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Rationing in IPOs ⋆

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Abstract. We provide a model of bookbuilding in IPOs, in which the issuer can choose to ration shares. Before informed investors submit their bids, they know that, in the aggregate, winning bidders will receive only a fraction of their demand. We demonstrate that this mitigates the winner's curse, that is, the incentive of bidders to shade their bids. It leads to more aggressive bidding, to the extent that rationing can be revenue-enhancing. In a parametric example, we characterize bid and revenue functions, and the optimal degree of rationing. We show that, when investors' information is diffuse, maximal rationing is optimal. Conversely, when their information is concentrated, the seller should not ration shares. We provide testable predictions on bid dispersion and the degree of rationing. Our model reconciles the documented anomaly that higher bidders in IPOs do not necessarily receive higher allocations.

1. Introduction

Rationing in IPOs has been extensively documented.[1] Typically, at the offer price there is excess demand, and shares are rationed to investors. All investors, both informed and uninformed are rationed. Although explanations for the rationing of uninformed investors have been offered,[2] empirically, we observe that informed investors are also rationed. This is particularly puzzling, since informed investors submit price-contingent bids. For example, Cornelli and Goldreich (2001) examine the bids and allocations of one bookrunner in the UK. They find that informed bidders (those who submit these price-contingent or limit bids) only receive between 48.2% and 54.2% of the amount they bid for. Further, allocations that are received by limit bidders do not appear to depend on the level of their bid (contingent on it being higher than the offer price).

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[1] See, for example, Cornelli and Goldreich (2001), and the survey paper by Ritter (1998).

[2] One argument is that the investment bank needs to reward informed investors to convince them to reveal their information (see Benveniste and Spindt, 1989, for example).

The observed rationing of informed investors is counterintuitive for two reasons. First, it contradicts the optimal IPO mechanism of Benveniste and Spindt (1989), who postulate that, amongst informed bidders, those with higher signals receive their full allocation before any shares are given to bidders with lower signals. Second, it seems to contradict profit-maximization on the part of the seller. An IPO often proceeds by a bookbuilding process, during which a demand curve is generated for the shares to be sold.[3] Yet, faced with this demand curve, the seller frequently chooses a price below the market-clearing price. Why not choose a higher price, reduce the degree of rationing, and hence increase revenue? These concerns have brought IPO allocations to the forefront of policy debate.[4] Is rationing direct evidence of mispricing? That is, could a seller simply choose a higher offer price and increase revenue? Further, if the interests of the investment bank and entrepreneur differ, how much is the entrepreneur hurt by the investment banker's ability to allocate shares?

In this paper, we construct a stylized version of the bookbuilding process for a common value asset. Our mechanism includes the standard auction format as a special case. Indeed, our view corresponds to that expressed by Benveniste and Busaba (1997) and Sherman (2004), that auctions are merely a special case of book-building. We consider different allocation rules the seller may use. We show that it can be revenue-enhancing to ration, rather than choose a market-clearing price. Rationing shares at the offer price mitigates informed investors' fear of the winner's curse, and may thus increase seller revenue. Thus, oversubscription and rationing are not prima facie evidence that the issue price was sub-optimal, and the issuer could have raised more revenue. Finally, rationing allows for discretionary allocations among investors, which may have long-term benefits for the seller. This suggests that the current debate over discretionary allocations is misplaced to the extent that seller revenue is not hurt by such allocations.

Our model has empirical implications. First, for a specific class of signals, we numerically solve for the optimal degree of rationing. Thus, within our model, we are able to characterize the observed dispersion of bids and offer prices, and relate these to the allocations. We find that the larger the range of submitted bids and also winning bids, the higher the optimal degree of rationing.

Second, since rationing can increase the proceeds of the IPO, "money left on the table" cannot be estimated by the difference between the offer price and the long-term value of the asset. The offer price is determined by the bids submitted by investors. We demonstrate that the observed demand curve depends on the mechanism offered by the seller: if investors anticipate a different allocation mechanism (in particular, a different degree of rationing), they will submit different bids. Thus,

[3] A survey of international IPO practices is presented in Sherman (2004).

[4] For example, see Ritter and Welch (2002) for a survey of the academic questions. In the U.S., the SEC has recently expressed in this issue (e.g., "Harvey Pitt wants people to look at IPO pricing and allocations," *Wall Street Journal*, Aug 23, 2002).

caution must be exercised in performing these thought experiments on the demand curve.

The intuition that drives our results is straightforward. Changing the degree of rationing potentially affects two elements of the winner's curse: the expected consumption value of the asset conditional on winning, and the price the winner expects to pay. If informed investors are not rationed, then they only get shares when they are among the highest bidders. Thus, if they win the asset, their ex ante estimate of the value of the asset is higher than that of any bidder who did not get the asset. They take this into account when they bid; i.e., they optimally reduce their bids to avoid this winner's curse.[5] By contrast, consider pro rata rationing. Now, an investor can win the asset even when many other investors have higher signals. This causes every bidder to bid higher. However, to increase the degree of rationing, a seller selects a bidder with a lower estimate of the value of the shares. The optimal degree of rationing is determined by this tradeoff.

While we do not explicitly distinguish between issuers (i.e., firms going public) and underwriters, it is reasonable to suppose that their interests may diverge.[6] Indeed, NASD has recently suggested explicit prohibitions on the preferential allocation of shares to investors, provided in consideration of future business. This practice (called spinning) suggests that investment bankers do provide preferential allocations to some investors. While not all preferential allocations are necessarily bad, it is important to recognize that such discretion cannot exist unless there is oversubscription. Thus, even if rationing is sub-optimal, issuers need not be substantially damaged by these idiosyncratic allocations. This view is supported empirically by Ljungqvist and Wilhelm (2002), who conclude that discretionary allocation does not harm issuers.

In our model, a bookrunner has k units for sale and commits to allocate them across t bidders at the $(t+1)^{st}$ highest bid. In particular, t may be greater than k, in which case the seller rations. For example, with pro rata rationing, all winners may be allocated an amount $\frac{k}{t}$. Bidders in our model all receive signals about the value of the asset. They then submit sealed bids for the asset. Thus, we are considering informed bidders who submit limit bids (in particular, their bid consists of a price they are willing to pay).

Our model is, of course, a highly stylized description of an IPO process. In the U.S., an IPO is typically preceded by a "road show", during which the underwriter makes presentations to groups of buyers in various cities, and often meets with important buyers one-on-one (see, for example, Ritter, 1998). During this road show, the lead investment banker also solicits information from the buyers on quantities they are interested in buying, and the associated prices at which they

5 Nyborg, Rydqvist and Sundaresan (2002) find evidence that bidders compensate for the winner's curse in Swedish Treasury auctions.

6 Biais, Bossaerts and Rochet (2002) present a model in which institutional investors collude with bankers against issuers. The recent \$100 million settlement between Credit Suisse First Boston and the SEC supports this view.

are willing to buy. In this book-building process, “a demand curve is constructed” (Ritter, 1998). In our model, we interpret the construction of this book or demand curve as analogous to soliciting sealed bids from potential buyers. To the extent that a buyer is unaware of the price and quantity pair submitted by another buyer, this is equivalent to the simultaneous submission of sealed bids.

There are several features of the IPO process that we omit from our model. First, we fix the demand for each buyer to be the same.[7] Second, we consider a simultaneous game, in which bidders bid only once. With IPOs, the bookbuilding process is usually followed by the seller announcing a price at which shares will be sold. Bidders are then allowed to re-submit quantities they wish to buy at this price. In other words, they may be given a chance to revise their bids, which we do not allow in our model. However, Welch (1999) mentions that “In reality, an institutional investor who backs out after such informal requests (especially if it is close to the effective date) may not receive shares in future offerings; consequently, such indications of interest are practically firm.”

A seminal theoretical piece on rationing, the winner’s curse, and IPO underpricing is Rock (1986). Though our model contains both rationing and a winner’s curse, it is starkly different. In Rock’s model, there are two states of the world. In the good state (i.e., when the value of a share exceeds its offer price), both informed and uninformed investors demand shares. In the bad state (when the value of a share is less than the offer price), informed bidders withdraw from the process, and uninformed bidders obtain an excessive number of shares. Hence, uninformed investors receive higher allocations in the bad state, and are therefore subject to the winner’s curse. To compensate them for this asymmetry in allocations across states, the offer price must be lowered. Though the probability of receiving an allocation in the good state is lower, reducing the price sufficiently increases their expected revenue enough to make them willing to participate in the mechanism.

In our model, all bidders are informed and the degree of rationing is known ex ante (that is, the seller chooses a rationing mechanism before any bids are submitted). Importantly, the seller commits to a rationing rule that does not depend on the state of the world. In contrast, in Rock (1986), rationing varies across states in equilibrium: investors experience greater rationing in the good state. The variation in ex post rationing hurts Rock’s seller (it leads to lower revenue); the commitment to ex ante rationing in our model benefits the seller. That is, in Rock’s model ex post rationing is the embodiment of the winner’s curse and generates underpricing, while in our model, ex ante rationing mitigates the winner’s curse.

The distinction between ex post and ex ante rationing also distinguishes our work from Benveniste and Spindt (1989).[8] In their framework, the underwriter

[7] In multi-unit settings, Back and Zender (1993), building on work by Wilson (1979), show that the uniform price auction has self-enforcing collusive equilibria, leading to lower revenue than discriminatory auctions.

[8] Biais and Faugeron-Crouzet (2002) demonstrate that, given a proper choice of parameters, the French *mise en vente* mechanism replicates the optimal Benveniste–Spindt one.

presells some of the issue to informed regular investors.[9] As an incentive to reveal information, investors with good information get the quantity they demand before any shares are allocated to investors with lower signals. Therefore, they predict that, across all investors who receive shares, investors with higher bids must be rationed less than investors with lower bids. As mentioned, this contradicts the empirical observations of Cornelli and Goldreich (2001), who argue that allocations are independent of bids above the offer price. In contrast, in our model, equilibrium allocations can be achieved by pro rata rationing, in which all investors who receive shares are equally rationed. An alternative is to randomly choose a subset of potential winning bidders that will receive shares. In this case, it is feasible in equilibrium to have some high bidders receive no shares, though some lower bidders receive shares.

