Review of Finance (2006) 10: 417–441
DOI 10.1007/s10679-006-9002-y

Takeover Timing, Implementation Uncertainty, and Embedded Divestment Options ⋆

LUIS H. R. ALVAREZ[1] and RUNE STENBACKA[2]
[1]*Turku School of Economics and Business Administration;* [2]*Swedish School of Economics, Helsinki*

Abstract. We design a compound real options model, which determines the timing of takeovers and characterizes the distribution of the associated surplus. We delineate a relation between the bargaining power of the acquiring firm and the takeover incentives. The takeover threshold is decreasing as a function of the expected primary takeover gain and the embedded divestment gain. Decreased implementation uncertainty stimulates takeover activity. This uncertainty concerns the delay until either primary takeover synergies or subsequent divestment gains are realized. We demonstrate how the relation between volatility and takeover timing depends on the functional form of the profit flow with implementation uncertainty.

1. Introduction

A takeover tends to promote economic efficiency if the bidder firm is able to reorganize the assets of the target firm so as to create synergy gains. The bidder might be able to create merger synergies through, for example, the exploitation of complementarities or economies of scale or scope. However, acquisition decisions are typically sizeable irreversible investments characterized by substantial uncertainty. In particular, the issue of whether the corporate cultures, the governance systems or the brands of the merger partners are compatible is the source of an important type of implementation uncertainty. This can be exemplified by the experiences associated with the merger leading to the formation of DaimlerChrysler, where we are still waiting for the synergies anticipated in the merger prospect to be realized.

A takeover decision can be seen as an irreversible investment. It creates a real option, which incorporates a substantial degree of implementation uncertainty. This implementation uncertainty can be analyzed by assuming that the takeover decision generates a conditional probability distribution regarding the timing of when the benefits from a merger materialise. Such an approach captures the impact

⋆ Financial support from the Foundation for the Promotion of the Actuarial Profession and the Finnish Insurance Society to Luis H. R. Alvarez is gratefully acknowledged. Both authors acknowledge the financial support from the Yrjö Jahnsson Foundation and the Research Unit of Economic Structures and Growth (RUESG) at the University of Helsinki. The authors are grateful to Josef Zechner and two anonymous referees for their constructive comments and suggested improvements on an earlier version of this study.

of unpredictability regarding the time needed to make different corporate cultures, governance systems or established brands fit together. Alternatively, the implementation uncertainty could also capture merger gains of stochastic magnitude. In this paper we focus on the first type of implementation uncertainty.

In many situations takeovers also incorporate an embedded option to divest those activities, which do not belong to the core business of the acquiring firm. We can exemplify this by Kone's acquisition of Partek in May 2002 at a price of 1,45 Billion Euro (see Kauppalehti Optio, 11 December 2003). Kone's core business is focused on elevators and escalators as well as cargotec. The acquisition target, Partek, was a conglomerate with a range of activities belonging to business areas like container and cargo handling, production of machines for forestry and agriculture like tractors as well as mining technology. In 2003 Kone divested most of the activities outside its core business and these divestment operations were estimated to yield 1,1 Billion Euro. At the end of 2003, the container and cargo handling business acquired from Partek was estimated to represent a value of 960–1140 Million Euro (see Kauppalehti Optio, 11 December 2003) and the profitability of this activity has developed very positively under the new owner (see Kauppalehti, 21 July 2004). Thus, Kone's takeover of Partek incorporated an embedded option of divesting the activities outside of Kone's core business, and this divestment was executed before the end of 2003. This case illustrates that the delay until the primary synergy gains are realized might typically differ from the delay until the divestment gains are realized. Moreover, in general, Mulherin and Boone (2000) have empirically demonstrated that takeovers and divestments represent restructuring events that are empirically comparable in magnitude. As Mulherin and Boone state: "The combined target and bidder return at the announcement of an acquisition averages 3.5%, while the announcement return for corporate divestitures averages 3.0%". In our analysis we assume the takeover to incorporate the opportunity of divesting non-core business activities as an embedded option.

In the present article we design a model that endogenously determines the timing of a takeover and characterizes the distribution of the surplus created through the asset restructuring. The timing of the takeover is determined in recognition of an embedded divestment option, whereby the acquiring firm can divest non-core activities. In particular, our model is able to characterize the relation between the timing of a takeover and the bargaining power of the bidder. In the analysis of financial markets Fan and Sundaresan (2000) have previously explored the implications of relative bargaining power of claimants on optimal reorganization and debt valuation. One could interpret the bargaining power of the bidder as a measure of its market power in the market for corporate control relevant from the point of view of the primary takeover. This bargaining power may be different from the bargaining power of the merged firm relative to outside firms in determining the terms at which the divestment takes place, as the bargaining power is likely to be industry-specific.

Our model is able to address the following questions. How will increased bargaining power of the acquiring firm impact on the equilibrium timing of a takeover? What is the effect of increased implementation uncertainty on the timing of a takeover as well as on the distribution between the bidder and the target of the gains from an asset restructuring? In our study implementation uncertainty refers either to the delay until primary synergy gains are realized or the delay until the divestment gains are realized. How will increased volatility of the underlying economic environment affect the timing of corporate restructuring and the distribution of gains from takeovers? What is the relation between the potential gains from a corporate restructuring and the equilibrium timing and price of a takeover?

The present study shares the real options approach to the analysis of the timing and terms of takeovers with two other recent studies, namely Lambrecht (2004) and Morellec and Zhdanov (2005). Both these studies find that economic expansions promote takeover activity. Lambrecht (2004) focuses on friendly mergers motivated by economies of scale and on acquiring firms with or without market power in the product market. Within such a framework he finds that increased market power of the acquiring firm speeds up the merger activity. However, in hostile takeovers the restructuring of assets is delayed in comparison with the equilibrium timing of mergers. Our analysis differs from that of Lambrecht (2004) along several dimensions. First, we focus on more general representations of the synergy gains and in our analysis the timing of the realization of these gains is subject to uncertainty. Second, we focus directly on the bargaining power of the acquiring firm in the market for corporate control rather than restricting ourselves to formulations based exclusively on market power in the product market. Third, we focus on compound takeover options, which include the embedded divestment options. This feature seems to be highly relevant in many conglomerate mergers.

Morellec and Zhdanov (2005) analyze the determination of timing and terms of takeovers in the presence of a stock market with incompletely informed investors. They find that part of the uncertainty will not be resolved until the timing of the takeover, which leads to abnormal announcement returns. They also find that bidding competition among acquiring firms, potentially endowed with different estimates regarding the synergies associated with a takeover, speeds up the takeover process and may generate negative returns to the shareholders of the bidding firms. Our analysis is focused on the takeover decisions and the interactions between the firms involved in the takeovers and not on the issue of how a stock market comprised of incompletely informed investors transmits inside information regarding potential synergy gains.

We find that there is an intimate relation between the bargaining power of the acquiring firm and the takeover incentives. Furthermore, favorable states of nature promote takeover activity. Overall, decreased implementation uncertainty, meaning a lower variance of the delay until either primary takeover synergies or subsequent divestment gains are realized, tends to stimulate the takeover activity. In addition, the presence of implementation uncertainty makes the relation between underlying market volatility and the optimal takeover timing dependent on the functional form of the profit flow. In the absence of implementation uncertainty, our model would unambiguously predict increased volatility to decelerate takeovers. Finally, the optimal threshold for the exercise of the takeover is determined in relation to a compound real option, which incorporates the divestment opportunity as an embedded option. In this respect, we find that increased bargaining power of the acquiring firm at both the takeover stage and at the divestment stage stimulates takeover activity.

Our theoretical predictions are broadly in line with existing empirical evidence. Maksimovic and Phillips (2001) present empirical evidence in support of the view that merger and acquisition (M&A) activities are more likely during booms and thereby procyclical. Furthermore, Mulherin and Boone (2000) report evidence for significant industry clustering for both acquisitions and divestitures, suggesting that both synergy gains and divestment gains represent empirically important explanations for merger and acquisition activities. Andrade, Mitchell and Stafford (2001) present evidence that the target firm is typically able to obtain a substantial share of the gains from mergers. This feature can be captured in a tractable way by the Nash bargaining approach, which we apply for the determination of the takeover terms.

