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Household Portfolio Choices in Taxable and Tax-Deferred Accounts: Another Puzzle? ⋆

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Abstract. This paper provides a survey of existing literature on portfolio allocations in conventional and tax-deferred investment habitats. A long-standing puzzle in this literature has been the dissonance between the theoretical prediction of tax-efficient portfolio choices and observed portfolio allocations. I clarify this prediction and offer a different perspective by emphasizing the importance of uninsurable labor income risk and restrictions on accessibility of tax-deferred assets. I identify the key factors in dual-habitat portfolio decisions and highlight the necessary ingredients for producing non-tax-efficient, or precautionary, allocations.

Key words: household portfolio choice, precautionary savings, tax-efficiency.

JEL classification codes: D12, G11, H24.

1. Introduction

Over the past decade, US households have experienced an unprecedented array of changes in the structure of their financial portfolios. The booming stock market of the 1990s fueled a significant increase in the number of equity owners. Stock options and home equity lines of credit have become commonplace. Perhaps most importantly, a record number of households have acquired the primary responsibility for investing their retirement savings, as companies increasingly moved to switch their pension coverage from defined-benefit (DB) to defined-contribution (DC) plans.

In the United States and most industrialized countries, accounts that contain assets set aside for retirement are granted favorable tax treatment.[1] Typically, tax-

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[1] In the US, examples of tax-favored retirement accounts include employer-sponsored defined contribution plans such as 401(k) and 403(b), similar plans for the self-employed such as Keogh and SEP-IRA, and individual retirement savings accounts such as Regular and Roth IRA.

ation of both contributions and earnings is deferred until withdrawal, enabling households to accumulate retirement assets at pre-tax rates of return. In order to ensure that these assets are used for their intended purpose, a variety of restrictions are placed on accessibility of funds in tax-deferred retirement accounts (TDAs). Consequently, ownership of TDAs, which are distinct both in terms of their tax treatment and liquidity, makes the investment environment of a typical household much more complex. The participating households have to decide not only on broad asset classes such as stocks and bonds, but also on how much of each type of asset to locate in each of the two investment habitats. This joint portfolio decision was originally outlined in Shoven (1999) and became known in the literature as the asset *location* and *allocation* problem.

Economists have a clear-cut and intuitive portfolio advice for households faced with this decision – to hold *all* higher-tax-burden assets (bonds, in the case of the US) inside tax-sheltered retirement accounts. Only in cases when desired holdings of high-tax assets exceed TDA capacity can some of them spill over into taxable accounts. This advice is based on the pioneering work of Tepper (1980) and Black (1981), who demonstrate the existence of tax arbitrage strategies whenever opportunities to shelter high-tax assets inside retirement accounts are not used fully. As a result, portfolio allocations in which it is not possible to replace low-tax assets in TDA with high-tax assets held elsewhere have acquired the label of "tax-efficient".[2] Tax-efficient portfolios appear to be optimal even in complex multi-period environments with borrowing constraints and timing choice for realization of capital gains (e.g., Dammon et al., 2003; henceforth DSZ).

Such advice, however, is sharply at odds with observed portfolio choices of American households. In spite of an admonition that there should never be any equities in TDAs as long as there are bonds in taxable accounts, households commonly keep both equities and bonds in each account type (i.e., hold "mixed" allocations). Moreover, they often maintain higher equity positions inside their retirement accounts (Poterba and Samwick, 1997; Bergstresser and Poterba, 2002). Table I presents a summary of households' portfolio choices in the 1998 Survey of Consumer Finances (SCF). It illustrates that "tax-*in*efficient" allocations are widespread even among households with sizable wealth in both account types. While equity participation rates among TDA owners are extremely high (88%), a significant percentage of such households (39%) have all of their equities inside retirement accounts. It is conceivable that individual agents fail to appreciate the opportunities for tax-arbitrage demonstrated by theoretical work. However, it seems equally likely that the canonical model of Black and Tepper fails to take

[2] For the remainder of this paper, "tax-efficiency" of portfolio allocations is defined in the very narrow sense of Tepper-Black. An allocation is said to be tax-efficient if a strict pecking order is observed – the highest-taxed asset is always located in the tax-preferred habitat before any lower-taxed assets can be placed there. *Relative* to this benchmark, any allocation in which this pecking order is violated is labeled "tax-inefficient".

Table I. Summary of dual-habitat household portfolio choices for different wealth levels in tax-deferred (TDA) and taxable accounts

	Number of households	No TDA assets	Tax-efficient portfolios	All-bond portfolios	Equities only in TDA[b]	Mixed tax-ineff portfolios[b]
All households	102,501,995	53.0%				
Total fin. wealth > 0[a]	75,529,835	42.6%				
TDA > 0 & Taxable > 0	43,573,968		20%	12%	39%	28%
TDA > 10K & Taxable > 10K	19,361,287		30%	8%	19%	43%
TDA > 25K & Taxable > 25K	11,797,688		34%	5%	13%	47%

SOURCE: 1998 Survey of Consumer Finances, all figures are population-weighted.
Note: TDA (tax-deferred account) wealth includes assets in IRA, Keogh, 401k and 403b plans. Taxable account wealth excludes checking, but contains almost all other financial assets such as stocks, taxable and tax-exempt bonds, U.S. savings bonds, mutual funds, CDs, and money market accounts. By contrast, Bergstresser and Poterba (2003) also exclude money market and savings accounts from their definition of investable taxable assets.
[a]Financial wealth is the sum of tax-deferred and taxable account assets as defined above.
[b]Represents a "tax-inefficient" asset allocation.

into account certain aspects of household decision-making environment that make "tax-inefficient" allocations quite rational.

Recent studies of the dual-habitat portfolio problem have all recognized the discord between theory and observed portfolio choices and have attempted to resolve this tension in a variety of ways. For instance, Shoven and Sialm (2004) extend the asset set of the household to tax-exempt municipal bonds. Huang (2001) uses lumpy expenditure needs such as education and housing that are unavoidable at certain points over the lifecycle. DSZ (2003) add uncertain labor income and lumpy consumption shocks to their model. Yet, as will be discussed in detail in Section 2, these models either produce only tax-efficient corner solutions or, when mixed allocations are advocated, they often have empirically implausible properties.

This paper evaluates an alternative version of the dual-habitat portfolio model that emphasizes the joint importance of uninsurable labor income risk and imperfect liquidity of TDA assets for habitat-specific portfolio choices. In the presence of uninsurable background risk, certain borrowing-constrained households have precautionary savings motives. Such households may find it wise to build up buffer stocks of safe and easily accessible assets (Carroll, 1997). Given limited TDA accessibility, safety *and* liquidity can often be attained only by holding bonds in a taxable account, which violates tax-efficiency rules as long as precautionary motives push some equities into TDA.[3] The resulting tax-inefficient portfolios can thus be labeled "precautionary". In essence, labor income risk and accessibility restrictions create tension between one's desire to maintain tax-efficient allocations and concern about making costly TDA withdrawals in the event of bad income draws.[4] This paradigm would hold true in any country that (a) imposes accessibility restrictions on tax-favored retirement accounts and (b) has lower tax burdens on equities, as compared with less risky bonds. One example of such setting is the United Kingdom, where personal pension schemes allow tax-exempt earnings but restrict pre-retirement access, and where dividend-paying equities have lower tax burdens than interest-paying bonds.[5]

The costs of institutionally mandated accessibility restrictions alter the payoffs from tax-efficient portfolio reallocations and obviate opportunities for Tepper-Black arbitrage. The challenge, therefore, is to assess whether a plausibly parameterized theoretical model with precautionary savings motives can produce tax-inefficient portfolio choices. I find that with realistic withdrawal penalties, such precautionary model can generate substantial portfolio heterogeneity, provided that households face at least some amount of catastrophic labor income risk. The reliance on catastrophic risk, while not without precedent, is somewhat troubling and demands a close look. To address this concern, the paper analyses the relative effects of changes in modeling choices for risky income flows and accessibility restrictions on precautionary portfolio decisions. In particular, the model is evaluated at different levels of fixed TDA access costs, tax rate differentials, correlation between labor income shocks and stock marker returns, and likelihood and severity of unemployment spells. The emerging conclusion is that it is quite difficult to generate tax-inefficient allocations in the absence of catastrophic labor income shocks. Moreover, even with strict access restrictions, the precautionary model cannot produce tax-inefficient allocations on the scale commensurate with portfolio choices observed in the data.

The robustness analysis highlights similarities between potential resolutions of the equity premium puzzle and of lack of tax-efficiency in observed household portfolios. The combination of risky labor income and accessibility restrictions

[3] Another alternative is to build up liquid assets by withholding TDA contributions. As will be shown later in the paper, a sufficient level of liquid savings allows households to maintain tax-efficient portfolios.

[4] However, if there are no restrictions on accessibility of assets in retirement accounts, the precautionary motives can be satisfied by assets in either habitat and tax efficiency of allocations need not be violated. This should be the case with the Canadian system, which openly treats tax-favored "registered retirement savings plans" (RRSP) as just another means to smooth consumption. Unfortunately, the Canadian Survey of Financial Security does not collect information on RRSP asset composition. I am grateful to Michael Smart for pointing out this example.

[5] In the UK, both interest payments and capital gains are subject to the same tax rate of 40%. However, dividends receive a tax credit, intended to mitigate the taxation of profits at the corporate level. As a result, equity returns, which are comprised of dividend payments and capital gains, have a lower effective tax burden than bonds.

emphasizes states with high values of the pricing kernel (high marginal utility of consumption) and low market returns, just like some of the models seeking to explain the equity premium puzzle.[6] In the case of asset-pricing models, the increased importance of bad market states may help to explain high returns required for owning equity, while in dual-habitat portfolio models it may produce precautionary allocations.

While the precautionary model is similar in spirit to concurrently developed models of lumpy liquidity needs, it is set up to better accommodate some current institutional features of TDAs. The liquidity need models – whether deterministic (Huang, 2001) or stochastic (DSZ, 2003) – are motivated by lifecycle expenditures, many of which (e.g., house purchase and college tuition) are partially exempt from early withdrawal penalties.[7] The same isn't true in the case of withdrawals made to smooth out labor income shocks, making labor income risk the modeling choice with more plausible empirical implications. The precautionary model also produces a number of novel testable empirical predictions. In particular, it suggests that stronger precautionary savings motives are associated with lower equity shares in taxable accounts but higher equity shares in retirement accounts. In other words, stronger precautionary motives move a household farther away from a tax-efficient portfolio. The companion paper (Amromin, 2002) finds some empirical support for these predictions.

The paper proceeds as follows. Section 2 reviews the theoretical literature on dual habitat portfolio decisions and highlights its empirical implications in the cross-section and over the lifecycle. Section 3 outlines the decision-theoretic model with precautionary savings motives and discusses the properties of its numerical solutions. Section 4 conducts various robustness checks and tests alternative mechanisms for producing non-corner portfolio allocations. Section 5 summarizes the results and offers directions for future research.

2. Related Literature

The problem of optimal portfolio allocation has received a great deal of attention in financial economics. The seminal papers of Merton (1969, 1971), Mossin (1968) and Samuelson (1969) produced elegant analytical solutions for consumption and portfolio choices in a complete market setting with *i.i.d* returns for a

[6] This, of course, is a grossly oversimplified description of efforts to resolve the equity premium puzzle. The pricing kernel must not only be volatile, but also have a stable conditional expectation and co-vary in a particular way with market returns. Yet, it is fair to say that modification of a pricing kernel to amplify the importance of bad market outcomes is a hallmark of many asset-pricing models (e.g., Constantinides and Duffie, 1996; Campbell and Cochrane, 1999).

