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Corporate Walkout Decisions and the Value of Default

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Abstract. We present a continuous-time asset pricing model of the levered firm where shareholders select not only the *timing* but also the *form* of abandonment. Shareholders can *walk out* of the firm either by (i) *defaulting* on their debt obligations or (ii) selling their shares to alternative operators of the technologies, as in a *corporation sale*. The structural model relates shareholders' ex-post choice to both technological and financial factors. Considering that operators' technological supremacy is not universal, we obtain that whereas default necessarily involves an inefficient timing of ownership transfer, corporation sales do not. Then, the likelihood of default being chosen instead of a corporation sale increases with (i) the degree of leverage displayed by the firm and (ii) its technological supremacy. By ignoring corporation sales, existing defaultable bond pricing models have thus a tendency to exaggerate risk premia and underestimate the borrowing ability (debt capacity) of firms.

When a levered firm is in financial distress, ownership rights over productive technologies can pass from one set of operators to another either indirectly or directly:

1. The transfer is indirect when the existing shareholders exercise their limited liability option, *defaulting* on their debt obligations, or more generally *repudiate* the outstanding contract they have with their creditors. Creditors take control invoking debt collection law, and the firm's technologies are then sold through a liquidation sale to the best offering alternative operators who then take over operations.
2. Conversely, the transfer is direct when the existing shareholders simply *sell* their shares directly to the best offering alternative operators, as in a *corporation sale*.

The corporate finance literature has since Stiglitz (1972) mainly been concerned with a series of agency problems (and resulting inefficiencies) that the limited liability protection of shareholders in bankruptcy gives rise to.[1] In contrast, our un-

[1] Myers (1977) identified the underinvestment problem: firms in financial distress will forgo positive net present value investment opportunities because benefits are likely to accrue to debtholders rather than shareholders. Jensen and Meckling (1976) discussed the overinvestment or asset substitution problem: shareholders will want to increase the riskiness of a firm's activities so as to transfer

derstanding of the desirability of corporation sales for financially distressed firms and the conditions under which they will be preferred to the exercise of the limited liability option remains limited.[2] Does the fact that shareholders cannot commit ex-ante to relinquish ownership one way or the other have consequences for the efficient financing of projects?

The pricing literature on corporate debt initiated by Merton (1974) interprets risky debt as a portfolio comprising the safe asset plus a short position in a put option written on the value of the firm's underlying assets. Recent applications of the contingent claim approach study the impact on the price of corporate debt of additional shareholder suboptimal decisions which are also related to their limited liability protection.[3] But again, the corporation sale alternative has received little attention even though it is frequently used by shareholders of financially distressed firms.

The magnitude of credit spreads observed in debt markets are known to be substantial.[4] This suggests that self-serving decisions taken by shareholders when the firm is in financial distress deserve particular attention as they greatly affect the value of corporate assets. It does not however imply that the shareholders' ultimate decision is necessarily related to the limited liability option. Empirical evidence actually suggests that direct sales of ownership rights occur much more frequently than indirect transfers.[5] Default only constitutes one form of voluntary abandonment available to shareholders, and one ought to speak more generally of the shareholders' "*walkout*" decision, referring to their option to either default, or sell their shares to alternative operators.

This paper explicitly studies the value of this shareholders' entitlement to relinquish ownership either directly or indirectly, i.e., the value of their *walkout option*. It also examines the implications of this choice of exit form for a firm's ability to

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value from debt to equity. Aghion and Bolton (1992) emphasize that contracts granting control exclusively to one class of agents may not be efficient and show that contracts with contingent transfer of control rights may minimize inefficiencies, which provides a rationale for debt contracts.

[2] The choice between default and a corporation sale displays a degree of resemblance to the choice between bankruptcy and a merger dealt with in studies such as Higgins and Schall (1975), Bulow and Shoven (1978), and Shrieves and Stevens (1979). These studies, however, stress the importance of institutional features such as tax codes in determining the choice of exit form.

[3] Several directions have been explored since the work of Black and Cox (1976). Leland (1994) focuses on the under-investment problem identified by Myers (1977). In related models which allow for ex-post renegotiation, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) examine the value of strategic default, and Mella-Barral (1999) studies dynamic debt forgiveness and endogenous departures from APR. Ericsson (1997), Decamps and Faure-Grimaud (1997) and Leland (1998) provide models which account for the asset substitution effect or risk shifting incentive identified by Jensen and Meckling (1976).

[4] Kim, Ramaswamy and Sundaresan (1993) report 77 basis points as the average spread for investment-grade corporate bonds, and Litterman and Iben (1991) report historical ranges for par spreads between 20 and 130 basis points.

[5] In the Compustat data set used by Pastena and Ruland (1986), for instance, 531 manufacturing firms merged during 1970-1983, whereas only 56 filed for bankruptcy in the same period.

efficiently raise the outside finance required to undertake worthwhile projects. We present a continuous-time pricing model of the levered firm where shareholders select not only the *timing* but also the *form* of ownership abandonment. They relinquish control either (i) repudiating the debt contract (default) or (ii) selling their ownership rights (corporation sale). Shareholders behave non-cooperatively throughout, so as to maximize the value of their holdings.

The paper contributes to both corporate finance and valuation theory, incorporating insights of the corporate finance literature into a valuation framework. Our structural model enables us to jointly obtain efficiency and pricing results with directly testable implications. The set-up explicitly considers the technological characteristics of the firm as well as of its competitors and we derive simple closed-form solutions for the value of shares and bonds throughout the firm's existence.

We consider a set-up where, although the innovators are initially superior at operating the firm, there are circumstances under which it becomes ex-ante optimal for them to abandon operations and transfer control to alternative operators. Essentially, we consider that an operator's technological supremacy is never universal. The friction we then consider is that innovators have limited wealth and must seek external financing to set-up the firm. There is incomplete contracting and only simple debt contracts are available.

We find that default induces an inefficiently *early* transfer of ownership to alternative operators. Conversely corporation sales induce the correct timing of ownership transfer, because the ex-post interests of share-holders are aligned with the ex-ante interest of all asset-holders. The intuition for this difference is as follows:

In the case of default, introducing debt has no effect on the shareholders' stake in the residual value, but reduces the stream of revenues accruing to equityholders in current operations. Therefore, increasing levels of debt induce increasingly early equityholder defaults. These are increasingly departing from the time it is ex-ante optimal to transfer control to alternative operators, hence evermore inefficient. In the case of a corporation sale, introducing debt still reduces the stream of revenues accruing to equityholders in current operations, but also reduces the value of the shares in a corporation sale. Furthermore, both these reductions correspond to exactly the same promised payments to bondholders. Hence, debt does not affect the corporation sale time preferred by equityholders, which is therefore always equal to the ex-ante optimal, efficient one.

Note that this is general result: When operators' technological supremacy is not universal, corporation sales are an opportunity for avoiding the agency costs associated with debt, which in many circumstances, shareholders' find preferable pursuing ex-post.

We obtain that the shareholders' non-cooperative optimum policy bears direct relations to the firm's (i) capital structure and (ii) relative technological characteristics: First, shareholders of firms with low levels of leverage tend to exit by means of corporation sales, whereas those of highly levered firms tend to default. Secondly, the likelihood of default being chosen instead of a corporation sale increases with the technological supremacy of the firm.

One implication for valuation theory is that, given that existing defaultable bond pricing models relate shareholders' walkout decisions exclusively to default, they have a tendency to exaggerate risk premia and underestimate the borrowing ability (debt capacity) of firms. Such models are likely to perform at their worst when applied to (i) industries where the different operators have fairly similar abilities to generate revenues and (ii) the innovators' need for credit finance is relatively modest.

We present numerical simulations, showing how changes in firm parameters affect risk premia and credit spreads. Not surprisingly, the direct sale alternative has a major influence on corporate debt pricing, as for typical parameter values, the difference with a "default-only" optimization can reach 50%.

The paper is structured as follows. Section 1 presents the model. Section 2 derives asset values and the first-best policy. Section 3 introduce the friction and discusses the resulting moral hazard problem. Section 4 determines shareholders' ex-post behavior and examine the resulting ex-ante contracting. Section 5 consolidates our results providing (i) closed-form pricing solutions, (ii) measures of the relative price impact of our analysis and (iii) numerical estimates. Section 6 concludes highlighting the main testable implications.