While both Benveniste and Spindt (1989) and the model in this paper consider financial assets to be common value objects, a critical difference is that our investors receive correlated signals. Benveniste and Spindt provide an optimal IPO mechanism when investors have independent signals. Since we have correlated signals, their mechanism is no longer optimal. The optimal mechanism with correlated signals is exhibited by Cremer and McLean (1988) and McAfee et al. (1989), who show that it is possible for the seller to extract the full surplus from buyers (i.e., eliminate all underpricing). However, surplus extraction mechanisms include a strong individual rationality constraint on the buyers. A seller must have the ability to potentially inflict large punishments on the buyers. Such mechanisms are not observed in practice, partly because the seller's ability to penalize the buyers is limited.

Our model draws upon previous work in auction theory, notably Milgrom (1981) and Pesendorfer and Swinkels (1997). Milgrom considers an auction for a common-value object, where the seller has t units for sale and each buyer demands one unit. The t highest bidders are each given one unit, at the $(t+1)^{st}$ highest bid. Pesendorfer and Swinkels (1997) show that the t-unit auction has a unique symmetric equilibrium (albeit under conditions that are stronger than those in our paper) and examine properties of the convergence of the price to the true value of the object. We show that the equilibria in this auction are identical to those in our rationing mechanism.

To the extent that bidders bid one price in our model, the framework is different from Wilson (1979). In the latter, bidders submit demand functions.[10] We assume constant marginal valuation for each fractional amount of one unit, or flat demand curves. Hence, our bidders submit a single price.

[9] We note that, in our model, we consider only informed bidders, and hence (unlike Benveniste and Spindt), we provide no predictions on the relative allocations between informed and uninformed bidders.

[10] Other papers that consider models in which bidders submit demand (or supply) functions include Klemperer and Meyer (1989), Kyle (1989), Back and Zender (1993), and Kremer and Nyborg (2004).

In a model with independent signals and (almost) common values, Bulow and Klemperer (2002) demonstrate conditions under which rationing is the optimal mechanism for the seller. A sufficient condition they identify is that the hazard rate of signals must be decreasing. Under this condition, the optimal mechanism involves the seller posting a price, at which all buyers are willing to buy the good. While this condition is violated in our example with a uniform distribution in Section 4, Bulow and Klemperer offer an intuition for our results in terms of a game they introduce called the "Maximum Game." We comment on this in greater detail in Section 5.

The rest of this paper is organized as follows. Section 2 outlines our general model. Section 3 examines some properties of bid and revenue functions in the general case. Section 4 discusses a special case of the signal distribution that proves more tractable and allows for explicit revenue comparisons, and is followed by a discussion of our results in Section 5 and some concluding remarks in Section 6. All proofs are relegated to the Appendix.

2. Model

We model the bidding behavior of informed investors in an IPO. The seller wishes to sell k shares to $n > k$ investors, each of whom demands the same number of shares which we normalize to one. Following Benveniste and Busaba (1997) and Sherman (2004), we model the bookbuilding mechanism as a generalization of a multi-unit common value auction. In practice, of course there are many distinctive features of bookbuilding, including information sharing and gathering.[11] However, once bids have been entered in the book, they are essentially firm. We thus view them as sealed bids.

In the bookbuilding mechanism, the seller announces a rationing rule, which has two components. First, the seller chooses a t, where $k \leq t \leq n-1$. He promises that the IPO offer price will be equal to the $(t+1)^{st}$ highest bid.[12] If $t > k$, he further announces an allocation rule, to divide the shares across the t highest bidders. An investor then receives a private signal about the value of the asset, and submits a bid for one share. Finally, based on the book the seller announces an offer price and the k shares are allocated to investors.

The offer price thus depends on, and reflects information in, the received bids. In addition, the seller must decide on the allocations to be given to investors who bid more than the offer price. When $t = k$, each of the highest k bidders is allocated

[11] Spatt and Srivastava (1991) discuss some of these features, including the communication between the underwriter and the investors.

[12] Consistent with the mechanism design literature (including Benveniste and Spindt, 1989), we assume the seller can commit to a mechanism. Suppose, instead, the seller cheated and were to set the offer price equal to the expected value of the asset conditional on all bids. This would violate incentive compatibility for the bidders. No bidder (even a bidder with a high signal) would have an incentive to make a high bid.

one share. This effectively represents market-clearing. That is, there is no rationing, and the allocation received by the winning bidders is 100% of what they requested. In contrast, for $t > k$, there is rationing. Each of the t bidders have indicated a willingness to buy the item at strictly greater than the offer price. Hence, in the aggregate, potential winners are rationed to a degree $1 - \frac{k}{t}$.

DEFINITION 1. Consider some integer $t \geq k$. In a (k, t)*-bookbuilding mechanism*:

(i) the offer price is set to the $(t+1)^{st}$ highest bid. Each bidder who receives a positive allocation pays this price.
(ii) the k shares are allocated amongst the t highest bidders. No bidder receives more than her demand (i.e., one share), and,
(iii) conditional on being amongst the t highest bidders, the allocation received by an agent is independent of her bid.

Many allocation rules are consistent with this definition, including:

(i) pro rata rationing: each investor in the set of potential winners receives $\frac{k}{t}$ shares,
(ii) random allocation: the seller randomly chooses k of the t potential winners; each of these k bidders receives their full demand (one share), and the others receive nothing.
(iii) discretionary allocation: the seller wishes to reward a set of long-term customers, investor 1 through i. These customers (preferred investors) receive their full quota (i.e., one share) if they are in the set of potential winners, and nothing otherwise. Other winners (regular investors) have their allocations reduced accordingly, either in pro rata or random fashion.

The shares have a common value to all investors, V, which is drawn from an atomless distribution, $F(\cdot)$, on $[v_\ell, v_h]$. V represents the long-term value of the asset.[13] We emphasize that we do not expect V to be represented by the price at the end of the first day (or the first week) of trading in the secondary market.

Investors have private information about the long-term value of the asset. The information of investor i is represented by a signal, S_i. The signals of different investors are conditionally independent (given V), but all depend on V in the following manner. Each S_i, for $i = 1, \ldots, n$, is independently drawn from the same atomless distribution $G(\cdot \mid V = v)$, with support $[v - \epsilon, v + \epsilon]$ for some $\epsilon > 0$. Here, ϵ represents dispersion of opinion about the value of the asset. We assume that $v_h - v_\ell > 2\epsilon$, so that informed investors' beliefs are more precise than the prior over V.

[13] Notationally, we use upper case letters denote random variables, and lower case ones to denote particular realizations of random variables. Thus, v denotes a realization of random variable V.

Hence, given V, the height of the conditional signal distribution at any signal, s, depends only on the position of the signal, relative to the lowest possible value $(v - \epsilon)$. Therefore, a higher value of v leads to a shift in the support of the signal distribution, but the distribution has the same shape, given the support.

Formally,

ASSUMPTION 1.

(i) $G(s \mid v)$ is an atomless distribution with support $[v - \epsilon, v + \epsilon]$, and density $g(s \mid v)$.

(ii) For any pairs (s, v) and $(\tilde{s}, \tilde{v})$, if $s - v = \tilde{s} - \tilde{v}$, then $G(s \mid v) = G(\tilde{s} \mid \tilde{v})$.

(iii) (MLRP): $\frac{g(s|v)}{g(s|v')} \geq \frac{g(s'|v)}{g(s'|v')}$ for all $s > s', \quad v > v'$ such that s, s' are both in the support of $G(\cdot \mid v)$ and $G(\cdot \mid v')$ respectively.

Part (ii) of the assumption further implies that $g(s \mid v) = g(\tilde{s} \mid \tilde{v})$ when $s - v = \tilde{s} - \tilde{v}$. Part (iii) is a variant of the Monotone Likelihood Ratio Property.[14]

Note that, in our model, ϵ is independent of V. Intuitively, this means that the dispersion of investors' opinions over value does not depend on whether V is high or low. That is, ϵ does not depend on the long-term per share price of the asset. Empirically, ϵ can be inferred from the range of analyst forecasts over the value of the asset.[15]

We consider one-shot equilibria of the bookbuilding game. In practice, the set of bidders in an IPO varies from issue to issue. For example, in the 39 issues that Cornelli and Goldreich (2001) analyze, only 16.8% bid in at least 10 issues. There are, therefore, several bidders who rarely participate in more than one deal. These players will perforce bid as if in the one-shot game. Further, amongst the long-term players, absent explicit collusion, none knows the specific set of bidders participating in a specific IPO. Since long-term bidders also pick and choose transactions, the usual punishments seen in repeated games are difficult to enforce. Hence, we consider equilibria of the one-shot game among investors, and ignore repeated game effects in bidding.

2.1. EQUILIBRIUM IN THE BOOKBUILDING MECHANISM

Given an allocation rule, a bidder observes her own signal s, and chooses a bid. We consider a symmetric Bayesian-Nash equilibrium, in which all bidders choose the same bid function, and a bidder with signal s bids $b(s)$. It is natural to consider

14 In our model, MLRP cannot hold for all s, s', v, v' because a particular s can lie outside the support of $G(\cdot \mid v)$, and hence have a density of zero.

15 Analyst forecasts have been used to proxy for the dispersion of beliefs over asset values in the accounting literature; see, for example, Ajinkya, Atiase, and Gift (1991), and the subsequent literature.

equilibria in which $b(\cdot)$ is strictly increasing in s; that is, bidders with higher signals submit higher bids.

Let $Y_{j,n}$ be a random variable representing the j^{th} highest order statistic of bidders' signals, where n signals are drawn. Hence, $Y_{j,n} \geq Y_{j+1,n}$ for all $j = 1, \ldots, n-1$.

We first show that the equilibrium bids (and, by extension, seller revenue) do not depend on the particular allocation rule used, provided it satisfies the property that the allocations are independent of the actual bids of agents who bid more than the offer price. The intuition is that, conditional on being amongst the t highest bids, the allocation rule exposes each agent to a lottery. If the agent does not have one of the t highest bids, she gets zero allocation, with no access to the lottery. As long as the probability of receiving an allocation in the lottery is independent of the bids of any agent in the set of potential winners (as required by part (iii) of Definition 1), the behavior of a risk-neutral agent will be unaffected.