[7] Both exemptions became law in 1997, as a part of the Taxpayer Relief Act. The education withdrawals can be used towards tuition, as well as room and board and can be taken out for oneself, one's children or even grandchildren and have no fixed dollar limit. The housing exemption applies to first-time homebuyers and is capped at $20,000 per household. Both exemptions apply only to assets held in non-employer-sponsored TDAs, such as IRAs and rollover IRAs.

number of commonly used preference functions. Yet, these canonical models of portfolio choice were lacking in several important respects. First, their assumption of complete markets effectively obviated the need to analyze additional sources of background risk, such as stochastic labor income processes. Such processes, however, are an integral part of household decision-making environment since moral hazard renders them uninsurable for individual households. A number of recent papers, including Bertaut and Halliassos (1997), Bodie et al. (1991), Cocco et al. (1999), Gakidis (1998), Heaton and Lucas (1997, 2000), Koo (1999), and Viceira (2001), among others, study the effect of labor income risk on portfolio allocations when markets are incomplete. The major insight of this literature is that the effect of labor income on portfolio choices is primarily determined by the extent to which it resembles a risk-free asset. As long as labor income is bounded from below, it functions as a risk-free asset and thus brings about higher equity holdings. However, Heaton and Lucas (2000) show that if labor income innovations are sufficiently positively correlated with equity returns, as they are found to be for entrepreneurial households, equity holdings are reduced. Either way, there exists a systematic relationship between household portfolio composition and riskiness of its labor income process.

Second, the canonical models made no distinction between various lifecycle motives for savings, such as retirement, bequest, precautionary reasons, etc.[8] Each of these motives requires an evaluation of a different subset of risks that pertain to different investment horizons (e.g., mortality, labor income, or health risks). Yet, all of these motives shape the overall portfolio choice while interacting with each other, often in non-separable ways. Campbell et al. (2001) study the interaction between asset choices for retirement savings and taxable accounts. The authors use numerical methods to evaluate optimal consumption and portfolio choices when a fixed share of risky labor income is automatically invested in a retirement account with an exogenously mandated asset mix. They show that even when TDA assets are inaccessible until retirement, their composition influences the choices made in the taxable account throughout the lifecycle. For example, when retirement account composition is changed from 100% bonds to 50/50 bonds and equities (holding the expected income replacement rate at retirement fixed) the households respond by holding fewer equities in their taxable accounts and by increasing their precaution-

[8] Data from the 1998 SCF indicate that about 1/3 of US households save primarily for precautionary motives and 1/3 for retirement/bequest purposes. About 20% save primarily to finance targeted expenditures like education and housing. Interestingly, in comparison with the 1989 survey, the share of “retirement-saving” households rose dramatically (from 1/5 to 1/3), mostly at the expense of precautionary savers.

ary saving, especially in midlife.[9] Moreover, these effects are much stronger for households with a higher degree of labor income risk.

Finally, the models of Merton (1969, 1971) and Samuelson (1969) assumed that all assets have identical tax treatment and that there are no tax differences across investment environments available to households. As discussed earlier, such differences are the defining feature of tax-deferred accounts, which represent nearly a third of total household financial assets in the US.[10] Thus, portfolio models that seek to make qualitative or quantitative predictions about household portfolio allocations must pay attention to tax features of investment habitats.

In sum, the existing theoretical literature demonstrates that labor income risk, interactions between various savings motives, and tax treatment systematically influence portfolio choices. By expanding the set of tax and liquidity characteristics of assets available to households, tax-deferred retirement accounts (TDA) provide an interesting setting for studying all three of these effects. Tax advantages and limited liquidity of TDAs imply that labor income shocks pose asymmetric risks to taxable and tax-deferred holdings, since it is less costly to smooth such shocks with taxable assets. Once TDAs are added to portfolio models, contributions and allocations to the two account types need to be determined jointly, which permits analysis of interactions between the retirement and other savings motives. The study of competing savings motives is helped further by expiration of TDA liquidity restrictions at retirement age.

The pioneering contributions in portfolio theory with both asset- and habitat-specific tax differences were made by Black (1980) and Tepper (1981) who studied the optimal portfolio choice of a corporation interested in funding its defined-benefit pension plan. Assuming unrestricted borrowing by corporations, their papers demonstrate risk-free strategies to increase after-tax wealth by replacing TDA equities with bonds and taxable bonds with equities to maintain the desired overall asset mix. The assumption of unlimited borrowing is not tenable for typical households. Yet, the relationship between optimal portfolio choice and household tax environment has started to receive research attention only recently. The next

[9] The lifecycle implications in Campbell et al. (2001) largely follow the intuition of Jagannathan and Kocherlakota (1996). Young households have high future labor income, which implicitly constitutes a riskless asset. They are also borrowing-constrained and so their relatively small savings are invested 100% in equities. As investors grow older, their accumulated savings increase relative to the stock of future labor income, and their holdings of risky assets decline. When a retirement system is switched from bonds-only to a mixed equities-bonds allocation, the same income replacement rate can be achieved with a lower contribution rate. In response, the young borrowing-constrained households consume most of the extra income and still keep an all-equity portfolio. By contrast, the middle-aged households increase the level of their precautionary savings and decrease equity holdings.

[10] Author's tabulation from the 1998 Survey of Consumer Finances. Total financial assets are defined as the sum of "investment" assets like mutual funds, CDs, money market accounts, etc. The definition specifically excludes transaction accounts, housing and proprietary business wealth, as well as imputed values of future guaranteed pension income (Social Security, defined benefit plans) and human capital wealth.

section takes a closer look at recent theoretical models of dual-habitat portfolio choice.

2.1. GENERAL MODEL OF DUAL-HABITAT PORTFOLIO CHOICE

It is convenient to present models of dual-habitat portfolio choice within a common framework. There are two necessary ingredients: tax differences across assets and between investment habitats. When there are no borrowing restrictions in the taxable account and no additional sources of risk, the problem lends itself to analytical treatment, whether in a static two-period framework or in a dynamic multi-period setting (Huang, 2001). The analytical results suggest that there should never be any holdings of low-tax assets in retirement accounts as long as there are high-tax assets in regular accounts. In the US, equities are tax-favored in several respects. They are taxed at lower rates than interest-paying assets, receive tax breaks when used for bequests, and afford a timing choice for realization of capital gains or losses. For all these reasons, it is believed to be better to locate bonds in TDAs which defer taxation, have no use for timing capital gains or losses, and are ill-suited for bequest planning.

Once the implicit assumption of complete markets is relaxed (or uninsurable risks are introduced) the two components of this portfolio problem – location and allocation choices – are no longer separable (see Huang, 2001) and one has to rely on numerical solutions for optimal portfolio rules. The general setup of a decision-theoretic dual-habitat portfolio model was first proposed in DSZ (1999) and it has served as the basis for subsequent work.

Consider then the problem of a household that lives for a finite number of periods. In each period a household chooses consumption, given the draw from its stochastic labor income and wealth accumulated in the two available savings habitats – a tax-deferred retirement account (TDA) and a conventional savings account (CSA). At the same time, a household also decides on the level of contributions and portfolio composition for each account.

The novelty of dual-habitat portfolio models derives from their explicit differentiation of taxable and tax-deferred investment environments. Therefore, the description below focuses on tax and liquidity differences in the two account types, while the detailed setup of the general model is relegated to the Appendix.

Each of the two accounts can be invested in a broadly diversified equity portfolio with a pre-tax return of r, a tax-exempt municipal bond that returns r^{m}, or a risk-free bond that returns r^{f}, with $\mathrm{E}[r] > r^{\mathrm{f}} > r^{\mathrm{m}}$. The total return on equities is comprised of dividends and capital gains. Dividends d_{t} (and interest payments) are realized automatically and are taxed at τ^{d}, while realization of capital gains g_{t} depends on timing choices of the investor. When realized, capital gains[11] are

[11] In the US, a distinction is made between short-term capital gains (on equities held less than a year), which are taxed at τ^{d}, and long-term gains taxed at τ^{c}. The model assumes that all capital gains are long-term.

taxed at τ^{c}, such that $\tau^{c} < \tau^{d}$. Therefore, the after-tax return on equity is given by: $r_t^{AT} = (1 - \tau^{d})d_t + (1 - \tau^{c})\, \mathrm{h}(gt)$, where $\mathrm{h}(\cdot)$ denotes capital gains realization choice of an investor. Note that tax-exempt bonds are subject to an implicit tax rate τ^{m}, defined through $r^{m} = (1 - \tau^{m})r^{f}$.

The TDA account defers taxation on returns that accumulate on pre-tax contributions.[12] The CSA account taxes all earnings as soon as they are realized. Consequently, one-period returns on a \$1 TDA portfolio with α^{TDA} in equities and a \$1 portfolio in CSA, with α^{CSA} in equities are given by:

$$\begin{aligned} R^{TDA}(\alpha^{TDA}, r) &= \alpha^{TDA}(1 + r) + (1 - \alpha^{TDA})(1 + r^{f}) = (1 + r^{f}) \\ &\quad + \alpha^{TDA}(r - r^{f}) \end{aligned} \tag{1}$$

$$\begin{aligned} R^{CSA}(\alpha^{CSA}, r) &= \alpha^{CSA}(1 + r^{AT}) + (1 - \alpha^{CSA})(1 + r^{b}(1 - \tau^{b})) \\ &= (1 + r^{b}(1 - \tau^{b})) + \alpha^{CSA}(r^{AT} - r^{b}(1 - \tau^{b})) \end{aligned} \tag{2}$$

where $\tau^{b} = \min(\tau^{m}, \tau^{d})$ and $r^{b} = r^{f}$ if $\tau^{m} > \tau^{d}$ and $r^{b} = r^{m}$ otherwise.

The above expressions for asset returns are based on the following two features of bond investments. The first is that tax-exempt bonds will never be held inside TDA, since they have a lower before-tax return than taxable bonds. The second is that only one of the two bond types will be held in the CSA: munis if $\tau^{m} < \tau^{d}$ or taxable bonds if $\tau^{m} > \tau^{d}$.

TDA contributions can be made only during working years and cannot exceed a fixed fraction of stochastic wages. While CSA assets can be accessed without cost at any time, withdrawing TDA funds prior to retirement requires paying both fixed and proportional penalties. Neither borrowing nor short-selling is allowed in either account.

2.2. SPECIFIC MODELS

Shoven and Sialm (2004) adapt the Tepper-Black tax arbitrage arguments to the portfolio problem of individual households in the presence of an additional asset class – tax-exempt municipal bonds (munis). The risk properties of such bonds resemble those of taxable government notes, but their tax treatment is more similar to non-dividend-paying stocks for which the tax rate remains implicit prior to realization of capital gains. Thus, munis may have a lower tax burden than actively managed equity mutual funds that realize most of their returns as short-term capital gains or funds with high dividend yields. In order to highlight the effect of municipal bonds on optimal location and allocation choices the model disallows intermediate consumption and labor income flows. The specialization of the general model (see Appendix) to the setup of Shoven and Sialm (2004) is summarized in the leftmost column of Table II.

[12] Equivalently, one can consider a TDA setup where *after*-tax contributions escape further taxation altogether as is the case with Roth IRA accounts in the US.