1. The Model

1.1. MAIN COMPONENTS AND RANDOM TIME LINE OF THE MODEL

Our continuous-time pricing model is intended to capture in a structural fashion the following features of firms operating in a competitive environment:

- *Firms* are created by innovators who develop worthwhile *technologies*. Profiting from a technology typically requires the combination of their *human capital*, which is inalienable by nature, and *physical assets* that need to be acquired against an initial *investment*.[6]
- Apart from the innovators, there exists a number of alternative operators who, after acquiring the technology, could use their own human capital and generate profits.
- There is however an element of uniqueness to the technological ability of operators, and new ones cannot fully capture their predecessors' know-how through an acquisition. Another way to say this is that reversing an abandonment decision is *costly*. The withdrawal of the innovators' human capital involves at least a partial irreversible loss of their know-how.

[6] Our construction can be thought of in terms of the inalienable human capital theory of debt of Hart and Moore (1994).

- Although the innovators are initially superior at generating cash operating the technology, there most often exist other economic circumstances under which either (i) they are inferior to alternative operators, or (ii) it is better to simply scrap the physical assets. That is, there are circumstances under which it becomes ex-ante optimal for innovators to *abandon* operations and transfer control.
- Operations can be abandoned at any time. This is here followed either by (i) *continuation*, whereby the technology is operated by someone else or ultimately (ii) *termination*, whereby the physical assets simply cease to serve a productive purpose, as no operator finds it profitable to operate the technology.
- Finally, firms are initially created when operating the technology is more positive NPV to the innovators than selling it to alternative operators for them to operate it instead.

To capture these elements, the random time line of the model, illustrated in Figure 1, is as follows:

1. In the beginning, a technology is developed by a set of innovators and it is worthwhile for them to operate it, i.e., it offers a positive NPV. Because of cash constrains and incomplete contracting (the friction is detailed in Section III.A), they issue debt to finance the required initial investment. We use ex-ante and ex-post with respect to this date, when debt is issued.
2. The innovators, then as shareholders of the newly created firm, have *ownership* rights. In the terminology of Grossman and Hart (1986) and Hart and Moore (1990), these rights consist of (i) residual control rights and (ii) income rights:
   (a) *Residual control rights* imply that shareholders have the right to decide all usages of the physical assets, as long as they do not violate the outstanding debt contract.
   (b) Both shareholders and bondholders have *income rights*: These are provisions of the contract which stipulate the sharing of the generated revenues between the two classes of claimants.

   While operating the technology, the shareholders are constantly aware that (i) it has some value to alternative operators and (ii) physical assets can always be scrapped. These two possibilities determine the *reservation value* of the firm. By construction there will be circumstances when operations should be abandoned by the innovators.
3. Shareholders have the right to abandon operations whenever they want, and importantly, they are the only ones who control the *timing* of this decision. That is, this random time is non-cooperatively optimized over by the shareholders, ex-post. But shareholders also control the *form* of abandonment: They can abandon operations either
   (a) repudiating the debt contracts. They essentially *default* on their debt service obligations, which entitles creditor to take over control of the firm;
   (b) or selling their shares (hence ownership rights) to alternative operators. This is referred to as a *corporation sale*.

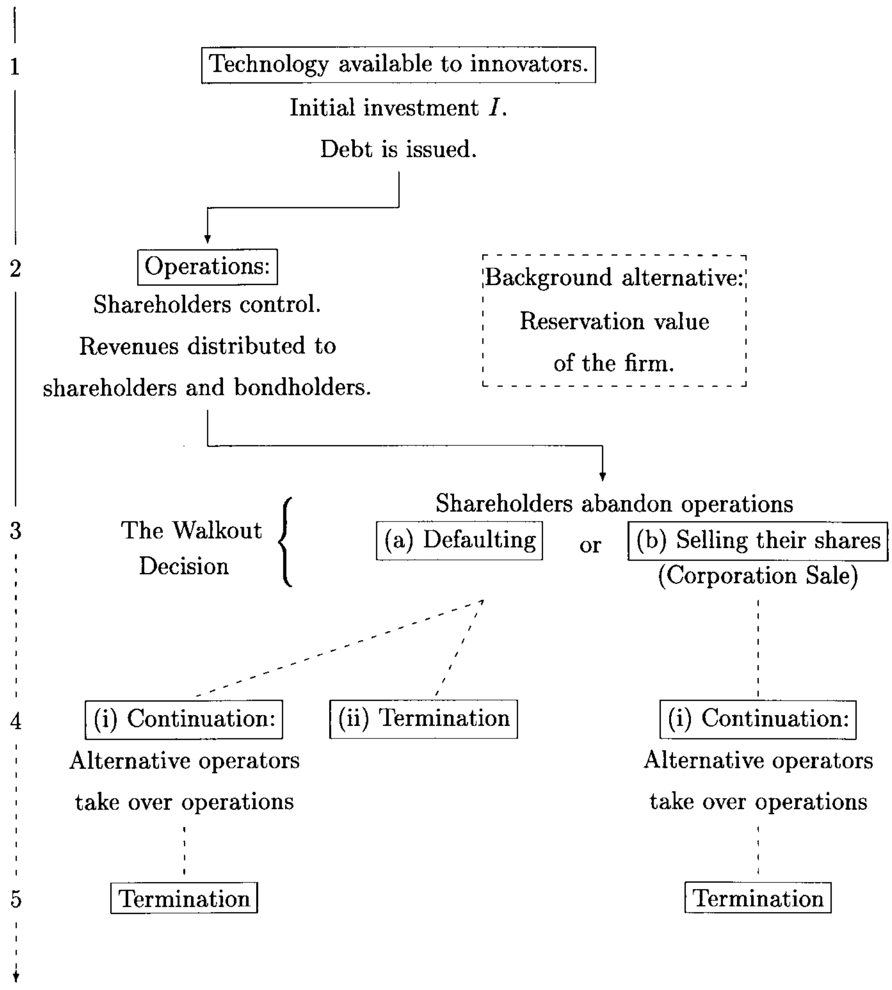


Figure 1. Random time line of the model.

We call the *walkout* decision, this double choice of timing and form of abandonment.

4. A walkout may be followed either by (i) *continuation*, as alternative operators are interested in purchasing and operating the technology, or (ii) *termination*, as scrapping is just best. Clearly whereas default can be followed either by (i) or (ii), a corporation sale can only be followed by *continuation*: Alternative operators only purchase the shares if the value of the revenues they can generate exceeds their liquidation value.
5. When alternative operators take over operations, they then control the firm until, in turn, they decide to abandon operations. This process ultimately finishes with termination.

In Figure 1, the dashed lines highlight the fact that the (i) continuation and (ii) termination alternatives completely determine the *reservation value* of the firm. Steps 4 and 5 feed backwards in time the alternative available to shareholders during operations in step 2.

1.2. BASIC ASSUMPTIONS AND EXOGENOUS PARAMETERS

The technology we consider is developed by a set of innovators at date $t = 0$. To operate the technology, it is necessary for them to acquire a set of physical assets against an immediate investment I. The uncertain factors influencing the profitability of subsequent operations are summarized by a single state variable which follows a diffusion process:

$$dx_t = \mu(x_t)\,dt + \sigma(x_t)\,dB_t\,, \tag{1}$$

where B_t is a standard Brownian motion.

Employing their human capital in the technology, the innovators can generate indefinitely a stream of uncertain period income flows. We denote $\Pi(x)$ the unlimited liability value of a perpetual claim on these operations' income flow, which depends on the current state x.

Alternative operators can generate different revenues using their own human capital. We denote $\Pi^*(x)$ the unlimited liability value of a perpetual claim on these alternative operations' income flow. We assume that both $\Pi(x)$ and $\Pi^*(x)$ are continuous and twice differentiable functions of x.[7]

Operating losses are not ruled out; consequently $\Pi(x)$ and $\Pi^*(x)$ may be negative over some range. It may therefore (and will by Assumption 1) be worthwhile to *terminate* operations altogether at some point, scrapping the physical assets. To simplify, we will normalize the scrap value of the assets to zero.

