Review of Finance (2006) 10: 353–387
DOI 10.1007/s10679-006-9000-0

Structuring the Initial Offering: Who to Sell To and How to Do It ⋆

VOJISLAV MAKSIMOVIC[1] and PEGARET PICHLER[2]
[1]*Robert H. Smith School, University of Maryland and Tanaka School, Imperial College, London;*
[2]*Institute for Advanced Studies, Vienna and Vienna Graduate School of Finance*

Abstract. We develop a unified model of the issuer's decisions that takes into account both mechanism design and adverse selection risk. The model enables us to determine the optimal amount of information gathering prior to setting the offer price, and to understand what does and does not cause underpricing. The flexibility to allocate securities between a pool of investors who provide pricing-relevant information and investors who do not provide information is key to controlling underpricing. Policies that guarantee a minimum allocation to investors in the pool result in underpricing; policies that cap the allocations to such investors do not. The optimal number of investors in the pool, and thus the amount of information acquired, generally increases with the riskiness of the issue. However, this relation breaks down if pool members are guaranteed minimum allocations.

1. Introduction

Issuers of unseasoned securities who wish to sell the securities publicly must make a number of decisions when determining how to price and place their offerings. In this paper we develop a unified model of the issuer's decisions that takes into account both mechanism design and adverse selection risk. Doing so enables us to determine the optimal amount of information gathering prior to setting the offer price, and to understand what does and does not cause underpricing in a mechanism for pricing a *public* offering.

We use our model to answer the following questions: Should an issuer gather information from investors prior to setting the issue price? If so, what is the optimal amount of information gathering? What sorts of constraints on the information gathering process do, and do not, lead to underpricing? Under what conditions is it optimal for an issuer to stage an IPO? Our model enables us to obtain empirically testable relations between underpricing, issue method, and issue characteristics

⋆ We are grateful to the referees and the editor, Marco Pagano, for their insightful suggestions. We thank Sugato Bhattacharyya, Jos von Bommel, Zhoahui Chen, Robert Hauswald, Dilip Madan, Kristian Rydqvist, Alex Stomper, Meg Van De Weghe, Ivo Welch and participants at Carnegie Mellon, Indiana University, the University of Maryland, Tulane University and Yale University finance seminars for their helpful comments. This paper has evolved from an earlier paper titled Private versus Public Offerings: Optimal Selling Mechanisms with Adverse Selection, that was presented at the 1999 Western Finance Association meetings.

such as the issue size and the amount of prior (before gathering any information) uncertainty about share value.

We explicitly take into account the roles of two different types of investors. "Regular investors" are sophisticated investors who have private information about the value of the offering, and have the resources to purchase large numbers of shares. Issuers may elicit information from these investors through the process of bookbuilding. "Retail investors" include both informed and uninformed investors. These investors differ from regular investors in that a large fraction of retail investors do not have private information and the issuer is unable to differentiate between retail investors with information and those without.

The mechanisms that we design take into account a key benefit of engaging in a public offering: the ability to allocate shares to retail investors. That is, we allow the issuer to allocate shares to investors other than those from whom information is directly elicited. In contrast, much of the existing literature assumes that if a mechanism is used to elicit information from regular investors, then shares are allocated only to those investors from whom information is elicited.[1] We demonstrate the value of including in the mechanism both investors who provide information, and retail investors who do not. At the extreme we show that if there are no restrictions on the way in which shares are allocated between these types of investors then it is not necessary to underprice in order to induce truth-telling on the part of informed investors.

There exists, however, a rather extensive literature that documents significant underpricing in IPOs.[2] Our result thus implies that constraints, apart from just the need to induce truth-telling, are likely to be present in the bookbuilding process. For example, sellers may follow a policy of allocating a minimum fraction of the issue to institutional (regular) investors in order to both provide validation for the issue and to encourage analyst coverage after the offering is completed.[3] Or, the seller may feel obligated to allocate securities to every regular investor who participates actively in the pricing process, regardless of the nature of the information provided. We show that the use of a mechanism to elicit information results in underpricing if the issuer is faced with either of these types of constraints. We also consider the possibility that there are upper bounds on the number of shares that can be allocated to regular investors, either due to individual budget constraints, or due to a requirement to allocate securities to retail investors. These types of constraints do not require underpricing in order to elicit truthful reports. Finally, individual regular investors may find it costly to participate in the mechanism, perhaps due to costs of obtaining information. In this case, each regular investor will demand a strictly positive expected return for participating in the mechanism.

[1] Exceptions to this are discussed at the end of the Introduction.

[2] Ibbotson, Sindelar and Ritter(1988), Loughran and Ritter (2002b), and Ritter (1987) all provide evidence of significant underpricing, defined as positive returns from the offer price to the first day.

[3] Analysts are evaluated by institutional investors, and so prefer to cover firms that have institutional ownership.

It is well known that, if no information is gathered prior to pricing an issue, then the total wealth loss to the issuer, due to underpricing, is increasing in both issue size and uncertainty about the issue value. As such, one might expect that the optimal amount of information gathering, prior to setting the offer price, will be increasing in both of these factors. Our work conrm s that the optimal amount of information gathering is always increasing in issue size, but we show that it may be independent of the level of uncertainty about the issue value. In bookbuilding the issuer can increase the amount of information gathering by increasing the number of regular investors who are invited to participate in the mechanism. Both a strictly positive participation constraint and a constraint requiring that individual regular investors receive positive allocations result in these investors obtaining informational rents. Both of these types of constraints thus place limits on the optimal level of information gathering, but the constraints differ in the nature of the limits. For example, if regular investors require a xed expected return for participating, then the optimal amount of information gathering increases with uncertainty about issue value. If instead, regular investors obtain informational rents through minimum allocation constraints, then the optimal amount of information gathering is independent of this uncertainty.

Even after taking into account the optimal amount of information gathering, we show that total wealth loss is always increasing in both issue size and uncertainty about issue value. Thus, regardless of the means by which investors obtain informational rents, the bene t to staging an IPO, that is breaking the IPO into parts, is always increasing in both issue size and uncertainty about issue value.

The empirical evidence linking allocation policies and information gathering is mixed. Cornelli and Goldreich (2001) examine bookbuilding by one European investment bank and nd that investors who post more informative bids receive more favorable allocations of IPO shares, and earn on average higher prots .[4] Jenkinson and Jones (2004), on the other hand, n d that while some bidders are favored in the allocation of IPO shares, this favorable treatment does not appear to be a reward for information contained in their orders. But, Jenkinson and Jones also state (p. 2311) that investors may reveal information prior to the of cial bookbuilding period, and may be rewarded for this information with allocations.

Our work builds on that of Benveniste and Spindt (1989) who rst modeled bookbuilding as a direct mechanism. Benveniste and Wilhelm (1990), in their comparison of differential and uniform pricing policies, model a mechanism that allows for the sale of shares to retail investors, as well as regular investors. They do not address the question of optimal information gathering, and their solution of the basic model with a xed amount of information gathering also differs markedly from ours.[5] Benveniste and Busaba (1997) point out the potential value of gath-

[4] Cornelli and Goldreich (2003) also n d that the offer price tends to reect information that is contained in the bids of larger and more frequent investors. These may be the “regular” investors.

[5] This difference may be due to their implicit assumption that retail investors do not update information rationally.

ering information through the use of a mechanism before setting an offer price. In their mechanism shares are allocated, by assumption, only to regular investors. Benveniste, Busaba and Wilhelm (1996) examine the role that an underwriter's commitment to price stabilization plays in mechanism design. Their mechanism allows for the allocation of shares to retail investors, but there is no adverse selection risk in their model, and they assume a fixed amount of information gathering. Sherman and Titman (2001) model the moral hazard problem of providing incentives for regular investors to collect information. They consider the optimal amount of information gathering, given that polled investors require a strictly positive expected return for participating in the mechanism. In their mechanism shares are allocated only to regular investors, and there is no adverse selection risk.[6]

Our work ties together much of this other work by developing a model that explicitly takes into account the role of retail investors in mechanism design *and* determines the optimal amount of information gathering. Our approach has the advantage of starting out in a very simple and very structured manner that allows us to add a number of complexities. We are thus able to go beyond the previous work and examine the effects of different types of constraints in the information gathering process. We endogenize both the benefit and the cost of gathering information through a mechanism. This approach extends the existing understanding of mechanism design, and enables us to develop predictions on the functional forms of the relations between uncertainty, IPO pricing strategies and underpricing.

The rest of the paper is organized as follows. In the following section we describe the basic model. In Section 3 we derive the expected underpricing in two polar cases: when the issuer gathers no information and when the issuer is able to gather all information from just two polled investors. This work sets the foundation for the more general case presented in Section 4. Section 5 addresses the question of staging an IPO. In Section 6 we conclude. The Appendix includes all proofs and a list of notation.

2. The Model

We compare three alternative selling strategies that correspond to existing institutional practices. For each of the strategies we derive a selling mechanism that

[6] Our work is also related to the following papers: Back and Zender (1993) solve for an optimal sale mechanism for Treasury auctions with two regular investors. The solution of our base-case mechanism follows closely from their solution, except that in their model securities are allocated only to regular investors. Habib and Ljungqvist (2001) model promotion activities for selling IPO shares in the presence of adverse selection risk, but they do not consider a mechanism for gathering information. Biais and Faugeron-Crouzet (2002) expands on the Benveniste and Spindt (1989) model in order to compare bookbuilding with the French mise-en-vente. This paper does not consider adverse selection risk in the retail market, or address the question of optimal information gathering. Malakhov (2004) points out that for our first result to hold it is necessary that there be a sufficiently large number of retail investors who are willing to participate in the issue.

maximizes the expected value for an entrepreneur who wishes to issue securities.[7] The market for the securities consists of a potentially large number of investors, all of whom have a common knowledge prior belief about the value of the issuing firm. Some of the investors, who we call "informed", have additional relevant information about the value of the firm. The issuer and the remaining investors do not possess this additional information.

The first selling strategy that we consider is the Retail strategy (R) in which the issuer sets a price and sells to a large number of investors, without attempting to pregather information possessed by informed investors. This corresponds to a best efforts offering and is the case analyzed by Rock (1986). Because no information is gathered prior to setting the issue price, and because retail investors include a mix of informed and uninformed investors, there will be an adverse selection risk for the uninformed investors.

In the pool-only strategy (P) the issuer first elicits information from some informed investors (the pool). The issuer then sets the offer price and allocates the securities. In the pool-only strategy securities are allocated only to members of the pool. This strategy is similar to the mechanism design problem typically modeled in the economic literature, such as Myerson (1981).

In the third strategy, pool and retail (PR), the issuer also forms a pool of investors from whom he solicits information before setting the price of the issue. The issuer may grant these investors preferential access to shares in return for information, but the issuer retains the option to sell a fraction of the issue to the retail market, i.e., to investors who are not members of the pool. The proportion of the issue sold to the pool may be a function of the information received from the pool. This selling strategy corresponds to the bookbuilding method used for pricing and distributing most IPOs in the United States and in much of Europe.