Table II. Summary of models of dual-habitat portfolio choice

Model Component	Shoven & Sialm (2004)	DSZ (2003) [a]	Huang (2001)	Campbell et. al. (2001)
Control variables	$\alpha^{\mathrm{CSA}}, \alpha^{\mathrm{TDA}}$	$c, \alpha^{\mathrm{CSA}}, \alpha^{\mathrm{TDA}}, \mathrm{s}^{\mathrm{TDA}}$	$c, \alpha^{\mathrm{CSA}}, \alpha^{\mathrm{TDA}}, \mathrm{s}^{\mathrm{TDA}}$	c, α^{CSA}
Labor income	none	**prop. to wealth:** $\mathbf{w}g_t = lW_t$, *l = 0.15*	prop. to wealth: $wg_t = lW_t$,	wg_t = f(dem$_t$) + shocks$_t$
Liquidity shock	none	none/ **consumption shock prop. to wealth:** $shock_t$ **= 0.5**$^* W_t$	**\$100K @ age 35** **\$100K @ age 55**	None
TDA access. restrictions	Completely inaccessible	Inaccessible before ret./ 10% penalty on withdrawals	10% penalty on withdrawals	**Completely inaccessible**
TDA contrib. for $t < R$	No	$\mathbf{s}^{\mathbf{TDA}} = kwg = 0.2^* lW/$ $s^{TDA} \le kwg = 0.2^* lW$	$s^{TDA} \le \$10{,}000$	$\mathbf{s}^{\mathbf{TDA}} = \theta^* wg$ $\theta = 0.1$
CSA alloc. restrictions	$\alpha^{CSA} \in [0, 1]$	$\alpha^{CSA} \in [0, 1]$	$\alpha^{CSA} \in [0, 1]$	$\alpha^{CSA} \in [0, 1]$
TDA alloc. restrictions	$\alpha^{TDA} \in [0, 1]$	$\alpha^{TDA} \in [0, 1]$	$\alpha^{TDA} \in [0, 1]$	$\alpha^{TDA} = 0$ or $\alpha^{TDA} = 0.5$
Asset menu (tax rates)	**Stocks/bonds/ municipal bonds**	Stocks/bonds	Stocks/bonds	Stocks/bonds
Cap. gain/ Div. tax rates	(0.4/0.2/ $/\tau^m$ varies)	(0.36/0.2)	(0.4/0.2)	**(0/0)**
Equity premium	3%	5.18%	3%	4%

Table II. Continued

Model Component	Shoven & Sialm (2004)	DSZ (2003) [a]	Huang (2001)	Campbell et. al. (2001)
Capital gains realization	Varies with tax efficiency of mutual funds	Chosen by HHs/automatic	Chosen by HHs	Automatic
State variables	W_0	TDA/W, α_{t-1}^{CSA}, tb_{t-1}/p_t	TDA/W, α_{t-1}^{CSA}, tb_{t-1}/p_t	TDA/v, CSA/v; v = perm. comp. of wg
Time horizon	**2 periods**	Finite, $T_{max} = 100$	Finite, $T_{max} = 100$	Finite, $T_{max} = 100$
Income in retirement	W_2		Annuitized WR Annuitized W_R	Annuitized W_R
Bequest treatment	No bequests	Discounted utility flow for an H-per. annuitized W	Discounted utility flow for an H-per. annuitized W	No bequests

[a]DSZ(2003) consider two main formulations – with and without consumption shocks. The assumptions for these setups are separated by "/" in the table above.

Common notation:

TDA = TDA wealth holdings, CSA = wealth holdings in taxable (or conventional savings) accounts, W = CSA + $(1-\tau^d)$TDA = total after-tax wealth; α^{TDA} = share of TDA held in equities, α^{CSA} = share of CSA held in equities; s^{TDA} = contribution (withdrawal) to TDA, s^{CSA} = contribution (withdrawal) to CSA; c = consumption; wg = labor income flow; p_t = price of equity at time t; tb_t = tax basis at time t ($tb_t = p_t$ if $tb_{t-1} > p_t$).

bold = assumption that has a key effect on model predictions.

Using both analytical arguments and numerical methods, the authors show that when municipal bonds are available, tax-efficient portfolios can simultaneously contain equities in TDA and bonds in taxable accounts. The catch is that tax-exempt munis are the only type of bonds that can be held in CSA.[13] The intuition for this result is as follows: in order to be optimally located in TDA, equities must have higher tax burden than bonds. However, as long as equity returns contain any long-term capital gains, their effective tax burden (defined as $\tau^{\mathrm{s}} = 1 - r^{\mathrm{AT}}/r$) will be lower than τ^{d}. In contrast, munis can have an implicit tax rate that is lower than τ^{s}. This would necessarily imply that $\tau^{\mathrm{m}} < \tau^{\mathrm{d}}$, which, as argued before, is a condition for holding only municipal (and not taxable) bonds in CSA. Therefore, if municipal bonds are available and if equity mutual funds carry a sufficiently high tax burden ($\tau^{\mathrm{m}} < \tau^{\mathrm{s}}$), it is possible to have mixed holdings in both accounts.

However, this finding encounters two critical empirical difficulties. The first is due to the immediate implication of the above argument that TDA equity holdings are optimal only if CSA bond holdings are either zero or consist exclusively of tax-exempt munis. All *taxable* bonds should still be located inside TDA in order to achieve tax-efficiency. The data from the 1998 Survey of Consumer Finances suggest that this is not the case. Ninety-three percent of households that own equities in TDA also own taxable bonds or bond-like assets in their conventional savings accounts.[14] While tax-exempt bonds are held almost exclusively in CSA, 97% of households that own such securities also keep taxable bonds or bond-like assets in their CSA.

The second empirical difficulty lies in the assumption of high tax burdens of available equity mutual funds. Imagine that there exist two mutual funds that differ in their asset turnover and thus in effective tax rates on asset returns – an actively managed fund and a passive index fund. A valid question then is whether a household is better off choosing a high-turnover equity fund for TDA and munis for CSA, or taxable bonds for TDA and a low-turnover equity fund for CSA. As Shoven and Sialm (2004) note, the former choice is preferred only if the actively managed mutual fund has significantly higher expected returns than an index fund.[15] The extensive evidence on relative performance of actively managed mutual funds points to the contrary (see, for example, Carhart, 1997 and Daniel et al., 1997, among others).

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[13] The only exception occurs for very risk-averse investors if after-tax returns of bonds in CSA are assumed to be more volatile than muni returns. In this case, CSA bonds holdings may contain taxable as well as tax-exempt bonds.

[14] The definition of "bonds" is extended to include other assets that provide safe return and face largely the same tax treatment on earnings, as do conventional bonds. Thus, "bond-like assets" include money market and savings accounts, while "bonds" account for both direct and mutual fund holdings of corporate and government bonds.

[15] A calibration in DSZ (2003) suggests that an actively managed fund has to outperform an index fund by as much as 165 b.p. per annum, assuming that returns of these two funds are perfectly correlated.

While re-defining Tepper-Black tax-efficiency for environments that contain tax-exempt municipal bonds is highly informative, it is based on the same pecking order strategy and it does not help explain existing portfolio choices. An avenue that was left unexplored in the two-period model of Shoven and Sialm (2004) is the effect of accessibility restrictions on portfolio choice. The presence of such restrictions changes the implicit payoffs of TDA assets and takes the guarantee of higher after-tax wealth out of Tepper-Black asset reshuffling. With limited TDA accessibility, replacing CSA bonds with equities and TDA equities with bonds still yields higher after-tax returns. However, such reshuffling also makes the CSA portfolio more risky and thus increases the likelihood of early TDA withdrawals. In some states of the world, costs of TDA withdrawals will more than offset gains from tax-efficiency.

DSZ (2003) solve a multi-period dual-habitat portfolio model that incorporates stochastic labor income, consumption shocks, and limited TDA accessibility before retirement.[16] The model has two asset classes – stocks and bonds[17] – and it allows investors to choose when to realize their capital gains. Since the model allows optimal tax timing, its state space has to include variables that track unrealized capital gains (tax basis and the current market price). In order to reduce the dimensionality of the problem and to be able to exploit homogeneity of the utility function, the authors assume a labor income process that is strictly proportional to total after-tax financial wealth (15%). Similarly, consumption shocks are defined as a constant fraction (50%) of total after-tax wealth. These shocks are motivated by sizable expenditures like house purchase or college tuition. For convenience, the DSZ model with stochastic labor income and consumption shocks is summarized in Table II.

DSZ (2003) first evaluates the model without consumption shocks, but with TDA assets completely inaccessible prior to retirement. Intuitively, uncertain labor income and inaccessible TDAs should generate positive CSA bond holdings during working years as a means of ensuring some minimum level of consumption. However, the numerical solutions of the model indicate that Tepper-Black tax efficiency remains the optimal policy. Stocks can be held in retirement accounts if total desired bond holdings do not exhaust TDA capacity and CSA is invested 100% in equities. Conversely, if the overall desired bond holdings exceed TDA limits, the spillover goes in the other direction – the "surplus" bonds are held in taxable accounts. Both cases, however, rule out keeping equities in retirement accounts while simultaneously holding bonds in taxable accounts.[18]

[16] In addition to numerical analysis, DSZ (2003) contains an analytical section that makes a number of insightful points.

[17] DSZ setup can also easily accommodate tax-exempt bonds, which would be only be held in CSA by high-tax households (those with $\tau^{\mathrm{d}} > \tau^{\mathrm{m}}$ if their TDAs are fully invested in taxable bonds and if passive index mutual funds are available.

[18] These results are unchanged if one eliminates the timing choice for capital gains or modifies the way in which bequests affect utility (see Appendix). While these features of the model are quite

The absence of "tax-inefficient" portfolio allocations owes mainly to the choice of labor income process. The proportionality of labor income to total wealth means that one of the most effective ways to insure against labor income shocks is to maximize after-tax wealth, which is exactly the objective of Tepper-Black efficient portfolio policies. Moreover, the riskiness of labor income in this setting is endogenous and as long as enough bonds are held overall, the wage stream is fairly safe. Tax-efficient allocations produce enough wealth and, by extension, labor income to ensure that consumption can be financed from wages and CSA holdings. Thus, the absolute (in)accessibility constraint on TDA assets never binds and it is effectively irrelevant.

DSZ (2003) next considers a setting with consumption shocks and TDA assets accessible at a 10% penalty before retirement. The shocks are equal to 50% of total after-tax wealth and occur with probability of 2% when equity returns are positive, and 18% when they are negative. There are two main results: (1) households that are close to retirement and that have slightly less than 50% of their wealth in the taxable account hold non-tax-efficient portfolios; and (2) all other households try to prevent early penalized withdrawals by reducing their TDA contributions.

Why do young households prefer to incur a sizable cost of not contributing to TDA as opposed to a much smaller cost of tax-inefficient (but safe) allocation?[19] The answer appears to be due to the way in which consumption shocks are defined. Since shocks result in withdrawal penalties only when CSAs contain less than 50% of total wealth, the relative size of taxable wealth matters more than its composition. Tax-efficient allocations maximize total after-tax wealth, *but* they also increase the relative share of CSA wealth since equities, on average, have higher returns. Hence, young households postpone their TDA contributions to get themselves over the 50% threshold quickly, and over the longer run they rely on tax-efficient allocations to stay above this threshold.