Abandoning a project, whether it leads to actual termination or not, is assumed to be an *irreversible* decision. We here consider for simplicity that liquidation *fully* avoids future use of the incumbents' inalienable human capital, as in Hart and Moore (1994). With the irreversibility assumption, technology transfer costs if innovators abandon at x are measured by $\Pi(x) - \Pi^*(x)$, precisely because it is assumed that the human capital which generates $\Pi(x)$ is not available afterwards.[8]

We assume risk neutrality and a constant identical borrowing and lending safe interest rate, ρ.[9] Trading of assets is assumed to occur continuously in perfect and frictionless markets with no asymmetry of information or transaction costs. Firms'

[7] This is reasonable, given that these are integrals over time of perpetual streams of discounted period income flows.

[8] Actually Assumption 2 will essentially impose that $\Pi(x) - \Pi^*(x)$ becomes *negative* in the lowest interval of states x. When innovators do abandon, there is then a technology transfer *benefit*.

[9] Harrison and Kreps (1979) show how to extend the results of the paper to a world without risk-neutrality by using an equivalent martingale measure.

management act in the best interest of their shareholders, ignoring the insiders-outsiders principal-agent conflict of interest discussed in Hart (1993). We finally also ignore risk-shifting incentives highlighted by Jensen and Meckling (1976).[10]

1.3. ASSET VALUATION

Asset prices are time-homogeneous, because (i) the uncertain state variable and the safe interest rate are time independent, and (ii) the problem is parametrized in terms of unlimited liability values of perpetual claims on different income flows available – $\Pi(x)$, $\Pi^*(x)$ and zero – that are just state dependent. Pricing formulas can therefore be expressed in extremely simple fashion, introducing a single pricing operator, $\mathcal{P}(x_t \rhd y)$:

Let $T_y \equiv \inf\{\tau \mid x_\tau = y\}$ denote the first time at which the state variable x_t hits the level y, and $f_t(T_y)$ the density of T_y conditional on information at t. The Laplace transform of $f_t(T_y)$,

$$\mathcal{P}(x_t \rhd y) \equiv \int_t^\infty e^{-\rho(T_y - t)} f_t(T_y) dT_y \,, \tag{2}$$

is then very intuitively a probability-weighted discount factor for events that will occur the first time the state variable, equal to x_t, reaches the level y. This operator allows a direct derivation of all the pricing expressions contained in this paper, and will crucially help us to develop an intuitive understanding of their dynamics.

2. First-Best Value of the Technology: The Unlevered Firm

In this section we determine the first-best policy that would be implemented in the absence of frictions. We first establish the value of the technology to alternative operators. Working backwards in time, we secondly obtain the first-best value of the firm, by feeding this reservation value of the firm in the optimization of the innovators. The following Sections, will introduce a friction and examine the resulting second-best policy.

2.1. RESERVATION VALUE OF THE FIRM

As we mentioned earlier, alternative operators can at any time bid for the technology (including the set of physical assets). If they purchase the firm, they can

[10] Firm and equity values will be convex functions of the state, hence there is an incentive to control the volatility of the income generating process. This leads to another moral hazard problem, as there is excessive risk shifting by non-cooperative share-holders relative to the cooperatively optimal level. This further problem is simply ignored here as it is in Leland (1994), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999), for tractability. Papers by Ericsson (1997), Decamps and Faure-Grimaud (1997) and Leland (1998) however provide a specific analysis of asset substitution effects.

operate the technology using their own human capital (which is the support of $\Pi^*(.)$), and have furthermore the option to later *abandon* these operations and simply scrap the physical assets. The value of the technology in the hands of alternative operators can therefore readily be obtained as follows:

If alternative operators were to ultimately abandon their operations the first time the state variable, currently at x_t, reaches a *given* lower level y, the assets would be worth

$$\Pi^*(x_t) - \Pi^*(y)\mathcal{P}(x_t \rhd y) . \tag{3}$$

In the above expression, the first term is the value of a perpetual entitlement on the competitors' flow of income, $\Pi^*(x_t)$. The second term is the product of (i) the change in asset value, $\left[0 - \Pi^*(y)\right]$, intervening when the irreversible regime switch occurs (termination), and (ii) the probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd y)$.

Clearly, instead of taking their irreversible abandonment trigger level, y, as given, the alternative operators' optimal policy consists of selecting the one that maximizes the value of their claim, $V^*(x_t \mid y)$. The *optimal* alternative operators' termination trigger level, which we denote $\underline{x}^*$, therefore solves

$$\underline{x}^* \equiv \arg\max_y \left[\Pi^*(x_t) - \Pi^*(y)\mathcal{P}(x_t \rhd y)\right] . \tag{4}$$

Note that nothing in the structure we have so far introduced actually guarantees the eventual desirability of this decision. The existence and uniqueness of an optimal alternative operators' termination trigger level is ensured as follows:

ASSUMPTION 1. At the entry state x_0, the option value of the alternative operators' decision to trigger termination at y,

$$-\Pi^*(y)\mathcal{P}(x_0 \rhd y) ,$$

is a strictly concave function in y, maximized at a trigger level, $\underline{x}^*$, strictly smaller than x_0.

Under Assumption 1, if alternative operators' acquire the firm when the economic fundamental x_t is greater than $\underline{x}^*$, they will initially find it worthwhile to operate the firm. They will then eventually prefer to abandon these operations when x_t falls to $\underline{x}^*$. That is, for high states x_t the alternative operators' operations dominate scrapping, but for lowest states this is reversed.

We denote $U^*(x_t)$ the value of the technology in the hands of alternative users:

$$U^*(x_t) \equiv \Pi^*(x_t) - \Pi^*(\underline{x}^*)\mathcal{P}(x_t \rhd \underline{x}^*) \tag{5}$$

where $\underline{x}^*$ solves the first order condition

$$\frac{\partial}{\partial \underline{x}^*}\left[\Pi^*(x_t) - \Pi^*(\underline{x}^*)\mathcal{P}(x_t \rhd \underline{x}^*)\right] = 0\,0 \tag{6}$$

We consider for simplicity that $U^*(x_t)$ also represents the *bid* made by the highest bidding set of alternative operators, for ownership of the technology. We essentially consider a competitive industry where, to acquire a technology, each potential set of alternative operators' has to offer what it expects to generate with it.

Working backwards in time, $U^*(x_t)$ can now be used to derive the value of the technology to the innovators themselves, as a compound option: While operating the assets, the innovators know that they can at any time abandon their operations and sell the firm for $U^*(x_t)$. $U^*(x_t)$ is therefore also the *reservation value* of the firm initially implementing the technology.

2.2. VALUE OF THE FIRM UNDER THE FIRST-BEST POLICY

The value of the firm to begin with (in the hands of the innovators) is obtained as follows: If operations are abandoned the first time the state variable x_t reaches a lower level y, the assets are worth

$$V(x_t \mid y) \equiv \Pi(x_t) + \left[U^*(y) - \Pi(y)\right]\mathcal{P}(x_t \rhd y)\,. \tag{7}$$

In the expression of $V(x_t \mid y)$, the first term on the right-hand side is the value of a perpetual entitlement on the flow of income the innovators can generate, $\Pi(x_t)$. The second term is the product of (i) the change in asset value intervening when these operations are abandoned and the assets of the firm are optimally either purchased by alternative operators or scrapped, $\left[U^*(y) - \Pi(y)\right]$, and (ii) the probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd y)$.

The innovators' first-best policy involves an irreversible abandonment trigger level, y, which maximizes the value of the firm, $V(x_t \mid y)$. The *ex-ante optimal* firm abandonment trigger level, which we denote $\underline{x}$, therefore solves

$$\frac{\partial V(x_t \mid \underline{x})}{\partial \underline{x}} = 0\,. \tag{8}$$

The existence and uniqueness of an optimal firm abandonment trigger level is similarly guaranteed as follows:

ASSUMPTION 2. At the entry state x_0, the option value of the firm's decision to trigger abandonment at y,

$$\left[U^*(y) - \Pi(y)\right]\mathcal{P}(x_0 \rhd y)\,,$$

is a strictly concave function in y, maximized at a trigger level, $\underline{x}$, strictly smaller than x_0.