PROPOSITION 1. Consider any two allocation rules within a (k, t)-bookbuilding mechanism. These rules result in the same set of equilibria.

Proposition 1 implies that all allocation rules allowed for by the mechanism are equivalent (in terms of bids and revenues) to the pro rata rationing rule with no discretionary allocation, though each of these implies a very different set of final allocations. Thus, if a seller allocates shares in a way that benefits clients with whom he has a long term relationship, this is not to the detriment of the issuer.

Within this class of allocation rules, there are several advantages to pro rata rationing, in particular. Many exchanges have requirements on the distribution of shares across investors.[16] Thus, one of the goals of an IPO must be to generate a dispersed shareholder base. It has also been suggested that share dispersion per se increases the value of a firm.[17] In addition, Brennan and Franks (1996) argue that rationing is used because current owners want to reduce the block size of new shareholders.

Markets which exhibit pro rata rationing include Singapore (Koh and Walter, 1989), Israel (Amihud et al., 2003), and the UK (Levis, 1990). Ljungqvist and Wilhelm (2002) describe the allocation methods used in the UK, Germany, France and the US. In Germany, the June 7, 2000, guidelines promulgated by the Federal Ministry of Finance tries to rule out "subjective" criteria for allocating shares to retail investors. It recommends that issuers draw lots, allocate pro rata either within certain order sizes or across the whole offer, or allocate according to time priority or some other "objective" criteria. One of the mechanisms adopted in France, the

[16] For example, the NYSE requires at least 500 holders of round lots, while the NASD requires 400.

[17] Booth and Chua (1996) consider a model in which dispersed ownership increases secondary market liquidity and hence the value of the firm. Sherman (2000) comments on benefits to the seller and Stoughton and Zechner (1998) on benefits to the issuer.

offre à prix ferme, has pro rata allocations at a fixed price. Further, the recent legal troubles of Salomon, Smith Barney over preferential allocation of shares in IPOs[18] enhance the appeal of pro rata rationing, with no discretionary allocation, in the US.

We show that the equilibrium bids of the (k, t)- bookbuilding mechanism (in which the seller has k shares for sale, and rations across $t > k$ investors), is identical to the corresponding set when the seller actually sells t shares with no rationing. For a fixed t, in either of these mechanisms, a winning bidder has the same information: she knows she has one of the t highest signals. Hence, her bids in the two mechanisms are the same.

The symmetric equilibrium of the t-unit common-value auction is characterized by Milgrom (1981). Without loss of generality, consider the behavior of bidder 1. As Milgrom (1981) shows, bidder 1, with signal s, bids as if her signal is equal to the t^{th} highest (or pivotal) signal amongst the remaining $(n - 1)$ bidders. This is the price at which she is indifferent between winning and losing the asset. In a symmetric equilibrium, all bidders choose this strategy.

PROPOSITION 2. In a (k, t)-bookbuilding mechanism,

(i) the set of equilibria is equivalent to the set of equilibria in the $t-$unit auction.
(ii) there is a symmetric equilibrium in which a bidder with signal s chooses a bidding function

$$b(s; t) = E[V \mid S = Y_{t,n-1} = s].$$

The offer price is set by the $(t + 1)^{st}$ highest bid. Since the bidding function is monotone, this is determined by the bidder with the $(t + 1)^{st}$ highest signal, who bids as if she has same signal as the t^{th} highest bidder. In practice, since the $(t+1)^{st}$ highest bidder is a losing bidder, the t^{th} highest bidder has an even higher signal. Hence, the expected value of the asset is always higher than the $(t + 1)^{st}$ highest bid (that is, the offer price). Therefore, in equilibrium there is no winner's curse. Bidder's rationally anticipate the information content in winning, and adjust their bids accordingly. Since the offer price is less than the expected value of the asset, conditional on the $(t+1)^{st}$ highest bid, there is no ex post regret, and each winner is happy to receive an allocation. We show that for a fixed k, the bid function, $b(s; t)$ is strictly increasing in t.[19] Since t is directly related to the degree of rationing $(1-\frac{k}{t})$, the bid function increases with rationing. This implies that the seller can benefit from rationing – bidders bid more aggressively. There is, of course, the obvious

[18] See, for example, "Ex-Broker Says Salomon Gave IPOs to CEOs to Win Business," *Wall Street Journal*, July 18, 2002.

[19] Except, of course, for the extreme signal values, $v_\ell - \epsilon$ and $v_h + \epsilon$. At these values, which occur with zero probability, the true value of the asset is known with certainty.

cost of choosing a lower ranked bid to set the offer price. However, we show later that, under some conditions, the seller can increase her revenue by rationing.

PROPOSITION 3. In the (k, t)-bookbuilding mechanism, bid functions are strictly increasing in the degree of rationing. That is, $b(s; t + 1) > b(s, t)$ for all $s \in (v_\ell - \epsilon, v_h + \epsilon)$ and all $t = k, \ldots, n - 2$.

Intuitively, a higher value of t implies that a bidder may win the object even if he did not receive the highest signal. Thus, the "winner's curse" is mitigated: conditional on winning, a bidder in the (k, t)-bookbuilding mechanism knows only that her signal was among the t highest. Hence, the greater the degree of rationing in the mechanism, the more aggressive is each bidder.

Thus, even when the number of shares being sold is held fixed, the equilibrium bids in the book will vary with the seller's commitment to rationing. What can we infer about v from realized bids (i.e., the book)? We show that, for low t, bidders shade their bids, in the sense of bidding less than the conditional value of the asset, given their signal. However, for high t, they increment their bids, by bidding more than this conditional asset value. This effect, therefore, must be taken into account in inferring bidders' beliefs on the conditional value of the asset from an observed book.

PROPOSITION 4. Consider the (k, t)-bookbuilding mechanism. For all $s \in (v_\ell - \epsilon, v_h + \epsilon)$, there exists a $\hat{t}(s)$ such that, for $t \leq \hat{t}(s)$, $b(s; t) \leq E(V \mid s)$, and for $t > \hat{t}(s)$, $b(s; t) > E(V \mid s)$.

In Section 4, in a parametric model, we characterize the dispersion of bids submitted to the book and the dispersion of bids that receive positive allocations, and tie these to the degree of rationing.

3. Bid and Revenue Functions with Uniform Prior

3.1. BIDS

To characterize the bid function further, we assume for the rest of the paper that $F(\cdot)$, the prior over V, is uniform over $[v_\ell, v_h]$. Because this prior has finite support, the inference drawn from any signal depends on its size relative to the endpoints. Consider a signal in the range $[v_\ell + \epsilon, v_h - \epsilon]$. For any such signal s, the posterior over V has support $[s - \epsilon, s + \epsilon]$. We term such signals interior signals. For a signal $\tilde{s}$ in the range $[v_\ell - \epsilon, v_\ell + \epsilon)$, the posterior over V has a truncated support $[v_\ell, s + \epsilon]$ because the lowest possible value of V is v_ℓ. Such signals, and those in the range $(v_h - \epsilon, v_h + \epsilon]$, are dubbed corner signals.

We first show that, for interior signals, bids are linear in signal. In particular, the bid function can be written as the signal plus an adjustment term that compensates for the size of the winner's curse. This adjustment term is increasing in the number of investors, n, but decreasing in the degree of rationing.

PROPOSITION 5. Consider the (k, t)-bookbuilding mechanism. Suppose $s \in [v_\ell + \epsilon, v_h - \epsilon]$. Then, in equilibrium, the bid function is linear in signal. In particular, $b(s; t) = s + \epsilon(1 - 2\delta(n, t))$, where δ is increasing in n, decreasing in t, and is independent of s and ϵ.

In other words, regardless of the shape of $G(\cdot)$, the bid function over interior signals is linear, and takes the form of the signal plus a constant (which may be positive or negative). Further, a change in t leads to a parallel shift of the bid function. The term representing the winner's curse adjustment, $\delta(\cdot)$, may be greater or less than $\frac{1}{2}$. It is greater than $\frac{1}{2}$ for small values of t, and less than $\frac{1}{2}$ for large values of t. Hence, when t is low, bidders bid less than their signal, and, when t is high, they bid more than their signal.

For signal ranges in the corners, $s \in [v_\ell - \epsilon, v_\ell + \epsilon)$ and $s \in (v_h - \epsilon, v_h + \epsilon]$, the bid function is non-linear. It is still increasing in n and decreasing in t, but depends on signal as well. In Section 4, we exhibit the bid function over this range for a particular signal distribution.

3.2. REVENUE

We next examine the effect of rationing on revenue. As the degree of rationing increases, the seller sets the offer price at the bid of a bidder with a lower signal. Thus, the seller trades off the increase in the bid function against the fact that he is awarding shares to lower signal bidders. In this section, we characterize the revenue function of the seller. We provide intuition about our results in terms of bidders' marginal revenues (along the lines of Bulow and Klemperer, 2002) in Section 5.

Let $R(v; t)$ denote the seller's expected revenue per share in the (k, t)-bookbuilding mechanism, if the realization of V is v.[20] The offer price is the expected $(t + 1)^{th}$ highest bid, out of n signal draws. That is, $R(v; t) = E\,[b(Y_{t+1,n};\ t) \mid V = v]$. As the seller in our model is uninformed, he earns an ex ante revenue of $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v; t) dv$. He optimally chooses a t that maximizes $\hat{R}(t)$. Market-clearing corresponds to choosing $t = k$, whereas larger values of t imply rationing.

A necessary condition for seller revenue, $\hat{R}(t)$, to increase in t (over any range of t) is that $R(v; t)$ be increasing in t for some t and some v. Hence, we focus initially on $R(v; t)$. When signals are in the interior, the bid function is linear. When $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, all signals are interior. Hence, it follows immediately that the revenue function is linear in v over this range. In particular, the revenue function can be written in terms of the winner's curse adjustment, $\delta(n, t)$.

[20] Since the number of shares to be sold is fixed at k, maximizing revenue per share is equivalent to maximizing total revenue.