The three strategies that we consider expose the issuer to different costs. In the P strategy the issuer sells only to a prespecified pool of investors and incurs the cost of motivating these investors to bid aggressively for shares of the issue, as well as any participation cost demanded by these investors. In our mechanism all information that is held by pool members is elicited from them before the issue price is set. Thus, since the entire issue is placed with the pool, the issuer does not have to underprice the issue in order to compensate purchasers for adverse selection risk. By contrast, an issuer who follows the R strategy (sells an IPO directly to the retail market) does not bear a participation cost, or the cost of motivating informed investors to bid aggressively. But, the issuer also does not pregather information from investors and so investors are subject to adverse selection risk. In the PR strategy the issuer grants preferential access to a pool of informed investors *and* retains the option to sell to retail investors. The issue thus potentially incurs all of the above types of costs, but each may be incurred to a lesser degree.

[7] While there may be an invesment banker facilitating the sale, we will assume that the incentives of the entrepreneur and the investment banker are perfectly aligned.

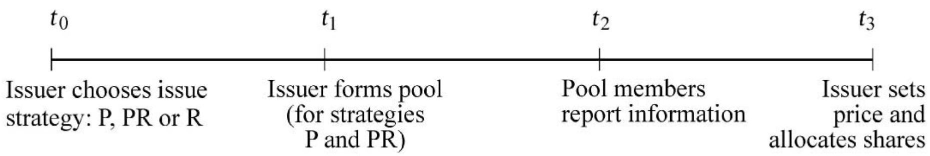

Figure 1. Timeline for security issurance.

The sequence of events is depicted in Figure 1. At time t_0 the issuer decides which of the three strategies to follow. At time t_1, if the issuer has chosen strategy P or PR then he decides how large the pool should be and he forms the pool. In strategies P and PR we will design a mechanism such that each pool member agrees to provide information to the issuer, on the understanding that she will receive an allocation of shares that depends on the information reported by herself and other pool members. If a pool is formed, pool members report their information to the issuer at time t_2. At time t_3 the issuer sets the issue price and allocates shares. The shares are allocated to each pool member (strategies P and PR) and to the retail market (strategies R and PR) according to the allocation rules.

For most of the analysis we assume that a fixed number Q of shares will be sold. In order to keep the algebra simple so that we can focus on information effects, we assume that the offering is a pure secondary offering.[8] These two assumptions imply that the objective of maximizing the issuer's value is equivalent to minimizing expected underpricing. In Section 5 we will drop the assumption of a fixed number of shares in order to examine the value of staging an IPO.

The secondary market value of one share is

$$\tilde{v} = v_0 + \tilde{v}_1 + \tilde{v}_2 \,, \tag{1}$$

where v_0 is the prior expected share value, and $\tilde{v}_1$ and $\tilde{v}_2$ are valuation factors. Each of these factors can take on a value of either $w/2$ or $-w/2$, with each outcome equally likely. The outcomes of these two factors are distributed independently of each other. Thus, $\tilde{v}$ can take on one of three values:

$$\tilde{v} \in \{v_0 - w, \;\; v_0, \;\; v_0 + w\} \,, \tag{2}$$

with the probability distribution $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$. w is the most that the secondary market price can differ from the prior expected share value. This parameter reflects the level of prior uncertainty about the issue value.

A fraction z_I of all potential investors are informed. Each informed investor has correctly observed exactly one of the two valuation factors. Each informed investor has a probability one-half of observing a given factor. Thus, in expected value a fraction $z_I/2$ of all potential investors will have observed each factor. Each of the

[8] The analysis can be extended to a combination primary and secondary offering so as to take into account the effect of dilution when an issue is underpriced. This extension is fairly straightforward, but greatly adds to the algebraic complexity without significantly altering the intuition.

parameters v_0, w, z_I, as well as the probabilities described above, are exogenously given and are common knowledge.

Any two informed investors chosen at random will, with probability 1/2, have observed the same factor, and with probability 1/2 have observed different factors. Thus, with probability 1/2 their information is perfectly correlated and with probability 1/2 their information is independent. This information structure captures, in a simple way, the realistic characteristic that informed investors' signals are positively, but not perfectly, correlated. One effect of this is that, if informed investors are selected randomly, then the expected benefit to polling an additional informed investor is positive, but this expected benefit is decreasing in the number of investors who have already been polled.

3. Underpricing in Two Polar Cases

In this section we solve two polar cases: the case where no information is gathered and the case where all information can be gathered by polling just two informed investors. The first of these cases is the R strategy. In the second case we determine solutions for the P and the PR stratgies under a simplifying assumption regarding the selection of polled investors. The purpose of doing this is to understand how the mechanisms work in these two strategies, without the added complexity of determining the optimal amount of information gathering. We reserve the investigation of optimal information gathering for Section 4.

3.1. THE R STRATEGY: NO INFORMATION GATHERING PRIOR TO PRICING

Under the R strategy the issuer sets a fixed price, without pregathering information from regular investors. After observing this price, retail investors decide whether to participate in the offering. A retail investor participates by entering into a queue to obtain shares. Arrival order in the queue is random and shares are allocated on a first-come first-served basis. The size of an individual retail allocation is e shares, where $\frac{e}{Q}$ is very small. As described in the previous section, a fraction z_I of all potential investors, including investors who may participate in the retail market, are informed. If all of the informed investors participate, on average a fraction z_I of the shares will go to informed investors. However, informed investors will participate only if the expected share value, conditioned on their information, is at least as high as the offer price. As shown below, they will participate only if they have positive information. Thus, if the realizations of both valuation factors are positive, then all informed investors will participate in the retail market. If the realizations of both are negative, then no informed investors will participate. If one factor has a positive realization and the other is negative, then in expected value half of the informed investors will participate. An uninformed investor who participates in an offering, thus has a higher probability of receiving an allocation when the share value is lower.

Because of this adverse selection risk, in order to ensure the participation of uninformed investors, and thus ensure the success of the offering, the issuer must on average underprice the issue. That is, the shares must be priced below v_0, the a priori expected value of the shares. The expected per-share underpricing when information is not gathered and the IPO shares are sold entirely to the retail market is:[9]

$$Eu_R = Eu_{Retail} = \frac{wz_I}{2(2 - z_I)} . \tag{3}$$

3.2. THE P AND PR STRATEGIES WHEN ALL INFORMATION IS GATHERED PRIOR TO PRICING

We now solve for the P and PR strategies, assuming that the issuer forms a pool with two informed investors who have observed different valuation factors. We assume in this section that the issuer can not only identify informed investors, but can also identify one investor who has observed the realization of $\tilde{v}_1$ and one investor who has observed the realization of $\tilde{v}_2$. Thus, the two members of this pool have between them, with probability one, all of the relevant available information. We also assume that information obtained by the issuer is revealed in the issue price. As a result, if the issuer sells shares to the retail market after obtaining the pool's information, there will be no adverse selection risk. This assumption enables us to analyze the differences between mechanism design under the P and the PR strategies, without the added complexity of determining the optimal amount of information gathering. We will eliminate this assumption in Sections 4 and 5.

The problem faced by the issuer is to design a mechanism that minimizes the expected underpricing. (In our model this is equivalent to maximizing expected issue proceeds.) Prior to communicating with the pool members, the issuer (implicitly) commits to a mechanism that specifies the issue price and the allocation that each pool member receives, contingent on the information that the pool members provide to the issuer. After the pool members report their information to the issuer, the issuer sets the issue price and allocates shares to pool members according to this mechanism. Any shares that are not allocated to the pool are then sold to retail investors on a first-come, first-served basis. In the P strategy all of the shares must be allocated to the pool.

In solving for the optimal mechanism, we proceed in two stages. First, we assume that, under the PR strategy, the issuer is not constrained in how he allocates the issue between retail investors and pool members, and that the issuer does not need to compensate the pool members for any cost of becoming informed. We refer to this as the benchmark, or unconstrained, case. Following the analysis of

[9] See the Appendix for the derivation. The value of $z_I/(2 - z_I)$ is less than one. Thus, the offer price of $v_0 - Eu_R$ is greater than $v_0 - w/2$ and, as stated above, informed investors will not participate if they have observed negative information.

the benchmark case, we will generalize the model to take into account a number of possible constraints that can limit the issuer's flexibility. In all cases we invoke the Revelation Principle that says that the issuer can do at least as well with a mechanism that induces the polled investors to truthfully report their signals, as with any mechanism that does not.[10] We also assume limited liability on the part of investors, so that in each outcome (set of realized reports from the polled investors), the issuer must price the issue so that all participants at least break even in expected value. Thus, the issuer will not deliberately overprice in any state.

3.2.1. *The Benchmark Case*

The mechanism design problem is to choose a selling mechanism (pricing and allocation rules) that minimizes the following function, subject to satisfying a number of constraints:

$$\min \; Eu = \frac{1}{4}(u^{++} + 2u^{+-} + u^{--})$$

where $+$ $(-)$ signifies positive (negative) information and u^{ab} = underpricing when one pool member reports a and the other reports b. This function is minimized subject to four sets of constraints.

(i) Participation of pool members:

$$EV^{+} = \frac{1}{2}\left(u^{++}q^{++} + u^{+-}q^{+-}\right) \geq 0 \tag{4}$$

$$EV^{-} = \frac{1}{2}\left(u^{+-}q^{-+} + u^{--}q^{--}\right) \geq 0 \tag{5}$$

where EV^{a} = the expected excess return to an investor who sees and reports a, and q^{ab} = the quantity received by a pool member who says a when the other says b.

(ii) Incentive Compatibility for pool members:[11]

$$EV^{+} \geq EV^{-} + \frac{w}{2}(q^{-+} + q^{--}) \tag{IC$^+$}$$

$$EV^{-} \geq EV^{+} - \frac{w}{2}(q^{++} + q^{+-}) \tag{IC$^-$}$$

[10] See Myerson (1981) for a discussion of this principle.

[11] An investor who sees a positive signal and reports a negative signal lowers v_p (the expected share value given the pool's reported information) by an amount w and expects an allocation of $\frac{q^{-+}+q^{--}}{2}$. Similarly, an investor who sees a negative signal and reports a positive signal raises v_p by an amount w and expects an allocation of $\frac{q^{++}+q^{+-}}{2}$.

(iii) Allocation constraints:

$$2q^{++},\ 2q^{--},\ q^{+-}+q^{-+}\le Q \quad \text{and} \quad q^{++},\ q^{--},\ q^{+-},\ q^{-+}\ge 0.$$

In the P strategy the first set of allocation constraints must be satisfied with equality.

(iv) Participation of the retail market:

$$u^{++}(Q-2q^{++})\ge 0\ \ 2u^{+-}(Q-q^{+-}-q^{-+})\ge 0\ \ u^{--}(Q-2q^{--})\ge 0.$$

This problem may also be referred to as the "unconstrained case" for the PR strategy, because the participation constraints require only that all investors expect a nonnegative return, and the allocation constraints require only that allocations be nonnegative and that they add up to the total issue size. (The constraint that the allocation to retail investors is equal to Q minus the allocations to pool members is implicitly included in the last set of constraints.) In the PR strategy the issuer may allocate shares, possibly all of the shares in this unrestricted problem, to the retail market after eliciting information from the pool. In the P strategy the issuer is constrained to sell all of the shares to pool members, regardless of the information learned. In the following proposition we present the expected underpricing for both strategies. The optimal allocation and pricing rules for each strategy are given in the Appendix.

PROPOSITION 1. *Zero Expected Underpricing.* When the issuer forms a pool of two informed investors who between them have all the available information, if there are no direct costs of information, then

(i) Expected underpricing in a public offering with no allocation restrictions is zero.
(ii) Expected underpricing if the issuer is constrained to sell shares only to pool members is strictly positive: $Eu_P = \frac{w}{4}$.