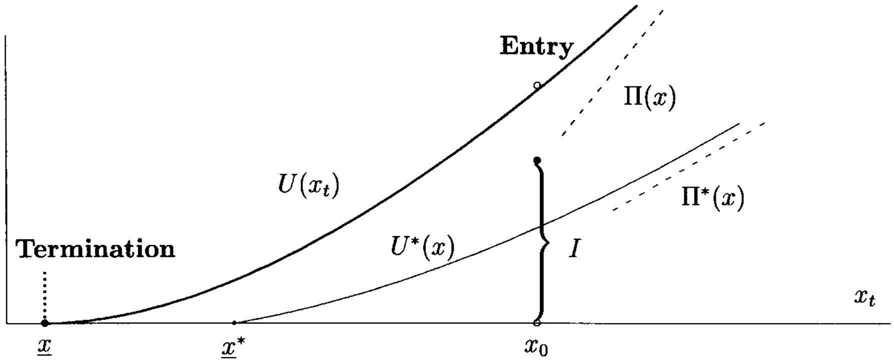


Figure 2. First-best value of the firm consisting 'termination" ($\underline{x} \leq \underline{x}^*$).

Under Assumption 2, the innovators' ex-ante optimal policy is as follows: They should first operate the technology, and eventually abandon these operations when (the first time) the economic fundamental x_t falls reaches $\underline{x}$. We denote $U(x_t) \equiv V(x_t \mid \underline{x})$ the value of the innovative firm under this first-best policy. We consider situations where it is profitable for the innovators to implement the technology, otherwise they do not set up the firm in the first place:

At the entry state x_0, it is worthwhile for the innovators to implement the technology: The initial investment, I, necessary to purchase the required set of physical assets is less than $U(x_0)$.

Under Assumptions 1 to 3, the first-best policy consists of the having the innovators implementing the technology against an initial investment I at date $t = 0$. The innovators then operate the technology, and eventually abandon its operations at $\underline{x}$.

With this structure, two cases emerge depending on whether, when the innovators abandon operations, the value of the technology to alternative operators, $U^*(\underline{x})$, is strictly positive or not:

1. If $\underline{x} \leq \underline{x}^*$, we have $U^*(\underline{x}) = 0$ and the assets are scrapped. The end of the innovators' operations will then be referred to as a "*termination*".
2. If $\underline{x}^* < \underline{x}$, we have $U^*(\underline{x}) > 0$ and it is worthwhile for alternative operators to purchase the technology in order to operate it. The end of the innovators' operations will then be referred to as a "*continuation*".

Figures 2 and 3 give a graphical representation of the entire set-up: Depending on the values of the exogenous parameters describing (i) the innovators and the technology, $\{\Pi(x), I, x_0\}$, (ii) alternative operators, $\{\Pi^*(x)\}$, and (iii) the economic environment, $\{x; \mu(x); \sigma(x); \rho\}$, one of the two cases just mentioned above (termination or continuation) corresponds to the first-best policy.

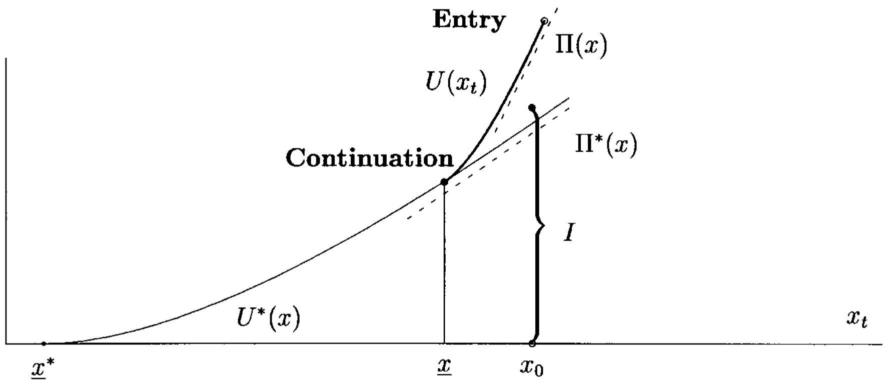


Figure 3. First-best value of the firm consisting of a "continuation" ($\underline{x}^* \leq \underline{x}$).

3. Leverage

3.1. THE FRICTION AND AVAILABLE CONTRACTS

The problem we consider is that the innovators have limited wealth and must seek external financing in order to acquire the physical assets necessary to operate the technology. The innovators can only contribute to the initial investment up to a level which is strictly less than I. We denote I_D the amount they need to raise at date $t = 0$.

Furthermore there is incomplete contracting, in that the state is observable to both contracting parties but not verifiable to outsiders, as in Hart and Moore (1989). The state therefore cannot form part of an enforceable contract. Consequently only simple debt contracts are available to the innovators.

This is the one friction we focus on. It is only because the innovators are wealth constrained that the first best policy previously determined will not be implemented and inefficiencies will arise. We furthermore only introduce this friction. That is, for simplicity, we consider that alternative operators do not need external financing to acquire the firm.

The debt contracts available to the innovators first consist of a promise to an infinite stream of instantaneous flow of constant coupon payments δ to creditors. The remaining part of the operating income accrues to the shareholders. Here, we consider infinite-maturity debt for simplicity.

Secondly, debt contracts are written under the Absolute Priority Rule (APR): If the shareholders repudiate the debt contract, creditors can invoke debt collection law and impose a liquidation sale of the firm. In the absence of renegotiation, the liquidation sale is creditors only available response to a shareholders' default. It therefore immediately follows a default. It is not triggered by a corporation sale. Under the APR, debtholders are to be paid first out of the proceeds of this sale, up to a par value, P. The par value P should be such that immediate default is not optimal. This is satisfied with P greater or equal to δ/ρ, the value the debt would have if it were riskless.

We consider (Section 2.2) that at any time the alternative operators competitively bid the reservation value $U^*(.)$ for the technology. Therefore, if the shareholders repudiate at y, the expected liquidation value of the firm is $U^*(y)$, hence the debt is worth $U^*(y) \wedge P$.[11]

The debt is assumed to be held by a large number of non-cohesive creditors (as with publicly traded debt), hence *ex-post renegotiation is impossible*, because of the hold-out problem: No individual creditor is ever willing to make a first concession, even if it would serve creditors' collective interests, as other creditors would then benefit from a reduced default risk without having to make any concessions themselves.

3.2. MORAL HAZARD PROBLEMS ASSOCIATED WITH DEBT

In terms of corporate governance, the main feature differentiating equity from debt is *ownership*: Although both shareholders and bondholders have *income rights*, the shareholders are the only ones entitled to decide all usages of the physical assets, as long as they do not violate the contract: They hold *residual control rights*.

Upon *default*, shareholders forfeit these rights, precisely because they violate the debt contract. In the absence of renegotiation, creditors invoke debt collection law. Any decisions concerning the allocation and use of the assets are thereafter made solely by the creditors.

The right of shareholders to exercise their limited liability option whenever they wish is central in the theory of corporate asset pricing. This paper however underlines that default only constitutes one form of voluntary abandonment available to shareholders.

One ought to speak more generally of a "*walkout*" decision, because in addition to default, shareholders can alternatively choose to simply sell their shares (the firm with outstanding debt obligations) directly to the best offering alternative operators, which we referred to as a *corporation sale*. In our context, innovators/shareholders' walkout decisions consist of the following two dimensions:

I. Innovators/shareholders control the *timing* of their walkout, selecting non-cooperatively the trigger level of their decision, y. They can do so at any time, hence y can be selected anywhere in the state space.
II. Innovators/shareholders also control the *form* of their walkout, selecting between *two* possible *equity reservation value functions*:
    (a) They can decide to violate their ownership rights, *defaulting* on their debt obligation. As we have just described in Section III.A., given that the expected liquidation value of the firm is $U^*(y)$, the debt is worth $U^*(y) \wedge P$,

[11] The APR proves to be the best sharing rule of liquidation right, in this context. That is, deviations the APR would only induce even earlier time of default, hence worsen inefficiencies.

under the APR. The equity reservation value is therefore equal to $U^*(y)$ minus $U^*(y) \wedge P$.