As in the proof of Proposition 5, we can define $\delta(n,t)$ as follows. First, let $x = \frac{s-(v-\epsilon)}{2\epsilon}$. Then, x is a variable defined over the interval [0, 1], which has the property that $H(x) = G(s|v)$ for all $s \in [v-\epsilon, v+\epsilon]$ (since the distribution of s given v depends only on $s-(v-\epsilon)$). Let $\tilde{x}$ represent a single draw of X, and let $X_{t,n}$ be the t^{th} highest of an independent sample of n draws from X. Then, $\delta(n,t)$ is defined as $E(\tilde{x} \mid \tilde{s} = X_{t,n-1})$.

Now, the revenue function can be written in terms of $\delta(n,t)$ and $X_{t+1,n}$, the $(t+1)^{st}$ highest order statistic from a sample of n draws of X.

PROPOSITION 6. Consider the (k,t)-bookbuilding mechanism, and an interior value of v, so that $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then, for all $t \leq n-1$, $R(v;t) = v - 2\epsilon(\delta(n,t) - E[X_{t+1,n}])$.

This form of the revenue function illustrates the tradeoff for the seller. For each value of V in the relevant range, increasing t leads to a lower winner's curse, and hence a lower adjustment $(\delta(n,t))$ and a higher revenue. However, increasing t leads to a reduced order statistic being used to set the price of the item, captured by the $E(X_{t+1,n})$ term (which is declining in t).

The linear revenue function allows us to determine when rationing increases revenue. Indeed, the seller compares the increase in revenue (in going from a (k,t)-to a $(k,\tilde{t})$-bookbuilding mechanism) obtained as a result of the increase in the bid functions to the decrease in revenue as a result of choosing the $(\tilde{t}+1)^{st}$ bid to set the price of the item, rather than the $(t+1)^{st}$ one. The left-hand side of Equation (1) below is the increase in revenue and the right-hand side is the decrease.

COROLLARY 6.1. Consider an interior value of v, so that $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then, for any $t, \tilde{t} = k, \ldots, n-2$, $R(v;\tilde{t}) > R(v;t)$ if and only if

$$\delta(n,t) - \delta(n,\tilde{t}) > E[X_{t+1,n}] - E[X_{\tilde{t}+1,n}]. \tag{1}$$

Notice, when $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, the comparison of $R(v;t)$ and $R(v;\tilde{t})$ is independent of the actual value of V in this range. That is, if (1) holds for some $t, \tilde{t}$, then $R(v;\tilde{t}) > R(v;t)$ for all $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. This condition is easy to check for any distribution.

While (1) is a necessary condition for higher degrees of rationing to yield higher revenue, clearly there are values of ϵ (given v_ℓ and v_h) for which it is also a sufficient condition. In particular, if (1) holds and ϵ is small relative to $v_h - v_\ell$, then ex ante revenue must be increasing in the degree of rationing. To see this, suppose (1) holds for some distribution G and some $\tilde{t} > k$, so that $R(v;\tilde{t}) > R(v;t)$ for all $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. In the corner regions, $V \in [v_\ell, v_\ell + 2\epsilon)$, and $V \in (v_h - 2\epsilon, v_h]$, we may have $R(v;\tilde{t}) < R(v;t)$ over some range. However, the difference, $R(v;t) - R(v;\tilde{t})$ is bounded. Hence, it follows that, if ϵ is small enough relative to $(v_h - v_\ell)$, then $\hat{R}(\tilde{t}) > \hat{R}(t)$. Thus, if signals are precise relative to the prior over V, (i.e., ϵ small), then (1) is also a sufficient condition.

4. Parametric Signal Distribution

From the tradeoff explored in the previous section, it is clear that the optimal choice of t depends on the signal distribution. To explore this further, and to provide some economic intuition for this tradeoff, we consider a broad class of signal distributions that has the uniform as a special case. We determine the bid and revenue functions, and numerically solve for the optimal t, given a signal distribution. This allows us to compute endogenous bid ranges, and generate empirical predictions. First, assume that the signal distribution is generalized uniform. The extra parameter c allows for strict convexity of the distribution function.

ASSUMPTION 2. For $s \in [v - \epsilon, v + \epsilon]$, and some $c \geq 1$,

$$G(s \mid v) = \left(\frac{s - v + \epsilon}{2\epsilon}\right)^{c}, \text{ and } g(s \mid v) = \frac{c}{2\epsilon} \left(\frac{s - v + \epsilon}{2\epsilon}\right)^{c-1}.$$

Figure 1 plots the density and distribution functions for different values of c. When $c = 1$, the distribution is uniform and signals are most diffuse; that is, all signals in the range are equally likely. For $c > 1$ it is strictly convex. As c becomes large, the distribution becomes more concentrated. Indeed, as $c \to \infty$, the signal distribution converges to a point mass at $v + \epsilon$. Increases in c thus represent first-order stochastic shifts in the underlying density function. For all $c \geq 1$, this distribution satisfies Assumption 1 (MLRP).

Given this signal distribution, we determine closed form expressions for the bid functions. For interior signals, that is $s \in [v_\ell + \epsilon, v_h - \epsilon]$, the bid function is linear whereas for corner signals, the bid function is more complicated.

PROPOSITION 7. In the (k, t)-bookbuilding mechanism, with generalized uniform signals,

(i) for interior signals, $s \in [v_\ell + \epsilon, v_h - \epsilon]$, the equilibrium bid function is

$$b(s; t) = s + \epsilon \left(1 - 2 \prod_{j=0}^{t-1} \frac{n - j - \frac{1}{c}}{n - j}\right).$$

(ii) For corner signals, $s \in [v_\ell - \epsilon, v_\ell + \epsilon)$ or $s \in (v_h - \epsilon, v_h + \epsilon]$, the equilibrium bid function is $b(s; t) = s + \epsilon(1 - 2\tilde{\delta}(s, n, t, \epsilon))$, where

$$\tilde{\delta}(s, n, t, \epsilon) = \begin{cases} \dfrac{\phi(\bar{x}(s)/2\epsilon, 0)}{\phi(\bar{x}(s)/2\epsilon, \frac{1}{c})} & \text{if } s < v_\ell + \epsilon \\[2ex] \dfrac{\phi(1, 0) - \phi(\underline{x}(s)/2\epsilon, 0)}{\phi(1, \frac{1}{c}) - \phi(\underline{x}(s)/2\epsilon, \frac{1}{c})} & \text{if } s > v_h - \epsilon \end{cases}$$

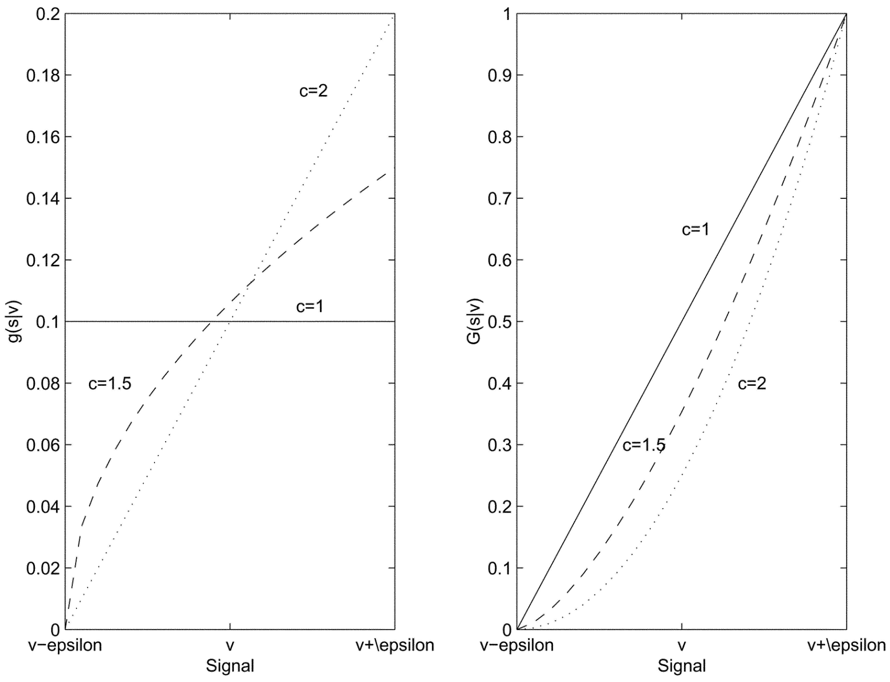

Figure 1. Density (left) and distribution (right) functions, for different values of c.

and

$$\phi(x, y) = \sum_{i=0}^{t-1} \frac{x^{c(n-i-y)}\ (1 - x^c)^i}{i!\ \prod_{j=i}^{t-1}(n - j - y)}.$$

We provide a numerical characterization of the bid functions based on the results of Cornelli and Goldreich (2001). They document that (i) there are an average of 38.9 limit bids and 9.1 step bids[21] per issue (Table III), (ii) the average rationing across limit and step bids is approximately 50% (Table III), (iii) 21.5% of limit bids and 9.8% of step bids are allocated no shares, since the prices are below the offer price (page 2343), and (iv) the mean IPO offer price is \$23.6 (Table I). For our numeric example, therefore, we assume $n = 50$. As approximately 20% of all bidders get no shares, and those that do are rationed by about 50%, we use $t = 40$ and $k = \frac{t}{2} = 20$. Thus, $t = 20$ represents the case of 100% allocation, and $t = 40$ the case of 50% allocation. We further use $v_\ell = 10$, $v_h = 40$, and $\epsilon = 5$.

Figure 2 plots the bid functions for the case of $c = 1.5$ when $t = 20$ and $t = 40$. The vertical difference between the two functions represents the difference in the winner's curse adjustment between the two rationing levels.