In order to minimize the cost of ensuring truth-telling the issuer should employ a mechanism that allocates as few shares as possible to any pool member who says that the firm has a low value. Part i) of Proposition 1 refers to the PR strategy. In this unconstrained case it is possible to allocate no shares at all to pool members. As a result, pool members do not expect to profit from providing false negative information, and it is thus unnecessary to provide strictly positive incentives in order to induce the pool members to truthfully report. That is, the incentive compatibility constraints are nonbinding. Part ii) of the Proposition refers to the P strategy in which the issuer is constrained to sell shares only within the pool. Hence, if a pool member deviates and reports a negative signal after observing a positive signal, while the other pool member also reports a negative signal, then each receives half of the shares. As a result, pool members have strictly positive incentives to provide false negative reports, and strictly positive rents must be paid in order to induce truthtelling.

Part (ii) of Proposition 1 follows directly from Lemma 2 of Myerson (1981). Part (i) goes beyond Myerson (1981) in that we demonstrate the effect of being able to allocate shares to buyers other than those who provide information in the mechanism. Part (i) of Proposition 1 presents an important benchmark for understanding the role that mechanism design plays in the pricing of IPOs. As described in the Introduction, there is much empirical evidence of significant average underpricing in bookbuilt IPOs. The zero underpricing result is thus not posited as a realistic prediction of what we expect to find in actual markets. What we learn from Proposition 1 is that, in a *public* offering, in which there is a large number of potential buyers, beyond those who participate directly in the mechanism, the need to induce truthtelling does not *by itself* lead to underpricing. This understanding provides us with a starting point to determine what it is about a mechanism that may cause underpricing.

3.2.2. *The Constrained Case*

Proposition 1, together with the empirical literature cited in the Introduction, suggests that allocation and/or participation constraints are likely to be important factors in the underpricing of public offerings. (An inability to gather all of the information for certain from a small number of investors also results in underpricing, but as we will see later, in the absence of significant allocation or participation constraints, this source of underpricing can be largely eliminated.) We consider in this section several possible constraints that may affect the mechanism design. First, we allow for the possibility that each pool member requires a minimum nonnegative expected excess return, $\gamma/2$, in order to be induced to participate. While we allow the value of γ to equal zero, we expect that if potential pool members have a cost to obtaining information, then they will require strictly positive rents.[12]

Next, we allow for four different types of allocation constraints. The first two constraints place lower limits on the allocations to pool members. (i) Large regular investors potentially play a role not only in providing information that is useful for pricing IPOs, but also in validating the IPO offer price. By purchasing shares at the offer price these investors signal to other investors that the issue is not overpriced. Arguments have also been put forward that institutional participation at the IPO may be advantageous.[13] We thus assume that the issuer must allocate at least q_P shares to the pool members as a group, where $q_P \in [0, Q)$. By definition, q_P is equal to Q in the P strategy. ii) We also allow for the possibility that the issuer is constrained to offer each individual pool member a minimum number of shares of the issue. The intuition for this comes from discussions with an investment banker.

[12] We model the per-investor cost as $\gamma/2$, rather than γ purely for ease of notation: in most constraints the per-investor cost is multiplied by 2.

[13] See for example, Mello and Parsons (1998) and Stoughton and Zechner (1998).

Large investors who are involved in discussions concerning the value of an IPO typically are included when the IPO is sold. An underwriter may be unwilling to assign a zero allocation to an investor who has been directly involved in the information gathering process and has placed a positive order at the issue price. This is the case even if the optimal mechanism calls for only the most enthusiastic pool members to receive positive allocations. We model this possible loss of flexibility by assuming that there is a lower limit, q_L, on each *individual* pool member's allocation, where $q_L \in [0, Q/2)$. The second constraint is potentially more restrictive than the first. For example, if $2q_L \geq q_P$, then the first constraint is irrelevant. In our analysis we allow for the possibility that q_L and/or q_P are equal to zero.[14]

The third and fourth allocation constraints place upper limits on the allocations to pool members. (iii) An issuer may be constrained to allocate a minimum number of shares to the retail market. In some countries a constraint requiring allocations to the retail market may be legally mandated. This is especially true for privatizations. Even if such allocations are not legally mandated, investment banks that specialize in distributing IPO shares to retail investors may negotiate up front to receive a certain minimum number of shares. We thus allow for a minimum total allocation, q_R, to the retail market, where $q_R \in [0, Q)$. By definition, q_R is equal to zero in the P strategy. iv) We also allow for the possibility that individual pool members have an upper limit, q_U, on the number of shares they can purchase, perhaps due to budget constraints. This upper limit may take on any value between $Q/2$ and Q.[15]

The problem is unchanged except for the participation and allocation constraints:

$$EV^+ + EV^- \geq \gamma \tag{PC}$$

where γ is an exogenously given constant. We assume that $0 \leq \gamma < \frac{wQ}{2}$; otherwise information gathering is never economical. The allocation constraints are now:

$$2q^{++},\ 2q^{--},\ q^{+-} + q^{-+} \geq q_P \tag{AC1}$$

$$q^{++},\ q^{--},\ q^{+-},\ q^{-+} \geq q_L \tag{AC2}$$

$$2q^{++},\ 2q^{--},\ q^{+-} + q^{-+} \leq Q - q_R \tag{AC3}$$

$$q^{++},\ q^{--},\ q^{+-},\ q^{-+} \leq q_U \tag{AC4}$$

where q_L, q_U, q_P and q_R are exogenously given constants. Constraints (AC1) and (AC2) place lower limits on the allocations to pool members, (AC1) for the

[14] The model could also be extended to allow for the possibility that investors with access to different types of information have different values of q_L, some strictly positive and some equal to zero. The resulting mechanism and expected underpricing would be a bit more complicated than what we present here, but it would be qualitatively very similar.

[15] We restrict the upper limit to be at least $Q/2$ so that the pool as a whole can purchase the entire issue. Placing a stricter upper limit on share purchases, given a fixed pool size, is similar to setting $q_R > 0$.

pool as a group and (AC2) for individual pool members. Constraints (AC3) and (AC4) place upper limits on the allocations to pool members, (AC3) for the pool as a group and (AC4) for individual pool members. Due to the interaction of constraints, the parameter limits given above are somewhat refined: $q_L \in \left[0, \frac{Q-q_R}{2}\right)$, $q_U \in \left[Q/2, Q\right]$, $q_P \in \left[0, Q - q_R\right)$ and $q_R \in \left[0, Q - \max[q_P, 2q_L]\right)$. Apart from the limits given here, which are determined only by feasibility considerations, we make no claims regarding the sizes of these parameters. In each case we allow for the possibility that the parameter takes on its least restrictive feasible value (zero for q_L, q_P and q_R, and Q for q_U). Our objective is to show how each of these constraints affects the design of the optimal mechanism for pricing and allocating IPO shares, and thus the expected underpricing.

In modeling the P strategy we model a typical mechanism in which allocations are made only to the participants in the mechanism. In the PR strategy we take into account the role that the retail market can play in the mechanism for a public offering. The following proposition presents the expected underpricing for each strategy, as a function of the constraint parameters. An intuitive explanation follows the proposition.

PROPOSITION 2. *Expected Underpricing, with constraints.* When the issuer forms a pool of two informed investors who have all the available information:

(i) The expected underpricing in an offering that uses the PR strategy is

$$Eu_{PR} = \max\left[\frac{w}{2}\left(\frac{\max\left[q_L, q_P/2\right] + \max\left[q_L, q_P - q_U\right]}{Q - q_R}\right), \frac{\gamma}{Q - q_R}\right]. \quad (6)$$

(ii) The expected underpricing in an offering that uses the P strategy is

$$Eu_P = \max\left[\frac{w}{2}\left(\frac{1}{2} + \max\left[\frac{q_L}{Q}, 1 - \frac{q_U}{Q}\right]\right), \frac{\gamma}{Q}\right]. \quad (7)$$

Positive expected underpricing is caused either by the cost of inducing pool members to truthfully report (when the left-hand term in the maximand is largest), or by the cost of ensuring the participation of pool members (when the right-hand term in the maximand is largest). In both cases, the per-share expected underpricing is decreasing in the size of the issue, as represented by Q. This is because at least some part of the underpricing that results from information gathering takes the form of a fixed cost that is distributed across the shares that may be allocated to pool members (Q or $Q - q_R$).

In order to compare Equations (6) and (7) we first assume that γ is small enough so that the left-hand term is the largest in both maximands. As shown by Proposition 1, inducing truthtelling has a strictly positive cost only if the issuer is faced with binding allocation constraints. Of the four allocation constraints considered, only the minimum allocation constraints ($q_L > 0$ and/or $q_P > 0$) cause positive expected underpricing. These constraints cause the incentive compatibility constraint for investors who have observed positive information to be strictly binding, requiring the issuer to pay informational rents in order to elicit truthful reports. Incentive compatibility is strictly binding in the P strategy because this strategy implicitly incorporates a very restrictive allocation constraint: q_P in the P strategy is equal to Q. In contrast, placing an *upper* limit on the allocations to pool members in the PR strategy, either by requiring a minimum allocation for retail investors ($q_R > 0$), or by placing an upper limit on individual allocations ($q_U < Q$), does not by itself cause positive expected underpricing. Such constraints can only cause an increase in expected underpricing that is due to some other constraint.

Proposition 1 showed that, in the unconstrained problem, the PR strategy clearly dominates the P and the R strategies. We now revisit this question for the constrained case. The following proposition presents conditions such that the P strategy is dominated, either by the PR strategy or the R strategy. In order to simplify the analysis we eliminate the constraint (AC4) from this point forward. This is equivalent to assuming that $q_U = Q$.

PROPOSITION 3. *Choice of Issue Method.*

1. The PR strategy dominates the P strategy ($Eu_{PR} < Eu_P$) if:

   (a) $q_R + \max[2q_L, q_P] + 2q_R q_L/Q \leq Q$ and

   (b) $\gamma \leq \frac{w}{2}(Q - q_R)\left(\frac{1}{2} + \frac{q_L}{Q}\right)$

2. The R strategy dominates the P strategy ($Eu_R < Eu_P$) if: $z_I < 2/3$.

Proposition 3 presents sufficient conditions such that an offering strategy that excludes the retail market (strategy P) is dominated by a strategy that includes retail investors. By including retail investors, the PR strategy generally allows for more discretion in allocations than the P strategy, and so should require a lower expected payment to pool members in order to induce truthtelling. Condition 1a) of Proposition 3 ensures that the first term of the maximand in Equation (7) is at least as large as the first term in Equation (6). Condition 1b) of Proposition 3 states that the second term of the maximand in Equation (6) is not larger than the first term in Equation (7). That is, the participation cost, γ, is not the dominant factor in determining expected underpricing. Thus, part 1 of Proposition 3 can be qualitatively summarized as: the PR strategy will dominate the P strategy, unless either q_R and q_L are so large that the issuer does not retain much allocational flexibility in the PR strategy, or the cost of participation is so high that condition 1b) is violated. The condition in part 2. of Proposition 3 is that fewer than two-thirds of all potential investors are privately informed. This condition almost certainly holds for a public offering. Thus, part 2. of Proposition 3 points out that, if the parameter values are such that the P strategy dominates the PR strategy, then the R strategy (no direct information gathering) will, for reasonable parameter values, dominate both. That is, explicitly including the retail market in the design of the pricing mechanism is beneficial for the issuer. In what follows we will focus only on the issuing strategies that include the retail market: PR and R.