(b) They can decide to *sell* their ownership rights, selling their shares to alternative operators, as in a *corporation sale*. Here, the alternative operators do not just acquire the technology, but also the obligations on the outstanding debt.

Now, the alternative operators are still willing to competitively bid, $U^*(.)$, for the *full* ownership rights, i.e. for *both* the innovators's shares and the debt. This is not different in a corporation or liquidation sale, because the alternative operators are not wealth constrained, hence will find it optimal to purchase back the debt and abandon at the ex-ante optimal $\underline{x}^*$.

Clearly, creditors are not in any way obliged to sell their debt in the process. Hence the value of their claim is at least that of the contracted flow of coupon payments δ, until the alternative operators intend to terminate the technology (which has a scrap value of zero), the first time the state $\underline{x}^*$ is reached. This reservation value of the debt therefore equals $\delta\rho[1 - \mathcal{P}(y \triangleright \underline{x}^*)]$.

It turns out that it would be suboptimal to guarantee the creditors a higher value than the above debt reservation value with protective debt covenants: Alternative operators would not implement the ex-ante optimal termination policy. This would then (feeding backwards in time) not induce the innovators to choose ex-post the time of corporate sales in an ex-ante efficient way, as later obtained in Corollary 1. Our model has little to say about protective debt covenants, because corporation sales are efficient in our model, hence adding debt protection against them would only yield inefficiencies.

Therefore the equity reservation value at the time of the corporation sale is equal to $U^*(y)$ minus $\delta\rho[1 - \mathcal{P}(y \triangleright \underline{x}^*)]$.

We can actually relate these two forms of abandonment to their associated equity reservation value functions:

LEMMA 1. When shareholders default, bond-holders are not compensated up to their par value. Consequently, shareholders' ex-post optimal equity reservation value function, which we denote $S^*(y)$, is therefore

$$S^*(y) = \max\left\{0;\, U^*(y) - \frac{\delta}{\rho}[1 - \mathcal{P}(y \triangleright \underline{x}^*)]\right\}. \tag{9}$$

Lemma 1 states that it is only optimal for shareholders to default at y when in doing so their claim becomes worthless, i.e., $S^*(y) = 0$. It rules out the possibility that shareholders could receive part of the residual value in conjunction with default.

It also captures the fundamental shareholders' incentive problem in the *default* region: whereas the value of the firm reacts to the state, shareholders' rewards do not. Conversely, in the *corporation sale* region, both firm value and shareholders' rewards react to the state. The former will generate inefficiencies, and we will show that the latter does not.

3.3. VALUATION OF SHARES AND BONDS

In order to price innovators/shareholders' and debtholders' claims, we first establish the values they would have if the innovators' operations were abandoned the first time the state variable x_t reached a *given* level y. Although they are conditional on the walkout occurring at y, these calculations will nevertheless embed the shareholders' optimization over the form of walkout. Here, we use the fact that the optimal walkout is implicitly given by the shareholders' ex-post optimal equity reservation value function, $S^*(y)$, and that this is just a function of the (single) control variable y (Lemma 1).

We therefore directly obtain that the values of the shares and the debt prior to abandonment, for a given couple y and δ, are respectively

$$S(x_t \mid y, \delta) = \Pi(x_t) - \frac{\delta}{\rho} + \left[S^*(y) - \Pi(y) + \frac{\delta}{\rho} \right] \mathcal{P}(x_t \rhd y)\,, \tag{10}$$

$$D(x_t \mid y, \delta) = \frac{\delta}{\rho} + \left[U^*(y) - S^*(y) - \frac{\delta}{\rho} \right] \mathcal{P}(x_t \rhd y)\,. \tag{11}$$

The structure of these expressions is similar to Equations (3) and (5): the first term on the right-hand side is the value of a perpetual entitlement on the flow of income to the particular asset class; $\Pi(x) - \delta/\rho$ for innovators' equity and δ/ρ for bonds. The second term is the product of (i) the change in asset value intervening when the innovators/shareholders abandon operations (the expression in the square brackets), and (ii) the probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd y)$.

Shareholders choose the timing and form of their walkout in order to maximize the value of their claim. Assuming they select it in an unconstrained fashion,[12] the *shareholders' ex-post optimal* walkout trigger level, $\underline{x}_{\mathcal{S}}$, therefore solves

$$\underline{x}_{\mathcal{S}} = \arg\max_{y} S(x_t \mid y, \delta)\,. \tag{12}$$

The derivation of $\underline{x}_{\mathcal{S}}$ is the result of an optimization over (a) walkout time and (b) walkout form. This double optimization is easiest carried out in two steps: First consider the value of equity *conditional* on one particular form of walkout (default or corporation sale) and optimize over the *time* of this decision (its trigger level). Secondly compare conditional equity values to determine which *form* of walkout is actually the optimal one. The procedure is as follows:

[12] This is the Endogenous Closure Rule assumed in Leland (1994, 1998), Leland and Toft (1996), Fries et al. (1997), Mella-Barral and Perraudin (1997) and Mella-Barral (1999).

1. Define $S_{\text{def}}(x_t \mid y, \delta)$ and $S_{\text{corp}}(x_t \mid y, \delta)$ to be the *conditional* value of the equity assuming that shareholders choose a *default* and a *corporation sale*, respectively, as the walkout form and set the timing of abandonment as if this walkout form were the only one available. Algebraically, this consists of fixing $S^*(y) = 0$ for the former and $S^*(y) = U^*(y) - \delta/\rho[1 - \mathcal{P}(y \rhd \underline{x}^*)]$ for the latter:

$$S_{\text{def}}(x_t \mid y, \delta) \;\equiv\; \Pi(x_t) - \frac{\delta}{\rho} + \left[-\Pi(y) + \frac{\delta}{\rho}\right]\mathcal{P}(x_t \rhd y)\,, \tag{13}$$

$$\begin{aligned} S_{\text{corp}}(x_t \mid y, \delta) \;\equiv\; & \Pi(x_t) - \frac{\delta}{\rho} + \left[U^*(y) - \Pi(y)\right]\mathcal{P}(x_t \rhd y) \\ & + \frac{\delta}{\rho}\mathcal{P}(x_t \rhd \underline{x}^*)\,. \end{aligned} \tag{14}$$

Derive the shareholders' ex-post optimal *default* and *corporation sale* trigger levels, $\underline{x}_{\text{def}}$ and $\underline{x}_{\text{corp}}$, solving respectively the first-order optimality conditions:

$$\frac{\partial S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)}{\partial \underline{x}_{\text{def}}} = 0\,, \quad \text{and} \quad \frac{\partial S_{\text{corp}}(x_t \mid \underline{x}_{\text{corp}}, \delta)}{\partial \underline{x}_{\text{corp}}} = 0\,. \tag{15}$$

2. Compare $S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)$ and $S_{\text{corp}}(x_t \mid \underline{x}_{\text{corp}}, \delta)$ to determine which of the two forms of walkout corresponds to the unconditional optimal form of walkout. The actual walkout trigger level is then

$$\underline{x}_{\mathcal{S}} = \begin{cases} \underline{x}_{\text{def}} & \text{when} \quad S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta) \;\geq S_{\text{corp}}(x_t \mid \underline{x}_{\text{corp}}, \delta) \\ \underline{x}_{\text{corp}} & \text{when} \quad S_{\text{corp}}(x_t \mid \underline{x}_{\text{corp}}, \delta) \;> S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)\,. \end{cases} \tag{16}$$

All existing defaultable bond pricing models associate the shareholders' walkout decision exclusively with default. They ignore the shareholders' option to sell directly their shares to alternative operators. In our set-up, this amounts to imposing $S^*(y) = 0$ and $\underline{x}_{\mathcal{S}} = \underline{x}_{\text{def}}$. The values for equity and debt such a restriction would generate are $S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)$ and $D_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta) \equiv V(x_t \mid \underline{x}_{\text{def}}) - S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)$.

To simplify, we denote from now on $S(x_t \mid \delta) \equiv S(x_t \mid \underline{x}_{\mathcal{S}}, \delta)$ and $D(x_t \mid \delta) \equiv D(x_t \mid \underline{x}_{\mathcal{S}}, \delta)$ the ex-post value of the shares and the debt, respectively, under this second-best policy.