21 These are the two kinds of bids that are price-contingent in their study.

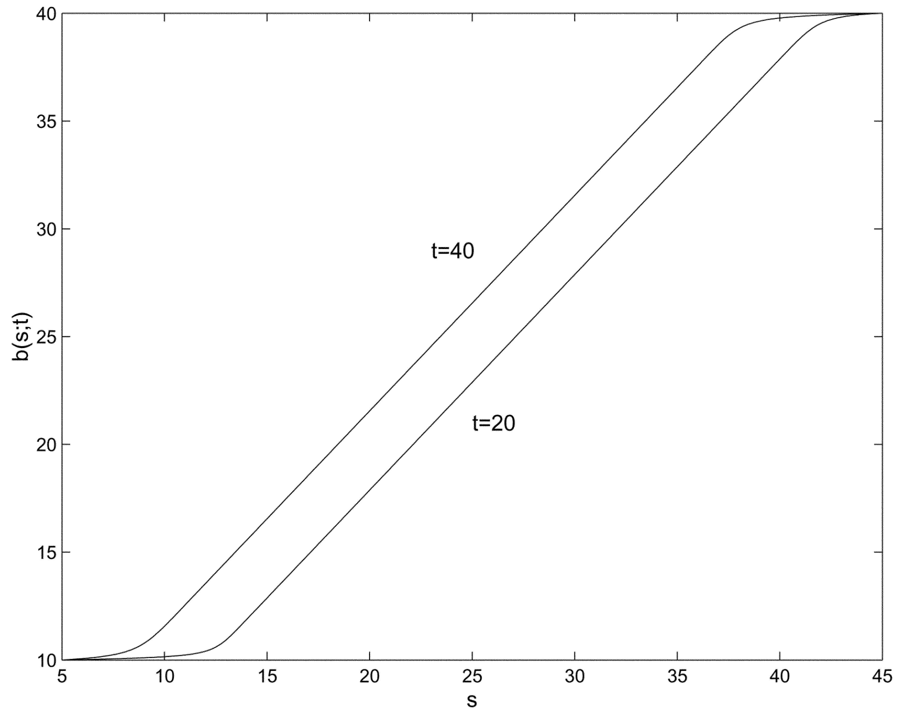

Figure 2. Bid functions for $c = 1.5$, $t = 20$, $t = 40$.

While it is immediate from Proposition 7 that $b(s;t)$ is increasing in t, the extent of the increase depends on the winner's curse adjustment and hence c. For $c = 1$, increasing t has a large effect on bids. As c gets large, changing t has a smaller effect on bid functions. When c is large and t is low, there is potentially a large winner's curse. Consider, for example, the case of $c = 2$ and $t = k$. If a bidder wins, since the distribution function is steep at high s, her signal is likely to be further away from other bidder's signals than if c were, say, 1. Hence, for low values of t, bidders should shade their bids (relative to their signals) more for higher values of c. When t is high, the reverse logic holds: there is a probability that some bidders have higher signals, and, when the distribution function is steep, other bidder's signals can be significantly higher than that of the winning bidder.

The t that we expect to observe in IPO deals is the t that maximizes seller revenue. Given a value of c, i.e., dispersion of investors' beliefs, the seller chooses an optimal degree of rationing to maximize ex ante revenue $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v;t)dv$. However, as argued before, if rationing leads to higher revenue for all v in the interior, and if ϵ is small relative to $(v_h - v_\ell)$, then ex ante revenue $\hat{R}(t)$ is also higher for the seller with rationing. Thus, we first characterize the revenue for a given value of v in the interior with a degree of rationing, t. We provide a closed form expression for condition (1).

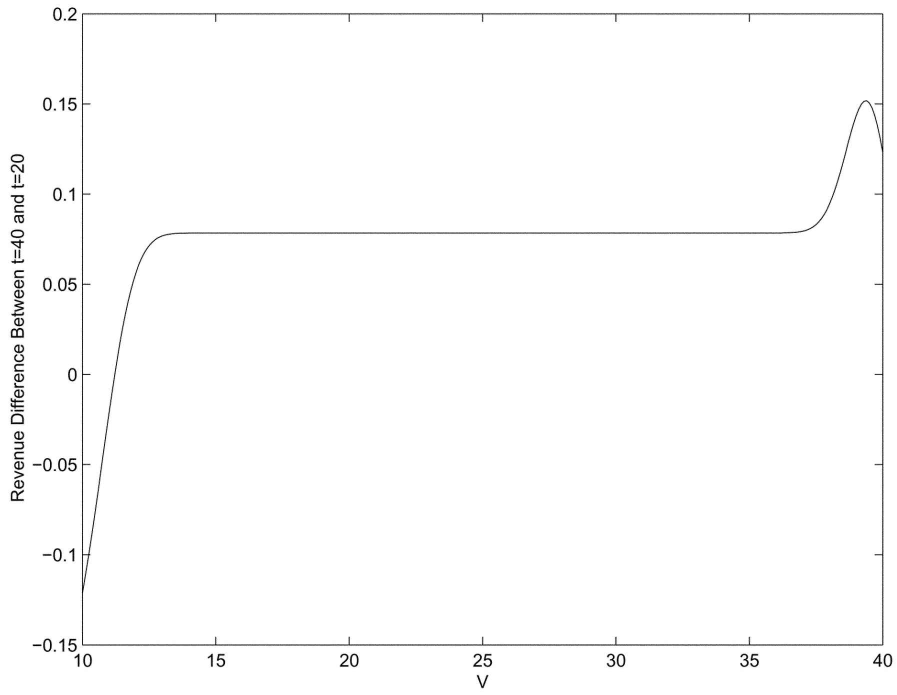

Figure 3. Revenue difference between $t = 40$ and $t = 20$, for $c = 1$.

PROPOSITION 8. Suppose v is in the interior; that is, $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then, in equilibrium,

(i) $R(v; t) = v - 2\epsilon \left\{ \prod_{j=0}^{t-1} \frac{n-j-\frac{1}{c}}{n-j} - \prod_{j=0}^{t} \frac{n-j}{(n-j+\frac{1}{c})} \right\}$

(ii) $R(v; t+1) > R(v; t)$ if and only if

$$\left(\frac{n - t - 1 + \frac{1}{c}}{n - t} \right) \prod_{j=0}^{t-1} \frac{n - j - \frac{1}{c}}{n - j} > \prod_{j=0}^{t} \frac{n - j}{n - j + \frac{1}{c}}. \tag{2}$$

Next, we consider the effects of rationing when V is in the corners; that is, $V < v_\ell + \epsilon$ or $V > v_h - \epsilon$. For v in this range, with positive probability signals will lie outside the interior segment, so that the linear bid function no longer applies. In this case, an analytical examination of the change in revenue due to rationing is difficult. Instead, we numerically evaluate the effects of rationing. Figure 3 below provides the difference in revenues, in going from $t = 20$ to $t = 40$ for all values of v (including the corners) in our numeric example.

Note that, as v approaches v_ℓ, rationing loses its lustre, and leads to lower revenue. From the seller's point of view, the benefit to rationing is that, by mitigating the winners' curse, it induces bidders to bid more aggressively. This effect is at its strongest when a bidder's posterior belief over v is diffuse, given his own signal. In our context, since the prior over v is diffuse, a diffuse signal distribution translates to a diffuse posterior belief over v. In general, as v approaches v_ℓ or v_h, the posterior belief becomes more concentrated (at least, its support shrinks in size), potentially reducing the benefit of rationing. This leads to the downward segment of the curve for values of v close to v_h.

This intuition applies both to a posterior that is concentrated because the signal is close to one of the endpoints and to a posterior that is concentrated because the signal distribution was concentrated (large (c)). To see this intuition consider the special case of $c = 1$, or $G(\cdot \mid v)$ is uniform. Here, the bid and revenue functions simplify considerably. As shown by Harstad and Bordley (1996), the bid function for interior signals reduces to a function of the ratio $\frac{t}{n}$, and the revenue function for interior values of v is clearly increasing in t.

COROLLARY 8.1. Suppose $c = 1$; that is, the signal distribution is uniform. Then, in equilibrium,

(i) for $s \in [v_\ell + \epsilon, v_h - \epsilon]$, $b(s; t) = s + \epsilon(\frac{2t}{n} - 1)$.
(ii) for $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, $R(v; t) = v - \frac{2\epsilon(n-t)}{n(n+1)}$.

Clearly, in this case revenue increases in t so $R(v; t+1) > R(v; t)$ for all $t = k, \ldots, n-2$. That is, the revenue unambiguously increases as the degree of rationing increases. Condition (2) reduces to $1 > \frac{n}{n+1}$. This holds for all t, and thus maximum rationing ($t = n - 1$) is optimal when $c = 1$. This suggests that the oversubscription in IPOs of firms in new industries (where investor beliefs are diffuse) should be high.

As c increases, the optimal level of rationing decreases,[22] and when c is large, market-clearing ($t = k$) is optimal. We first demonstrate analytically, for interior values of v, that market-clearing is optimal when c becomes large enough.

PROPOSITION 9. Consider interior values $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then,

(i) if $c = 1$, maximal rationing is optimal
(ii) there exists a $\bar{c} > 1$ such that, for $c < \bar{c}$, rationing (to some degree) yields higher revenue than market-clearing, while for $c \geq \bar{c}$, market-clearing is preferable to rationing.

Since further analytic results on the corners are difficult to obtain, we use our numeric example to compute the optimal allocation proportion when the seller maximizes ex ante revenue, $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v; t)dv$. Figure 4 demonstrates the results.

[22] Since t must be an integer, this decrease is in steps, rather than continuous.

