{1}{\sigma_g^{*2}} - \frac{1}{\sigma_b^{*2}}\right)\log\left(\frac{\sigma_b^*}{\sigma_g^*}\rho\right) \tag{15}$$

Let

$$h(\sigma_g, \sigma_b) = \left(\frac{1}{\sigma_g^2} - \frac{1}{\sigma_b^2}\right)\log\left(\frac{\sigma_b}{\sigma_g}\rho\right)$$

and

$$\hat{h}(\rho) = \min_{(\sigma_g,\sigma_b),\sigma_b \geq \sigma_g} h(\sigma_g, \sigma_b, \rho).$$

$\hat{h}(\rho)$ is continuous in ρ and $\hat{h}(1) \geq 0$. Therefore, by continuity, there exists $\overline{\rho} < 1$ such that if $\rho > \overline{\rho}$, then $\Delta > 0$ and (13) has two real solutions. Furthermore, we note that for $\sigma_g^* = \sigma_b^*$, the coefficient on the lead quadratic term becomes zero. The solutions then take the form $R_h = \infty$, $R_l = \underline{R}(\sigma_g^*)$, where

$$\underline{R}(\sigma_g^*) = \frac{\mu_g + \mu_b}{2} - \frac{\sigma_g^{*2}}{\mu_g - \mu_b}\log\left(\frac{\psi(1-\tau)}{(1-\psi)\tau}\right) \tag{16}$$

A First-Order Condition for the Fund Manager

With (12), the objective (5) of the fund manager with quality $x = g, b$ can be rewritten as

$$\max_\sigma f(\sigma; \sigma_g^*, \sigma_b^*) \tag{17}$$

where $f(\sigma; \sigma_g^*, \sigma_b^*)$ is the integral

$$f(\sigma; \sigma_g^*, \sigma_b^*) = \int_{R_l(\sigma_g^*, \sigma_b^*)}^{R_h(\sigma_g^*, \sigma_b^*)} \frac{1}{\sqrt{2\pi}\sigma} e^{-(R-\mu_x)^2/(2\sigma^2)} dR.$$

To find the optimum, it is useful to differentiate f with respect to σ,

$$\frac{df}{d\sigma} = \frac{1}{\sigma} I(\sigma; R_l(\sigma_g^*, \sigma_b^*), R_h(\sigma_g^*, \sigma_b^*))$$

where $I(\sigma; R_l, R_h)$ is the integral

$$I(\sigma; R_l, R_h) = \int_{\frac{R_l-\mu_x}{\sigma}}^{\frac{R_h-\mu_x}{\sigma}} (\epsilon^2 - 1) \frac{1}{\sqrt{2\pi}} e^{-\epsilon^2/2} d\epsilon. \tag{18}$$

Standard results about normal distributions imply immediately that

$$I(\sigma; -\infty, \infty) = I(\sigma; 0, \infty) = I(\sigma; -\infty, 0) = 0. \tag{19}$$

Furthermore, note that the integrand $(\epsilon^2 - 1)$ in (18) is negative if $|\epsilon| < 1$. These relationships will prove useful in the next step.

When Do We Have $\sigma_g^* = \underline{\sigma}$?

From the preceeding analysis, we see that (17) is solved at $\sigma = \underline{\sigma}$, if

$$I(\sigma; R_l(\sigma_g^*, \sigma_b^*), R_h(\sigma_g^*, \sigma_b^*)) < 0$$

for all $\sigma \geq \underline{\sigma}$. With (19) and the remark following it, it is straightforward to check that this is the case if $R_l \leq \mu_x \leq R_h$. This in turn is true iff $q(\mu_x; \sigma_g^*, \sigma_b^*) \leq 0$. For $x = g$, this can be rewritten as

$$q(\mu_g) = -\frac{(\mu_g - \mu_b)^2}{2\sigma_b^{*2}} - \log\left(\frac{\sigma_b^*}{\sigma_g^*} \frac{\psi(1-\tau)}{(1-\psi)\tau}\right) \leq 0.$$

The second inequality of (6) is sufficient for this inequality to hold. This shows, that $\sigma_g^* = \underline{\sigma}$, as claimed.

When Do We Have $\underline{\sigma} < \sigma_b^* < \infty$?