The policy resulting from (10) is second best, because it has no reason to yield the first-best timing of switching operations we derived in Section 2. For most coupon levels, δ, the resulting ex-post value of the firm, $V(x_t \mid \underline{x}_{\mathcal{S}}) = S(x_t \mid \delta) + D(x_t \mid \delta)$, is less than the ex-ante optimal one, $U(x_t)$.

4. Second-Best Policy

In this Section, we determine the second-best policy actually chosen, because of the presence of debt: We first study the ex-post behavior of shareholders, assuming a given debt contract. Working backward in time, we then examine the ex-ante optimal debt contract issued to complete the initial financing of the firm.

4.1. EX-POST BEHAVIOR

We analyze the ex-post behavior of shareholders and the resulting second-best usage of assets, considering all *algebraically* possible coupon levels without limiting their range. We assume that the project was financed at time $t = 0$ by issuing a coupon δ paying debt contract, and examine at dates $t > 0$ the timing and form of shareholders' preferred walkout.

We derive the following observations by simply examining the non-cooperative solutions obtained in the unconstrained ex-post optimization problem of Equations (10) and (12), for a *given* debt coupon δ:

LEMMA 2. If ex-post shareholders default, their walkout trigger level, $\underline{x}_{\mathcal{S}}$, is strictly increasing in the coupon level, δ. If they walk out with a corporation sale instead, the trigger level is independent of the coupon level.

$$\text{If } S^*(\underline{x}_{\mathcal{S}}) \;=\; 0, \quad \text{then} \quad \frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta} > 0\,. \tag{17}$$

$$\text{If } S^*(\underline{x}_{\mathcal{S}}) \;=\; U^*(\underline{x}_{\mathcal{S}}) - \frac{\delta}{\rho}[1 - \mathcal{P}(\underline{x}_{\mathcal{S}} \rhd \underline{x}^*)], \quad \text{then} \quad \frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta} = 0\,. \tag{18}$$

In the case of default, increasing the debt has no effect on the shareholders' stake in the residual value (which remains at zero by Lemma 1). Increasing the debt however reduces the stream of revenues accruing to equityholders in current operations. This modification of the balance yields an *earlier* equityholder abandonment. These dynamics of default are similar to those in Leland (1994), but are particular to default:

In the case of a corporation sale, increasing the debt still reduces the stream of revenues accruing to equityholders in current operations. However, increasing the debt also reduces the value of the shares in a corporation sale. Furthermore, both these reductions correspond to exactly the same promised payments to bondholders. Hence, at any time (and in particular at the time of the sale), the present value of the former equals the latter. The monotonous relationship between the degree of indebtedness and exit timing therefore does not hold when abandonment occurs through a corporation sale. Contrary to default, the timing of such sales is *independent* of the capital structure of the firm.

The nesting result that the dynamics of walkout (shareholders' abandonment time versus leverage) differ across *forms* of walkout is at the heart of the efficiency results we will derive. To begin with, relating these observations to the firm value yields:

Whereas default results in a suboptimally early abandonment of the firm's operations, a corporation sale leads to the ex-ante optimal timing of technology transfer.

$$\text{If } S^*(\underline{x}_{\mathcal{S}}) = 0, \quad \text{then} \underline{x}_{\mathcal{S}} > \underline{x} \quad \text{and} \quad V(x_t \mid \underline{x}_{\mathcal{S}}) < U(x_t)\,. \tag{19}$$

$$\text{If } S^*(\underline{x}_{\mathcal{S}}) = U^*(\underline{x}_{\mathcal{S}}) - \frac{\delta}{\rho}[1 - \mathcal{P}(\underline{x}_{\mathcal{S}} \rhd \underline{x}^*)] \quad \text{then} \quad \underline{x}_{\mathcal{S}} = \underline{x} \quad \text{and} \quad V(x_t \mid \underline{x}_{\mathcal{S}}) = U(x_t)\,. \tag{20}$$

Corollary 1 establishes the differing welfare implications of the two walkout forms:

1. Default triggers-off an agency problem inherent to debt contracts and causes superior technologies to be utilized insufficiently: the innovative firm should operate the assets until x_t hits $\underline{x}$; instead it chooses ex-post to abandon the assets at an *earlier* point in time when x_t reaches $\underline{x}_{\mathcal{S}}$, where $\underline{x} < \underline{x}_{\mathcal{S}}$.
2. Corporation sales, on the other hand, do not suffer from this moral hazard problem as the interests of all asset holders are aligned.

4.2. SUBOPTIMAL WALKOUT FORM

It is convenient to pursue the analysis distinguishing between the two types of situations ("termination" or "continuation") which emerged from our structural model in Section 2.2. The differences in the dynamics of the ex-post timing of abandonment identified in Lemma 2, and the first-order optimality conditions which determine $\underline{x}^*$ and $\underline{x}_{\mathcal{S}}$ give:

LEMMA 3. When the first-best policy involves termination, i.e., $\underline{x} < \underline{x}^*$, shareholders walk out ex-post always defaulting.

After default, for lower levels of coupons, $\delta \in (0; \hat{\delta}^*]$ where

$$\hat{\delta}^* \equiv \rho\Pi(\underline{x}^*) - \rho\Pi^*(\underline{x}^*)\frac{d\Pi(\underline{x}^*)}{d\underline{x}^*}\left[\frac{d\Pi^*(\underline{x}^*)}{d\underline{x}^*}\right]^{-1}, \tag{21}$$

the technology is terminated, whereas for higher coupon levels, it is continued.

Figure 4 gives a graphical representation of the latter case (higher coupon levels) considered in Lemma 3. The values of the exogenous parameters characterizing the setup are exactly those used in Figure 2, where we illustrated the value the firm would take under the first-best policy.

When the first-best policy involves continuation, i.e., $\underline{x}^* < \underline{x}$, shareholders walk out ex-post with a corporation sale for lowest coupon levels, $\delta \in (0; \hat{\delta}]$, where $\hat{\delta}$ solves