By contrast, for old households close to the 50% threshold, the gains from TDA contributions (even over a few years) are more than the tax cost of holding bonds in taxable accounts. For them, holding some bonds in CSA can eliminate the danger of penalized withdrawals; especially since consumption shocks are much more likely to occur when equity returns are negative and thus the relative value of TDA is below the 50% threshold. However, even for these households non-tax-efficient portfolios are short-lived. As soon as they reach the retirement age, early

important (especially the effect of step-up in basis provision on lifecycle portfolio allocations), they do not bear directly on portfolio tax-efficiency.

[19] The cost of forgoing tax-efficiency is proportional to the difference between tax rates on dividends and capital gains, and to the risk-free rate. Assuming $\tau^{\mathrm{d}} = 36\%$, $\tau^{\mathrm{c}} = 20\%$, and r^{f} of 6%, the annual forgone return is (0.36–0.2) *0.06 = 96 b.p. (see Shoven and Sialm, 2004). By contrast, the cost of keeping assets in a taxable, as opposed to a tax-deferred account, is given by the difference between before- and after-tax returns. With the assumptions above, the forgone one year return on a bond investment is 216 b.p. Note that after the dividend tax cut passed recently by the US Congress ($\tau^{\mathrm{d}} = 35\%$, $\tau^{\mathrm{c}} = 15\%$) the cost of tax inefficiency rose to 120 b.p. per year.

withdrawal penalties disappear and there is no reason to sacrifice tax-efficiency.[20] The "transience" of the response to consumption shock is driven by the exogenous change in institutional characteristics of investment habitats.

Huang (2001) suggests that mixed stock and bond holdings in both accounts are due to predictable and unavoidable lumpy expenditures at certain points in household lifecycle.[21] These unavoidable liquidity needs are modeled as a $100,000 tax on wealth occurring at ages 35 and 55 for housing and education, respectively (see Table II for detailed model description). The numerical solutions of the model are quite similar to those in DSZ (2003). Mixed portfolios are optimal only for households with CSA wealth roughly equal to the required expenditure and only in the few periods before such expenditure occurs. For example, households that have $105,000 in CSA and $45,000 in TDA opt to hold some bonds in CSA starting at age 30. After the need for liquidity passes, they return to Tepper-Black tax-efficient allocations. Here, again, the "transience" of non-tax-efficient portfolios is due to the exogenous features of the model, which turns off liquidity shocks in periods other than 35 and 55.

As described in the introduction, using lumpy housing or education expenses to highlight the importance of TDA accessibility restrictions ignores existing institutional exemptions. For most households such liquidity needs can be satisfied without sacrificing tax-efficiency. Moreover, certain empirical predictions of the liquidity needs models are not supported by the data. Since lifecycle events have a strong demographic component, one such prediction is that for certain ages and family compositions one should observe uniformly tax-efficient behavior. In particular, households that do not have foreseeable lumpy expenditures have no reason to forgo tax-efficiency. Table III summarizes the data from the 1995–1998 Surveys of Consumer Finances on portfolio allocations of US households between the ages of 25 and 55 that had at least $10,000 in both TDA and CSA.[22] Households in this age group account for nearly 80% of all those who save to buy their first house or finance their children's education. The data show that tax-inefficient non-corner portfolios are equally common among households with and without foreseeable housing or education expenses. Moreover, the degree of tax-inefficiency of portfo-

[20] In their analytical section, DSZ (2003) show that it may be optimal to hold bonds in CSA even in the absence of withdrawal penalties, since smoothing consumption with TDA assets always carries a cost of forgone tax-deferred wealth accumulation. However, the numerical results in their paper and those presented below indicate that penalties play the key role.

[21] The main focus of Huang (2001) is on novel theoretical proofs of optimality of Tepper-Black tax-efficient portfolios in a multi-period setting. The proofs rely on the ability to replicate a unit of equity in a tax-deferred account by a portfolio consisting of a unit bond holding in a TDA and a leveraged position in equities in the taxable account. When borrowing at the risk-free rate in taxable accounts is prohibited, the proof still carries through with replication replaced by option pricing methods. The underlying model thus discards uninsurable labor income risk.

[22] The wealth threshold serves to identify households that have non-trivial savings in both account types and are thus concerned with tax-efficiency of their portfolios (Bergstresser and Poterba, 2002).

Table III. Household portfolio choices by presence of foreseeable housing or education expenditures

		No plans to buy a house or finance education	Plan to buy a house or finance child's education in 5 yrs
Portfolio choice	Median % of CSA in equity	47.5%	47.6%
	Median % of TDA in equity	89.9%	94.3%
Tax-efficiency	No equities at all	4.5%	4.1%
	No equities in CSA[a]	21.0%	23.3%
	Mixed tax-inefficient portfolios[a]	44.3%	46.5%
	Tax-efficient portfolios	27.8%	25.2%
Savings	Median TDA	\$55,500	\$55,000
	Median CSA	\$46,800	\$43,000
	Median age	46	43

SOURCE: 1998 Survey of Consumer Finances, all figures are population-weighted.
Note: sample is limited to households between 25 and 55 years of age with at least \$10,000 in both TDA and CSA.
[a]Represents a "tax-inefficient" asset allocation.

lio choices (i.e., the share of bonds in CSA and stocks in TDA) and account-specific wealth levels are virtually identical for both household types.

Alternative approaches to resolving the alleged tax-inefficiency puzzle can be categorized into three different sets, all of which stress the joint importance of limited TDA liquidity and additional risk sources. The first is to exploit preferences that exhibit a higher and/or time-varying degree of risk aversion. The second approach is based on introduction of additional sources of risk motivated by empirical research, such as highly variable entrepreneurial income that is positively correlated with stock market returns (see Heaton and Lucas, 2000).[23] The final set of changes involves alternative ways to model liquidity restrictions on TDA assets, using existing institutional arrangements as a guide. In addition to proportional early withdrawal penalties, such restrictions can also take the form of fixed access costs such as time delays for processing claims and administrative fees.

The model analyzed in the next section concentrates on risks posed by catastrophic labor income shocks. It also considers the consequences of adding fixed costs of TDA access. Finally, it evaluates sensitivity of portfolio and savings choices to assumptions on the magnitude of risk aversion and tax rate differentials.

[23] In the 1998 Survey of Consumer Finances, 15.3% of households between the ages of 25 and 55 had at least one self-employed member working full-time. According to estimates in Heaton and Lucas (2000), the growth rate of proprietary income is more volatile than the growth rate of wage income and is positively correlated with equity returns ($\rho = 0.14$ with CRSP value-weighted index).

3. Model with Precautionary Savings Motives

The defining characteristic of the model in this section is its choice of presenting background risk through an uninsurable labor income process. This approach is similar in spirit to the concurrent liquidity risk models, but it has several distinct advantages. First, this setting is better aligned with the existing institutional rules on pre-retirement asset withdrawal. Second, using labor income risk embeds the dual-habitat portfolio choice model within an established class of precautionary savings models and expands the set of their portfolio predictions. Importantly, it also allows formulation of a greater number of empirical restrictions that can be used to assess the quality of model's predictions.

The wage process is defined as $w_t = f(age, educ, \ldots) * (1 - s\mathrm{I}_u)$, where $\mathrm{I}_u = 1$ with an exogenously given probability p. Here $f(\cdot)$ denotes the predictable component of household wages that is a function of demographic factors such as age, education and occupation. In the unemployment state the household receives only a fraction $1 - s$ of its trend income. Undoubtedly, realistic labor income processes have a much richer structure that accommodates both transitory and permanent shocks, as well as jump-like unemployment shocks. Still, in this purposely simple model, a binary wage process is the most direct way to generate labor income uncertainty. The model also eliminates any additional demographically-driven liquidity shocks.

In the presence of such uncertainty and credit market imperfections, prudent households choose to hold buffer stocks of assets, which is commonly referred to as precautionary savings behavior (Kimball, 1990; Carroll, 1997).[24] In the context of the dual-habitat portfolio allocation problem, such buffer stocks can be manifested through holdings of safe and liquid assets – that is, bonds outside of retirement accounts.[25]

It is possible to describe "precautionary portfolio effects" for the dual-habitat problem similar to the definition of precautionary savings. Such effects are defined here as changes in habitat-specific portfolios that a liquidity-constrained household makes when it faces labor income risk, as compared to the benchmark of Tepper-Black tax-efficiency that is obtained in the certain income scenario. As will be shown later, the interaction between accessibility restrictions and risky labor income is needed for producing habitat-specific precautionary portfolio effects.

This "precautionary" model of dual-habitat investment choices can peacefully co-exist with the "tax-efficient" model outlined above. In fact, tax-efficient behavior is optimal for sufficiently wealthy or retired households. When taxable wealth

[24] Precautionary savings are commonly defined as the incremental savings that a liquidity-constrained household makes when it faces labor income risk, compared to the certain income scenario.

[25] Simulation results indicate that under plausible specifications of stochastic income processes, buffer stocks need to be not only liquid, but also include safe assets like bonds. Still, one needs stronger theoretical results to establish conditions under which buffer stocks necessarily contain riskless assets.

is high, the risk of not being able to smooth wage shocks is small even if CSA is invested in equities. Hence, bonds can be optimally stored inside TDA. With this reasoning, CSA composition depends on the level of wealth: the household wants to "buffer" with safe and liquid assets unless liquidity of safe assets costs too much relative to the risks posed by equity buffers. Similarly, households not subject to labor risk can afford to concentrate on making tax-efficient location choices. The intuition is analogous to that of the precautionary savings literature, where even with uninsurable income risk there may be agents that are either not exposed to that risk or are wealthy enough not to be affected by credit constraints (Carroll, 1997; Carroll et al., 1999).

3.1. NUMERICAL SETUP

This problem has a recursive representation, with state space given by $\{TDA_t, CSA_t, t\}$. The model is solved by backward induction with MATLAB's constrained optimization routines, using shape-preserving bivariate interpolation (Costantini and Fontanella, 1990) and evaluating expected values by quadrature methods. A solution consists of a set of policy functions for consumption, savings and portfolio composition in each account habitat.

Although the model is calibrated on annual data, the discussion of its solutions focuses on just three periods that capture the distinct lifecycle stages that differ in terms of risks and TDA restrictions. These stages are: retirement, which has no labor income or TDA access restrictions, the penultimate period in which there are still TDA access restrictions but no remaining labor risk, and the working period that has both labor risk and limited TDA liquidity.[26] This shift in emphasis comes at the expense of detailed description of evolution of portfolio choices over the entire lifecycle (which was highlighted in DSZ (2003) and Huang (2001)). Instead, I concentrate on identifying model components that elicit precautionary portfolios at a point in time by varying settings for background risk, liquidity restrictions, tax differentials, etc.

The labor income process in the model is calibrated by selecting the values of p and s. I follow Carroll (1992) in assuming a very small probability (0.5%) of a catastrophic loss defined as losing 99% of one's labor income. In retirement, the household receives no labor income and consumes an annuitized flow of total wealth available at the start of last period. While the assumption of catastrophic labor income shocks may seem extreme, the above choice of (p, s) maintains a direct link with earlier precautionary savings models and allows their evaluation in the context of tax-efficient portfolio decisions. Moreover, there exists some empirical evidence that households do experience significant income losses that are unmitigated by social insurance programs. For example, Gakidis (1998) reports

[26] The retirement period, denoted as R, can be of arbitrary length. The working period is $R - k$, where k is usually set at 2 to economize on calculations, although the model was evaluated for larger values of k as well.

that in his sample of PSID data from 1977–1983, 6% of all observations experienced "catastrophic drops in income" defined as declines below 50% of the 7-year average annual income. The median household in this group received just 10% of previous year's income.