4. Optimal Selling Mechanism and Underpricing: The General Case

In this section we continue to assume that the issuer can identify informed investors. However, we now assume that the issuer cannot determine which factor any given investor has observed. Thus, with a limited pool size there is a positive probability that all members of the pool have observed the same factor. As a result, the issuer cannot be certain that all of the relevant information is held by pool members. This changes the issuer's problem in a number of ways. First, in a public offering uninformed retail investors face a "residual" adverse selection risk, because of the possibility that only some of the available information has been learned before setting the issue price. The issuer must underprice the issue sufficiently in states in which he sells to the retail market in order to compensate uninformed investors for this risk. Second, pool members' information is now positively correlated: any two informed investors chosen at random have a positive probability of having observed the same information. This correlation helps the issuer to detect false reports, and thus affects the design of the optimal mechanism. Third, the issuer may wish to increase the pool size so as to increase the probability that the pool contains all available information. The probability tree for a pool of size M is illustrated in Figure 2.

We can define the following variables, where their values are derived from Figure 2:

$$v_p^+ \equiv E[\hat{v}|\text{all } M \text{ pool members report +}] = v_0 + \frac{2^{M-1}w}{1+2^{M-1}} < v_0 + w \qquad (8)$$

$$v_p^- \equiv E[\hat{v}|\text{all } M \text{ pool members report } -] = v_0 - \frac{2^{M-1}w}{1+2^{M-1}} > v_0 - w \qquad (9)$$

If all pool members report identical information (positive or negative), then there will be residual adverse selection risk. This is because some investors in the retail market may have information that is not known by the pool. If instead, some pool members report positive information and some report negative, and there is truthful reporting, then all of the information has been captured in the pool. In this case there is no residual adverse selection risk. The probability of zero residual adverse selection risk is $\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{M-1}\right)$. This probability is increasing in the number of pool members, M.

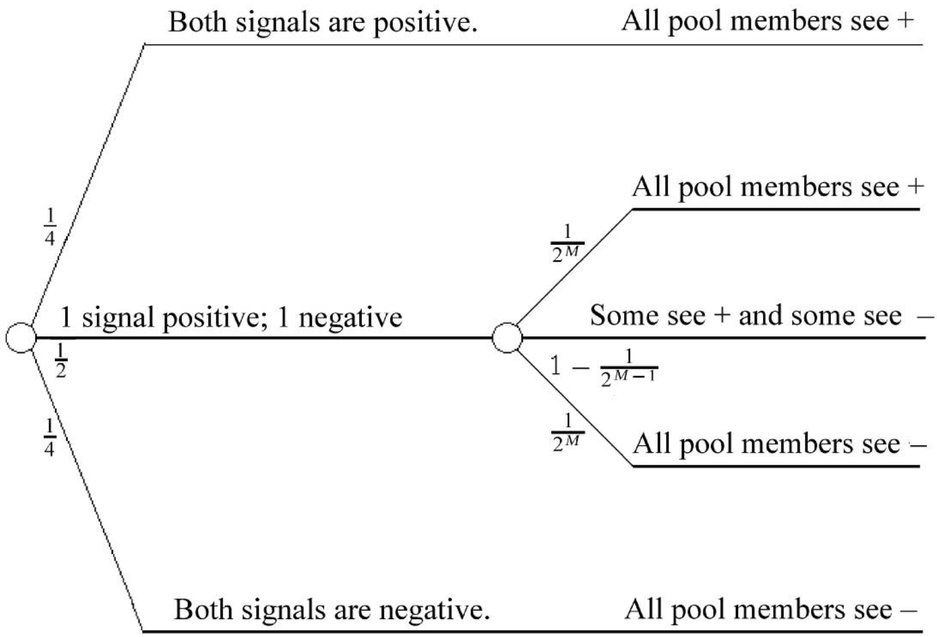

Figure 2. Probability tree when pool may not have all information. M = number of pool members.

If each pool member reports positive information, then the underpricing due to residual adverse selection risk is:

$$u_{AS}^{+} = \frac{2^{M-2} w z_I}{(1+2^{M-1})(1+2^{M-1}-(\frac{1}{2}+2^{M-1})z_I)} .$$

If each pool member reports negative information, then the underpricing due to residual adverse selection risk is:

$$u_{AS}^{-} = \frac{2^{M-2} w z_I}{(1+2^{M-1})(1+2^{M-1}-z_I/2)} .$$

u_{AS}^{+} and u_{AS}^{-} are both decreasing in M.

The absolute value of the per-share impact of a lie on $v_p = E[\hat{v}|\text{all pool members' reports}]$ is:

$$w_M = \frac{2^{M-1} w}{1+2^{M-1}} \text{Prob}\{M-1 \text{ others agree with each other}\} = \frac{(1+2^{M-2})w}{1+2^{M-1}} .$$

For $M \geq 2$, w_M is in the interval $(\frac{w}{2}, \frac{2w}{3}]$. w_M is decreasing in M. We would thus expect that with a larger pool size the issuer could design an optimal selling mechanism that results in less underpricing. However, as the pool size increases the issuer may lose allocational flexibility. This latter effect works to increase expected underpricing. In addition, the second derivative of w_M with respect to M is positive, meaning that the incremental informational benefit obtained from adding a pool member is a decreasing function of pool size. In other words, there are decreasing informational returns to pool size.

The following proposition gives the expected underpricing when a pool of size M is formed to obtain information. As discussed at the end of the last section, we calculate this only for the PR strategy. The form that the underpricing takes depends on the nature of the constraints that the issuer faces in designing the optimal mechanism. We present the expected underpricing for the three possible cases that can occur, given the constraints that are considered. The revised mechanism design problem is specified in the proof of Proposition 4.

PROPOSITION 4. If the issuer forms a pool of size $M \geq 2$ that may not contain all information, then expected underpricing in the PR strategy depends on the nature of the constraints:

1. The "mostly" unconstrained case: If q_L, q_P and γ are close enough to zero so that neither the incentive compatibility constraint for investors who have observed positive information, (IC^+), nor the participation constraint, (PC), is strictly binding, then

$$Eu_{PR} = \frac{w(z^- + J \times z^+)}{4} \tag{10}$$

where

$$z^- = \frac{z_I}{2 + 2^M - z_I}, \quad z^+ = \frac{z_I}{2 + 2^M - (1 + 2^M)z_I},$$

$$J = \begin{cases} 1 & \text{if } q_R > 0 \\ 0 & \text{if } q_R = 0 \end{cases}$$

2. If $\max[q_L, q_P]$ is large enough so that (IC^+) is strictly binding (and $\max[Mq_L, q_P]/Q \leq 1 - z_I$), then[16]

$$\begin{aligned} Eu_{PR} = {} & \frac{w(1 + 2^{M-2})(2^M - 1)Mq_L}{2^{M+1}(1 + 2^{M-1})(Q - q_R)} \\ & + \frac{wz^-}{4} + \frac{w(1 + 2^{M-2} + 2^{M-1}z^-)\max[Mq_L, q_P]}{2^{M+1}(1 + 2^{M-1})(Q - q_R)} \end{aligned} \tag{11}$$

[16] If $\max[Mq_L, q_P]/Q \leq 1 - z_I$, then it is optimal to sell shares to the retail market when all pool members report negative information. If $\max[Mq_L, q_P]/Q > 1 - z_I$ and $q_R = 0$, then the allocation constraints are so stringent that it may be optimal not to sell to the retail market in this state. In this case the expression for Eu_{PR} is a bit different from that in (11). The exact expression is given in the proof of Proposition 4.

Table I. Constraints on the mechanism design

Constraint on design of mechanism:	Effect on underpricing:	Effect on information gathering: $M^* \equiv$ optimal pool size
Upper limit on allocation to pool as a group: 1. $\sum_{i=1}^{M} q_i \leq Q - q_R$	May increase underpricing; does not by itself cause underpricing	Higher q_R can lead to higher or lower M^*
Lower limit on allocations to pool members:		
2. Individual: $q_i \geq q_L$	Causes underpricing	Higher $q_L \Longrightarrow$ lower M^*
3. Group: $\sum_{i=1}^{M} q_i \geq q_P$	Causes underpricing, but only if pool size is limited	Higher $q_P \Longrightarrow$higher M^*
Participation constraints:		
4. Individual pool mbr: $(EV^+ + EV^-)/2 \geq \gamma/2$	Causes underpricing	Higher $\gamma \Longrightarrow$ lower M^*
5. Retail investor: $EV^R \geq 0$	Causes underpricing, if there is residual adverse selection risk	Higher M^* is wanted to decrease residual adverse selection risk

3. If γ is large enough so that (PC) is strictly binding, then

$$Eu_{PR} = \frac{M\gamma}{2Q} + \max\left[\frac{M\gamma}{2(Q - q_R)}, \frac{wz^-}{4}\right]\frac{q_R}{Q} \tag{12}$$

The results in Proposition 4 are directly comparable to those of the previous section. There we found that, in the case such that allocations to pool members are constrained only from above, there is zero underpricing in the PR strategy. In part 1. of Proposition 4 expected underpricing is positive in this case, but only because of residual adverse selection risk. Underpricing caused by residual adverse selection risk is found in the terms that contain either z^- or z^+, both of which are decreasing in pool size, M. The effects of different constraints on the mechanism design problem are summarized in Table I.[17] The first row of the table corresponds to part 1. of Proposition 4.

If allocations to pool members are constrained from below, as in in part 2. of Proposition 4, then underpricing is needed in order to induce truthtelling by investors who have observed positive information; i.e., (IC$^+$) is strictly binding. Looking at Equation (11) we see that, if there is an individual minimum allocation requirement ($q_L > 0$), then increasing the pool size will, for large enough M, cause

[17] We include in this table only constraints that are included in the mechanism design problem of Proposition 4. We thus do not include the constraint that $q_i \leq q_U$. This constraint does not by itself cause underpricing, but may increase underpricing from other sources. Lower q_U leads to higher M^*.

an increase in expected underpricing. This is not the case if $q_P > 0$. The middle row of Table 1 corresponds to part 2. of Proposition 4.

Part 3. of Proposition 4 presents the case such that each informed investor's required expected return for participating in the mechanism is so high that the incentive compatibility constraints are nonbinding.[18] Equation (12) indicates that expected underpricing due to this constraint is increasing in pool size. The last row of Table I presents the effects of participation constraints on the mechanism design.

By increasing pool size, and thus collecting more information, the issuer lowers the residual adverse selection risk. However, an increase in pool size may also increase the cost of information gathering. The following proposition summarizes the relation between optimal pool size and constraints on the mechanism design.

PROPOSITION 5. *Pool Size.*

(a) Optimal pool size (M^*) is decreasing in the minimum individual allocation, q_L, and the individual cost of participation, γ, and increasing in issue size, Q. If $q_L = \gamma = 0$, then the optimal pool size is infinite.
(b) If γ is zero and q_L is strictly positive, then the optimal pool size is independent of w, prior uncertainty about issue value.
(c) If γ is strictly positive and q_L is zero, then the optimal pool size is increasing in w.