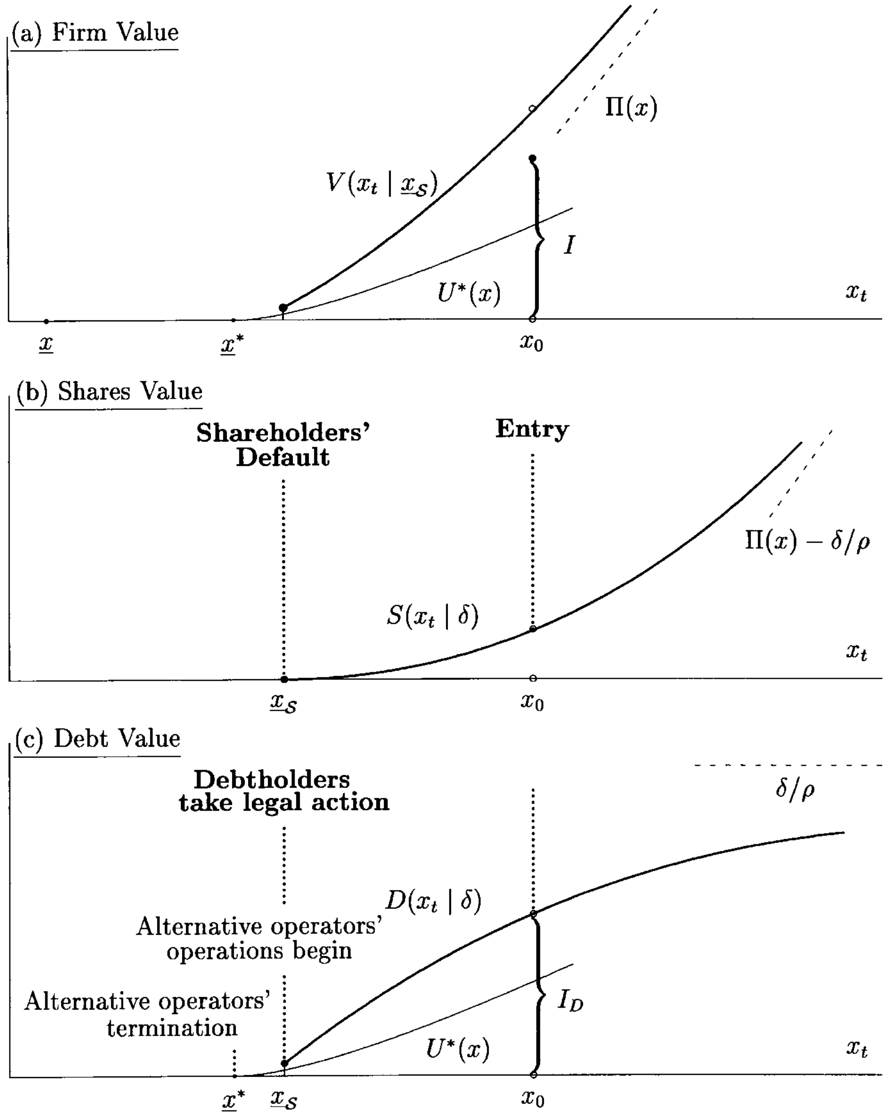


Figure 4. Second-best firm, shares and debt values, when (i) the the first-best policy involves "termination" ($\underline{x} \leq \underline{x}^*$), and (ii) for highest levels of borrowings ($\delta > \hat{\delta}^*$).

$$\hat{\delta} = \rho \frac{\Pi(\underline{x}_{\text{def}}) + [U^*(\underline{x}) - \Pi(\underline{x})]\, \mathcal{P}(\underline{x}_{\text{def}} \rhd \underline{x})}{1 - \mathcal{P}(\underline{x}_{\text{def}} \rhd \underline{x}^*)}. \tag{22}$$

and defaulting for higher ones. The technology is continued in all cases.

Figure 5 illustrates the case with a corporation sale (lower coupon levels); the setup corresponds exactly to that of Figure 3, and the graphs gain in being compared.

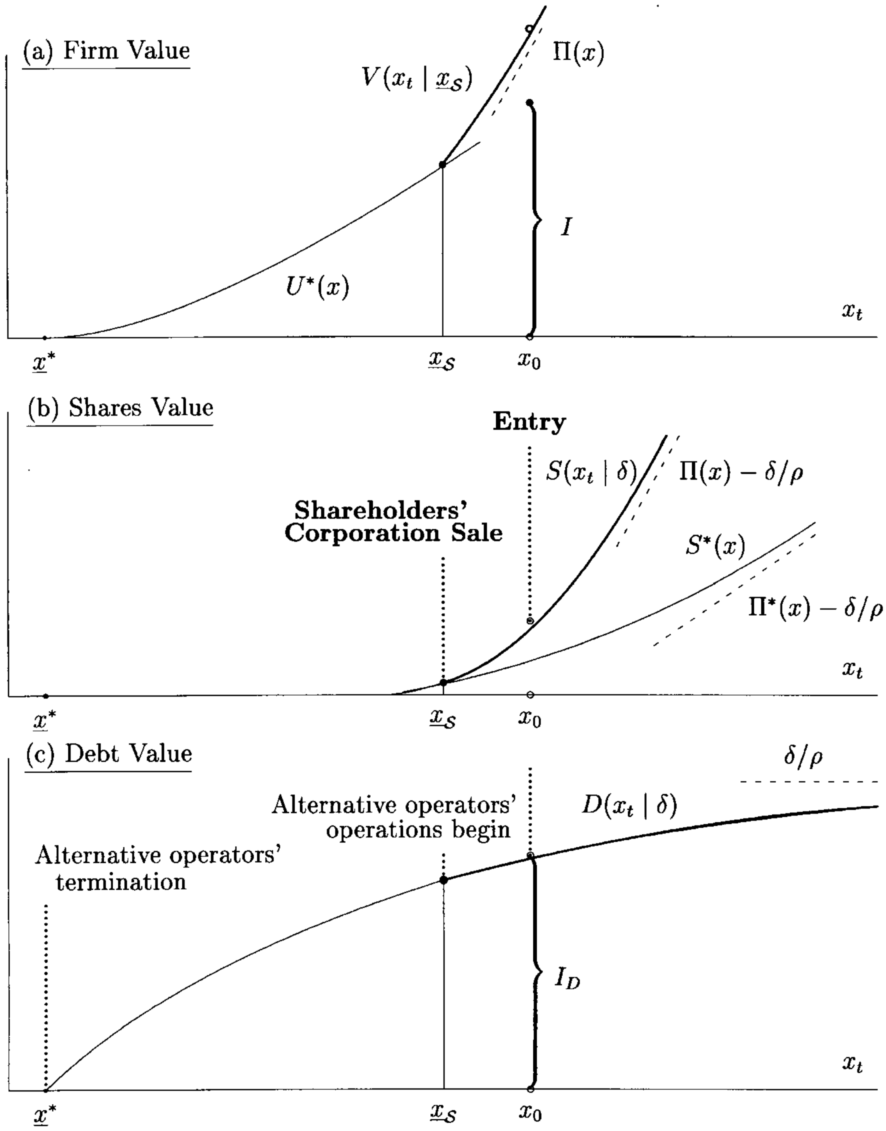


Figure 5. Second-best firm, shares and debt values, when (i) the first-best policy involves "continuation" ($\underline{x}^* \leq \underline{x}$), and (ii) for lower levels of borrowings ($\delta < \hat{delta}$).

Lemmas 3 and 4 highlight the fact that ex-post not only the (random) time of the walkout may be too early (Corollary 1), but also the type of usage of the technology (termination or continuation) may also differ from the ex-ante optimal one.

4.3. EX-ANTE OPTIMAL DEBT CONTRACT

We now examine the ex-ante optimal coupon, among all the algebraically possible, contracted upon at the date of entry, $t = 0$. That is, we determine the coupon level δ creditors demand ex ante, given that the innovators need to raise a total of I_D. We then study the circumstances under which there is no contract allowing the implementation of the technology.

We begin simply studying the response of debt values to coupon levels: Inserting the shareholders' ex-post optimal walkout trigger level, $\underline{x}_{\mathcal{S}}$, derived from (12), into the debt value function (11) in place of y, we express the ex-post value of debt, $D(x_t \mid \delta)$, solely as a function of the coupon level. Then, using Lemma 1 we obtain

LEMMA 5. If ex-post shareholders walk out defaulting, the prior value of debt, $D(x) \mid \delta)$, is increasing, then decreasing in the coupon level δ. Therefore, there exists a coupon

$$\overline{\delta} \equiv [1 - \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})] \left[\frac{\partial \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})}{\partial \underline{x}_{\mathcal{S}}} \frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta} \right]^{-1}, \tag{23}$$

at which the value function of debt reaches a maximum. As a consequence, coupon levels issued at entry must be in the interval $\delta \in (0; \overline{\delta}]$

LEMMA 6. If ex-post shareholders walk out with a corporation sale, the ex-ante value of debt, $D(x \mid \delta)$, is increasing in the coupon level δ.

We can now proceed to examine the ex-ante financing problem: At the date of entry, $t = 0$, the innovators/shareholders, need to issue a debt contract whose market value, $D(x_0 \mid \delta)$, equals the initial outlay they are unable to cover by internal funding, I_D. Sufficient debt financing is found, if at the entry state, x_0, there exists a coupon δ, such that

$$I_D = D(x_0 \mid \delta). \tag{24}$$

Here, we assume that debt is not issued at a discount.[13] To establish whether financing is possible, we rather study the inverse function

$$\delta = \delta(x_0; I_D); \tag{25}$$

which is a mapping from the financial requirement space to the contract space. The case where termination-abandonment is ex-ante optimal presents us with little difficulty in this respect. Combining Lemmas 3 and 5 yields:

[13] If debt was issued at a discount Δ, the above financing Equation (24) would have to be replaced by $I_D + \Delta = D(x_0 \mid \delta)$. All results of the paper would be applicable replacing I_D by $I_D + \Delta$.

LEMMA 7. When the first-best policy involves termination, i.e., $\underline{x} < \underline{x}^*$, a unique coupon level δ and associated debt contract can be found to finance the project as long as the required external finance, I_D, does not exceed $D(x_0 \mid \bar{\delta})$.