The model is solved for the benchmark case of power utility with the coefficient of relative risk aversion equal to 3 and the annual rate of time preference of 0.96. The rate of log return on equities is assumed to be i.i.d. normally distributed with mean annual nominal return of 0.101 and standard deviation of 0.192. These figures are computed from 1926–2000 CRSP data on cum dividend returns on the value-weighted equity index. Inflation is assumed to be constant at 3.5%. The riskless asset is assumed to have a nominal return of 6.8%, so that the annual pre-tax equity risk premium is 3.9%. This is markedly lower than the historical risk premium of 6.9%, but it approximates the expected equity risk premium argued for in Fama and French (2002). Capital gains and losses are realized automatically each period. Dividends, wages, and interest income are taxed at τ^{d} of 40%, while the capital gains rate, τ^{c} is set at 20%.

In the base case scenario, households can contribute up to 20% of pre-tax wages to TDA, but they are assessed a 10% penalty for pre-retirement withdrawals of TDA assets. This penalty effectively applies even in the case of borrowing against TDA, which is a common feature of many retirement plans. Under the current law, a household that loses its job while carrying a loan balance has to repay the entire loan immediately in order to avoid withdrawal penalties. If, as postulated in the model, unemployment risk is the main source of labor income uncertainty, both ways of accessing TDA assets for financing current consumption trigger penalties. Consequently, the model makes no distinction between borrowing and early withdrawals.

3.2. PROPERTIES OF NUMERICAL SOLUTIONS

3.2.1. *Policy functions*

It is easier to analyze the relationships between controls and state variables on cross-sections of three-dimensional policy surfaces. Therefore, in this section, I will present the results holding one of the state variables fixed (e.g., the level of TDA wealth), while varying the other.

The top panel in Figure 1 shows optimal portfolio choices in the last working period. At the time of decision, the wage income is already known and the household also knows the values of its two state variables – CSA_{T-1} and TDA_{T-1}. The household understands that it will not face any more wage uncertainty and will have full access to its retirement assets next period. Thus there exist account and asset tax differences but no labor market risk or liquidity restrictions.[27]

[27] In the absence of either background risk or accessibility restrictions it can be shown analytically that a household will always choose a tax-efficient allocation. When both risk and restrictions are

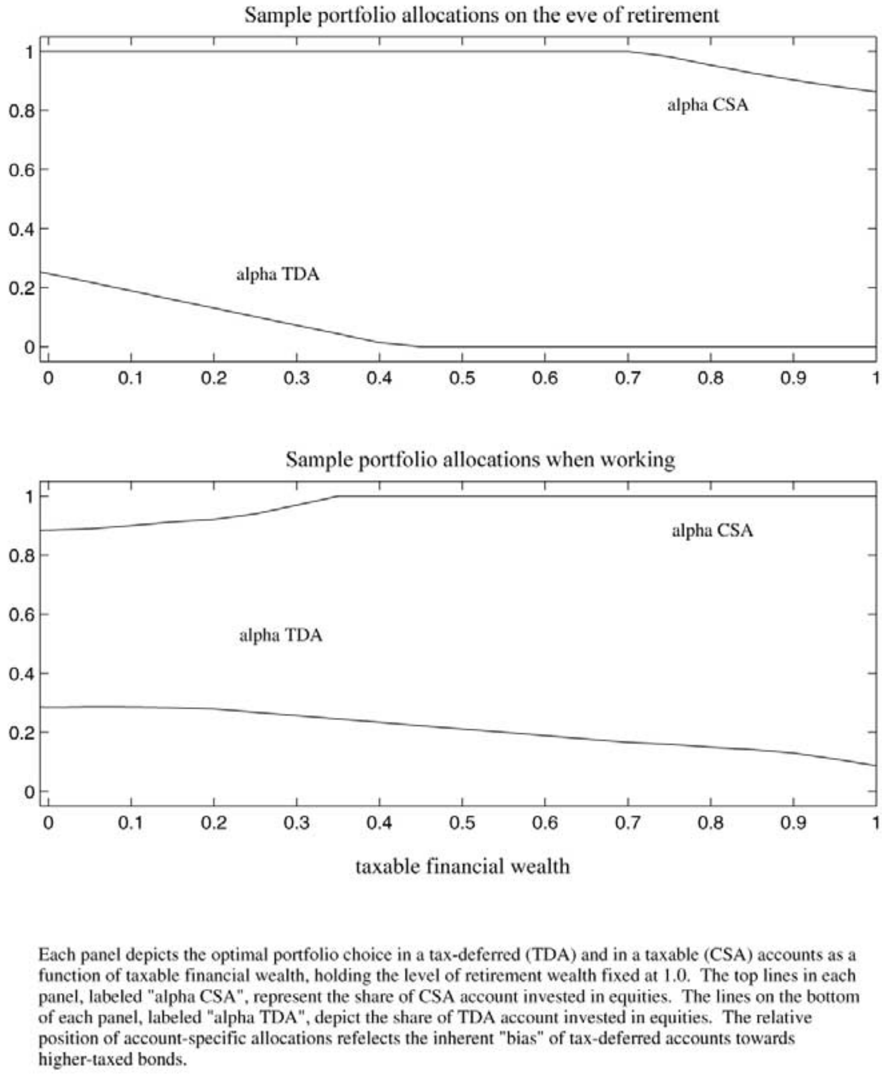


Figure 1. Policy functions for portfolio allocations.

The top panel confirms optimality of tax-efficient portfolio allocations. Just as in the DSZ model discussed earlier, the allocations consist of tax-efficient asset "spillovers". The direction of spillover (equities from CSA to TDA or bonds from TDA to CSA) depends on the relative sizes of the two accounts. In sum, an asset

introduced, it is possible to get mixed portfolio allocations. These results are obtained from a simple two-period version of the model and are available upon request. As mentioned in footnote 20, DSZ (2003) show that in a multi-period setting it is analytically possible to obtain mixed allocations in the absence of accessibility restrictions, as long as market returns and incidence of early TDA withdrawals are negatively correlated.

spills over from its preferred account type if the account is not large enough to accommodate this asset fully.

The location component of household decision in the last working period is also straightforward. Most households contribute the maximum amount to TDA and finance current consumption out of remaining wages and CSA assets. Only when the amount of taxable assets is small relative to TDA wealth, do households cut back on retirement contributions or even withdraw TDA assets at a penalty. Since there is no reason to maintain a buffer stock of liquid assets in the penultimate period, there are no interesting interactions between location and allocation decisions – whatever is not consumed gets saved in the retirement account up to the contribution limit and an appropriate tax-efficient allocation is chosen.

The nature of decisions changes significantly in all preceding periods, during which the next period wage realization is uncertain and TDA withdrawals are penalized. Mixed allocations in both account habitats can now be observed, and the degree of such precautionary portfolio effects varies systematically with the composition of household endowment. In particular, households that have less taxable wealth resort to holding bonds in their taxable accounts as a way to decrease the risk of early TDA withdrawals. An example of such allocation can be seen in the bottom panel of Figure 1 for low CSA values. For example, when taxable wealth is equal to 0.2, a household optimally chooses to hold 92% of its taxable account and 27% of its TDA in equities. However, the scope of precautionary allocations depicted in the bottom panel is quite limited – the households put a relatively small share of their CSA wealth in bonds.[28]

The main message of the bottom panel of Figure 1 is that there exists a strong inverse relationship between equity shares in two account habitats. As the level of taxable assets declines, so does the optimal equity share in the taxable portfolio. This change is “compensated” by an increase in equity share inside TDA. The household wants to be prudent in its taxable allocations, but does not want to completely forgo the higher return potential of equity investments. However, since equity returns are more variable inside TDA accounts, the TDA equity share increases slower than the CSA equity share declines.

The intuition for such opposing effects can be gleaned from the first-order conditions of the optimization problem. In the dual habitat portfolio problem, Euler equations incorporate a number of Lagrange multipliers associated with no-borrowing and accessibility constraints (see Appendix). For example, the Euler

[28] Note that the model predicts sizable holdings of risk-free bonds in contrast to many earlier papers with labor income. Recently, Heaton and Lucas (2000) demonstrated that significant bond holdings can be obtained if labor income shocks are positively correlated with equity returns. Similarly, the current model relies on catastrophic wage shocks to elicit bond holdings, since such shocks make labor income an inherently risky asset and thus provide an incentive to maintain at least some risk-free financial assets. This result is further reinforced by the absence of an exogenous income floor in retirement, which turns retirement into the most severe of all labor income shocks.

equations for contribution choices in each of the two accounts ($\mathrm{s}_{\mathrm{t}}^{\mathrm{TDA}}$ and $\mathrm{s}_{\mathrm{t}}^{\mathrm{CSA}}$) have the form:

$$[\mathrm{s}_{\mathrm{t}}^{\mathrm{TDA}}] : (1 - \tau - pen_{\mathrm{t}})\mathrm{U}'(c_{\mathrm{t}}) - (\phi_{\mathrm{t}} - \psi_{\mathrm{t}}) = \beta \mathrm{E}[\{(1 - \tau - pen_{\mathrm{t+1}})\mathrm{U}'(c_{\mathrm{t+1}}) + \phi_{\mathrm{t+1}}\} R_{\mathrm{t+1}}(\alpha_{\mathrm{t}}^{\mathrm{TDA}})] \tag{3}$$

$$[\mathrm{s}_{\mathrm{t}}^{\mathrm{CSA}}] : \mathrm{U}'(c_{\mathrm{t}}) - \phi_{\mathrm{t}} = \beta \mathrm{E}[\{\mathrm{U}'(c_{\mathrm{t+1}}) + \phi_{\mathrm{t+1}}\} R_{\mathrm{t+1}}(\alpha_{\mathrm{t}}^{\mathrm{CSA}})] \tag{4}$$