Our study proceeds as follows. Section 2 presents the model. In Section 3 we analyze the takeover timing and terms for general profit flows. In Section 4 we present explicit characterizations of the exercise threshold for the class of profit flows with a multiplicatively separable stochastic component. Finally, we offer some concluding comments in Section 5.

2. The Model

Consider a situation where an acquiring firm, denoted A, faces the opportunity to buy out a target firm, denoted T. Firm T could be a rival firm as in the case of a horizontal merger, but it could also be a conglomerate with some activities in industries different from that or those of firm A. These firms operate in an environment where the market uncertainty is captured by an underlying state variable which evolves on $\mathbb{R}_+$ according to a geometric Brownian motion described by the Itô-stochastic differential equation

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x, \tag{2.1}$$

where both the drift coefficient $\mu \in \mathbb{R}$ and the diffusion coefficient $\sigma \in \mathbb{R}_+$ are assumed to be exogenously determined constants and W_t denotes a standard Brownian motion.

Prior to the takeover, the business activities of firm A generate a profit flow $\pi^A(x)$, whereas those of firm T yield the profit flow $\pi^T(x)$. Through the takeover, firm A acquires the right to reorganize the assets of T so as to create synergy gains. For example, the merger might make it possible for firm A to exploit complementarities or to exploit economies of scale or scope. Through the reorganization of the assets the merged firm can achieve the profit flow

$$\pi_c^A(x) + \pi_d^A(x) \geq \pi^A(x) + \pi^T(x), \tag{2.2}$$

which captures the idea that the synergies associated with the merger increases the profit flows from $\pi^A(x) + \pi^T(x)$ to $\pi_c^A(x) + \pi_d^A(x)$. The post-merger profit flow is decomposed into two parts. $\pi_c^A(x)$ captures the profit flow associated with the core business of the merged firm, whereas $\pi_d^A(x)$ denotes the profit flow associated with the activities outside the core competence of the merged firm. Condition (2.2) captures merger synergies in a very general way and it covers, for example, efficiency benefits associated with scale economies or exploitation of market power. Thus, condition (2.2) is general enough to describe the synergy gains associated with horizontal, vertical or conglomerate mergers. Generally, condition (2.2) can be seen as an analytical representation whereby M&A-activities serve as a facilitator of productivity gains and redeployment of assets from firms with a lower ability to firms with a higher ability to exploit assets for productive purposes.[1]

We assume that the activities outside the firm's core business can potentially be sold to an outside firm O, which can make more efficient use of these resources. Thus, firm A has the option to divest the activities outside its core business and the outside firm can transform these assets into a profit flow $\pi^0(x)$. Formally, we assume that

$$\pi^0(x) \geq \pi_d^A(x). \tag{2.3}$$

Consequently, the takeover makes it possible to exploit synergy gains, and in addition to this primary benefit, it also incorporates the embedded option represented by the potential divestment gains. In this respect, our model captures the synergistic view advocated by Mulherin and Boone (2000), according to which acquisitions and divestment will both have strong positive effects on the profitability of an acquiring firm.

We assume that the expected cumulative present values of all the relevant cash flows exist and are well-defined. In the subsequent analysis we denote, for the sake of notational simplicity, the expected cumulative present value of a cash flow $f(x)$ by $(R_r f)(x)$, i.e.,

$$(R_r f)(x) = \mathbf{E}_x \int_0^\infty e^{-rt} f(X_t) dt.$$

The acquisition decisions are typically sizeable irreversible investments to which the bidding firm has to commit itself in the presence of substantial uncertainty. In particular, the issue of whether the corporate cultures, the governance systems or the produced brands of the merger partners are compatible represents

[1] This interpretation of M&A activities has been empirically emphasized by Maksimovic and Phillips (2001).

the source of an important type of implementation uncertainty. In this model we formally capture the implementation uncertainty associated with takeovers in the following way. Suppose that the takeover takes place at time τ. Then a consolidation phase of random length T_1 is needed until the synergies captured by (2.2) can be realized. We will assume that T_1 is exponentially distributed with parameter λ_1, i.e. T_1 is randomly distributed with the density function $f_1(t) = \lambda_1 e^{-\lambda_1 t}$, $t \geq 0$. Thus, the synergies associated with a merger require an expected implementation lag of λ_1^{-1}.

As shown by (2.2), the takeover yields a surplus, which has to be divided by the owners of the acquiring firm A and the target firm T. We model the determination of the takeover price as the outcome of bargaining, which takes place at the time of the acquisition. For reasons of tractability, the outcome of the negotiation is represented by the traditional Nash bargaining solution. We assume that the bargaining power of firm A is β_A ($0 \leq \beta_A \leq 1$), whereas that of firm T is $1 - \beta_A$.

Once the primary merger synergies are realized, the merged firm has the option to divest those assets, which are associated with activities outside the firm's core business. The inequality (2.3) describes the surplus, which can potentially be achieved through the divestment. The divestment gains cannot be realized until the merged firm has found an outside firm that can make more efficient use of the non-core assets. We can view this as a search process resulting in a delay until the divestment gains can be realized. We assume that the length T_2 of this delay is exponentially distributed with parameter λ_2. This means that T_2 is randomly distributed with the density function $f_2(t) = \lambda_2 e^{-\lambda_2 t}$, $t \geq 0$. Consequently, the realization of the divestment gains requires an expected implementation lag of λ_2^{-1}.

We assume that the price at which the ownership of the divested assets is shifted to the outside firm (O) is determined through bargaining between the merged firm (A) and the outside firm (O). This bargaining takes place at time $t = \tau + T_1 + T_2$. Again, for reasons of tractability, we apply the Nash bargaining solution also to model this round of negotiation. We assume that at the divestment phase of the bargaining power firm A is β_d ($0 \leq \beta_d \leq 1$), whereas that of firm O is $1 - \beta_d$.

In our analysis of the timing and terms of the takeover and of the terms of the divestment we apply the principles of dynamic programming. Thus, we initially characterize the outcome at the last stage, the price whereby the non-core business activities are divested. This equilibrium is then anticipated when the price of the primary takeover is determined. This price determination is, in its turn, anticipated by firm A in its timing decision of when to take over firm T. The sequence of decisions and the associated profit flows are illustrated in Table I.

Table I. The sequence of decisions and associated profit flows

Phase	Pre-takeover $[0, \tau)$	Consolidation $[\tau, \tau + T_1)$	Synergy $[\tau + T_1, \tau + T_1 + T_2)$	Divestment $[\tau + T_1 + T_2, \infty)$
Profit flow of A	$\pi^A(x)$	$\pi^A(x) + \pi^T(x)$	$\pi_c^A(x) + \pi_d^A(x)$	$\pi_c^A(x)$
Profit flow of T	$\pi^T(x)$	–	–	–
Profit flow of 0	–	–	–	$\pi^0(x)$

3. Takeover Timing and Terms

3.1. DIVESTMENT PHASE

Assume that the takeover was initiated at time τ and that we have reached time $\tau + T_1 + T_2$ so that both the synergy and divestment gains have been realized. At this stage the expected cumulative present value of the future revenues for firm A is

$$V_4(x) = (R_r\pi_c^A)(x) + D^*(x), \tag{3.1}$$

where $D^*(x)$ denotes the negotiated price of the divested assets. The divestment process associated with the potential divestment of parts of the acquired firm is modelled as a Nash bargaining game. Thus, the price of the divestment are determined as the solution of the optimization problem

$$\sup_{D\in\mathbb{R}_+} \left[D - (R_r\pi_d^A)(x)\right]^{\beta_d} \left[(R_r\pi^0)(x) - D\right]^{1-\beta_d}, \tag{3.2}$$

where $\beta_d \in [0, 1]$ measures the bargaining power of the acquiring firm (firm A) and $\pi^0(x)$ denotes the profit flow of the outside firm (firm 0) once this has transformed the divested assets into its business operation. By solving (3.2) we find that the Nash bargaining solution is given by

$$D^*(x) = (R_r\pi_d^A)(x) + \beta_d[(R_r\pi^0)(x) - (R_r\pi_d^A)(x)]. \tag{3.3}$$

The Nash bargaining solution (3.3) says that the selling price of the divested assets is equal to the value of these assets under the current ownership plus the fraction β_d of the surplus generated by the asset sale. Furthermore, (3.3) exhibits the intuitively appealing feature that the negotiated price is a linearly increasing function of firm A's bargaining power at the divestment stage. Combining (3.3) with the definition of the expected cumulative present value of the future revenues for firm A (3.1) yields that

$$V_4(x) = (R_r\pi_c^A)(x) + (R_r\pi_d^A)(x) + \beta_d((R_r\pi^0)(x) - (R_r\pi_d^A)(x)). \tag{3.4}$$

Naturally, we find that $\partial V_4(x)/\partial\beta_d = \partial D^*(x)/\partial\beta_d = (R_r\pi^0)(x) - (R_r\pi_d^A)(x) > 0$ meaning that increased bargaining power of firm A translates into a more valuable divestment opportunity.