From the previous step, we deduce that we have an equilibrium with $\sigma_g^* = \underline{\sigma}$ and $\sigma_b^* > \underline{\sigma}$ if we can rule rule out $\sigma_b^* = \underline{\sigma}$ via

$$I(\underline{\sigma}, R_l(\underline{\sigma}, \underline{\sigma}), R_h(\underline{\sigma}, \underline{\sigma})) > 0 \tag{20}$$

and show, that for some $\sigma > \underline{\sigma}$, we have

$$I(\sigma, R_l(\underline{\sigma}, \sigma), R_h(\underline{\sigma}, \sigma)) < 0. \tag{21}$$

An interior solution must then exist by the mean value theorem.

Inequality (20) is equivalent to $\underline{R}(\underline{\sigma}) > \mu_b$ which is in turn equivalent to the first inequality of (6). For inequality (21), note that the integrand in Equation (18) is negative for $|\epsilon| < 1$. Thus, (21) follows if

$$-1 < \frac{R_l(\underline{\sigma}, \sigma) - \mu_b}{\sigma} \leq \frac{R_h(\underline{\sigma}, \sigma) - \mu_b}{\sigma} < 1$$

for some σ. Rewrite this as

$$\mu_b - \sigma < R_l(\underline{\sigma}, \sigma) \leq R_h(\underline{\sigma}, \sigma) < \mu_b + \sigma$$

which, in turn, is equivalent to

$$q(\mu_b - \sigma, \underline{\sigma}, \sigma) > 0,\ q(\mu_b + \sigma, \underline{\sigma}, \sigma) > 0$$

for some σ. However, inspecting (11), it is easy to see that $q(\mu_b - \sigma, \underline{\sigma}, \sigma)$ and $q(\mu_b + \sigma, \underline{\sigma}, \sigma)$ go to infinity as $\sigma \to \infty$, completing the proof.

Proof of (ii)

We consider the three possible types of pure-strategy equilibria: $\sigma_g < \sigma_b$, $\sigma_g > \sigma_b$ and $\sigma_g = \sigma_b$.

Let's consider first the case $\sigma_g < \sigma_b$. From the proof of (i), we know that if $\rho > \overline{\rho}$, then $\Delta > 0$ and (13) has two real solutions. Hence, there is no equilibrium, in which a rational investor will invest if $R \geq \underline{R}$ for some $\underline{R}$.

Consider now the case $\sigma_g > \sigma_b$. Proceeding as in the proof of (i), one obtains that if the beliefs of the investor are such that $\sigma_g^* > \sigma_b^*$, then the objective of a manager is to minimize $f(\sigma; \sigma_g^*, \sigma_b^*)$ over σ, where $f(\sigma; \sigma_g^*, \sigma_b^*)$ is the integral

$$f(\sigma; \sigma_g^*, \sigma_b^*) = \int_{R_l(\sigma_g^*, \sigma_b^*)}^{R_h(\sigma_g^*, \sigma_b^*)} \frac{1}{\sqrt{2\pi}\sigma} e^{-(R-\mu_x)^2/(2\sigma^2)} dR$$

where $x = g$ if the manager is good and $x = b$ if the manager is bad. It is straightforward that $f(\sigma; \sigma_g^*, \sigma_b^*)$ converges to 0 when σ goes to $+\infty$. Hence, there is no equilibrium in which the smart investor will invest if $R \geq \underline{R}$ for some $\underline{R}$.

Last, consider the case $\sigma_g = \sigma_b$. Assume that the investor's beliefs are such that $\sigma_g^* = \sigma_b^*$. The objective of a manager is to maximize

$$Prob\left(R > -\frac{\sigma^2}{\mu_g - \mu_b}\log(\rho) + \frac{\mu_g + \mu_b}{2}\right)$$

over σ. From the proof of part (i), we know that a good manager chooses $\sigma_g = \underline{\sigma}$. The condition $\log(\rho) < 0$ implies that

$$-\frac{\sigma^2}{\mu_g - \mu_b}\log(\rho) + \frac{\mu_g + \mu_b}{2} > \mu_u.$$

As a consequence, a bad manager chooses $\sigma_b > \underline{\sigma}$. Hence, there is no equilibrium, in which the smart investor will invest if $R \geq \underline{R}$ for some $\underline{R}$.

■

Proof of Theorem 2. If Inequality (7) holds then inequality (14) holds. Therefore, we have the desired result.