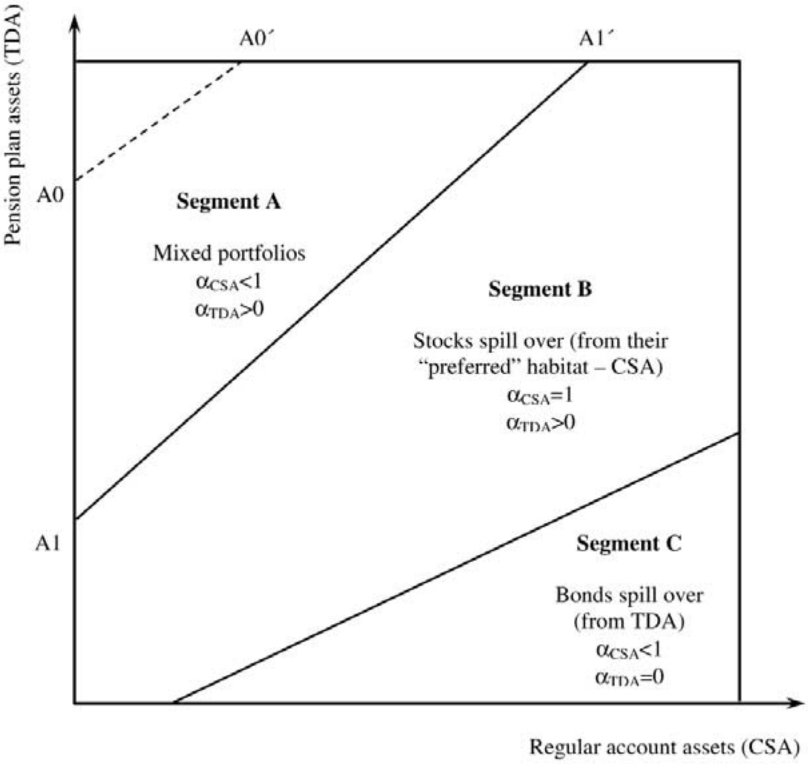
where ϕ_{t} is a Lagrange multiplier on the CSA no-borrowing restriction ($\mathrm{s}_{\mathrm{t}}^{\mathrm{CSA}} + CSA_t \geq 0$), ϕ_{t} is a Lagrange multiplier on the TDA no-borrowing restriction ($\mathrm{s}_{\mathrm{t}}^{\mathrm{TDA}} + TDA_t \geq 0$), and ψ_{t} is a Lagrange multiplier on the maximum TDA contribution restriction ($\mathrm{s}_{\mathrm{t}}^{\mathrm{TDA}} \leq q w_t$). TDA accounts are never drained completely in working years so that $\phi_{\mathrm{t+1}} = 0$. However, during periods of unemployment $\mathrm{pen}_{\mathrm{t+1}}$ is often positive, as households liquidate some of their TDA assets. Liquidation of TDA assets implies that the CSA no-borrowing constraint is binding ($\phi_{\mathrm{t+1}} > 0$) and the shadow price of relaxing the constraint is likely to be significant. Thus, in states that require TDA withdrawals, the part of the Euler equation in curly brackets is higher in the FOC for $[\mathrm{s}_{\mathrm{t}}^{\mathrm{CSA}}]$ and lower in the FOC for $[\mathrm{s}_{\mathrm{t}}^{\mathrm{TDA}}]$. Since R( ) is monotonically increasing in α, catastrophic shocks can produce lower optimal ($\alpha_{\mathrm{t}}^{\mathrm{CSA}}$) and higher optimal ($\alpha_{\mathrm{t}}^{\mathrm{TDA}}$).

Simultaneously with adjustment in portfolio allocations, the household changes its TDA contribution choice (not shown). As the level of taxable assets relative to TDA wealth declines, the household gradually decreases its TDA contribution from the maximum amount allowed to zero. Because withdrawal penalties produce a kink in the marginal cost schedule of TDA funds, households remain at the zero-contribution level for a certain range of taxable asset holdings (the "inaction region"). When taxable assets are very low, the household may choose to withdraw TDA assets at a penalty to finance current consumption. Such withdrawals are made only when all taxable wealth is exhausted. This is precisely the type of situation that the household tries to avoid *ex ante* by keeping more assets in the taxable account and investing them in bonds.

In sum, holding all else equal a household with a lower level of taxable wealth will be described by the following characteristics: (1) smaller contribution to its retirement account; (2) lower current consumption; (3) safer allocation of its taxable wealth portfolio; and (4) riskier allocation of assets in its retirement account.

Figure 2 depicts the general relationship between the type of optimal portfolio choice and the state variables of the problem. Segments B and C represent combinations of state variables (levels of CSA and TDA wealth) that result in tax-efficient allocations of two different spillover types described earlier. By contrast, segment A contains pairs of state variables that produce mixed portfolio allocations.[29] As seen from the diagram, segment A describes households that hold much of their

[29] Examples of typical portfolio allocations within each these segments are presented in panels A and B of Figure 1.



The axes represent levels of household saving in each of the two accounts at the beginning of the period. These are the state variables of the problem. α_{CSA} and α_{TDA} are equity shares in CSA and TDA, respectively.

Segment A denotes those combinations of state variables that result in an optimal mixed portfolio allocation. Such combinations typically consist of low liquid wealth levels and high pension wealth levels. Segment B represents pairs of state variables that result in stock spillover allocations described in section 3.2, and segment C represents bond spillovers. The relative positions of the regions do not change with exogenous variables of the problem, but their relative sizes do. For example, segment A shrinks from A1A1′ to A0A0′ in response to a decrease in withdrawal penalty.

Figure 2. Diagram of portfolio allocation choices as a function of state variables.

overall savings in their retirement accounts. The relative size of these segments evolves over the lifecycle and also varies with the exogenous parameters of the model, such as unemployment rate, liquidity restrictions, etc. The comparative statics exercises of the following section focus on how the size of segment A depends on household environment.

4. Model Modifications

4.1. CHANGES IN RETIREMENT WEALTH

The top panel of Figure 3 depicts working period portfolio choices as a function of share of total wealth held inside TDA. In these graphs, TDA holdings are fixed at two different levels (0.8 and 1.0), and the share of total wealth in TDA varies along with taxable financial wealth. The first feature of these graphs is that portfolio allocations in both accounts vary monotonically with the share of wealth held inside TDA. As this share increases, the fraction of the taxable account held in equities declines, while the similar fraction in the TDA goes up. Put differently, households with a higher share of wealth confined to TDA have stronger precautionary motives, since less of their overall wealth is readily accessible (i.e., the liquidity constraint is tighter). Hence, they respond by deviating more from the Tepper-Black portfolios.

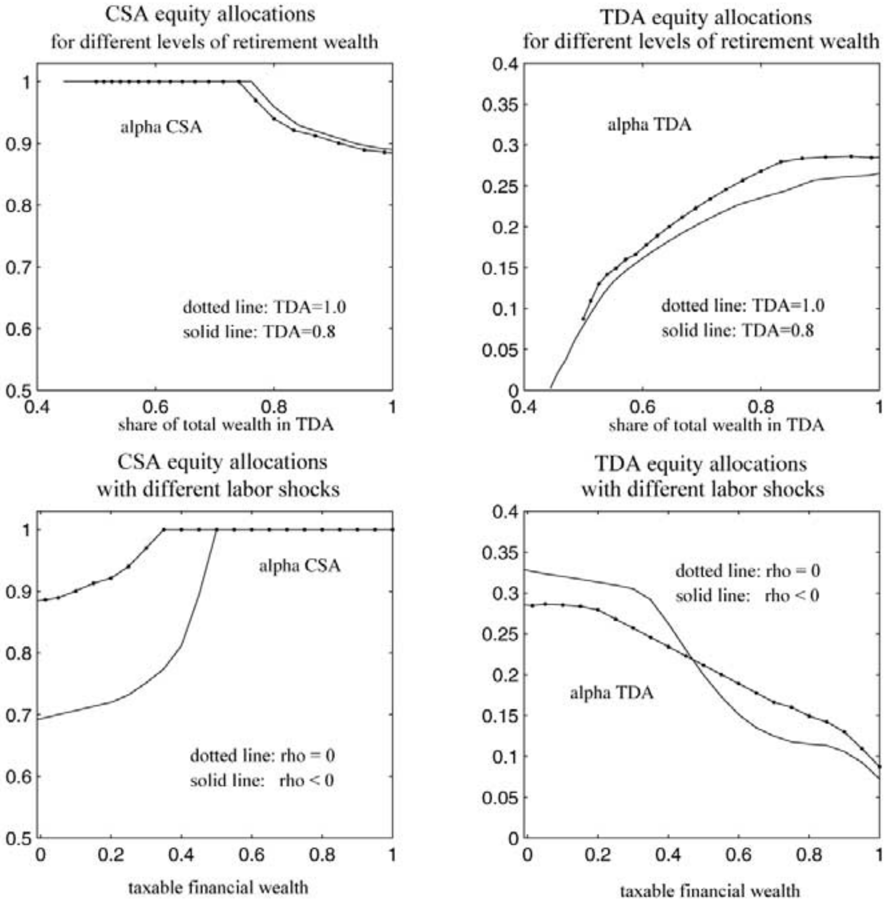
Another feature of the top panel is that as the *level* of TDA wealth increases (holding the share of TDA wealth fixed) households choose more strongly precautionary allocations. That is, higher TDA wealth results in a lower equity position in CSA and a higher equity position in TDA. The explanation is due to the properties of the optimal consumption function, which increases monotonically in both CSA and TDA wealth. As a result, for the same value of share of total wealth in TDA, households with more TDA wealth consume more. Since consumption is financed from wages and taxable wealth, these households have less net taxable assets to invest and so choose a more cautious CSA allocation.[30]

4.2. NEGATIVE CORRELATION BETWEEN UNEMPLOYMENT SHOCKS AND EQUITY RETURNS

One approach to measuring the effect of unemployment shocks is to allow them to be correlated with equity returns, while maintaining the same unconditional probability of experiencing such a shock. In particular, assume that a catastrophic labor income shock occurs with probability of 0.9% when equity returns are negative, and 0.1% when equity returns are positive. Using the 2-node quadrature approximation for convenience, the above assumption keeps the unconditional shock probability at 0.5%, but it considerably increases the likelihood of the worst-case scenario.

The lower panel in Figure 3 shows the resulting portfolio choices. As shown in the lower left-hand graph, the share of taxable account held in equities declines

[30] In the model of DSZ (2003), CRRA preferences and the assumption of labor income proportional to total wealth allow restating the problem with share of wealth held in TDA as its state variable. Even though consumption also increases with total wealth, portfolio allocations vary only with the relative share of wealth in TDA. In contrast with the current model, in the DSZ setup wages rise along with total wealth and so the effect of higher consumption on the amount of taxable wealth left for investment is smaller.



The top panels depict optimal portfolio choices as a function of share of total financial wealth held inside TDA. TDA wealth if fixed at a different level in each of the lines (TDA=1.0 for the dotted line and TDA=0.8 for the solid line), and the share of wealth in TDA varies along with holdings of taxable financial wealth. As can be seen in the upper left graph, equity allocation in CSA decreases as the share of wealth held in TDA grows and as the level of TDA wealth increases. The latter result is due to monotonicity of consumption function with respect to both TDA and CSA wealth levels. The equity allocation in TDA (upper right graph) increases with rises in either share or level of TDA wealth.

The lower panels depict optimal portfolio choices as a function of taxable financial wealth, holding TDA wealth fixed at 1. The solid line represents the case of negatively correlated unemployment shocks and stock market returns (higher prob(unempl) when r < 0). The dotted line shows the case with equial probability of unemployment shocks across market returns states. Equity allocations in CSA are lower in the case of negatvely correlated shocks. The effect of more severe unemployment shocks on equity allocations in TDA is mixed.

Figure 3. Portfolio choices for different TDA wealth levels and labor shock specifications.

significantly when probability of unemployment is assumed to be higher in periods of low equity returns. However, the changes in TDA allocations are mixed. For low values of taxable financial wealth, the share of TDA in equities increases when $\rho < 0$, but the opposite occurs at high taxable wealth values.