3.2. SYNERGY PHASE

Having determined the expected cumulative present value of the revenues associated with the divestment phase for firm A, we now proceed to analyze the synergy phase $[\tau + T_1, \tau + T_1 + T_2]$. During this phase the primary takeover has taken place and the associated synergy gains are already realized. However, the search process of finding a divestment target transforming the non-core assets into more efficient use is still ongoing. During this phase, and contingent on the state variable at the date $(\tau + T_1)$ when the primary synergies are realized, the takeover value reads as[2]

$$V_3(x) = \mathbf{E}_x \left[\int_0^{T_2} e^{-rs} (\pi_c^A(X_s) + \pi_d^A(X_s)) ds + e^{-rT_2} V_4(X_{T_2}) \right] \tag{3.5}$$

where $T_2 \sim \exp(\lambda_2)$ is an exponentially distributed random date capturing the delay until the divestment gains are realized. In (3.5) time runs from 0 to T_2 (and not from $\tau + T_1$ to $\tau + T_1 + T_2$), reflect the strong Markov property of diffusions. Applying this principle implies that the expected cumulative present value (3.5) can be expressed as

$$\begin{aligned} V_3(x) &= (R_r \pi_c^A)(x) + (R_r \pi_d^A)(x) \\ &\quad + \beta_d \left((R_r \Delta_2)(x) - (R_{r+\lambda_2} \Delta_2)(x) \right), \end{aligned} \tag{3.6}$$

where $\Delta_2(x) = \pi^0(x) - \pi_d^A(x)$ measures the expected divestment gain.

Standard differentiation implies that

$$\frac{\partial V_3}{\partial \lambda_2}(x) = -\beta_d \frac{\partial (R_{r+\lambda_2} \Delta_2)}{\partial \lambda_2}(x) > 0.$$

Intuitively, firm A can benefit more from the divestment option if this divestment is associated with a shorter expected delay or, if the distribution of this delay is characterized by a lower variance. Formally, an increase in the intensity λ_2 increases the risk-adjusted discount rate and, therefore, increases the expected cumulative risk-adjusted present value of the profit flow associated with the divestment gain. In particular, by taking the limit we observe that

$$\lim_{\lambda_2 \to \infty} V_3(x) = (R_r \pi_c^A)(x) + (R_r \pi_d^A)(x) + \beta_d (R_r \Delta_2)(x),$$

which captures the value in the limiting case where the divestment gain is expected to be realized immediately after the decomposition of the post-merger profits into core profits and non-core profits has taken place. At the other extreme, if the divestment gains cannot be expected to be realized within finite time the value is

$$\lim_{\lambda_2 \downarrow 0} V_3(x) = (R_r \pi_c^A)(x) + (R_r \pi_d^A)(x).$$

[2] The values during the consolidation and the synergy phases are derived analytically in Appendix A.

3.3. CONSOLIDATION PHASE

We now turn to the consolidation phase where the profit flow is $\pi^A(x)+\pi^T(x)$, but where the primary synergy gain $\pi_c^A(c)+\pi_d^A(c)$ has not yet been realized. During this phase the value of the firm contingent on the state variable at the takeover timing τ can be expressed as

$$V_2(x) = \mathbf{E}_x\left[\int_0^{T_1} e^{-rs}(\pi^A(X_s)+\pi^T(X_s))ds + e^{-rT_1}V_3(X_{T_1})\right], \tag{3.7}$$

where $T_1 \sim \exp(\lambda_1)$ is the exponentially distributed delay until the primary merger gains are realized. Applying again the strong Markov property of diffusions implies that this expected cumulative present value can be written as

$$V_2(x) = (R_{r+\lambda_1}\pi^A)(x) + (R_{r+\lambda_1}\pi^T)(x) + \lambda_1(R_{r+\lambda_1}V_3)(x). \tag{3.8}$$

Applying Dynkin's theorem (cf. Karlin and Taylor 1981, pp. 297–299) to the value $V_3(x)$ implies that (3.8) can be rewritten as

$$\begin{aligned} V_2(x) \;=\; & \beta_d\left((R_r\Delta_2)(x) - (R_{r+\lambda_2}\Delta_2)(x)\right) + (R_r\Delta_1)(x) - (R_{r+\lambda_1}\theta)(x) \\ & -\beta_d\lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x), \end{aligned}$$

where $\Delta_1(x) = \pi_c^A(x)+\pi_d^A(x)$ denotes the profit flow during the synergy phase and $\theta(x) = \pi_c^A(x)+\pi_d^A(x)-\pi^A(x)-\pi^T(x)$ measures the primary synergy gains generated by the acquisition of the target firm. Standard differentiation shows that

$$\frac{\partial V_2}{\partial \lambda_1}(x) = -\frac{\partial(R_{r+\lambda_1}\theta)}{\partial\lambda_1}(x) - \beta_d\lambda_2\frac{\partial(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)}{\partial\lambda_1}(x).$$

The first term captures how an accelerated realization of the primary synergy gains affect the associated expected cumulative present value. The second term, in turn, measures how an accelerated realization of these synergy gains affects the expected cumulative present value of the embedded divestment gain. This embedded divestment gain is weighted by the acquiring firm's bargaining power at the divestment stage. If the synergy gains $\theta(x)$ generated by the acquisition of the target firm are positive, as we have assumed in (2.2), the sign of this expression is positive, meaning that an increased arrival intensity of the primary synergy gains increases the value of the takeover option. This holds true because an increase in the intensity λ_1 increases the risk adjusted discount rate and, therefore, decreases the expected cumulative present values.

In particular, we find that

$$\lim_{\lambda_1\downarrow 0} V_2(x) = (R_r\pi^A)(x) + (R_r\pi^T)(x).$$

This value represents the limit when the primary takeover gain cannot be expected to be realized within finite time. This would also postpone the embedded divestment to an unreachable future. If the primary takeover gain can be expected to be realized without a delay the value approaches the limit

$$\lim_{\lambda_1\to\infty} V_2(x) = (R_r\Delta_1)(x) + \beta_d\left((R_r\Delta_2)(x) - (R_{r+\lambda_2}\Delta_2)(x)\right).$$

3.4. TAKEOVER PHASE: TIMING AND PRICE

Having derived the value associated with an optimal post-takeover management of the acquiring firm, we now proceed to characterize the timing and price of the takeover. We first consider the determination of the takeover price. We assume that the acquiring firm has to bear sunk costs C in order to prepare the takeover bid. This may capture, for example, management focus, consultant reports characterizing the profitability of the target firm, and the design of legal contracts associated with the takeover. These sunk costs cannot be recovered in case the takeover negotiations are unsuccessful.[3]

The takeover price is determined through Nash bargaining. Thus, the takeover price constitutes the solution of the optimization problem

$$\sup_{P\in\mathbb{R}_+} \left[P - (R_r\pi^T)(x)\right]^{1-\beta_A}\left[V_2(x) - P - (R_r\pi^A)(x)\right]^{\beta_A}, \tag{3.9}$$

where $\beta_A \in [0, 1]$ denotes the bargaining power of the acquiring firm. The solution of this optimization problem reads as

$$\begin{aligned} P^*(x) &= (R_r\pi^T)(x) \\ &\quad +(1-\beta_A)(V_2(x) - (R_r\pi^T)(x) - (R_r\pi^A)(x)). \end{aligned} \tag{3.10}$$

The Nash bargaining solution (3.10) says that the selling price of the target firm is equal to the value of these assets under the current ownership plus the fraction $1-\beta_A$ of the surplus generated by the control transaction. From (3.10) we can also directly conclude that the negotiated takeover price is a weighted average of the expected cumulative present value of the target firm's future revenues and the expected value associated with an optimal post-takeover management of the acquiring firm's assets.[4]

It is worth observing that for all possible states $x \in \mathbb{R}_+$ it holds that

$$\frac{\partial P^*}{\partial\lambda_2}(x) = -(1-\beta_A)\beta_d\lambda_1\frac{\partial(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)}{\partial\lambda_2}(x) > 0,$$

[3] For a similar justification regarding the treatment of sunk costs to prepare a takeover bid, see Lambrecht (2004).