The results of Proposition 5 follow directly from the equations in Proposition 4. If the optimal pool size is unbounded, then the expected underpricing is zero. We do not suggest this solution as an economically realistic solution, but rather as a second benchmark case that helps us to understand the source(s) of underpricing.[19]

As the amount of information gathering (pool size) increases, the underpricing that is due to adverse selection risk decreases, while the underpricing that is due to information gathering costs increases. The optimal amount of information gathering is the amount that equates expected underpricing due to adverse selection risk with expected underpricing caused by information gathering.[20] Wealth loss from underpricing caused by adverse selection risk is a variable cost, in that it increases proportionally with issue size. In contrast, for any given amount of information gathering, the wealth loss due to information gathering is a fixed cost; it is independent of issue size. Thus, the optimal amount of information gathering is increasing in issue size.

[18] Incentive compatibility and the participation constraint are never simultaneously strictly binding.

[19] It is also true that if the investment bank has its own administrative costs that are increasing in pool size, then the bank would wish to limit the pool size. Such a cost is effectively equivalent to $\gamma > 0$.

[20] Assuming that this amount is ≥ 0 and $< \infty$.

The pool members are able to capture informational rents, in the form of positive allocations of underpriced shares, if either γ or q_L is strictly positive. Other constraints on the mechanism design may increase the level of rents that informed investors can expect, but they do not by themselves lead to the payment of rents.[21] The two means of obtaining rents (positive allocation requirement or positive participation cost) differ qualitatively from each other in that an allocation restriction directly affects the cost of inducing truthtelling, while the participation constraint does not. As indicated by Equation (11), the cost of inducing truthtelling is directly increasing in prior uncertainty, w. That is, the more valuable the investors' information, the more the issuer must pay for the information. As indicated by Equation (10), the benefit of gathering more information is also increasing directly in w. Thus, if the cost of gathering information is determined by a minimum allocation requirement, then the optimal amount of information gathering is independent of the prior level of uncertainty. A direct result of this is that the level of residual uncertainty (remaining uncertainty at the time of pricing) will be proportionally related to the level of prior uncertainty. In contrast, if there are no allocation restrictions, but investors require a fixed positive expected return for participating ($\gamma > 0$), then information gathering costs are not directly linked to the value of the information. This is seen by the absence of w in equation (12). Thus, more information gathering will be done as the level of prior uncertainty increases, and as a result, the residual uncertainty will be less than proportionally related to the level of prior uncertainty.

In Figure 3 we present a number of numerical examples that illustrate the results of Proposition 5 and also shed light on the choice of issue method. Each graph in Figure 3 shows expected underpricing versus pool size for the pool and retail (PR) and retail only (R) strategies.[22] In Figures 3a, 3c and 3e, $q_L > 0$ and $\gamma = 0$. In Figures 3b, 3d and 3f, $q_L = 0$ and $\gamma > 0$. In 3a and 3b, the level of prior uncertainty about issue value (w) and the fraction of informed investors (z_I) are low; in 3c and 3d w is low and z_I is high; in 3e and 3f both w and z_I are high. In all of the graphs Q is normalized to one and $q_R = q_P = 0$.

As is illustrated in Figure 3, Eu_{PR} is a convex function of pool size, M. The optimal pool size is the value of M at which the expected underpricing is lowest. For a wide range of parameter values the optimal pool size (number of investors from whom to gather information) is greater than two and less than $\frac{Q}{q_L}$ (the maximum allowed by the allocation constraints). By comparing Figures 3c and 3e we see that if $q_L > 0$ and $\gamma = 0$, then expected underpricing increases proportionally with the increase in uncertainty, w. (The scales on the vertical axes differ across graphs.) Also, the optimal pool size is unaffected by a change in w. In contrast, Figures 3d and 3f show that if $q_L = 0$ and $\gamma > 0$, then while an increase in w does

[21] $q_P > 0$ enables pool members to obtain informational rents if the pool size is limited, but not otherwise.

[22] Although not graphed here, expected underpricing for the P strategy is greater than Eu_{PR} for all parameters considered.

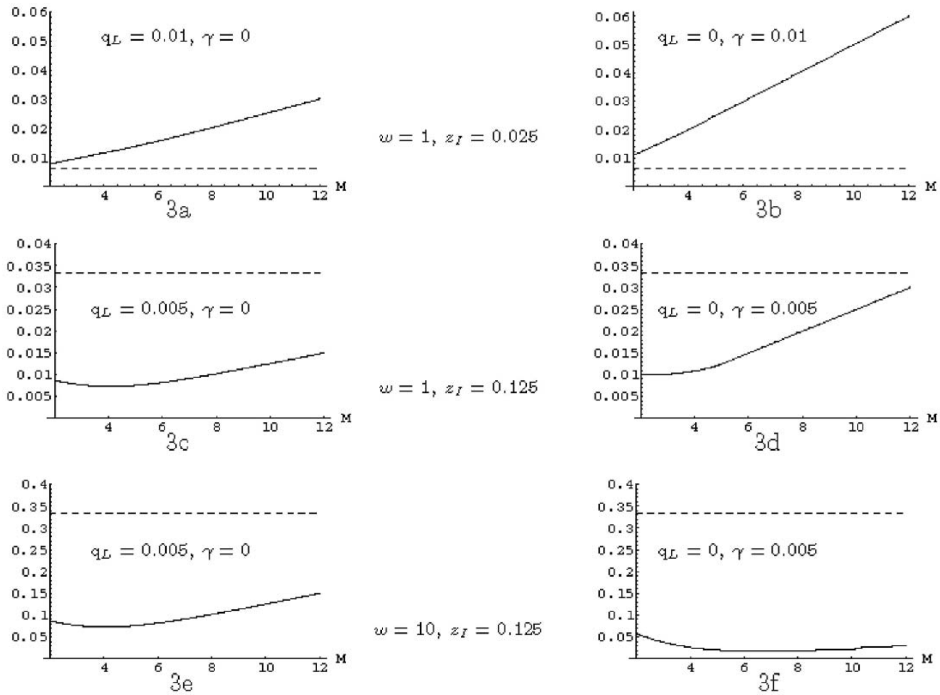

Figure 3. Expected underpricing versus pool size. The solid curves represent Eu_{PR} and the horizontal dashed lines represent Eu_R. In Figures 3a, 3c, and 3e pool members require a strictly positive minimum allocation. In Figures 3b, 3d and 3f pool members require a strictly positive return for participating. In Figures 3e and 3f the prior uncertainty about issue value is ten times higher than in Figures 3a through 3d.

cause expected underpricing to increase, the increase is less than proportional. In addition, the optimal pool size is larger with a larger value for w.

The results of this section provide some additional empirical predictions. Total wealth loss at the optimal level of information gathering will be a combination of a fixed and a variable cost. This is because some, but not all, adverse selection risk will be eliminated through information gathering. Thus, total wealth loss will be increasing in the number of shares sold, but less than proportionally. Translating this to underpricing, that is typically defined relative to the issue size, our model predicts that percent underpricing will be decreasing in issue size.

5. Wealth Loss and Staging of IPOs

Up to this point we have taken issue size as given. In this section we will continue to assume that the issuer wishes to ultimately issue Q shares, but we allow the issuer to "stage" the IPO. If the issuer stages an IPO, then some of the Q shares are sold at the IPO and the remainder are sold in a follow-up offering. The advantage of staging the offering is that a secondary market for the shares is created between the two offerings, thus eliminating the need to underprice in the second offering.[23] If wealth loss due to underpricing is a variable cost, in that it increases with issue size, then staging the IPO will result in lower wealth loss due to underpricing. In fact, it will be in the issuer's best interest to sell as few shares as possible at the IPO. This strategy is limited, however, by the fact that a well-functioning secondary market requires that a sufficient number of shares be sold at the IPO.

We model the staging decision by assuming that the issuer may either sell all Q shares at the IPO, or Q_1 at the IPO and $Q - Q_1$ shares in a second offering. Q_1 is the minimum issue size necessary for a well functioning secondary market. If $Q \leq Q_1$, then the issue cannot be staged. The benefit to staging is:

$$\begin{aligned} B_S &\equiv Eu(Q) \times Q - Eu(Q_1) \times Q_1 \\ &= \min\left[Eu^*_{PR}(Q), Eu_R\right] \times Q - \min\left[Eu^*_{PR}(Q_1), Eu_R\right] \times Q_1 \end{aligned} \tag{13}$$

where $Eu^*_{PR}(\cdot)$ is the expected underpricing that results from the PR strategy when the pool size is set to its optimal value. It is assumed that, for any given issue size, the issuer chooses the strategy, PR or R, that results in lower expected underpricing. The first term in B_S is thus the expected wealth loss due to underpricing in an IPO of size Q, and the second term is the expected wealth loss in an IPO of size Q_1. The benefit, B_S, will be offset by costs of going to the market a second time to sell shares. For example, there will be accountants' fees, investment bankers' fees and/or the announcement effect as modeled by Myers and Majluf (1984). Rather than model these costs, which would be outside the scope of this study, we focus only on B_S, with the idea that the larger is B_S, the more likely it is that the benefit to staging an IPO will outweigh the costs.

From Equation (3) we know that Eu_R, the expected per-share underpricing with no information gathering, is independent of issue size. From Propositions 4 and 5 we know that $Eu_{PR}(Q)$ is decreasing in Q, but $Eu_{PR}(Q) \times Q$ is increasing in Q. We also know from the analysis of the last section that, if $q_L = \gamma = 0$, then the optimal pool size is unlimited, and the issuer can drive underpricing to zero. Thus, staging is potentially beneficial only if $q_L > 0$ and/or $\gamma > 0$. It also follows from Proposition 5 that: If $q_L > 0$ and $\gamma = 0$, then wealth loss increases proportionally with w. But, if $\gamma > 0$ and $q_L = 0$, then wealth loss increases less than proportionally with w. We thus have the following Proposition.

PROPOSITION 6. *Benefit of Staging.*

(a) $\partial B_S/\partial Q > 0$. $\partial^2 B_S/\partial Q^2 < 0$.
(b) If $q_L > 0$ and $\gamma = 0$, then B_S increases proportionally with prior uncertainty, w.
(c) If $\gamma > 0$ and $q_L = 0$, then B_S increases less than proportionally with w.

[23] We abstract away from the Myers and Majluf (1984) announcement problem where share price typically drops at the announcement of an equity offering.