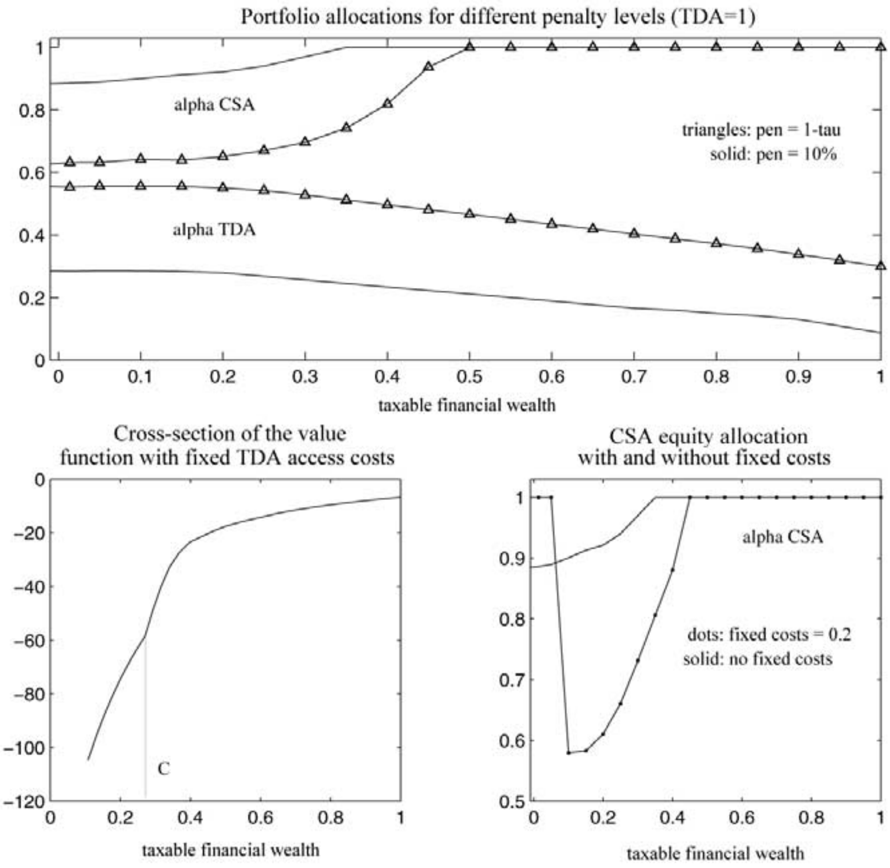
One possible explanation for this result has to do with which margins of adjustment are available for household's response to an increase in background risk. When unemployment risk becomes more severe, households change their portfolios and also cut back consumption and TDA contributions (not shown). At high taxable wealth levels these actions increase household's buffer holdings, which are still fully invested in equities. Since retirement wealth is now lower (given decreased TDA contributions), a decrease in the overall equity share in response to higher background risk is accomplished by decreasing TDA allocation to equities. However, households with little taxable wealth optimally choose to contribute nothing to TDA under both scenarios for unemployment risk. Their already low consumption has limited scope for further downward correction. As a result, most of the adjustment for these households takes place through changes in portfolio allocations, which deviate farther away from tax-efficiency.

4.3. SIZE OF THE EQUITY PREMIUM

In the scenario with the higher equity premium (of 6%), equity spillover choices are much more common at the expense of both bond spillovers and mixed portfolio choices. In terms of Figure 2, this change is represented by the expansion of segment B and shrinking of segments A and C. More attractive equities reduce overall bond holdings, so that smaller TDAs are sufficient to hold all desired bonds and bond spillovers are much less common. Furthermore, a higher equity premium means that one needs to move fewer bonds outside TDA to decrease the probability of an early withdrawal. Therefore, there exist more combinations of the beginning-of-period wealth levels that do not result in tax-inefficient allocation choices. In fact, a high enough equity premium may eliminate the precautionary allocation segment altogether.

4.4. PROPORTIONAL WITHDRAWAL PENALTIES

The top panel of Figure 4 shows the effect of tightening withdrawal restrictions on portfolio choices. When TDA assets are made inaccessible prior to retirement ($pen = 1 - \tau^{\mathrm{d}}$), the mixed nature of optimal allocations becomes much more pronounced. Since higher penalties increase the costs of maintaining tax-efficient portfolios, the range of wealth values for which tax-efficient reshuffling is avoided expands along with penalty values. In terms of Figure 2, heavier penalties expand segment A (from $\mathrm{A}_0\mathrm{A}_0'$ to $\mathrm{A}_1\mathrm{A}_1'$) so that more households in the state plane choose precautionary allocations. As expected, higher penalties also bring about reductions in TDA contributions and consumption, which are especially pronounced in regions where early TDA withdrawals were optimal under lower penalties. Finally, the "inaction region", where TDA contributions remain at zero, grows more extensive as penalties rise.



The top panel depicts optimal portfolio choices as a function of taxable financial wealth for different levels of TDA withdrawal penalties. The case of completely inaccessible TDA assets (prior to retirement) is shown by the triangle line. Heavier penalties are associated with lower equity allocations in CSA and higher equity allocations in TDA.

The two lower panels show a sample cross-section of the value function and CSA portfolio choices when fixed costs of access are added. As a result, the value function is no longer globally concave. With fixed costs, bonds are held in CSA at higher levels of taxable financial wealth and in greater quantity. However, at low taxable wealth values the CSA portfolio reverts to an all-equity allocation.

Figure 4. Different costs of TDA access.

4.5. FIXED COSTS OF TDA ACCESS

In reality, however, proportional penalties represent only one component of the total cost of accessing TDA assets. In addition to paperwork and administrative fees, receiving the proceeds of early TDA withdrawals often takes time. This is especially relevant in the case of hardship withdrawals from 401k plans or distribution of 401k assets after employment termination. In the latter case, typical distribution periods range from 45 days to several months. Another cost is related to the loss of contributing privileges for a year after the hardship withdrawal.[31] Since the cost of waiting is difficult to quantify in a discrete-time model, the fixed cost of access is set equal to the maximum one-period TDA contribution of 20% of trend wages.

When fixed costs are added to the model, the value function is no longer globally concave. There are now convex regions associated with pairs of state variables at which TDA withdrawals would have been optimal in the absence of fixed costs. Similar results are well documented in the extensive literature on adjustment costs. An example of a cross-section of the value function under a fixed-cost setting is depicted in the lower left panel of Figure 4.

There is a corresponding change in policy functions. The TDA contribution function has a more extensive inaction region, followed by a discontinuous jump to positive withdrawal levels once taxable wealth becomes very low. The discontinuity in withdrawal (and consumption) schedule produces a kink in the value function. The range of taxable wealth values over which the value function is convex can be broken into two parts. For taxable wealth values to the right of the kink point C, the household withdraws less (typically, 0) and consumes less than it would without fixed costs. For values to the left of C, the household withdraws more than it would without fixed costs but still consumes less. Unlike in the case with no fixed costs, working period TDA withdrawals are in excess of what would be required to finance current consumption. The household does not want to incur fixed costs twice, and so it uses a current TDA withdrawal to build a small buffer stock for the next period.

The portfolio choice in the taxable (CSA) account is shown in the lower right panel of Figure 4. As compared with the no-fixed-costs scenario, the household starts holding bonds in CSA at higher values of taxable wealth and stakes out larger bond positions. This is very similar to the comparison in the top panel of Figure 4, which contrasts scenarios with absolute and partial withdrawal penalties. Over this range of high taxable wealth values, fixed costs make TDA wealth effectively inaccessible, and so households respond in kind. However, for households with sufficiently low taxable wealth, optimal CSA equity share reverses trend and goes back up to 100%. This non-monotonicity with respect to the level of taxable wealth is attributable to the convex regions of the value function. The household values the upside of good returns and avoidance of TDA withdrawals next period more than the downside of poor returns and the need to access TDA in the event of a bad labor shock. Therefore, households that are in dire straits with respect to liquidity choose to invest all their taxable wealth in equities.

[31] Loans against 401k assets are cheaper to obtain by comparison, but they still come with a variety of costs. The costs of borrowing against one's TDA include administration fees, double taxation of loan payments, and the spread between the loan interest rate and the returns on non-borrowed assets. Tax-deferred accounts that are not employer-sponsored (including rollover IRAs) do not allow borrowing at all.

Table IV. Minimum probability of labor income shock required for precautionary portfolio allocations, by severity of shock

Income replacement in unemployment state	1%	5%	10%	20%	30%	40%
Minimum prob(unempl) needed for precautionary portfolios	0.03%	0.1%	2%	31%	–	–

Note: income replacement is measured relative to the baseline level of wages that obtain in the employment state. The model assumes completely inaccessible assets in TDA prior to retirement.

4.6. IMPORTANCE OF CATASTROPHIC RISK

All of the scenarios investigated above relied on the presence of catastrophic labor income risk to elicit precautionary portfolio responses. Recall that precautionary portfolios arise because of the interaction between labor risk and TDA accessibility restrictions. A natural question is to what extent tightening TDA accessibility restrictions can relax the extreme assumption on labor income risk. Consider the case of trading off an extreme assumption on labor income risk for an extreme assumption of completely inaccessible TDA wealth prior to retirement (i.e., setting withdrawal penalty to $1-\tau^d$). If non-catastrophic labor income risk fails to produce precautionary portfolios with TDAs inaccessible before retirement, setups with weaker withdrawal penalties will not work either. I evaluate shocks ranging from 60 to 99% of labor income and compute *lower* bounds on shock probability levels that trigger precautionary allocations for a given value of labor income loss. In all of these parameterizations, a household chooses the level of inflows and allocations in both accounts. Table IV summarizes the results.

In spite of disallowing pre-retirement TDA access altogether, one still needs to rely on extreme combinations of shock levels and probabilities to get precautionary portfolios. The willingness to hold bonds in taxable accounts is driven by the pain of the worst-case scenario – stock market "collapse" coupled with job loss. Even though equity investments can theoretically lose all their value, quadrature methods with a finite number of nodes keep the worst-case returns far from zero.[32] Therefore, when the magnitude of labor income shocks becomes truncated away from zero, the worst-case scenario can be managed through consumption and contribution channels, without resorting to holding bonds in CSA. In contrast, with catastrophic shocks considered in the base case model (and with TDA assets off-limits before retirement) the likelihood of a shock can become almost arbitrarily small – less than 0.1% probability of a 99% wage loss is enough to obtain precautionary portfolios (the leftmost column of Table IV). In other words, given the

[32] The worst-case value-weighted index return calibrated on 1926–2000 CRSP data is –0.512 with probability 0.013 under 5-node Gauss-Hermite quadrature approximation.

limited nature of stock market risk imposed by the model, precautionary portfolio responses are driven by increases in labor income risk to catastrophic levels.