[4] It should be emphasized that our assumption regarding the sunk cost C implies that it will not be shared through the bargaining process. If the acquirer's payoff in the Nash bargaining game were defined as $(V_2(x) - P - (R_r\pi^A)(x) - C)$, then this cost of preparing the takeover bid would affect the negotiated takeover price.

since an increase in the intensity λ_2 increases the risk-adjusted discount rate during the synergy phase and, therefore, decreases the expected cumulative risk-adjusted present value of the profit flow associated with the expected divestment gain. Likewise, we find that

$$\begin{aligned}\frac{\partial P^*}{\partial \lambda_1}(x) &= -(1-\beta_A)\frac{\partial (R_{r+\lambda_1}\theta)}{\partial \lambda_1}(x) \\ &\quad -\beta_d\lambda_2(1-\beta_A)\frac{\partial (R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)}{\partial \lambda_1}(x) > 0.\end{aligned}$$

Consequently, decreased implementation uncertainty increases the negotiated takeover price. As we have seen, this holds true irrespective of whether the implementation uncertainty is reduced with respect to the synergy gain or with respect to the divestment gain.

Given the negotiated takeover price (3.10), the optimal timing of the takeover is determined by the optimization problem

$$V_1(x) = \sup_\tau \mathbf{E}_x\left[\int_0^\tau e^{-rs}\pi^A(X_s)ds + e^{-r\tau}(V_2(X_\tau) - P^*(X_\tau) - C)\right],$$

which can be re-expressed as

$$\begin{aligned}V_1(x) &= (R_r\pi^A)(x) \\ &\quad + \sup_\tau \mathbf{E}_x\left[e^{-r\tau}(V_2(X_\tau) - P^*(X_\tau) - (R_r\pi^A)(X_\tau) - C)\right]. \quad (3.11)\end{aligned}$$

It follows from (3.11) that if x^* is a state at which the takeover opportunity is worth exercising, then the classical balance identity[5]

$$V_2(x^*) = V_1(x^*) + P^*(x^*) + C$$

holds. This balance identity states that, at the optimum, the value of the opportunity has to coincide with its full costs which, in the present case, are formed by the sum of the sunk cost C, the takeover price $P^*(x)$, and the lost option value $V_1(x^*)$.

Given (3.10) we find that the problem characterizing the optimal timing of the takeover reads as

$$\begin{aligned}V_1(x) &= (R_r\pi^A)(x) \\ &\quad + \sup_\tau \mathbf{E}_x\left[e^{-r\tau}\beta_A(V_2(X_\tau) - (R_r\pi^T)(X_\tau) - (R_r\pi^A)(X_\tau)) - C)\right]. (3.12)\end{aligned}$$

[5] This identity constitutes a value matching condition guaranteeing the continuity of the value at the exercise threshold.

Applying the definition of the value of the opportunities represented by the primary takeover synergy and the embedded divestment gain finally implies that the value of the takeover opportunity (3.12) can be re-expressed as

$$V_1(x) = (R_r\pi^A)(x) + \sup_{\tau} \mathbf{E}_x \left[e^{-r\tau}(\Pi(X_\tau) - C)\right], \tag{3.13}$$

where

$$\Pi(x) = \beta_A\big[(R_r\theta)(x) - (R_{r+\lambda_1}\theta)(x) + \beta_d\big((R_r\Delta_2)(x) - (R_{r+\lambda_2}\Delta_2)(x) \\ -\lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x)\big)\big]$$

denotes the expected cumulative present value of the profit flow associated with the compound real option constituted by the takeover and the subsequent divestment opportunities. The payoff $\Pi(x)$ consists of two components. The first part $(R_r\theta)(x) - (R_{r+\lambda_1}\theta)(x)$ captures the expected cumulative present value of the primary synergy gain. This primary synergy gain is weighted by the bargaining power of the acquiring firm. The second part $((R_r\Delta_2)(x) - (R_{r+\lambda_2}\Delta_2)(x) - \lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x))$ measures the expected cumulative present value of the divestment gain. This divestment gain is weighted by the product $\beta_A\beta_d$ of the bargaining powers of the acquiring firm at the two relevant phases.

Below we formulate a set of reasonable sufficient conditions under which an optimal takeover policy and its value can be characterized. This takeover policy is expressed in terms of a unique takeover threshold (with respect to the prevailing state of nature) above which the takeover is optimal.

THEOREM 3.1. Assume that $\Pi(x)$ is non-decreasing and that there is a single threshold $\bar{x} \in \mathbb{R}_+$ such that

$$\frac{1}{2}\sigma^2x^2\Pi''(x) + \mu x\Pi'(x) \gtreqless r(\Pi(x) - C), \quad x \lesseqgtr \bar{x}. \tag{3.14}$$

Then there is a unique optimal takeover threshold

$$x^* = \operatorname{argmax}\left\{(\Pi(x) - C)x^{-\psi}\right\} > \bar{x} \tag{3.15}$$

satisfying the ordinary first order condition $\Pi'(x^*)x^* = \psi(\Pi(x^*) - C)$, where

$$\psi = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 0$$

denotes the positive root of the fundamental quadratic equation $\sigma^2a(a-1)+2\mu a - 2r = 0$. Moreover, the value of the takeover option under the optimal exercise policy is given by

$$V_1(x) = \begin{cases} \Pi(x) - C & x \geq x^* \\ \frac{\Pi'(x^*)}{\psi}x^{*1-\psi}x^{\psi} & x < x^*. \end{cases} \tag{3.16}$$

Proof. See Appendix B.

The sufficient condition (3.14) included in Theorem 3.1 can be interpreted in terms of the expected rate of return (per unit invested) associated with the takeover. More precisely, since the expected rate of return from the takeover can be expressed as

$$\frac{1}{dt}\mathbf{E}_x\left[d(\Pi(X_t) - C)\right] = \frac{1}{2}\sigma^2 X_t^2 \Pi''(X_t) + \mu X_t \Pi'(X_t),$$

we can interpret the sufficient condition (3.14) as the requirement that the expected rate of return per unit invested exceeds (falls short of) the risk free rate of return at unfavorable (favorable) states of nature. This condition essentially rules out situations where the takeover opportunity would dominate risk free investments in all states and vice versa.

In line with both the theoretical and the empirical literature (for example, Lambrecht (2004), Morellec and Zhdanov (2005) and Maksimovic and Phillips (2001)), Theorem 3.1 establishes that favorable states of nature stimulate the takeover activities in the sense that a takeover takes place when the state variable exceeds an endogenously determined threshold. In this respect our model predicts procyclical takeover activities.[6]

Next, we explore the relation between the bargaining power of the acquiring firm and the takeover threshold.

THEOREM 3.2. Assume that the conditions of Theorem 3.1 are satisfied. Then, increased bargaining power for the acquiring firm at the takeover or divestment negotiations accelerates takeover activity by decreasing the optimal takeover threshold. More precisely, $\partial x^*/\partial \beta_A < 0$ and $\partial x^*/\partial \beta_d < 0$.

Proof. See Appendix C.

Theorem 3.2 shows that the greater the acquirer's fraction of the synergy or divestment surplus, the lower is the takeover threshold. Consequently, increased bargaining power for the acquiring firm both at the takeover and the divestment stage promote takeover activity by lowering the optimal takeover threshold.[7] From the proof of Theorem 3.2 we can infer that an increase in β_A lowers the takeover threshold more significantly than an increase in β_d as long as the condition

$$\beta_A - \beta_d < \frac{(R_{r+\lambda_1}\theta)(x)}{\lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x)}$$

[6] However, except for this general procyclical feature our model is not designed to investigate merger waves as we focus on a single representative merger. For updated studies of merger waves we simply refer to Holmström and Kaplan (2001) or Jovanovic and Rousseau (2004).

[7] A related result has been established in different contexts by Morellec and Zhdanov (2005) as well as by Lambrecht (2004).

holds for all states x. Since the right-hand side is always positive we know for sure that the impact on the threshold of an increase in β_A is always stronger than the impact of an increase in β_d if, for example, $\beta_A \leq \beta_d$. Intuitively, is is natural that the takeover threshold is often more sensitive to changes in β_A than to changes in β_d, because the divestment opportunity is embedded in the takeover option.