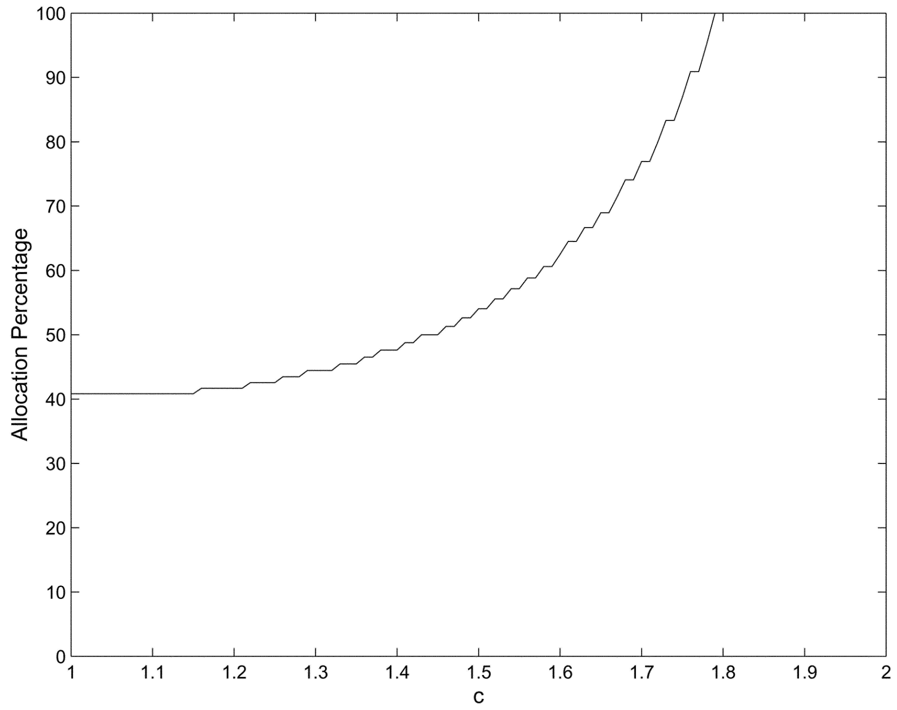

Figure 4. Optimal allocation percentage as c changes

Armed with an optimal t for each c, we can determine the endogenous distribution of bids in the book. In particular, we can compute the maximum and minimum bids (and hence the range), and the offer price (and thus the range of bids that received a positive allocation). This leads to empirical predictions linking the degree of rationing with observed properties of bids. We emphasize that these comparative statics predictions assume all else held constant; in particular, ϵ, n, k, and $[v_\ell, v_h]$. The optimal value of t is endogenously determined given these parameters and c.

OBSERVATION 1. *The range of submitted bids on the book is smaller if the degree of rationing is smaller.* The range of bids received (the maximum bid minus the minimum bid) declines with c.[23] Hence, this range should be directly related to the degree of rationing; i.e., inversely related to the allocation percentage across IPO deals.

OBSERVATION 2. *The range of bids above the offer price is smaller if the degree of rationing is smaller.* The observed range of bids that receive a positive allocation (i.e., the maximum bid minus the offer price) also declines with c (since t

[23] This is straightforwardly determined as the difference between the maximum and minimum order statistics.

declines with c). Hence, this range should also be positively related to the degree of rationing (or alternatively, inversely related to the aggregate allocation percentage).

Under what circumstances would we expect c to change in a predictable manner? Consider a sequence of IPOs of firms in the same industry. One would expect information on the first firm to be diffuse. Since information about other firms is likely to be correlated, as more firms go public, information should become more precise.[24] Thus, we would expect c to increase with each subsequent IPO.

The same comparative static holds when comparing IPOs to seasoned equity offerings (SEOs). If a firm has been publicly traded for a length of time, information about it is more precise, and hence c should be larger. Indeed, Cornelli and Goldreich (2003) find some support for this view. They find that, in their sample, the average elasticity of demand for SEOs is much larger than the average elasticity of demand for IPOs. This average elasticity measures the proportion of bids within 1% of the offer price, and hence proxies for the precision in the information of bidders.

Given a book with many price-contingent bids for a single IPO deal, the parameters c and ϵ can be structurally estimated. From this, signals can then be inferred from bids, and hence a more accurate measure of the true value of the asset (which depends on all signals) can be generated.

5. Discussion

It is useful to consider our results in the framework of Bulow and Klemperer (1996), who consider an English auction with no reserve price, and show that, even with affiliated signals, the expected revenue of the seller is just the expected marginal revenue of the winning bidder.[25] In our model, different rationing levels allocate the good to different sets of bidders, so variation in seller revenue can be interpreted in terms of variation in marginal revenues across bidders. It is important to keep in mind, however, that different levels of rationing reveal different information. As Bulow and Klemperer (1996) point out, to determine expected revenue, each bidder's marginal revenue must be calculated based on the information the auction will reveal.

Recall that in a (k, t)-bookbuilding mechanism, the price is the $(t+1)^{st}$ highest bid, so in equilibrium is set by the $(t+1)^{st}$ highest signal. Let $\Psi(\cdot \mid Y_{t+1,n})$ denote the conditional distribution of agent i's signal, given that the $(t+1)^{st}$ highest of all n signals is $Y_{t+1,n}$, and let $\psi(\cdot \mid Y_{t+1,n})$ be the associated density. Then, following Bulow and Klemperer (1996), the marginal revenue of bidder i in our setting is defined as

$$MR_i = v - \frac{1 - \Psi(s_i \mid Y_{t+1,n})}{\psi(s_i \mid Y_{t+1,n})} \frac{\partial v}{\partial s_i}, \tag{3}$$

[24] Alti (2002) suggests that such an information structure should be a feature of observed IPOs.

[25] We are grateful both to Paul Klemperer and to a referee for pointing us in this direction.

where v, the common value, is the same across all agents. The second term may be interpreted as the information rent captured by bidder i. Thus, maximizing marginal revenue is equivalent to minimizing a bidder's information rent in a common value setting.

Now, consider our result that maximal rationing is optimal when signals are uniformly distributed. Bulow and Klemperer (2002) offer an intuition for this result in terms of the "Maximum Game." In this game, signals are independently distributed, and the common consumption value to all agents is the maximum of all signals. For every bidder i except the bidder with the highest signal, $\frac{\partial v}{\partial s_i} = 0$, since her signal does not affect the common value. Hence, only the highest bidder captures any informational rent. As Bulow and Klemperer (2002) show, allocating the good (probabilistically) to as many bidders as possible minimizes the probability of the highest signal bidder winning and, hence, minimizes winners' information rents.

Now, consider our setting. Though we have affiliated signals, if the signal distribution is uniform, the maximum and minimum signals are jointly sufficient statistics for all other properties of the distribution. In particular, with a uniform signal distribution and interior signals, the expected consumption value given all signals is the mean of the maximum and minimum signals. Selling to the lowest signal bidder is not optimal, and eliminating this bidder recovers the intuition of the Maximum Game: maximal rationing minimizes the number of shares sold to the highest signal bidder. Further, as c approaches 1 from above, the model more closely resembles a Maximum Game, and hence maximal rationing becomes more valuable.

This intuition also extends into the region of corner signals. Consider Figure 3, for example. For signals in the lower corner, lower signal bidder have higher information rents (since they know that v must lie between v_ℓ and $s + \epsilon$, rather than $s - \epsilon$ and $s + \epsilon$). Hence, rationing is sub-optimal: the shares should be sold to the highest bidders. Conversely, for signals in the upper corner, it is the higher signal bidders who have higher information rents. Hence, the benefits of rationing are enhanced in this case.

Finally, we briefly consider the implications of different allocation rules. In our model, allocations to winning bidders do not depend on the their bids. A more general allocation rule would let the allocation to each winning bidder depend on the level or rank of her bid. A natural rule is to allocate more shares (or a greater proportion of their demand) to higher bidders. Intuitively, allocating more shares to higher bidders does encourage them to bid more. However, it also potentially exposes them to the winner's curse: they are more likely to get shares (equivalently, likely to get more shares) when their signal is amongst the highest. Determining the optimal allocation rule when allocations can depend on bids in this manner remains an open question. However, an extreme example of such a rule is market-clearing, in which all high bidders get the good. This is inferior to our rationing mechanism when information about the asset is diffuse.

We note that a rule allocating greater proportions to higher bidders is difficult for individual bidders to verify, unless there is either full disclosure of the book (which is not seen in practice) or truthful communication between bidders (which is unlikely in a private information setting). For example, suppose $t = 50$, so that the price should be set to the 51^{st} highest bid. If the 24^{th} highest bidder is receiving a different allocation from the 25^{th} highest one, there must be some way for a bidder to verify the exact ranking of her bid. Otherwise, the seller can costlessly deviate, by (say) setting the price to the 40^{th} highest bid, and giving a higher-than-stipulated allocation to bidders 1 through 39. In contrast, pro rata rationing is credibly verified once the price and aggregate degree of rationing are announced. Every bidder who has bid more than the price should receive her pro rata share, and no other bidder should receive any shares.

6. Conclusion

In this paper, we provide a model of rationing in IPOs. We demonstrate that the bids of the agents depend on the rationing rule: higher rationing mitigates the winner's curse, and leads to higher bids. This effect is beneficial to the seller; the cost of the rationing is that the bidder who sets the price has a lower signal.

We consider the case of a uniform prior over true value, and show that rationing leads to higher revenue than market-clearing when the signal distribution is diffuse enough. In other words, when an agent's posterior over V, conditional on his signal, is diffuse, there is a large winner's curse. In such cases, mitigating the winner's curse has a relatively significant impact on bids, and outweighs the effect of choosing a lower order bid to set the price. Conversely, when agents' signals are precise, the latter effect dominates, and increasing the degree of rationing leads to lower revenue.

This effect is in contrast to the earlier literature on rationing and the winner's curse, such as Rock (1986) and Benveniste and Spindt (1989). In this earlier work, rationing embodies the winner's curse, and hurts seller revenue. In our model, even within a one-shot game, rationing can be revenue-enhancing.

Our model allows realized allocations to vary widely across the set of winning bidders. Indeed, our results depend only on the aggregate rationing level chosen by the issuer, not on individual rationing across investors. This is also in contrast to Benveniste and Spindt (1989), whose model implies that investors with high bids should have their demands filled before investors with low bids receive any shares. The empirical literature suggests that departures from this latter paradigm are common (see, for example Cornelli and Goldreich, 2001).

Rationing affects the allocation of the good in the primary market. In a pure common value case, which we consider, it should have no effect on the secondary market (in fact, there should be no trade in the secondary market, by the Milgrom and Stokey, 1982, no-trade theorem). However, if the value of the asset to an agent includes a private component, the allocation in the primary market will, in general, affect volume and value in the secondary market. Such a model is an avenue for further research.

Appendix: Proofs

Proof of Proposition 1. Let ν_i define an allocation rule faced by agent i, conditional on i having one of the t highest bids (that is, being in the set of potential winners). ν_i can be a function of any variable observed by the seller (including, for example, the identity of agent i or some other agent j), except the bid of any agent.