4.7. SENSITIVITY TO TAX DIFFERENTIALS

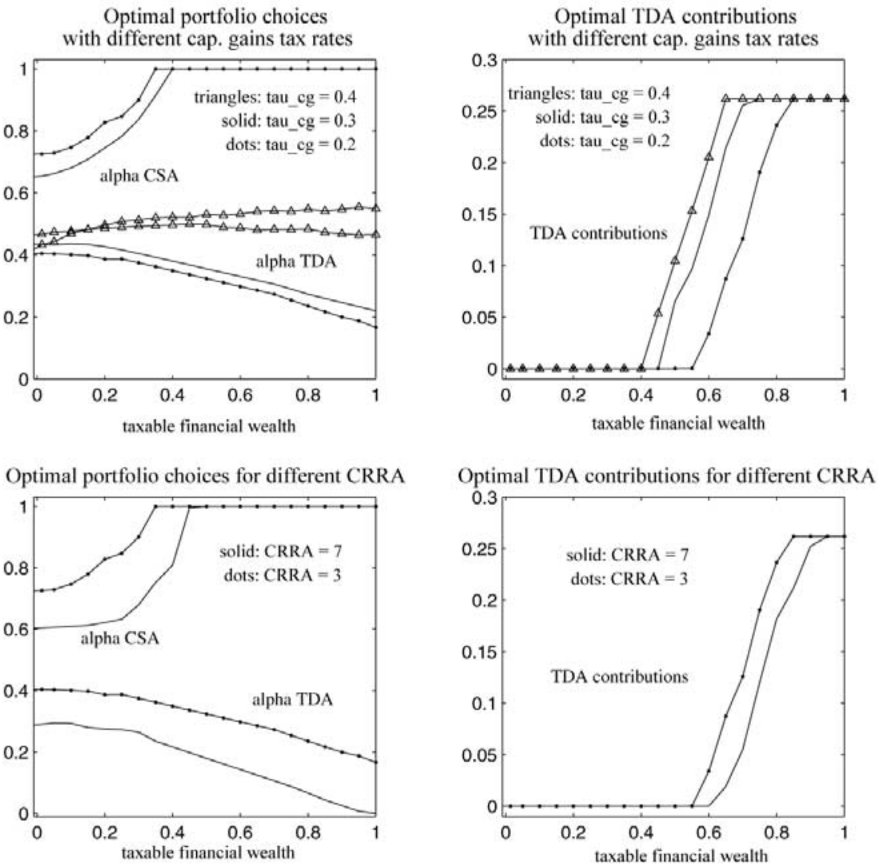
When taxes on capital gains are increased, the tax wedge between equities and bonds is reduced. A lower tax differential does not change the prescription for Tepper-Black tax-efficient location of assets, and so bonds should still be given priority in TDA. However, it does decrease both the mean and variance of equity returns held in the CSA and it makes holding equity in TDA relatively more risky. This can affect the degree of precautionary portfolio effects in response to a given level of labor income uncertainty.

The top two panels of Figure 5 present optimal portfolio and savings choices for scenarios with different capital gains tax rates. In these scenarios, the withdrawal penalty is set at 50% in order to make differences in policy functions more easily discernible on graphs.

Holding everything else fixed, higher taxes on capital gains (and hence lower tax differentials) have three distinct effects. They produce lower equity shares in CSA but higher equity shares inside TDA. They expand the range of wealth values for which precautionary portfolios are observed. They also result in (weakly) higher TDA contributions for same levels of CSA wealth.

In other words, with lower tax differentials households respond to labor income risk by increasing their TDA contributions and holding more bonds in their taxable accounts. One reason for this is that arbitrage gains from tax-efficient allocations are diminished when the tax wedge shrinks. In addition, when the tax wedge declines because of the rise in capital gains taxes, after-tax equity returns in CSA are lower while the advantages of tax-deferred savings are unchanged. Indeed, in this case the equity share in the overall after-tax portfolio declines, as the effect of lower mean returns trumps that of lower variance. It is not surprising then that the results are similar to those obtained with lower equity premia, which bring about an increase in the range of wealth values for which precautionary portfolios are observed.

Finally, note the similarity between portfolio changes in response to an increase in capital gains tax rates and an increase in withdrawal penalties. Both of these make tax-efficient choices less appealing, which accounts for stronger and more commonly observed precautionary portfolios. While the increase in penalties makes tax-efficient portfolios more costly, the decrease in tax differentials diminishes the very reason for tax-efficiency. In the limit, when $\tau^{c} = \tau^{d}$, “tax efficiency” is attained by any allocation choice. The household would still like to avoid premature TDA withdrawals, but there is now no tax cost associated with holding bonds in CSA. As a result, equity shares in CSA are much lower across all wealth levels. A sample portfolio allocation from this scenario is given by the triangle line in the upper left-hand panel of Figure 5.



The top two panels depict optimal portfolio and TDA contribution choices as a function of taxable financial wealth for different levels of capital gains taxation. The capital gains rate varies from 20% (dotted line) to 40% (trinagles), while the tax rate on interest, wages, and dividends remains fixed at 40%. As the capital gains rate increases, the tax wedge shrinks and households move farther away from Tepper-Black tax efficient allocations.

The two lower panels show optimal portfolio and TDA contribution choices as a function of taxable financial wealth for different levels of relative risk aversion. As risk aversion increases to 7 (dotted line), equity allocations in both accounts decline, as do contributions to the relatively illiquid TDA.

Figure 5. Optimal portfolio choices under different taxation and risk aversion settings.

4.8. VARIATION IN RISK AVERSION

The two lower panels of Figure 5 compare portfolio and savings choices under different assumptions for risk aversion. As expected, higher risk aversion results in lower overall equity holdings and smaller TDA contributions. In contrast to fluctuations in tax rates, which alter risk and return levels of assets, higher risk aversion brings about strictly lower equity exposure in both CSA and TDA portfolios. The cutbacks from equity are particularly pronounced in the taxable account, where the level of buffer stocks is also higher due to diminished consumption and TDA contributions.

4.9. SUMMARY

This section reviews a variety of modeling choices that affect the predictions of dual-habitat portfolio models. In particular, I consider the effect of withdrawal penalties, fixed TDA access costs, unemployment risk, attractiveness of equity returns, relative advantages of tax shelters, and varying levels of risk aversion on willingness of households to abandon Tepper-Black efficient portfolio choices. The conclusion that emerges from these exercises is twofold. On the one hand, it is difficult to obtain precautionary portfolio allocations without relying on harsh access penalties and severe labor income shocks. Yet, the model is able to demonstrate a clear link between precautionary portfolios and modeling choices that emphasize the importance of states with poor market returns and loss of labor income in household decision-making. This emphasis is quite similar to that found in several recent asset-pricing models that attempt to resolve the equity premium puzzle. As an example, consider the model of Constantinides and Duffie (1996) in which the authors show the way to support a given joint process of asset prices, dividends and aggregate income through "judiciously modeled income heterogeneity". This heterogeneity is manifested through an additional term in the pricing kernel related to the cross-sectional variance of consumption growth rates. As such variance is higher during recessions, the net effect of this new kernel component is to amplify the importance of bad market return/low wage states on equilibrium asset prices.

5. Conclusion

Observed portfolio allocations of US households are grossly inconsistent with predictions of theoretical models. This paper investigates the effect of various modeling choices on optimal portfolio location and allocation decisions. It reviews the extant literature and catalogues the evolution of model settings and their corresponding predictions from a simple two-period two-asset dual-habitat problem to a dynamic multi-asset model with exogenous liquidity needs. I argue that the existing models fall short of the goal to produce non-corner tax-inefficient solutions while satisfying other empirical facts about household portfolio choices and prevailing institutional restrictions. Among these facts are: low ownership levels of tax-exempt municipal bonds, simultaneous ownership of munis and regular bonds in taxable accounts, and lack of relationship between lumpy consumption expenditures at certain lifecycle stages and the degree of portfolio tax-inefficiency.

The present paper offers an alternative modeling approach for eliciting mixed portfolio allocations, which is based on the assumption of catastrophic labor income shocks. While the model produces "tax-inefficient" precautionary portfolios, its calibrating choices are open to criticism. A number of additional tests indicate that appealing to a different access cost structure is likely to preserve dependence on catastrophic labor income shocks.

However, by considering a wide range of modifications to the dual-habitat model, the paper clarifies the joint importance of background risk and TDA accessibility restrictions in producing precautionary portfolios. It also produces a number of novel empirical predictions that relate household portfolio choices to the degree of labor income risk, tax rates on capital gains, and strictness of borrowing constraints faced by the household. The predictions on the *degree* of tax-inefficiency receive a good measure of empirical support in the companion paper (Amromin, 2002). Contrary to the predictions of the model, it is still difficult to link the *existence* of tax-inefficient allocations with presence of accessibility restrictions or labor risk.

The qualified empirical support for model's predictions suggests that the interaction between labor income risk and accessibility restrictions is a key factor in explaining household behavior. This provides the impetus for understanding the nature of labor income shocks that influence portfolio choices and households' perceptions of inaccessibility of their TDA assets. In particular, how concerned are households with insuring against the type of catastrophic labor risk considered in this model? Are they aware of the rules that govern accessibility of TDA assets and of the costs associated with such withdrawals?

There remain several structural modifications left unexplored in the present paper that can relax the stringency of parameter choices while preserving the qualitative nature of the results. One such example is expansion of asset classes to housing, which also receives preferential tax treatment and greatly expands household borrowing opportunities. Another is incorporation of uncertainty in proprietary labor income into a set of risks faced by households.

The difficulty of obtaining empirically plausible portfolio predictions in models with taxable and tax-deferred accounts presents a puzzle. The resolution of this puzzle appears particularly important in light of the recent changes in tax treatment of dividends and the ongoing discussion on Social Security privatization reform in the US. Both of these reforms would have enormous implications for asset choices and welfare of American households. The same questions, however, have much broader applicability as demographic changes necessitate an ever more important role for individually financed retirement accounts in national pension systems throughout the industrialized world.

6. Appendix: General Model Setup

The intertemporal consumption-portfolio allocation problem of a finitely-lived household with uncertain labor income and access to both taxable and tax-deferred savings environments is stated as follows.

$$\max_{\{c_t, s_t^{TDA}, s_t^{CSA}, \alpha_t^{TDA}, \alpha_t^{CSA}\}} E\left(\sum_{t=1}^{T} \beta^{t-1}[F(t)U(\hat{c}_t) + (F(t-1) - F(t))B(CSA_t + (1-\tau^w)TDA_t)] \right)$$

$$s.t.\ c_t = w_t(1-\tau_t^w) - s_t^{CSA} - s_t^{TDA}(1 - \tau_t^w - pen * \mathrm{I}(s_t^{TDA} < 0, t \leq R))$$

$$CSA_{t+1} = (CSA_t + s_t^{CSA})R^{CSA}(\alpha^{CSA}, r)$$

$$TDA_{t+1} = (TDA_t + s_t^{TDA})R^{TDA}(\alpha^{TDA}, r)$$

$$s_t^{TDA} \leq \begin{cases} qw_t, & \text{if } t \leq R \\ 0, & \text{if } t > R \end{cases}$$

$$s_t^{CSA} + CSA_t \geq 0, s_t^{TDA} + TDA_t \geq 0, \forall t$$

$$0 \leq \alpha_t^{CSA}, \alpha_t^{TDA} \leq 1, \forall t$$

w_t is a given stochastic process for labor income.

In the beginning of each period, a household knows the values of its taxable and tax-deferred wealth, as well as the value of its current wage draw. A household then chooses consumption, the level of contributions and portfolio composition for each account. Household utility is defined over real consumption $\hat{c}_t = c_t/(1 + infl)^{\mathrm{t}}$ and $F(t)$ represents the cumulative probability of surviving through period t. The household also derives utility from bequests, defined as some function B of its real after-tax wealth at time of death. Net contributions to TDA and CSA are given by s_t^{TDA} and s_t^{CSA}, respectively and portfolio returns in the two accounts (R^{CSA}, R^{TDA}) are given by Equations (1) and (2) in Section 2.1. For convenience, the tax on wages, τ^{w} is set equal to τ^{d}. The presence of $(1-\tau^{\mathrm{w}})$ on TDA flows in the budget constraint reflects tax-deductibility of TDA contributions and accumulated tax liability on TDA withdrawals. The household can make positive contributions to TDA only during working years ($t < R$) and they cannot exceed $q\%$ of its stochastic wage w. While assets held in CSA can be accessed without cost at any time, withdrawing TDA funds prior to retirement requires paying a pre-specified penalty pen. Neither borrowing nor short-selling is allowed in either account.

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