The central interpretation of the sufficient condition (3.14) can also be extracted by reformulating (3.14) as the condition that the flow

$$H(x) = (r + \lambda_1)rC - \lambda_1\beta_A(\beta_d\lambda_2(R_{r+\lambda_2}\Delta_2)(x) + \theta(x))$$

is non-increasing and that there is a single threshold $\bar{x}$ such that $H(x) \gtreqless 0$ for $x \lesseqgtr \bar{x}$. Under these circumstances there is a unique optimal threshold x^* satisfying (3.15).

Finally, it should be emphasized that the impact of increased volatility σ on the optimal takeover timing is typically ambiguous. In general, it is hard to characterize the sign of the relation between the optimal timing for the exercise of a compound option and the volatility of the underlying stochastic process. The major complication explaining this ambiguity is that the values of the embedded options depend on the volatility due to the potential nonlinearity of the associated cash flows. In a somewhat related, but nevertheless simpler, setting Alvarez and Stenbacka (2004) show how the optimal timing for the exercise of a compound option represents the interplay between two effects: (1) the real option effect linked with the ability to postpone the exercise of an investment opportunity and (2) the Jensen's inequality effect associated with the compound real option constituted by the takeover and the subsequent divestment opportunities. We return to this issue in greater detail for a particular class of profit flows in the next section.

4. Multiplicatively Separable Profit Flows

We now present an explicit version of our analysis by focusing on the class of profit flows with a multiplicatively separable stochastic component. Assume therefore that the considered cash flows are of the form $\pi_d^A(x) = \pi_d^A x^\eta, \pi_c^A(x) = \pi_c^A x^\eta, \pi^A(x) = \pi^A x^\eta, \pi^T(x) = \pi^T x^\eta$, and $\pi^0(x) = \pi^0 x^\eta$, where $\eta \in \mathbb{R}_+$ is a known exogenously given parameter that captures whether the profit flow is convex or concave. If the condition $r > \mu\eta + \frac{1}{2}\eta(\eta - 1)\sigma^2$, guaranteeing the existence of the expected cumulative present values of the profit flows, is met, then the solution of the divestment game reads as

$$D^*(x) = \left[\beta_d\pi^0 + (1 - \beta_d)\pi_d^A\right]\frac{x^\eta}{r - \delta(\eta)},$$

where $\delta(\eta) = \mu\eta + \frac{1}{2}\eta(\eta-1)\sigma^2$ denotes the expected percentage growth rate of the cash flow x^η. Under these circumstances, the value of firm A at the divestment stage can be expressed as

$$V_4(x) = \left[\pi_c^A + \beta_d\pi^0 + (1-\beta_d)\pi_d^A\right]\frac{x^\eta}{r-\delta(\eta)}.$$

With this value of the corporation at the divestment stage, we can now demonstrate that the value of the firm at the synergy phase is

$$V_3(x) = \frac{(\pi_c^A + \pi_d^A)x^\eta}{(r-\delta(\eta))} + \frac{\lambda_2\beta_d(\pi^0 - \pi_d^A)x^\eta}{(r+\lambda_2-\delta(\eta))(r-\delta(\eta))}.$$

Along the lines of our general findings, we observe that the value of the firm at the synergy phase is increasing as a function of λ_2 (that is, $\partial V_3(x)/\partial\lambda_2 > 0$). Analogously, straightforward integration yields that the value of the firm at the consolidation stage is

$$\begin{aligned} V_2(x) &= \frac{(\pi^A + \pi^T)x^\eta}{r+\lambda_1-\delta(\eta)} \\ &\quad + \frac{\lambda_1}{r+\lambda_1-\delta(\eta)}\left[(\pi_c^A + \pi_d^A) + \frac{\lambda_2\beta_d(\pi^0 - \pi_d^A)}{(r+\lambda_2-\delta(\eta))}\right]\frac{x^\eta}{(r-\delta(\eta))}. \end{aligned}$$

We can now proceed to the analysis of the timing and terms of the takeover. We first determine the negotiated takeover price. In the present case, the Nash bargaining solution is

$$P^*(x) = \frac{\beta_A\pi^T x^\eta}{r-\delta(\eta)} + (1-\beta_A)\left(V_2(x) - \frac{\pi^A x^\eta}{r-\delta(\eta)}\right)$$

implying that

$$\begin{aligned} &V_2(x) - P^*(x) - \frac{\pi^A x^\eta}{r-\delta(\eta)} \\ &= \frac{\beta_A\lambda_1}{r+\lambda_1-\delta(\eta)}\left[\pi_c^A + \pi_d^A - \pi^A - \pi^T + \frac{\beta_d\lambda_2(\pi^0 - \pi_d^A)}{r+\lambda_2-\delta(\eta)}\right]\frac{x^\eta}{r-\delta(\eta)}. \end{aligned}$$

Hence, the optimal takeover timing solves the problem

$$V_1(x) = \frac{\pi^A x^\eta}{r-\delta(\eta)} + \sup_\tau \mathbf{E}_x\left[e^{-r\tau}\left(\frac{M}{r-\delta(\eta)}X_\tau^\eta - C\right)\right],$$

where the multiplier M is given by

$$M = \frac{\beta_A\lambda_1}{r+\lambda_1-\delta(\eta)}\left[\pi_c^A + \pi_d^A - \pi^A - \pi^T + \frac{\beta_d\lambda_2(\pi^0 - \pi_d^A)}{r+\lambda_2-\delta(\eta)}\right].$$

In contrast to standard real option application, the multiplier M depends on the bargaining powers (β_A, β_d) determining the distribution of synergy and divestment gains, the measures of implementation uncertainty (λ_1, λ_2), and the volatility (σ) of the underlying random state variable. As is intuitively clear, the takeover opportunity has value only when $M > 0$. Otherwise the exercise payoff associated with the takeover opportunity would always be negative and the opportunity would never be exercised. It is, however, worth emphasizing that the positivity of this multiplier does not necessarily require the positivity of the synergy gains. More precisely, the takeover opportunity has value as long as the condition

$$\frac{\beta_d \lambda_2(\pi^0 - \pi_d^A)}{r + \lambda_2 - \delta(\eta)} > \pi^A + \pi^T - \pi_c^A - \pi_d^A \tag{4.1}$$

is satisfied. Clearly, this condition is always satisfied whenever there are primary takeover gains, i.e. when $\pi_c^A + \pi_d^A \geq \pi^A + \pi^T$, as long as $\pi^0 \geq \pi_d^A$. Interestingly, (4.1) can also be met even in the absence of primary takeover gains as long as the embedded divestment gains are sufficiently significant. Furthermore, (4.1) is formulated independently of the bargaining power β_A.

Standard differentiation of M shows that this multiplier is an increasing function of the bargaining power of the acquiring firm both at the takeover stage (β_A) and at the divestment stage (β_d) as well as a decreasing function of the expected lag or the variance of the time required until the synergy gains (λ_1) and the divestment gains (λ_2) are realized. The impact of increased volatility on the multiplier depends on the convexity or concavity of the profit flows. If $\eta > 1$ (< 1) the profit flow is convex (concave) and increased volatility increases (decreases) the multiplier. Furthermore, with linear profit flows $(\eta = 1)$ the multiplier M is independent of σ.