Formally, ν_i is a probability distribution over [0, 1], where 0 is the minimum number of shares agent i can receive, and 1 (her demand) the maximum. The allocation rule has the property that agent i faces the probability distribution ν_i if her bid is among the t highest bids, and receives 0 shares otherwise.

Consider agent i's payoff when she is in the set of winners. Since all allocations pay the $(t+1)^{st}$ highest bid, this payoff is represented as

$$\int_0^1 E[V - b_{t+1} \mid S_i = s,\ b_i \geq b_{t+1}]\ \mathrm{Prob}(b_i \geq b_{t+1})\ d\nu_i$$

The expectation of V is with respect to the agent's posterior distribution over V. Applying Fubini's theorem to interchange the two expectations, and noting that ν_i is independent of all bids, this expected payoff can be written as

$$\begin{aligned} &E[V - b_{t+1} \mid S_i = s,\ b_i \geq b_{t+1}]\ \mathrm{Prob}(b_i \geq b_{t+1}) \int_0^1 d\nu_i \\ &\qquad = \bar{\nu}_i\ E[V - b_{t+1} \mid S_i = s,\ b_i \geq b_{t+1}]\ \mathrm{Prob}(b_i \geq b_{t+1}), \end{aligned}$$

where $\bar{\nu}_i$ is the mean of ν_i.

Now, $\bar{\nu}_i$ is a constant unaffected by agent i's signal, her bid, or the bids of any other agent. Hence, if all agents have the same bidding strategies across the two rationing rules, the optimal strategy of agent i too must be the same. Therefore, if the bidding functions $\{b_i(\cdot)\}_{i=1}^n$ constitute an equilibrium under one rationing rule, they must also comprise an equilibrium under any other rationing rule that satisfies Definition 1. □

Proof of Proposition 2.

(i) Consider the decision faced by agent 1 in the (k, t)-bookbuilding mechanism. Let $b_2, \ldots, b_n$ denote the bids of the other $(n-1)$ agents, in decreasing order. Then, agent 1 chooses a bid b_1 that maximizes

$$\bar{\nu}_1\ E(V - b_{t+1} \mid S_1 = s,\ b_1 \geq b_{t+1})\ \mathrm{Prob}(b_1 \geq b_{t+1}),$$

where $\bar{\nu}_1$ is the rationing rule faced by her, conditional on being among the t highest bidders.

Next, consider a t-unit auction. In this auction, bidder 1 chooses a bid $\tilde{b}_1$ to maximize

$$E(V - b_{t+1} \mid S_1 = s,\ b_1 \geq b_{t+1})\ \text{Prob}(b_1 \geq b_{t+1}).$$

That is, the payoffs differ only by a multiplicative constant, $\bar{v}_i$. This term is independent of any signals or bids. Clearly, the set of maximizers is the same in either case. Hence, the best response correspondences of the two mechanisms are identical, as are their equilibria.

(ii) Milgrom (1981) demonstrates that the t-unit auction has a symmetric equilibrium with $b(s;t)$ as defined. From (i) above, it follows that this is also an equilibrium of the (k, t)-bookbuilding mechanism. □

Proof of Proposition 3. Consider the signal S_1, and some $\tilde{t} > t$. By the definition of order statistics, the distribution of $Y_{\tilde{t},n+1}$ is first-order stochastically dominated by the distribution of $Y_{t,n+1}$. Hence, the joint distribution of $Y_{\tilde{t},n+1}$ and S_1 is first-order stochastically dominated by the joint distribution of $Y_{t,n+1}$ and S_1. Now, the equilibrium bid in the (k, t)-bookbuilding mechanism is $b(s;t) = E[V \mid Y_{t,n+1} = S_1 = s]$. It follows immediately that this is increasing in t. □

Proof of Proposition 4. Recall that $Y_{1,n-1}$ denotes the highest order statistic out of the signals of the $(n-1)$ bidders except bidder 1, and $Y_{n-1,n-1}$ denotes the lowest order statistic amongst these signals. Now,

$$\begin{aligned} E[V \mid S_1 = s] \; = \; & \text{Prob}(Y_1 \leq s)\ E\ \big\{E[V \mid S_1 = s, Y_{1,n-1} \leq s] \\ & +\text{Prob}(Y_1 > s)\ E[V \mid S_1 = s \mid Y_{1,n-1} > s]\ \big\} \end{aligned}$$

Since $E[V \mid S_1 = s, Y_{1,n-1} \leq s] < E[V \mid S_1 = s, Y_{1,n-1} > s]$, it follows that

$$E[V \mid S_1 = s] > E[V \mid S_1 = s, Y_{1,n-1} \leq s]$$

Similarly, it is straightforward to show that $E[V \mid S_1 = s] < E[V \mid S_1 = s, Y_{n-1,n-1} \geq s]$.

Hence, for $t = 1$, $b(s;t) < E[V \mid s]$, and for $t = n-1$, $b(s;t) > E[V \mid s]$. Since $b(s;t)$ is increasing in t, it follows that there exists a $\hat{t}(s) \in \{1, \ldots, n-1\}$ such that $b(s;t) \leq E[V \mid s]$ for $t < \hat{t}(s)$, and $b(s;t) > E[V \mid s]$ for $t > \hat{t}(s)$. □

Proof of Proposition 5. Let $\underline{v}(s)$ and $\bar{v}(s)$ be the possible bounds of v, given a signal s. Then, $\underline{v}(s) = \max\{v_\ell, s - \epsilon)\}$, and $\bar{v}(s) = \min\{v_h, s + \epsilon\}$. For interior signals, that is, $s \in [v_\ell + \epsilon, v_h - \epsilon]$, we have $\underline{v}(s) = s - \epsilon$ and $\bar{v}(s) = s + \epsilon$. Now, since V is uniformly distributed over $[v_\ell, v_h]$, we have

$$\begin{aligned} b(s;t) \; &= \; E[V \mid S = Y_{t,n-1} = s] \\ &= \; \frac{\int_{\underline{v}(s)}^{\bar{v}(s)} vG(s \mid v)^{n-t-1}(1 - G(s \mid v))^{t-1} g(s \mid v)^2 dv}{\int_{s-\epsilon}^{s+\epsilon} G(s \mid v)^{n-t-1}(1 - G(s \mid v))^{t-1} g(s \mid v)^2 dv}. \end{aligned}$$

We perform the following change of variables. Define $x = \frac{s-(v-\epsilon)}{2\epsilon}$. Then, $v = s + \epsilon - 2\epsilon x$, and $dv = -2\epsilon dx$. Further, let $\bar{x}(s) = \frac{s-(\underline{v}(s)-\epsilon)}{2\epsilon}$, and $\underline{x}(s) = \frac{s-(\bar{v}(s)-\epsilon)}{2\epsilon} < \bar{x}(s)$. Note that, for interior signals, $\bar{x}(s) = 0$ and $\underline{x}(s) = 1$. Finally, let H denote the distribution of x, and h the associated density. Note that $G(s \mid v)$ depends only on $s - (v - \epsilon)$ when the signal is interior. Hence, given s and v, we have $H(x) = G(s \mid v)$ for signals in this range, where $H(x)$ has support [0, 1]. Then,

$$\begin{aligned} b(s;t) &= \frac{-\int_1^0 (s+\epsilon-2\epsilon x)H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}{-\int_1^0 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx} \\ &= \frac{\int_0^1 (s+\epsilon-2\epsilon x)H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}{\int_0^1 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx} \\ &= s+\epsilon-2\epsilon\frac{\int_0^1 xH(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}{\int_0^1 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx} \\ &= s+\epsilon-2\epsilon\delta(n,t), \end{aligned}$$

where $\delta(n,t) = \frac{\int_0^1 xH(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}{\int_0^1 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}$. By inspection, δ is independent of s and ϵ.

Now, note that $\delta(n,t)$ may be written as $E[\tilde{S} \mid \tilde{S} = X_{t,n-1}]$, where $\tilde{S}$ has distribution H, and $X_{t,n-1}$ is the t^{th} highest signal from an independent sample of $n-1$ variables each with distribution H. Hence, δ is decreasing in t, and increasing in n. □

Proof of Proposition 6. When $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, all observed signals will lie in $[v_\ell + \epsilon, v_h - \epsilon]$. Hence, $\underline{x}(s) = 1$ and $\bar{x}(s) = 0$, so that each bidder bids $b(s;t) = s + \epsilon - 2\epsilon\delta(n,t)$. We have

$$\begin{aligned} R(v;t) &= E_{Y_{t+1,n}|v}\, b(s;t) = E_{Y_{t+1,n}|v}\,(s+\epsilon-2\epsilon\delta(n,t)) \\ &= E_{Y_{t+1,n}|v}(s+\epsilon) - 2\epsilon\delta(n,t). \end{aligned}$$

Noting that $x = \frac{s-(v-\epsilon)}{2\epsilon}$, we have $s + \epsilon = v + 2\epsilon x$. Therefore,

$$\begin{aligned} R(v;t) &= E_{Y_{t+1,n}}(v+2\epsilon x) - 2\epsilon\delta(n,t) \\ &= v + 2\epsilon E(X_{t+1,n}) - 2\epsilon\delta(n,t) \\ &= v - 2\epsilon(\delta(n,t) - E[X_{t+1,n}]). \end{aligned}$$ □

Proof of Corollary 6.1. Immediate. □

Proof of Proposition 7. The following two integration facts are used in this proof and the next two ones. Let α, β be real-valued, with $\alpha > -1$, and $\gamma > 0$ be an integer. Then, repeated integration by parts yields

$$\int x^{\alpha}(1-x^{\beta})^{\gamma}dx = \sum_{i=0}^{\gamma} \frac{\gamma!\,\beta^{i}\,x^{\alpha+1+i\beta}(1-x^{\beta})^{\gamma-i}}{(\gamma-i)!\,\prod_{j=0}^{i}(\alpha+1+j\beta)} + A \tag{4}$$

$$\int_0^1 x^{\alpha}(1-x^{\beta})^{\gamma}dx = \frac{\gamma!\,\beta^{\gamma}}{\prod_{j=0}^{\gamma}(\alpha+1+j\beta)}. \tag{5}$$

where A in Equation (4) is a constant of integration.