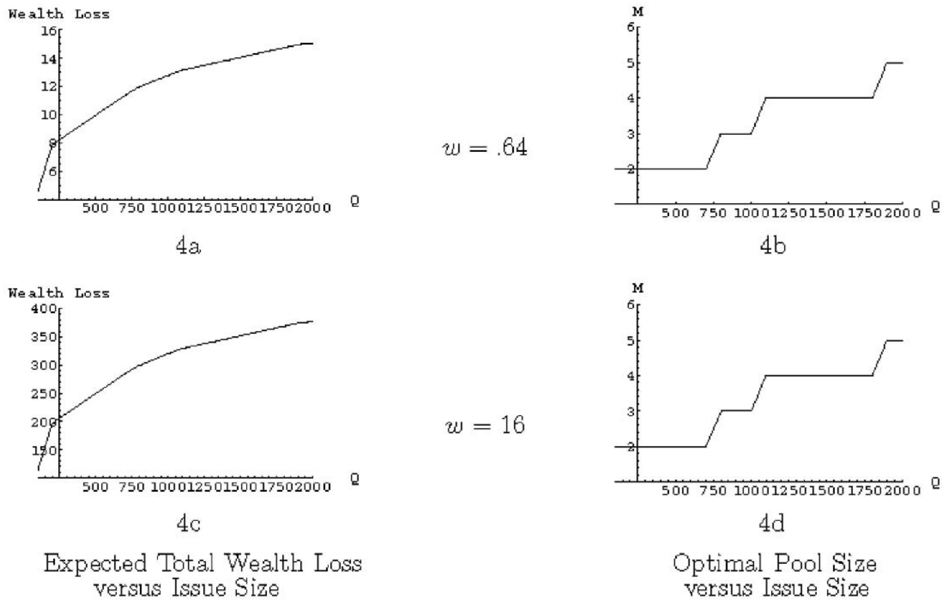

Figure 4. Wealth loss and information gathering versus issue size. Informational rents result from *allocation constraints*: $q_L > 0 \quad \gamma = 0$. The left-hand graphs show total wealth loss (per-share underpricing times issue size) vs. issue size; the right-hand graphs show optimal pool size vs. issue size. Uncertainty about issue value is 25 times higher in the bottom graphs ($w = 16$) than in the top graphs ($w = 0.64$). The vertical scale in graph 4c is also 25 times larger than in 4a; the vertical scales are the same in 4b and 4d.

The results of Proposition 6 are illustrated in Figures 4a, 4c, 5a and 5c, that show expected total wealth loss versus issue size. Consistent with part (a) of Proposition 6, expected wealth loss is an increasing, concave function of Q. Figures 4b, 4d, 5b and 5d illustrate a result of Proposition 5: the optimal amount of information gathering (pool size) is increasing in Q. It is this fact that causes the relation between expected wealth loss and issue size to be concave, so that the expected benefit to staging, B_S, increases in Q, but less than proportionally.

In Figure 4 the cost of gathering information is driven by minimum allocation restrictions: $q_L > 0$ and $\gamma = 0$. The only difference between the top two graphs and the bottom graphs is that the level of prior uncertainty about firm value, w, is 25 times higher in the bottom graphs.[24] Figures 4b and 4d illustrate the result of Proposition 5 part (b): If the cost of gathering information is driven by allocation restrictions, then the issuer's optimal amount of information gathering is independent of prior uncertainty. A direct result of this, as shown in Figures 4a and 4c, is that expected total wealth loss, and thus the benefit to staging, increases proportionally with uncertainty. (In Figures 4a and 4c the scales change proportionally with w.)

[24] For all examples in this section $z_I = 0.25$ and $q_R = q_P = Q - q_U = 0$.

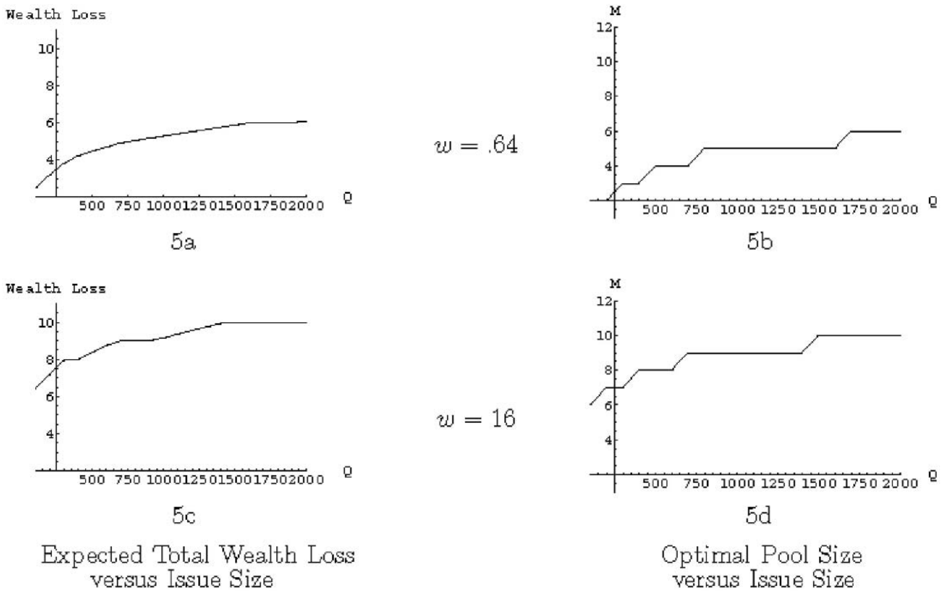

Figure 5. Wealth loss and information gathering versus issue size. Informational rents result from *participation constraint*: $\gamma > 0$, $q_L = 0$. The left-hand graphs show total wealth loss (per-share underpricing times issue size) vs. issue size; the right-hand graphs show optimal pool size vs. issue size. Uncertainty about issue value is 25 times higher in the bottom graphs ($w = 16$) than in the top graphs ($w = 0.64$). The vertical scales are the same in 5a and 5c, and in 5b and 5d.

In Figure 5 the cost of gathering information is driven by a fixed participation cost: $\gamma > 0$ and $q_L = 0$. Again, the only difference between the top two graphs and the bottom graphs is that w is 25 times higher in the bottom graphs. Figures 5b and 5d illustrate the result of Proposition 5 part (c): If the cost of gathering information is driven by a fixed participation cost, then the issuer optimally gathers more information when there is greater prior uncertainty. As a result, as shown in Figures 5a and 5c, while expected total wealth loss, and thus the benefit to staging, increases with uncertainty, it increases less than proportionally (by a factor of less than 25).

By comparing Figures 5a and 5c with Figures 4a and 4c, we observe an additional pattern. In Figures 5a and 5c, for large values of Q wealth loss is relatively constant. In addition, for any nontrivial Q_1 (minimum IPO size) and any fixed Q (total issue size), the value to staging is relatively independent of the level of prior uncertainty. The reason for this is that, if the cost of information gathering is driven by fixed participation costs, then the information gathering cost is largely a fixed cost. In Figures 5a and 5c we see that the fixed cost component increases with w, whereas the variable cost component appears to be largely independent of w. In contrast, in Figures 4a and 4c, the variable cost component increases proportionally with w. An implication of this pattern is that, if the cost of information gathering is driven by allocation restrictions, then there is generally a greater benefit to staging an IPO than if the cost of information gathering is driven by a fixed participation cost.

The analysis of this section highlights the fact that the means by which informed investors obtain informational rents affects the issuer's strategy for selling unseasoned equity shares. We have considered two constraints that enable an informed investor to obtain informational rents: a minimum allocation ($q_L > 0$) and a fixed expected return for particiption ($\gamma/2 > 0$). These two constraints are similar in that they both ensure that individual participants in the mechanism receive rents for their information. The two constraints differ in that they lead to different relations between information gathering and uncertainty, and between the benefit of staging and uncertainty.

6. Conclusion

We show that the IPO paradigm whereby an underwriter elicits information directly from a limited number of informed investors to whom he gives special access (bookbuilding) is optimal for a wide variety of security offerings. However, the amount of information that is optimally gathered prior to setting the issue price will vary across firms. We first solve for the optimal mechanism for extracting information from investors, and then apply this result to determine the optimal amount of information gathering as a function of characteristics of the firm and the bookbuilding process. We also determine conditions under which it is optimal to stage an IPO.

The issuer gains by including in the mechanism both investors who provide information, and retail investors who do not. At the extreme we show that if there are no restrictions on the way in which shares are allocated between these types of investors then it is not necessary to underprice in order to induce truth-telling on the part of informed investors. However, if investors who provide information expect minimum allocations, either individually or as a group, then underpricing results. We show that the optimal amount of information gathering is always increasing in issue size, but may be independent of the level of uncertainty about the issue value. In particular, if the issuer is constrained to always allocate shares to investors who provide information, then the optimal level of information gathering is independent of uncertainty.

Our analysis has implications both for issuers and for providers of information. It suggests, for example, that in order to acquire informational rents informed analysts should cooperate with or work for large institutional investors who can absorb large minimum allocations. One way such institutions may use their market power to extract informational rents is to negotiate large guaranteed minimum allocations from underwriters.

Appendix

NOTATION

Strategies: R $\equiv$ retail-only, no pregathering of information
P $\equiv$ pool–only, pregathering of information
PR $\equiv$ pool and retail, pregathering of information
Q = number of shares to be sold
$\tilde{v}$ = secondary market per-share value = $v_0 + \tilde{v}_1 + \tilde{v}_2$
$\tilde{v}_1$ and $\tilde{v}_2$ are independent valuation factors
z_I = fraction of retail investors who are informed
v_p = expected value of $\tilde{v}$, given all the information reported by pool members
w = possible revision in share value, due to private information, $\tilde{v}_1 + \tilde{v}_2 \in \{-w, 0, w\}$
u = per-share underpricing
Eu_X = expected per-share underpricing when following strategy X, $X \in \{R, P, PR\}$
M = number of pool members
$\frac{\gamma}{2}$ = cost of participation (per pool member), $0 \leq \gamma < \frac{w}{2}(Q - q_R)$
q_L = lower bound on allocation to each individual pool member $\in [0, (Q - q_R)/M)$
q_R = lower bound on total allocation to retail investors ≥ 0
q_P = lower bound on total allocation to the pool $\in [0, Q - q_R)$
q_U = upper bound on allocation to each individual pool member $\leq Q$
EV^k = expected return to a polled investor who sees and reports k, $k \in \{+, -\}$
EV^R = expected return to retail investors
u^k_{AS} = underpricing due to adverse selection risk, given all pool mbrs report k, $k \in \{+, -\}$
w_M = per-share impact of a lie by a single pool member, given M pool members

DERIVATION OF Eu_R. To simplify we assume that if the agents who have observed a given signal participate, then they receive fraction $z_I/2$, regardles of whether the other informed agents participate.

Secondary market value	$v_0 - w$	v_0	$v_0 + w$
Probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
Informed participation	0	$\frac{z_I}{2}$	z_I
Fraction for uninformed	1	$1 - \frac{z_I}{2}$	$1 - z_I$

Let s = IPO price. The expected return to uninformed investors must be nonnegative. When underpricing is minimized, this expected return will be zero:

$$EV_U = 0 = \frac{1}{4}(v_0 - w - s) + \frac{1}{2}(v_0 - s)\left(1 - \frac{z_I}{2}\right) + \frac{1}{4}(v_0 + w - s)(1 - z_I)$$

Solving the above: $Eu_R \equiv v_0 - s = \frac{wz_I}{4-2z_I}$.

PROOF OF PROPOSITION 1. (i) PR strategy. Set $2q^{++} = q^{+-} = Q$, $q^{-+} = q^{--} = 0$, and $u^{++} = u^{+-} = u^{--} = 0$. The constraints are satisfied and $Eu_{PR} = 0$.