Our main result for the class of profit flows under consideration is now summarized in the following:

THEOREM 4.1. Assume that inequality (4.1) is satisfied and that ψ is defined as in Theorem 3.1. Then there is a unique optimal takeover threshold

$$x^* = \left(\frac{\psi}{(\psi - \eta)} \frac{(r - \delta(\eta))C}{M}\right)^{1/\eta}$$

satisfying the ordinary first order condition

$$\frac{M\eta x^{*\eta}}{(r - \delta(\eta))} = \psi\left(\frac{Mx^{*\eta}}{(r - \delta(\eta))} - C\right).$$

This optimal takeover threshold features the following comparative statics properties:

$$\frac{\partial x^*}{\partial \lambda_1} < 0,\ \frac{\partial x^*}{\partial \lambda_2} < 0,\ \frac{\partial x^*}{\partial \beta_A} < 0,\ \frac{\partial x^*}{\partial \beta_d} < 0.$$

The value of the optimal takeover policy reads as

$$V_1(x) = \begin{cases} \frac{\pi^A x^\eta}{r-\delta(\eta)} + \frac{M}{r-\delta(\eta)} & \text{if } x \geq x^* \\ \frac{\pi^A x^\eta}{r-\delta(\eta)} + \frac{\eta C}{(\psi-\eta)} \left(\frac{x}{x^*}\right)^\psi & \text{if } x < x^*. \end{cases} \tag{4.2}$$

Theorem 4.1 demonstrates that the takeover threshold is a decreasing function of the acquiring firm's bargaining power at both the takeover and the divestment stage. Thus, increased bargaining power of the acquiring firm, no matter whether it refers to the takeover stage or the divestment stage, promotes the takeover activity. Also, from Theorem 4.1 we can infer that the takeover threshold is an increasing function of the expected lag or the variance of the time required until the primary synergy or the divestment gains are realized. Thus, increased difficulties or uncertainty with respect to the implementation of the potential takeover gains tend to slow down takeover activity.

From the definition of the multiplier M we can compare the impact on the takeover threshold of an increase in the primary synergy gain $\pi_c^A + \pi_d^A - \pi^A - \pi^T$ with that of an increase in the embedded divestment gain $\pi^0 - \pi_d^A$. The definition of M directly shows that a change in the primary takeover gain has a larger effect than a change in the embedded divestment gain since

$$\beta_d \frac{\lambda_2}{r + \lambda_2 - \delta(\eta)} < 1.$$

For this reason the takeover threshold is more sensitive with respect to changes in the primary takeover gain than in the divestment gain.

As we saw in the previous section it is in general very hard, if possible at all, to characterize the sign of the relation between increased volatility and the optimal takeover policy. For the particular class of profit flows considered in this section we can, however, establish the following.

LEMMA 4.2. Assume that inequality (4.1) is satisfied.

(i) If $\eta \geq 1$, then increased volatility increases the value of the takeover opportunity.
(ii) If $\eta < 1$ and $\delta(\eta) < r \leq \mu$, then increased volatility decreases the value of the takeover opportunity.

Proof. See Appendix D.

It should be observed that Lemma 4.2 does not characterize the sign of the relation between increased volatility and the value of the optimal takeover policy for all possible parameter configurations. More precisely, the impact of increased volatility on the value of the optimal takeover policy is ambiguous for $\eta < 1$ and $r > \mu$. Namely, under these conditions increased volatility decreases the factor

$M/(r - \delta(\eta))$, while it simultaneously increases the factor $(x/x^*)^{\psi}$ (measuring the price of a zero-coupon bond maturing at the takeover date), so that the overall effect is unclear.

In order to characterize the sensitivity of the optimal takeover threshold to changes in the volatility of the underlying state variable, we first observe that the optimal threshold can be re-expressed as

$$x^* = \left(\left(1 - \frac{\eta}{\varphi}\right)\frac{rC}{M}\right)^{1/\eta}, \tag{4.3}$$

where

$$\varphi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

denotes the negative root of the fundamental quadratic equation $\sigma^2 a(a-1)+2\mu a-2r = 0$. Thus, standard differentiation implies that

$$\frac{\partial x^*}{\partial \sigma} = \frac{rCx^{*1-\eta}}{\eta M}\left[\frac{\eta}{\varphi^2}\frac{\partial \varphi}{\partial \sigma} - \left(1 - \frac{\eta}{\varphi}\right)\frac{\partial M/\partial \sigma}{M}\right],$$

where

$$\frac{\partial \varphi}{\partial \sigma} = \frac{2\varphi(\varphi - 1)}{\sigma(\psi - \varphi)} > 0.$$

Consequently, we can decompose the total effect of increased volatility on the optimal takeover threshold as the sum of two separate effects. The first term is proportional to the positive effect ($\partial\varphi/\partial\sigma > 0$) of increased volatility on the standard option multiplier. The second term captures the effect on the volatility-dependent multiplier M. As we have pointed out prior to Theorem 4.1, the sign of $\partial M/\partial \sigma$ is determined by whether η exceeds or falls short of 1. Since the standard option multiplier $1-\eta/\varphi$ is always positive we can draw the following conclusion.

THEOREM 4.3. Assume that inequality (4.1) is satisfied. The total effect of increased volatility on the optimal threshold can be decomposed into two potentially offsetting effects: (1) an effect proportional to the effect on the standard option multiplier $1 - \eta/\varphi$ and (2) an effect proportional to the volatility-dependent multiplier M. In particular, increased volatility decelerates the takeover if

(a) the profit flow is concave ($\eta \leq 1$), or
(b) the implementation uncertainties disappear ($\lambda_1 \to \infty$ and $\lambda_2 \to \infty$).

From Theorem 4.3 we can conclude that the effect of increased volatility converges towards that predicted by conventional real option models in the sense of always decelerating takeover activity if implementation uncertainty disappears.

However, in the presence of implementation uncertainty we can see that the optimal takeover threshold is determined by the interplay between two effects. These effects operate in the same direction if the profit flow is concave, whereas they operate in different directions if the profit flow is strictly convex. Thus, the presence of implementation uncertainty, captured by the parameters λ_1 and λ_2, introduces a qualitatively important feature as it makes the effects of volatility dependent on the functional form (convexity or concavity) whereby the profit flow depends on the stochastic component. While increased volatility may increase the expected value of the primary takeover option, it may simultaneously increase the value of the subsequent divestment option. Since we cannot rule out cases where the latter effect dominates the former, the overall impact of increased volatility on the optimal takeover policy is, a priori, ambiguous. However, in our numerical experimentation we have not been able to find numerical configurations under which increased volatility would accelerate the takeover. Consequently, we conjecture that increased volatility decelerates the takeover in general and not only under the conditions reported in Theorem 4.3. In light of the explicit analysis presented in this section it should be emphasized that a delay in the realization of the primary takeover gains and the embedded divestment option, and the negotiation over the divestment surplus may result in situations where increased volatility reduces the expected profitability of a takeover despite its potentially positive impact on the value of waiting. To our best knowledge, this is a novel finding.

Based on the takeover threshold (4.3) we can also calculate the expected time until a takeover is realized. If the condition $2\mu > \sigma^2$ is satisfied, then for a given $x \leq x^*$ the expected duration until the takeover is

$$\mathbf{E}_x[\tau^*_{x^*}] = \frac{2\ln(x^*/x)}{2\mu - \sigma^2}.$$

In particular, we find that the expected time until a takeover is an increasing function of the variance of the time required until the primary synergy gains and divestment gains are realized. Formally, by differentiation we find that

$$\frac{\partial}{\partial \lambda_i}\mathbf{E}_x[\tau^*_{x^*}] = -\frac{2}{(2\mu - \sigma^2)\eta M}\frac{\partial M}{\partial \lambda_i} < 0, \quad i = 1, 2.$$

Further, we find that if the conditions of our Theorem 4.3 are satisfied, then increased volatility increases the expected time until the takeover is realized, because under such circumstances it holds that

$$\frac{\partial}{\partial \sigma}\mathbf{E}_x[\tau^*_{x^*}] = \frac{4\sigma\ln(x^*/x)}{(2\mu - \sigma^2)^2} + \frac{2}{(2\mu - \sigma^2)x^*}\frac{\partial x^*}{\partial \sigma} > 0.$$

Clearly, increased volatility and increased implementation uncertainty affect the expected optimal takeover timing in the same direction, but not with the same magnitude. In fact, the mechanisms whereby increased volatility affects the expected optimal takeover timing are more complex than those associated with implementation uncertainty. More precisely, since increased volatility affects the underlying state at all times, it also affects the valuation at all phases of a merger process and not only at the consolidation or the synergy phase. Furthermore, as we have seen earlier, the net impact of increased volatility on the expected takeover date is determined by a combination of effects operating potentially in opposite directions.

5. Conclusions

In this paper we have designed a compound real options model, which determines the timing of takeovers and characterizes the distribution of the surplus created through the asset restructuring. Relative to the existing literature we emphasized the relation between the bargaining power of the acquiring firm and its takeover incentives within the framework of a model with two distinguishable aspects, which have been neglected in the literature so far. First, the price and timing of the takeover is determined using a compound real option approach, which incorporates the divestment opportunity of non-core business activities as an embedded option. Second, our model incorporates implementation uncertainty with respect to the time required until the takeover synergies are realized. Furthermore, this implementation uncertainty is separated into two types: uncertainty regarding the delay for (1) primary synergy gains and (2) embedded divestment gains to realize.