Now, consider the bid function of the (k, t)-bookbuilding mechanism. $b(\cdot)$. This bid function is exhibited in Proposition 5; what remains is to evaluate $\delta(n, t)$.

Recall that $x = \frac{s-(v-\epsilon)}{2\epsilon}$. Since $G(s \mid v) = \left(\frac{s-v+\epsilon}{2\epsilon}\right)^{c}$, we directly have $H(x) = x^{c}$. Hence, $h(x) = cx^{c-1}$.

(i) For $s \in [v_\ell + \epsilon, v_h - \epsilon]$ we have

$$\delta(n,t) = \frac{\int_0^1 xH(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}{\int_0^1 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx} \tag{6}$$

Consider the denominator first. We have

$$\int_0^1 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx = c^2 \int_0^1 x^{c(n-t+1-\frac{2}{c})}(1-x^{c})^{t-1}dx.$$

Using (5), we have

$$\int_0^1 H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx = c^2\,\frac{(t-1)!\;c^{t-1}}{\prod_{j=0}^{t-1}c(n-t+j+1-\frac{1}{c})}$$

In a similar manner, the numerator reduces to

$$\begin{aligned}\int_0^1 xH(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx &= c^2 \int_0^1 x^{c(n-t+1-\frac{1}{c})}(1-x^{c})^{t-1}dx\\ &= c^2\,\frac{(t-1)!\,c^{t-1}}{\prod_{j=0}^{t-1}c(n-t+j+1)} \end{aligned}\tag{7}$$

Therefore,

$$\delta(n,t) = \prod_{j=0}^{t-1}\frac{n-t+j+1-\frac{1}{c}}{n-t+j+1} = \prod_{j=0}^{t-1}\frac{n-j-\frac{1}{c}}{n-j},$$

from which the bid function in the statement of the proposition follows.

(ii) $s \notin [v_\ell + \epsilon, v_h - \epsilon]$

The arguments from Proposition 5 can be replicated with corner signals, to yield that, for such signals,

$$b(s;t) = s + \epsilon(1 - 2\tilde{\delta}(s,n,t)),$$

where

$$\delta(s,n,t) = \frac{\int_{\underline{x}(s)}^{\bar{x}(s)} xH(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}{\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx}.$$

Consider the denominator first. Let $\kappa(y)$ denote the value of the integral $\int x^{c(n-t+1-\frac{2}{c})}(1-x^c)^{t-1}dy$ evaluated at y. Using Equation (4), we have $\alpha = c(n-t+1-\frac{2}{c})$, so that $\alpha + 1 = c(n-t+1-\frac{1}{c})$, $\beta = c$, and $\gamma = t-1$. Hence, ignoring the constant of integration A,

$$\begin{aligned}\kappa(y) &= \sum_{i=0}^{t-1} \frac{(t-1)!\, c^i y^{c(n-t+1+i-\frac{1}{c})}(1-y^c)^{t-1-i}}{(t-1-i)!\, \prod_{j=0}^{i} c(n-t+1+j-\frac{1}{c})} \\ &= \frac{(t-1)!}{c} \sum_{i=0}^{t-1} \frac{y^{c(n-i-\frac{1}{c})}(1-y^c)^i}{i!\, \prod_{j=i}^{t-1}(n-j-\frac{1}{c})} = \frac{(t-1)!}{c}\ \phi(z,\frac{1}{c}).\end{aligned}$$

Hence,

$$\begin{aligned}&\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1}(1-H(x))^{t-1}h(x)^2dx \\ &\quad = (t-1)!\ \left(\frac{c}{2\epsilon}\right) \left\{\phi\left(\bar{x}(s),\frac{1}{c}\right) - \phi\left(\underline{x}(s),\frac{1}{c}\right)\right\}.\end{aligned}$$

Next, consider the numerator. Let $\hat{\kappa}(y)$ denote the integral the integral $\int x^{c(n-t+1-\frac{1}{c})}(1-x^c)^{t-1}dy$, evaluated at z. Repeating the above steps, we have

$$\hat{\kappa}(y) = \frac{(t-1)!}{c}\ \phi(y,0)$$

Hence,

$$\delta(s,n,t) = \frac{\phi(\bar{x}(s),0) - \phi(\underline{x}(s),0)}{\phi(\bar{x}(s),\frac{1}{c}) - \phi(\underline{x}(s),\frac{1}{c})}.$$

Finally, note that (i) $\underline{x}(s) = 0$ for $x \leq v_h + \epsilon$, (ii) $\bar{x}(s) = 1$ for $x \geq v_\ell - \epsilon$, and (iii) $\phi(0,y) = 0$ for all y. The statement of the bid function follows. □

Proof of Proposition 8.

(i) When $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, we have $s \in [v_\ell + \epsilon, v_h - \epsilon]$. From Proposition 7, for signals in this range,

$$\delta(n, t) = \prod_{j=0}^{t-1} \left(\frac{n - j - \frac{1}{c}}{n - j} \right).$$

Next, consider $E[X_{t,n}]$. For s in this range, we have

$$\begin{aligned}
E[X_{t+1,n}] &= \frac{n!}{(n-t-1)!\,t!} \int_0^1 x H(x)^{n-t-1} (1 - H(x))^t h(x) dx \\
&= \frac{n!\,c}{(n-t-1)!\,t!} \int_0^1 x\, x^{c(n-t-1)}\, (1 - x^c)^t\, x^{c-1} dx \\
&= \frac{n!\,c}{(n-t-1)!\,t!} \int_0^1 y^{c(n-t)}\, (1 - y^c)^t dy \\
&= \frac{n!\,c}{(n-t-1)!\,t!} \left(\frac{t!\,c^t}{c^{t+1} \prod_{j=0}^{t} (n - t + j + \frac{1}{c})} \right) \\
&= \prod_{j=0}^{t} \frac{n - j}{(n - j + \frac{1}{c})},
\end{aligned}$$

where the second equality applies Equation (5), and the last step follows from $\prod_{j=0}^{t}(n - t + j + \frac{1}{c}) = \prod_{j=0}^{t}(n - j + \frac{1}{c})$.
Substitute the expressions for $\delta(s, n, t, \epsilon)$ and $EX_{t,n}$ into the revenue function in Proposition 6, and the statement of part (i) follows.

(ii) Consider condition (1) from part (ii) of Proposition 6, when $m = t + 1$. The left-hand side is

$$\begin{aligned}
\delta(n, t, \epsilon) - \delta(n, t+1, \epsilon) &= 2\epsilon \prod_{j=0}^{t-1} \left(\frac{n - j - \frac{1}{c}}{n - j} \right) - 2\epsilon \prod_{j=0}^{t} \left(\frac{n - j - \frac{1}{c}}{n - j} \right) \\
&= 2\epsilon \prod_{j=0}^{t-1} \left(\frac{n - j - \frac{1}{c}}{n - j} \right) \left(1 - \frac{n - t - \frac{1}{c}}{n - t} \right) \\
&= 2\epsilon \frac{1}{c(n - t)} \prod_{j=0}^{t-1} \left(\frac{n - j - \frac{1}{c}}{n - j} \right).
\end{aligned}$$

On the right-hand side, we have

$$E[X_{t+1,n}] - E[X_{t+2,n}] = 2\epsilon \prod_{j=0}^{t} \frac{n-j}{(n-j+\frac{1}{c})} - 2\epsilon \prod_{j=0}^{t+1} \frac{n-j}{(n-j+\frac{1}{c})}$$
$$= 2\epsilon \left(1 - \frac{n-t-1}{n-t-1+\frac{1}{c}}\right) \prod_{j=0}^{t} \frac{n-j}{n-j+\frac{1}{c}}$$
$$= 2\epsilon \frac{1}{c(n-t-1+\frac{1}{c})} \prod_{j=0}^{t} \frac{n-j}{n-j+\frac{1}{c}}.$$

Hence, $R(v; t+1) > R(v; t)$ if and only if

$$\left(\frac{n-t-1+\frac{1}{c}}{n-t}\right) \prod_{j=0}^{t-1} \frac{n-j-\frac{1}{c}}{n-j} > \prod_{j=0}^{t} \frac{n-j}{n-j+\frac{1}{c}}. \tag{8}$$

□

Proof of Corollary 8.1.

(i) Substitute $c = 1$ in the expression for the bid function in Proposition 7 when $s \in [v_\ell + \epsilon, v_h - \epsilon]$. This yields $\delta(n,t) = \prod_{j=0}^{t-1} \left(\frac{n-j-1}{n-j}\right) = (\frac{n-t}{n}) = (1 - \frac{t}{n})$. Hence, $b(s; t) = s + \epsilon(1 - 2(1 - \frac{t}{n})) = s + \epsilon(\frac{2t}{n} - 1)$.

(ii) This follows from substituting $c = 1$ in the expression for $R(v; t)$ in Proposition 8 (i), and simplifying. □

Proof of Proposition 9.

(i) This follows immediately from Corollary 8.1 (ii). The revenue per share is increasing linearly in t, so the optimal degree of rationing is obtained when $t = n - 1$.

(ii) Consider condition (2) for revenue comparison, expressed in Proposition 8 (ii). Reversing that inequality, we have $R(v; t) > R(v; t+1)$ if and only if

$$\left(\frac{n-t-1+\frac{1}{c}}{n-t}\right) \prod_{j=0}^{t-1} \frac{n-j-\frac{1}{c}}{n-j} < \prod_{j=0}^{t} \frac{n-j}{n-j+\frac{1}{c}}.$$

For market-clearing to be optimal, this inequality must hold for all $t \geq k$. Consider $c \to \infty$. In the limit, the condition reduces to $\frac{n-t-1}{n-t} < 1$, which clearly holds for all $t \geq k$. Since both sides of the inequality are continuous in c, it follows that there exists a $\bar{c}$ such that the condition holds for all $t \geq k$ and all $c \geq \bar{c}$. Since the difference between the two sides is decreasing in c, the condition is violated for some t (given k) when $c < \bar{c}$.

Hence, for $c < \bar{c}$, rationing (to an unspecified degree) must yield higher revenue than market-clearing, while for $c \geq \bar{c}$, market-clearing yields higher revenue. Finally, since maximal rationing is optimal at $c = 1$, it must be that $\bar{c} > 1$. □

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