(ii) P strategy. It is required that $2q^{--} = Q$. In order to satisfy (IC^+), it is necessary that $EV^+ \geq wQ/4$. Setting q^{++} and q^{+-} to their maximum values, this results in $Eu_P = w/4$. ■

PROOF OF PROPOSITION 2. (i) PR strategy:

$$\min Eu = \frac{1}{Q}\left[EV^R + EV^+ + EV^-\right]$$

subject to:

$$EV^+ - EV^- \geq \gamma - 2EV^- \tag{PC}$$

$$EV^+ - EV^- \geq \frac{w}{2}(q^{-+} + q^{--}) \tag{IC$^+$}$$

$$EV^+ - EV^- \leq \frac{w}{2}(q^{++} + q^{+-}) \tag{IC$^-$}$$

and, participation constraints for retail investors and the allocation constraints. We'll first assume that γ is small enough so that (PC) is nonbinding. If IC^+ is binding, then the objective function becomes:

$$\min \frac{w}{2}(q^{-+} + q^{--}) + u^{+-}q^{-+} + u^{--}q^{--} +$$

$$\frac{u^{++}}{4}(Q - 2q^{++}) + \frac{u^{+-}}{2}(Q - q^{+-} - q^{-+}) + \frac{u^{--}}{4}(Q - 2q^{--})$$

It is optimal to set q^{++} and q^{+-} as large as possible, and q^{-+} and q^{--} as small as possible. (Thus, if (IC^+) is binding, then (IC^-) will not be.) The optimal allocations are:

$$q^{++} = \min\left[q_U, \frac{Q - q_R}{2}\right] \qquad q^{--} = \max\left[q_L, \frac{q_P}{2}\right]$$

$$q^{+-} = \min\left[q_U, Q - q_R - q_L\right] \qquad q^{-+} = \max\left[q_L, q_P - q_U\right]$$

It is optimal to set $u^{--} = u^{+-} = 0$, so $EV^{-} = 0$. (IC^{+}) is binding. By Equation (4):

$$2EV^{+} = u^{++}q^{++} = w(q^{-+} + q^{--})$$

Thus, if (PC) is nonbinding:

$$Eu_{PR} = \frac{u^{++}}{4} = \frac{w\left(\max\left[q_L, q_P/2\right] + \max\left[q_L, q_P - q_U\right]\right)}{2(Q - q_R)} \tag{14}$$

(PC) is nonbinding iff $EV^{+} = w\left(\max\left[q_L, q_P/2\right] + \max\left[q_L, q_P - q_U\right]\right)/2 \geq \gamma$. Otherwise, (PC) is binding and $EV^{+} + EV^{-} = \gamma$; the incentive compatibility constraints are nonbinding, and:

$$Eu_{PR} = \frac{\gamma}{Q - q_R} \tag{15}$$

Expected underpricing is the maximum of (14) and (15). Thus, we obtain Equation (6).

(ii) Equation (7) obtains because in the P strategy $q_R = 0$ and $q_P = Q$. ■

PROOF OF PROPOSITION 3. (1a) $q_U = Q$ implies that $\max[q_L, q_P - q_U] = q_L$. Thus, the condition that the first term in the maximand of Equation (6) is no larger than the first term in the maximand of Equation (7) is:

$$Q - q_R \geq \max[2q_L, q_P] + \frac{2q_L q_R}{Q}.$$

(1b) From Equations (6) and (7): Eu_P is determined by the second term in the maximand, and if $q_R > 0$, then $Eu_P < Eu_{PR}$.

(2) Follows from a comparison of Equations (3) and (7). ■

DERIVATION OF u_{AS}^{+} AND u_{AS}^{-}, GENERAL VALUE OF M. Suppose all pool members report positive information: The IPO offer price is $v_p^{+} - u^{++}$. An informed retail investor who has seen a positive signal will believe a share is worth $v_p^{++} = v_0 + \frac{2^M w}{1+2^M} > v_p^{+}$, and will thus participate in the offering. An investor who has seen a negative signal will believe each share is worth v_0, and will not participate. (As shown below, underpricing due to residual adverse selection risk is strictly less than $v_p^{+} - v_0$.) The following table summarizes the case such that pool members all report positive information:

Secondary market value	v_0	$v_0 + w$
Probability	$\frac{1}{1+2^{M-1}}$	$\frac{2^{M-1}}{1+2^{M-1}}$
Informed participation	$\frac{z_I}{2}$	z_I
Fraction for uninformed	$1 - \frac{z_I}{2}$	$1 - z_I$

Using this table, underpricing due to residual adverse selection risk is calculated as: $u_{AS}^{+} = \frac{2^{M-2} w z_I}{(1+2^{M-1})(1+2^{M-1}-(\frac{1}{2}+2^{M-1}) z_I)}$.

Suppose that all pool members report negative information: The IPO offer price is $v_p^{-} - u^{--}$. An informed retail investor who has seen a positive signal will believe each share is worth v_0, and will participate. An investor who has seen a negative signal will believe each share is worth $v_p^{--} = v_0 - \frac{2^M w}{1+2^M}$, and will not participate. (Underpricing due to adverse selection risk will be strictly less than $v_p^{-} - v_p^{--}$.) The following table summarizes the case such that the pool members all report negative information:

Secondary market value	v_0	$v_0 - w$
Probability	$\frac{1}{1+2^{M-1}}$	$\frac{2^{M-1}}{1+2^{M-1}}$
Informed participation	$\frac{z_I}{2}$	0
Fraction for uninformed	$1 - \frac{z_I}{2}$	1

Underpricing due to residual adverse selection risk is:

$$u_{AS}^{-} = \frac{2^{M-2} w z_I}{(1+2^{M-1})(1+2^{M-1} - z_I/2)}.$$

PROOF OF PROPOSITION 4. Define the following variables:

$$\begin{aligned} u^{aa} &= \text{underpricing when all pool mbrs report } a \\ u_x^{+-} &= \text{underpricing when } 0 < x < M \text{ report } + \\ q^{aa} &= \text{qty to each pool mbr when all report } a \\ q_x^{ab} &= \text{qty to pool mbr who reports } a \text{ when } x \text{ report } +,\ 0 < x < M \end{aligned}$$

$$\text{Prob}\{\tilde{v}_1 \neq \tilde{v}_2\} \times \text{Prob}\{x \text{ see + and}(M-x) \text{ see} - \mid \tilde{v}_1 \neq \tilde{v}_2\} =$$

$$\text{Prob}\{\tilde{v}_1 \neq \tilde{v}_2\} \times \text{Prob}\{(M-x) \text{ see + and } x \text{ see} - \mid \tilde{v}_1 \neq \tilde{v}_2\} =$$

$$\frac{1}{2^{M+1}} \frac{M!}{x!\,(M-x)!}$$

The objective function is:

$$\begin{aligned} \min Eu &= \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{++} + \left(\frac{1}{2} - \frac{1}{2^M}\right) u^{+-} + \left(\frac{1}{4} + \frac{1}{2^{M+1}}\right) u^{--} \\ &= \frac{1}{Q}\left[EV^R + \frac{M}{2} EV^+ + \frac{M}{2} EV^-\right] \end{aligned}$$

where

$$u^{+-} = \frac{1}{2(2^{M-1}-1)} \sum_{x=1}^{M-1} \frac{M!\, u_x^{+-}}{x!\,(M-x)!}$$

$$EV^R = \left(\frac{1+2^{M-1}}{2^{M+1}}\right) u^{++}(Q - Mq^{++}) + \left(\frac{1+2^{M-1}}{2^{M+1}}\right) u^{--}(Q - Mq^{--})$$
$$+ \frac{1}{2^{M+1}} \sum_{x=1}^{M-1} \frac{M!\, u_x^{+-}(Q - xq_x^{+-} - (M-x)q_x^{-+})}{x!\,(M-x)!}$$

$$EV^+ = \left(\frac{1+2^{M-1}}{2^M}\right) u^{++}q^{++} + \frac{1}{2^M} \sum_{x=1}^{M-1} \frac{(M-1)!\, u_x^{+-} q_x^{+-}}{(x-1)!\,(M-x)!}$$

$$EV^- = \frac{1}{2^M} \sum_{x=1}^{M-1} \frac{(M-1)!\, u_x^{+-} q_x^{-+}}{(x)!\,(M-x-1)!} + \left(\frac{1+2^{M-1}}{2^M}\right) u^{--}q^{--}$$

The participation constraints are:

$$EV^+ + EV^- \geq \gamma \tag{PC}$$

$$u^{aa}(Q - Mq^{aa}) \geq u_{AS}^{a}(Q - Mq^{aa}), \;\; a \in \{+,-\} \quad \text{and} \quad u_x^{+-} \geq 0 \tag{PCR}$$

Incentive compatibility:

$$EV^+ \geq EV^- + \frac{u_{M-1}^{+-} q_{M-1}^{-+} - u^{--}q^{--}}{2}$$
$$+ \frac{w_M}{2^M}\left(2^{M-1} q_{M-1}^{-+} + \sum_{x=1}^{M-1} \frac{(M-1)!\, q_x^{-+}}{(x)!\,(M-x-1)!} + q^{--}\right) \tag{IC$^+$}$$

$$EV^- \geq EV^+ + \frac{u_1^{+-} q_1^{+-} - u^{++}q^{++}}{2}$$
$$- \frac{w_M}{2^M}\left(2^{M-1} q_1^{+-} + \sum_{x=1}^{M} \frac{(M-1)!\, q_x^{+-}}{(x-1)!\,(M-x)!}\right) \tag{IC$^-$}$$

$$w_M = \frac{(1+2^{M-2})w}{1+2^{M-1}}$$

Allocation constraints:

$$\max[q_L, q_P/M] \leq q^{++}, q^{--} \leq (Q - q_R)/M$$

and

$$q_P \leq xq_x^{+-} + (M-x)q_x^{-+} \leq Q - q_R$$

where $0 \leq Mq_L < Q - q_R$, $q_L < Q$, and $0 \leq q_P < Q - q_R$.

Solution. We solve the problem for different cases. Assume first that γ is small enough so that (PC) is not binding. If (IC^+) is binding, then the objective function is:

$$\begin{aligned}
\min\ & \frac{M}{2}(EV^+ - EV^-) + M\,EV^- + EV^R = \frac{M}{4}\left(u_{M-1}^{+-}q_{M-1}^{-+} - u^{--}q^{--}\right) \\
& + \frac{Mw_M}{2^{M+1}}\left(2^{M-1}q_{M-1}^{-+} + \sum_{x=1}^{M-1}\frac{(M-1)!\,q_x^{-+}}{x!\,(M-x-1)!} + q^{--}\right) \\
& + \frac{M}{2^M}\sum_{x=1}^{M-1}\frac{(M-1)!\,u_x^{+-}q_x^{-+}}{x!\,(M-x-1)!} + \left(\frac{1+2^{M-1}}{2^{M-1}}\right)\frac{Mu^{--}q^{--}}{2} \\
& + \left(\frac{1+2^{M-1}}{2^{M+1}}\right)u^{++}(Q - Mq^{++}) + \left(\frac{1+2^{M-1}}{2^{M+1}}\right)u^{--}(Q - Mq^{--}) \\
& + \frac{1}{2^{M+1}}\sum_{x=1}^{M-1}\frac{M!\,u_x^{+-}(Q - xq_x^{+-} - (M-x)q_x^{-+})}{x!\,(M-x)!}
\end{aligned}$$

subject to the allocation constraints. Also, according to (PCR), if

$$q^{--} < \frac{Q}{M}$$

then

$$u^{--} \geq u_{AS}^{-} = \frac{2^{M-1}wz^-}{1+2^{M-1}}$$

where

$$z^- = \frac{z_I}{2+2^M - z_I}.$$

and if

$$q^{++} < \frac{Q}{M},$$

then

$$u^{++} \geq u^{+}_{AS} = \frac{2^{M-1} w z^{+}}{1 + 2^{M-1}}$$

where

$$z^{+} = \frac{z_I}{2 + 2^M - (1 + 2^M) z_I}.$$

In each case (IC^-) is nonbinding. From the objective function it is clear that it is optimal to set q^{++} and q_x^{+-} as large as is feasible, and q_x^{-+} as small as is feasible. It is also optimal to set $u_x^{+-} = 0$. (In order to satisfy IC^+ it is most efficient to underprice only in state $^{++}$; underpricing in state $^{--}$ may occur due to adverse selection risk.) $\Longrightarrow$

$$Eu_{PR} = \left(\frac{1 + 2^{M-1}}{2^{M+1}}\right)\left(u^{--} + u^{++}\right) = \left(\frac{1 + 2^{M-1}}{2^{M+1}}\right) u^{--} + \frac{EV^{+}}{2q^{++}} \tag{16}$$