We establish a relation between takeover incentives and the bargaining power of the acquiring firm at the stages of the takeover and the divestment. In general, one could view the bargaining power of the acquiring firm as a measure of the imperfections in the market for corporate control.[8] Alternatively, the bargaining power could also in some circumstances be seen as capturing the intensity of competition in the market for corporate control. It should, however, be emphasized, that a change in the bargaining power of the acquiring firm need not be equivalent to a change in the competitiveness of the market for corporate control.[9]

We demonstrate that the presence of implementation uncertainty has an important qualitative impact on the relation between the optimal timing of takeovers and market volatility. Namely, with implementation uncertainty the effect of volatility on the optimal takeover timing depends on the way by which the stochastic component affects the profit flow. Further, we find takeovers to be a procyclical activity.

For empirical purposes it seems particularly interesting to think of proxies that could be used to capture implementation uncertainty. Factors determining the delay

[8] In the present analysis we focus on the optimal timing of a takeover and, in particular, on how it depends on the bargaining power of the acquiring firm. We assume that the bargaining powers of the takeover parties are exogenous. Croson, Gomes, McGinn, and Nöth (2004) apply an experimental method to determine these bargaining powers in the context of a takeover.

[9] For example, in the context of credit markets, Bester (1995) associates a change in the degree of competitiveness with a change in the outside option of the project holder. Thus, if the credit market becomes more competitive in response to a larger number of banks the outside option available to borrowers improves. But, this does not mean that the bargaining power of the borrowers would have increased.

until the primary synergy gains are realized could be geographical distance, the degree of homogeneity with respect to corporate governance and organizational structure, and homogeneity with respect to the products and services provided. Furthermore, the firms in the management consulting industry offer extensive lists of "best practices" with an ultimate purpose of shortening the delay until the primary synergy gains are realized. Moreover, the delay until divestment gains are realized essentially reflects how efficient the capital markets are in transforming assets from inefficient use in one firm to more efficient use in another firm. Empirically this might be captured by factors such as the liquidity of the acquired assets and the degree to which the merger is of a conglomerate character.

It should be noticed that our analysis does not encompass an institutional description of the capital market and the informational asymmetries prevailing therein. For that reason our study does not generate predictions concerning phenomena like merger waves and announcement premia associated with takeovers. As mentioned earlier, these phenomena have been studied by, for example, Holmström and Kaplan (2001) and Morellec and Zhdanov (2005). Likewise, our study concentrates on takeover thresholds, but does not consider closure thresholds. Lambrecht and Myers (2005) have recently analyzed closure thresholds within the framework of declining industries. Also, our analysis has abstracted from considerations related to how takeovers may serve as a device facilitating the replacement of inefficient managers. Such aspects have recently been studied by Morellec (2004) and Lambrecht and Myers (2005).

Throughout our analysis we assume that the acquiring firm has the exclusive right to take initiatives on the timing of takeover. Thus, in our model the target firm cannot influence the timing of a takeover by holding out for a higher share of the merger payoff. It could be an interesting extension of our model to incorporate this aspect. In this respect the studies of competitive bids in auctions developed by Mason and Weeds (2005) and of strategic merger waves by Toxvaerd (2004) offer promising approaches. An endogenous distribution of roles between that of an acquiring firm and that of a target firm may induce rent equalization. Consistent with such a conjecture an endogenous distribution of roles may lower the takeover threshold and thereby speed up the takeover process.

Appendix A. Derivation of the Value

In order to derive the value of the firm during the synergy and the consolidation phase, we first notice that if $T \sim \exp(\lambda)$ is an exponentially distributed random time which is independent of the underlying process X_t and the expected cumulative present value of the cash flow $f(x)$ exists, then Fubini's theorem (changing the order of integration) yields

$$\mathbf{E}_x \int_0^T e^{-rs} f(X_s)ds = \mathbf{E}_x \int_0^\infty e^{-(r+\lambda)s} f(X_s)ds = (R_{r+\lambda} f)(x). \qquad \text{(A.1)}$$

On the other hand, since $\mathbf{E}_x\left[e^{-rT}(R_r f)(X_T)\right] = \lambda(R_{r+\lambda}R_r f)(x)$ we find by applying the classical resolvent equation on $(R_r f)(x)$ (cf. Karlin and Taylor (1981), p. 293) that

$$\mathbf{E}_x\left[e^{-rT}(R_r f)(X_T)\right] = (R_r f)(x) - (R_{r+\lambda} f)(x). \tag{A.2}$$

Given the observations above, we find by applying (A.1) and (A.2) that during the synergy phase the value of the firm is

$$\begin{aligned} V_3(x) &= (R_{r+\lambda_2}\pi_c^A)(x) + (R_{r+\lambda_2}\pi_d^A)(x) + \lambda_2(R_{r+\lambda_2}R_r\pi_c^A)(x) \\ &\quad +\lambda_2(R_{r+\lambda_2}R_r\pi_d^A)(x) + \beta_d\lambda_2(R_{r+\lambda_2}R_r\Delta_2)(x) \\ &= (R_r\pi_c^A)(x) + (R_r\pi_d^A)(x) + \beta_d((R_r\Delta_2)(x) - (R_{r+\lambda_2}\Delta_2)(x)). \end{aligned}$$

Analogously, during the consolidation phase the value of the firm is

$$\begin{aligned} V_2(x) &= (R_{r+\lambda_1}\pi^A)(x) + (R_{r+\lambda_1}\pi^T)(x) + \lambda_1(R_{r+\lambda_1}R_r\pi_c^A)(x) \\ &\quad +\lambda_1(R_{r+\lambda_1}R_r\pi_d^A)(x) + \beta_d(\lambda_1(R_{r+\lambda_1}R_r\Delta_2)(x) - \lambda_1(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x)) \\ &= (R_r\pi_c^A)(x) + (R_r\pi_d^A)(x) - (R_{r+\lambda_1}\theta)(x) + \beta_d((R_r\Delta_2)(x) \\ &\quad -(R_{r+\lambda_1}\Delta_2)(x)) - \beta_d\lambda_1(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x). \end{aligned}$$

According to the classical resolvent equation $-\lambda_1(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x) = (R_{r+\lambda_1}\Delta_2)(x) - (R_{r+\lambda_2}\Delta_2)(x) - \lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x)$, from which the alleged result follows.

Appendix B. Proof of Theorem 3.1

Proof. Denote the proposed value function as $F(x)$. Since $F(x)$ can be re-expressed as

$$F(x) = \mathbf{E}_x\left[e^{-r\tau_{x^*}^*}(\Pi(X_{\tau_{x^*}^*}) - C)\right],$$

where $\tau_{x^*}^* = \inf\{t \geq 0 : X_t \geq x^*\}$, we naturally have that $V_1(x) \geq F(x)$. It is, therefore, sufficient to establish the opposite inequality. To accomplish this task, we first observe that the proposed value function is continuously differentiable on $\mathbb{R}_+$, twice continuously differentiable on $\mathbb{R}_+ \setminus \{x^*\}$, and satisfies the inequality $F''(x^*\pm) < \infty$. Moreover, $(\mathcal{A}F)(x) = rF(x)$ for all $x \in (0, x^*)$, where

$$\mathcal{A} = \frac{1}{2}\sigma^2 x^2 \frac{d^2}{dx^2} + \mu x \frac{d}{dx}$$

denotes the differential operator associated with the underlying geometric Brownian motion. It is, therefore, sufficient to establish that $x^* > \bar{x}$ since in that case $(\mathcal{A}F)(x) - rF(x) = (\mathcal{A}\Pi)(x) - r(\Pi(x) - C) \leq 0$ for all $x > x^* > \bar{x}$. To prove that this is indeed the case, consider the mapping

$$G(x) = \Pi'(x)x^{1-\varphi} - \psi x^{-\varphi}(\Pi(x) - C) = x^{1+\psi-\varphi}\frac{d}{dx}\left[x^{-\psi}(\Pi(x) - C)\right],$$