■

Proof of Theorem 3. Follow the proof of Theorem 1 above, except for Step 4. Instead of inequality (20), we now get $I(\underline{\sigma}, R_l(\underline{\sigma}, \underline{\sigma}), R_h(\underline{\sigma}, \underline{\sigma})) < 0$ as a consequence of (8). It follows that $\sigma_b^* = \underline{\sigma}$ also. Finally, combine (8) and (9) to see that $\underline{R} < \mu_b$.

■

Proof of Theorem 4. Suppose to the contrary, that there was an equilibrium, where the investor always invests, if $R \geq \underline{R}$ for some $\underline{R}$. We will show, that there is some $R \geq \underline{R}$ (perhaps requiring some sufficiently large $\overline{\sigma}$), so that $E_{\pi_R}[\mu] < \mu^*$, which is a contradiction.

As in the proof of Theorem 1, the fund manager will maximize the probability that $R \geq \underline{R}$ via his choice of σ. It is easy to check that he will choose $\sigma = \underline{\sigma}$ if $\mu \geq \underline{R}$, and $\sigma = \overline{\sigma}$, if $\mu < \underline{R}$. The posterior probability distribution therefore has the form

$$\pi_R(d\mu) = \phi(R, \overline{\sigma})\,(m_A(d\mu; R, \overline{\sigma}) + m_B(\mu; R, \overline{\sigma}))$$

where m_A is a measure with support $[\underline{\mu}, \underline{R})$ and given by

$$m_A(d\mu; R, \overline{\sigma}) = \frac{1}{\overline{\sigma}}\exp\left(-\frac{(\mu - R)^2}{2\overline{\sigma}^2}\right)\pi(d\mu),$$

where m_B is a measure with support $[\underline{R}, \overline{\mu}]$ and given by

$$m_B(d\mu; R, \overline{\sigma}) = \frac{1}{\underline{\sigma}}\exp\left(-\frac{(\mu - R)^2}{2\underline{\sigma}^2}\right)\pi(d\mu),$$

and where $\phi(R, \overline{\sigma})$ is chosen so that π_R is a probability measure.

We now distinguish two cases, $\underline{R} > \underline{\mu}$ and $\underline{R} \leq \underline{\mu}$.

CASE $\underline{R} > \underline{\mu}$. The measure μ_A has positive mass, since $\pi(\mu < \underline{R}) > 0$.

Note that the lead term in the exponential expression in m_A is $-R^2/(2\overline{\sigma}^2)$, whereas it is $-R^2/(2\underline{\sigma}^2)$ in the exponential expression for m_B. Since $\overline{\sigma} > \underline{\sigma}$, it follows that m_B vanishes relative to m_A as $R \to \infty, \overline{\sigma} \to \infty, R/\overline{\sigma} \equiv$ const.. More precisely, for any $\nu > 0$, one can find some sufficiently large R as well as some $\tilde{\sigma}$, so that for all $\overline{\sigma} > \tilde{\sigma}$, we have that $| E_{\phi(R,\overline{\sigma})m_A(\cdot;R,\overline{\sigma})}[\mu] - E_{\pi_R}[\mu] | < \nu$, holding $R/\overline{\sigma} \equiv$ const.

Fix $R/\overline{\sigma}$. One can see that $E_{\phi(R,\overline{\sigma})m_A(\cdot;R,\overline{\sigma})}[\mu] \to E_\pi[\mu \mid \mu < \underline{R}]$, as $\overline{\sigma} \to \infty$.

Put these two pieces together and use $E_\pi[\mu \mid \mu < \underline{R})] \leq E_\pi[\mu] < \mu^*$ to demonstrate that $E_{\pi_R}[\mu] < \mu^*$.

CASE $\underline{R} \leq \underline{\mu}$. The fund manager will choose $\sigma = \underline{\sigma}$, regardless of his type μ. Now suppose that the investor observes $R = \underline{\mu}$: note that he will invest. In that case,

$$E_{\pi_R}[\mu] = \frac{\int \mu e^{-(\mu-\underline{\mu})^2/(2\underline{\sigma}^2)} \pi(d\mu)}{\int e^{-(\mu-\underline{\mu})^2/(2\underline{\sigma}^2)} \pi(d\mu)} \leq E_\pi[\mu] < \mu^*,$$

(where the first inequality can be seen to hold, since the posterior puts larger weight on smaller μ's), yielding the contradiction. This contradiction does not require the probability-zero event of exactly observing $R = \underline{\mu}$: the inequality above also remains for R near $\underline{\mu}$ by continuity.

■

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