There may be a tradeoff in determining the optimal values for q^{--} and u^{--}. Define

$$q^{--}_{min} \equiv \max[q_L, q_P/M] = \text{lowest feasible value for } q^{--}$$

If $q_R > 0$, then $u^{--} = u^{-}_{AS}$, and q^{--} is optimally set equal to q^{--}_{min}. If $q_R = 0$, then it may be optimal to set $q^{--} = Q/M$ and $u^{--} = 0$. In the case that $q_R = 0$, q^{--} is optimally set equal to q^{--}_{min} iff

$$M q^{--}_{min} u^{-}_{AS} + Q(1 + 2^{M-1}) u^{-}_{AS} + M q^{--}_{min} w_M \;\leq\; Q w_M \Longrightarrow$$

$$\frac{M q^{--}_{min}}{Q} \;\leq\; \frac{w_M - u^{-}_{AS}(1 + 2^{M-1})}{w_M + u^{-}_{AS}} \tag{17}$$

The RHS of (17) is $> 1 - z_I$. Thus, sufficient for (17) is that $M q^{--}_{min}/Q \leq 1 - z_I$

CASE 1a. q_L, q_P and γ are close enough to zero so that neither (IC^+) nor (PC) is strictly binding. $q_R = 0$.

Solution. $q^{++} = Q/M$, $q^{--} = q_x^{-+} = 0$, $u^{++} = u^{+-} = 0$, $u^{--} = u^{-}_{AS}$. q_x^{+-} can take on any feasible value. $EV^{+} = EV^{-} = 0$. (PCR) for state $^{--}$ is strictly binding.

$$Eu_{PR} = \left(\frac{1 + 2^{M-1}}{2^{M+1}}\right) u^{-}_{AS} = \frac{w z^{-}}{4} \tag{18}$$

CASE 1b. q_L, q_P and γ are close enough to zero so that neither (IC^+) nor (PC) is strictly binding. $q_R > 0$.

Solution. $q^{++} = (Q - q_R)/M, q^{--} = q_x^{-+} = 0, u^{+-} = 0, u^{++} = u_{AS}^{+}, u^{--} = u_{AS}^{-}$. q_x^{+-} can take on any feasible value. $EV^{-} = 0$, $EV^{+} > 0$. (PCR) for states $^{--}$ and $^{++}$ are strictly binding.

$$Eu_{PR} = \left(\frac{1+2^{M-1}}{2^{M+1}}\right)\left(u_{AS}^{-} + u_{AS}^{+}\right) = \frac{w(z^{-}+z^{+})}{4} \tag{19}$$

CASE 2. $q_{min}^{--} \equiv \max[q_L, q_P/M]$ is large enough so that (IC^{+}) is binding. (If not, then underpricing is given by (19).) γ is small enough that (PC) is nonbinding. $q_R \geq 0$.

Solution. $q^{++} = (Q - q_R)/M$, $q_x^{-+} = q_L$, $u^{+-} = 0$. q_x^{+-} can take on any feasible value. If $q_R > 0$ or (17) holds, then $q^{--} = q_{min}^{--}$, $u^{--} = u_{AS}^{-}$, $EV^{-} = \left(1+2^{M-1}\right) u_{AS}^{-} q_{min}^{--}/2^{M}$,

$$\begin{aligned}
EV^{+} &= EV^{-} - \frac{u_{AS}^{-} q_{min}^{--}}{2} + \frac{w_M}{2^M}\left(2^{M-1} q_L + \sum_{x=1}^{M-1} \frac{(M-1)!\, q_L}{(x)!\,(M-x-1)!} + q_{min}^{--}\right) \\
&= \frac{w z^{-} q_{min}^{--}}{2(1+2^{M-1})} + w_M q_L + \frac{w_M\left(q_{min}^{--} - q_L\right)}{2^M} \\
&= \frac{w(1+2^{M-2})(2^M-1) q_L}{2^M(1+2^{M-1})} + \frac{w(1+2^{M-2}+2^{M-1}z^{-}) q_{min}^{--}}{2^M(1+2^{M-1})}
\end{aligned}$$

Inserting the above and Equation (18) into (16), we obtain Equation (11) of Proposition 4. If $q_R = 0$ and (17) doesn't hold, then $q^{--} = Q/M$, $u^{--} = 0$, $EV^{-} = 0$,

$$\begin{aligned}
EV^{+} &= \frac{w(1+2^{M-2})(2^M-1) q_L}{2^M(1+2^{M-1})} + \frac{w(1+2^{M-2}) Q/M}{2^M(1+2^{M-1})} \\
Eu_{PR} &= \frac{w(1+2^{M-2})(2^M-1) M q_L}{2^{M+1}(1+2^{M-1}) Q} \\
&\quad + \min\left[\frac{w z^{-}}{4} + \frac{w(1+2^{M-2}+2^{M-1}z^{-}) M q_{min}^{--}}{2^{M+1}(1+2^{M-1}) Q}, \frac{w(1+2^{M-2})}{2^{M+1}(1+2^{M-1})}\right].
\end{aligned} \tag{20}$$

CASE 3. If γ is large enough, so that (PC) is strictly binding, then (IC^{+}) is nonbinding and

$$E[\text{total wealth loss}] = \frac{M\gamma}{2} + \max\left[\frac{M\gamma}{2(Q-q_R)}, \frac{w z^{-}}{4}\right] q_R \tag{21}$$

Dividing (21) by Q, we obtain Equation (12) in Proposition 4. ■

PROOF OF PROPOSITION 5. (a) Follows from Proposition 4, and the fact that with larger Q, total wealth loss due to adverse selection risk is larger, and thus information is more valuable.

(b) Eu_{PR} in Equation (11) is linearly related to w, but the value of M for which $\partial Eu_{PR}/\partial M = 0$ is independent of w.

(c) Eu_{PR} in Equation (10) is decreasing in M. Because $q_L = 0$, Eu_{PR} in Equation (11) is decreasing in M. Because $\gamma > 0$, Eu_{PR} as given by Equation (12) is increasing in M. Because $q_L = 0$, the optimal value of M equates Eu_{PR} as given by Equation (12) with the maximum of Eu_{PR} as given by Equations (10) and (11). The first is independent of w and the latter two are increasing in w. Thus, the optimal value of M is increasing in w. ■

PROOF OF PROPOSITION 6. Follows from Propositions 4 and 5, and Equation (13). ■

References

Aussenegg, W., Pichler P., and Stomper, A. (2005) IPO pricing with bookbuilding and a when-issued market, *Journal of Financial and Quantitative Analysis* forthcoming.

Back, K. and Zender, J. F. (1993) Auctions of divisible goods: On the rationale for the treasury experiment, *The Review of Financial Studies* **6**, 733–764.

Benveniste L. M. and Busaba, W. (1997) Bookbuilding versus fixed price: An analysis of competing strategies for marketing IPOs, *Journal of Financial and Quantitative Analysis* **32**, 383–403.

Benveniste L. M., Busaba, W., and Wilhelm, W. J. (1996) Price stabilization as a bonding mechanism in new equity issues, *Journal of Financial Economics* **42**, 223–255.

Benveniste L. M. and Spindt P. A. (1989) How investment bankers determine the offer price and allocation of new issues, *Journal of Financial Economics* **24**, 343–362.

Benveniste L. M. and Wilhelm, W. J. (1990) A comparative analysis of IPO proceeds under alternative regulatory environments, *Journal of Financial Economics* **28**, 173–207.

Biais, B. and Faugeron-Crouzet, A. M. (2002), IPO auctions: English, Dutch, … French and internet, *Journal of Financial Intermediation* **11**, 9–36.

Bradley, D. J. and Jordan, B. D. (2002) Partial adjustment to public information and IPO underpricing, *Journal of Financial and Quantitative Analysis* **37**, 595–616.

Cornelli, F. and Goldreich, D. (2001) Bookbuilding and strategic allocation, *Journal of Finance* **56**, 2337–2369.

Cornelli, F. and Goldreich, D. (2003) Bookbuilding: How informative is the order book?, *Journal of Finance* **58**, 1415–1443.

Habib, M. A. and Ljungqvist, A. P. (2001) Underpricing and entrepreneurial wealth losses in IPOs, *Review of Financial Studies* **14**, 433–458.

Hanley, K. W. (1993) The underpricing of initial public offerings and the partial adjustment phenomenon, *Journal of Financial Economics* **34**, 231–250.

Ibbotson, R. G., Sindelar, J. L., and Ritter, J. R. (1988) Initial public offerings, *Journal of Applied Corporate Finance* **1**, 37–45.

Jenkinson, T. and Jones, H. (2004) Bids and allocations in European IPO bookbuilding, *Journal of Finance* **59**, 2309–2338.

Loughran, T. and Ritter, J. R. (2002) Why don't issuers get upset about leaving money on the table in IPOs?, *Review of Financial Studies* **15**, 413–443.

Loughran, T. and Ritter, J. R. (2004) Why has IPO underpricing changed over time?, *Financial Management* **33**, 5–37.

Lowry, M. and Schwert, G. W. (2002) IPO market cycles: Bubbles or sequential learning?, *Journal of Finance* **57**, 1171–1200.

Maksimovic, V. and Unal, H. (1993) Issue size choice and "underpricing" in thrift mutual-to-stock conversions, *Journal of Finance* **48**, 1659–1692.

Malakhov, A. (2004) The role of uninformed buyers in optimal IPO mechanisms, working paper, Northwestern University.

Mello, A. S. and Parsons, J. E. (1998) Going public and the ownership structure of the firm, *Journal of Financial Economics* **49**, 79–109.

Myers, S. C. and Majluf, N. S. (1984) Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics* **13**, 187–221.

Myerson, R. (1981) Optimal auction design, *Mathematics of Operations Research* **6**, 58–73.

Ritter, J. R. (1987) The costs of going public, *Journal of Financial Economics* **19**, 269–281.

Rock, K. (1986) Why new issues are underpriced, *Journal of Financial Economics* **15**, 187–212.

Sherman, A. E. and Titman, S. (2002) Building the IPO order book: underpricing and participation limits with costly information', *Journal of Financial Economics* **65**, 3–29.

Stoughton, N. M. and Zechner, J. (1998) IPO-mechanisms, monitoring and ownership structure, *Journal of Financial Economics* **49**, 45–77.