where ψ denotes the positive root and

$$\varphi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

denotes the negative root of the fundamental quadratic equation $\sigma^2 a(a-1) + 2\mu a - 2r = 0$. Since $x^{-\varphi} \downarrow 0$ as $x \downarrow 0$, our assumption on the monotonicity of $\Pi(x)$ implies that $\lim_{x\downarrow 0} G(x) \geq 0$. Moreover, standard differentiation yields

$$G'(x) = \frac{2}{\sigma^2}x^{-\varphi-1}\left[\frac{1}{2}\sigma^2 x^2\Pi''(x) + \mu x\Pi'(x) - r(\Pi(x) - C)\right] \gtreqqless 0 \quad x \lesseqqgtr \bar{x}$$

which, in turn, implies that if equation $G(x) = 0$ has a root it must be above the threshold $\bar{x}$. In order to establish that such a root indeed exists and is unique, we notice that if $x > K > \bar{x}$ then the mean value theorem for integrals implies that

$$\begin{aligned} G(x) &= G(K) + \frac{2}{\sigma^2}\int_K^x y^{-\varphi-1}\left((\mathcal{A}\Pi)(y) - r(\Pi(y) - C)\right)dy \\ &= G(K) + \frac{2\left((\mathcal{A}\Pi)(\xi) - r(\Pi(\xi) - C)\right)}{\sigma^2\varphi}\left[K^{-\varphi} - x^{-\varphi}\right], \end{aligned}$$

where $\xi \in (K, x)$. Letting $x \to \infty$ finally implies that $G(x) \downarrow -\infty$ and, therefore, equation $G(x) = 0$ has a unique root on the set $(\bar{x}, \infty)$. These observations imply that $F(x)$ constitutes a r-excessive majorant of the exercise payoff $\Pi(x) - C$. Since $V_1(x)$ is the least of these majorants, we find that $V_1(x) \leq F(x)$, which demonstrates that $V_1(x) = F(x)$.

Appendix C. Proof of Theorem 3.2

Proof. Since

$$\begin{aligned} &\Pi'(x)x^{1-\varphi} - (\Pi(x) - C)\psi x^{-\varphi} \\ &= \frac{2}{\sigma^2}\int_0^x y^{-1-\varphi}((\mathcal{A}\Pi)(y) - r(\Pi(y) - C))dy \end{aligned} \tag{C.1}$$

it is sufficient to investigate the dependence of the mapping $(\mathcal{A}\Pi)(x) - r(\Pi(x) - C)$ on the bargaining powers β_A and β_d. Since the expected cumulative present value $(R_r f)(x)$ of a cash flow $f(x)$ satisfies the ordinary second order differential equation $(\mathcal{A}R_r f)(x) - r(R_r f)(x) + f(x) = 0$, we observe that for any $\lambda \in \mathbb{R}_+$

we have $(\mathcal{A}R_{r+\lambda}f)(x) - r(R_{r+\lambda}f)(x) = \lambda(R_{r+\lambda}f)(x) - f(x)$. Applying this observation to $(\mathcal{A}\Pi)(x) - r(\Pi(x) - C)$ yields

$$
\begin{aligned}
&(\mathcal{A}\Pi)(x) - r(\Pi(x) - C) \\
&\quad = rC - \beta_A\lambda_1\left[\beta_d\lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x) + (R_{r+\lambda_1}\theta)(x)\right]
\end{aligned}
\tag{C.2}
$$

Hence, standard differentiation yields

$$
\frac{\partial}{\partial\beta_i}\left[rC - \beta_A\lambda_1\left[\beta_d\lambda_2(R_{r+\lambda_1}R_{r+\lambda_2}\Delta_2)(x) + (R_{r+\lambda_1}\theta)(x)\right]\right] < 0, i = A, d.
$$

Combining this observation with (C.1) then proves the alleged claim.

Appenedix D. Proof of Lemma 4.2

Proof. Standard differentiation yields

$$
\frac{\partial\Pi(x)}{\partial\sigma} = \frac{(r - \delta(\eta))\partial M/\partial\sigma + \sigma\eta(\eta - 1)M}{(r - \delta(\eta))^2}x^\eta \gtreqless 0, \quad \eta \gtreqless 1.
$$

Consider now the strategies characterized by the threshold policy stopping times $\tau_y = \inf\{t \geq 0 : X_t \geq y\}$, where $\Pi(y) \geq C$, and denote the value of such takeover strategy as $J(x, y) = \mathbf{E}_x\left[e^{-r\tau_y}(\Pi(X_{\tau_y}) - C)\right]$. It can now be established that this value can be re-expressed as

$$
J(x, y) = \begin{cases} \Pi(x) - C & x \geq y \\ (\Pi(y) - C)(x/y)^\psi & x < y. \end{cases}
$$

Standard differentiation yields that

$$
\frac{\partial J(x, y)}{\partial\sigma} = \begin{cases} \frac{\partial\Pi(x)}{\partial\sigma}, & \text{if } x \geq y \\ \frac{\partial\Pi(y)}{\partial\sigma}(x/y)^\psi + (\Pi(y) - C)(x/y)^\psi\ln(x/y)\frac{\partial\psi}{\partial\sigma}, & \text{if } x < y, \end{cases}
$$

where

$$
\frac{\partial\psi}{\partial\sigma} = \frac{2\psi(1 - \psi)}{\sigma(\psi - \varphi)} \gtreqless 0, \quad r \lesseqgtr \mu.
$$

If $\eta \geq 1$, then the absence of speculative bubbles condition $r > \delta(\eta)$ implies that $r > \mu$ and, therefore, that increased volatility increases the value $J(x, y)$. Since $J(x, x^*) = V_1(x)$, we find that increased volatility increases the value of the optimal takeover strategy as well. Analogous steps establish that increased volatility decreases the value of the optimal takeover strategy whenever the conditions $\eta < 1$ and $\delta(\eta) < r \leq \mu$ are satisfied.

References

Alvarez, L. and Stenbacka R. (2001) Adoption of uncertain multi-stage technology projects: A real options approach, *Journal of Mathematical Economics* **35**, 71–97.

Alvarez, L. and Stenbacka R. (2004) Optimal risk adoption: A real options approach, *Economic Theory* **23**, 123–148.

Andrade, G., Mitchell M., and Stafford, E. (2001) New evidence and perspectives on mergers, *Journal of Economic Perspectives* **15**, 103–120.

Bester, H. (1995) A bargaining model of financial intermediation, *European Economic Review* **39**, 211–228.

Croson, R. T. A., Gomes, A., McGinn, K. L., and Nöth, M. (2004) Mergers and acquisitions: An experimental analysis of synergies, externalities and dynamics, *Review of Finance* **8**, 481–514.

Fan, Hu and Sundaresan, S. S. (2004) Debt valuation, renegotiation, and optimal dividend policy, *The Review of Finanacial Studies* **13**, 1057–1099.

Holmström, B. and Kaplan, S. N. (2001) Corporate governance and merger activity in the U.S.: Making sense of the 1980's and 1990's, *Journal of Economic Perspectives* **15**, 121–144.

Jovanovic, B. and Rousseau, P. (2004) Mergers as reallocation, working paper, New York University.

Karlin, S. and Taylor, H. (1981) *A Second Course in Stochastic Processes*, Academic Press, Orlando.

Kauppalehti, 21 July, 2004.

Kauppalehti Optio, 11 December, 2003.

Lambrecht, B. (2004) The timing and terms of mergers motivated by economies of scale, *Journal of Financial Economics* **72**, 41–62.

Lambrecht, B. M. and Myers, S. C. (2006) A theory of takeovers and disinvestment, *Journal of Finance*, in press.

Maksimovic, V. and Phillips G. (2001) The market for corporate assets: Who engages in mergers and asset sales and are there efficiency gains, *The Journal of Finance* **56**, 2019–2065.

Mason, R. and Weeds, H. (2005) The timing of acquisitions, working paper, University of Southampton.

Morellec, E. (2004) Can managerial discretion explain observed leverage ratios, *The Review of Financial Studies* **17**, 257–294.

Morellec, E. and Zhdanov, A. (2005) The dynamics of mergers and acquisitions, *Journal of Financial Economics* **77**, 649–672.

Mulherin, J. H. and Boone, A. L. (2000) Comparing acquisitions and divestures, *Journal of Corporate Finance* **6**, 117–139.

Toxvaerd, F. (2004) Strategic merger waves: A theory of musical chairs, working paper, Faculty of Economics, University of Cambridge.