The function relating the coupon to the finance required, $\delta = \delta(x_0; I_D)$, is then strictly increasing in I_D over the interval $I_D \in [0; D(x_0 \mid \bar{\delta}))$.

Figure 6 gives a graphical representation of Lemma 7. When the first-best policy involves termination, we know that shareholders will ultimately default (Lemma 3), hence $D(x_0 \mid \bar{\delta}) = D_{\text{def}}(x_0 \mid \underline{x}_{\text{def}}, \delta)$. Then, the upper part of the figure simply shows the value of the debt at entry, $D(x_0 \mid \bar{\delta})$, for different coupon levels, δ. The lower part of the figure illustrates the implications for project financing, by simply inverting axis: If I_D is less than $D(x_0 \mid \bar{\delta}))$, the project can be financed. For a given I_D there exists one coupon, $\delta(x_0; I_D)$, which yields a debt value equal to I_D.

The case where the first-best policy involves continuation ($\underline{x}^* < \underline{x}$) is more complex. Combining in similar fashion Lemmas 4, 5 and 6 reveals that, as we consider gradually increased finance requirements, I_D, the chosen contracts may result in a switch from corporation sales to default at a critical coupon threshold $\hat{\delta}$. This generates a *discontinuity* in debt value as a function of coupon level:

When the first-best policy involves continuation, i.e., $\underline{x}^* < \underline{x}$, a unique coupon level δ and associated debt contract can be found to finance the project as long as the required external finance, I_D, does not exceed $D(x_0 \mid \hat{\delta}) \vee D(x_0 \mid \bar{\delta})$.

(i) If $D(x_0 \mid \bar{\delta}) < D(x_0 \mid \hat{\delta})$, the chosen coupon $\delta(x_0; I_D)$ is a continuous and strictly increasing function in I_D throughout the interval $I_D \in (0; D(x_0 \mid \hat{\delta}))$, with a constant slope $\partial\delta/\partial I_D = \rho[1 - \mathcal{P}(x_0 \rhd \underline{x})]^{-1}$.

(ii) If $D(x_0 \mid \hat{\delta}) < D(x_0 \mid \bar{\delta})$, the chosen coupon $\delta(x_0; I_D)$ is a continuous and strictly increasing function in I_D over the intervals $I_D \in (0; D(x_0 \mid \hat{\delta}))$ and $I_D \in (D(x_0 \mid \check{\delta}); D(x_0 \mid \bar{\delta}))$. At $I_D = D(x_0 \mid \hat{\delta})$, the chosen coupon function, $\delta(x_0; I_D)$, jumps however from $\hat{\delta}$ to $\check{\delta}$, where $\check{\delta}$ solves $D_{\text{corp}}(x_0 \mid \underline{x}_{\text{corp}}, \hat{\delta}) = D_{\text{def}}(x_0 \mid \underline{x}_{\text{def}}, \check{\delta})$. Denoting $l(x_t \mid \delta) \equiv D(x_t \mid \delta)/V(x_t \mid \underline{x}_{\mathcal{S}})$ the firm's level of leverage at time time t, there exists, therefore, a "leverage gap", an interval of leverage levels $(l(x_0 \mid \hat{\delta}); l(x_0 \mid \check{\delta}))$ that will never be observed at time $t = 0$.

The distinction between cases (i) and (ii) of Lemma 8 is best understood graphically:

Figure 7(i) depicts the situation in case (i): The (conditional) debt value associated with default, $D_{\text{def}}(x_0 \mid \underline{x}_{\text{def}}, \delta)$, fails to reach the level $D_{\text{corp}}(x_0 \mid \underline{x}_{\text{corp}}, \hat{\delta}) = D(x_0 \mid \hat{\delta})$ even at its maximum point, $\bar{\delta}$. It is therefore never optimal to issue a debt contract that would result in shareholders' walkout occurring through default, i.e., setting δ above $\hat{\delta}$. The lower part of the figure shows the implications for project financing: The inverse function $\delta(x_0; I_D)$ is cut off above at $\hat{\delta}$.

Figure 7(ii) tells the more complicated case (ii): Over the interval $\delta \in (0; \hat{\delta}]$, we observe the usual straight-line relationship associated with corporation sales.

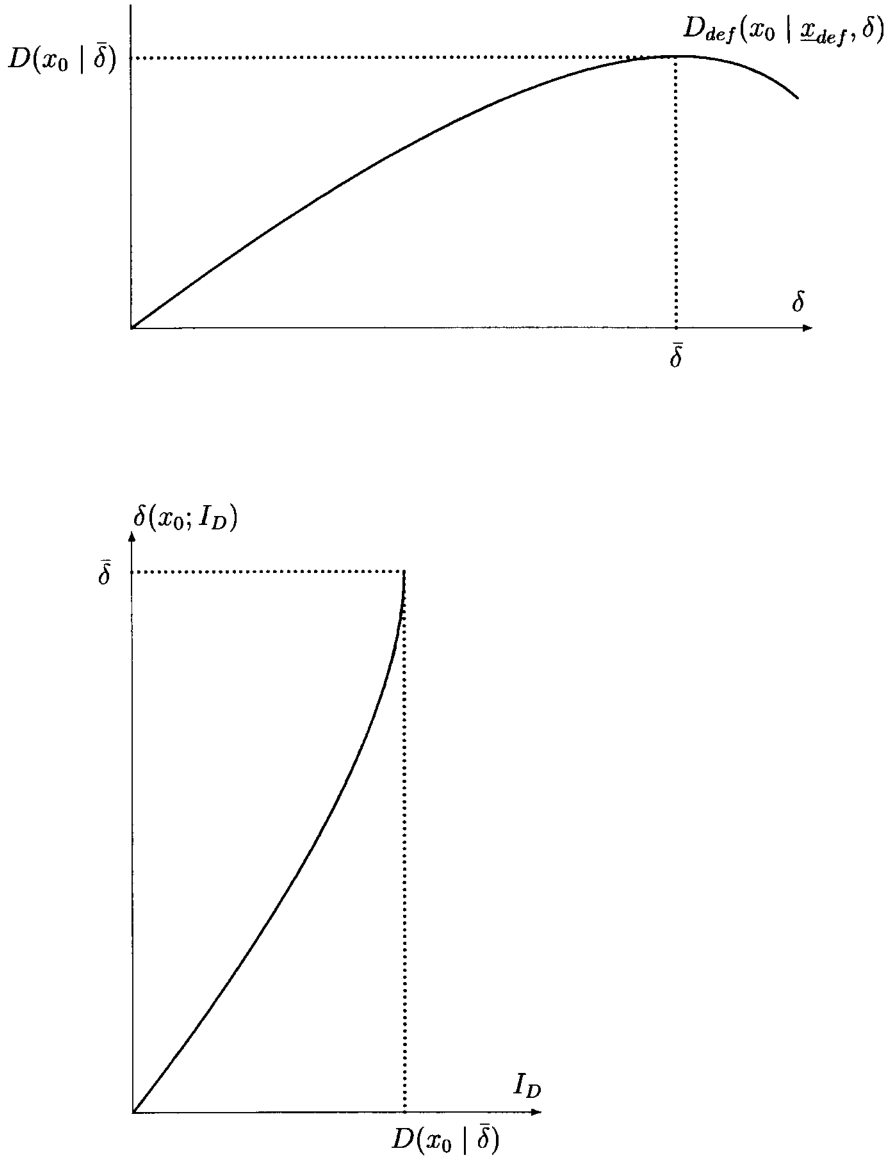


Figure 6. Project financing when the first-best policy involves "termination" ($\underline{x} \leq \underline{x}^*$).

Over the interval $\delta \in (\hat{\delta}; \check{\delta})$, default is optimal for shareholders, but because the market value of such contracts is less than $D(x_0 \mid \hat{\delta})$, they are dominated by the $\hat{\delta}$ coupon contract. Therefore, debt contracts offering a coupon in $\delta \in (\hat{\delta}; \check{\delta})$ will never be issued. At the high end, coupons $\delta \in (\check{\delta}; \bar{\delta}]$ yield higher debt values and are therefore necessary to cover the higher range of financing needs. Again, the lower part of the figure shows the implications for the inverse function $\delta(x_0; I_D)$.

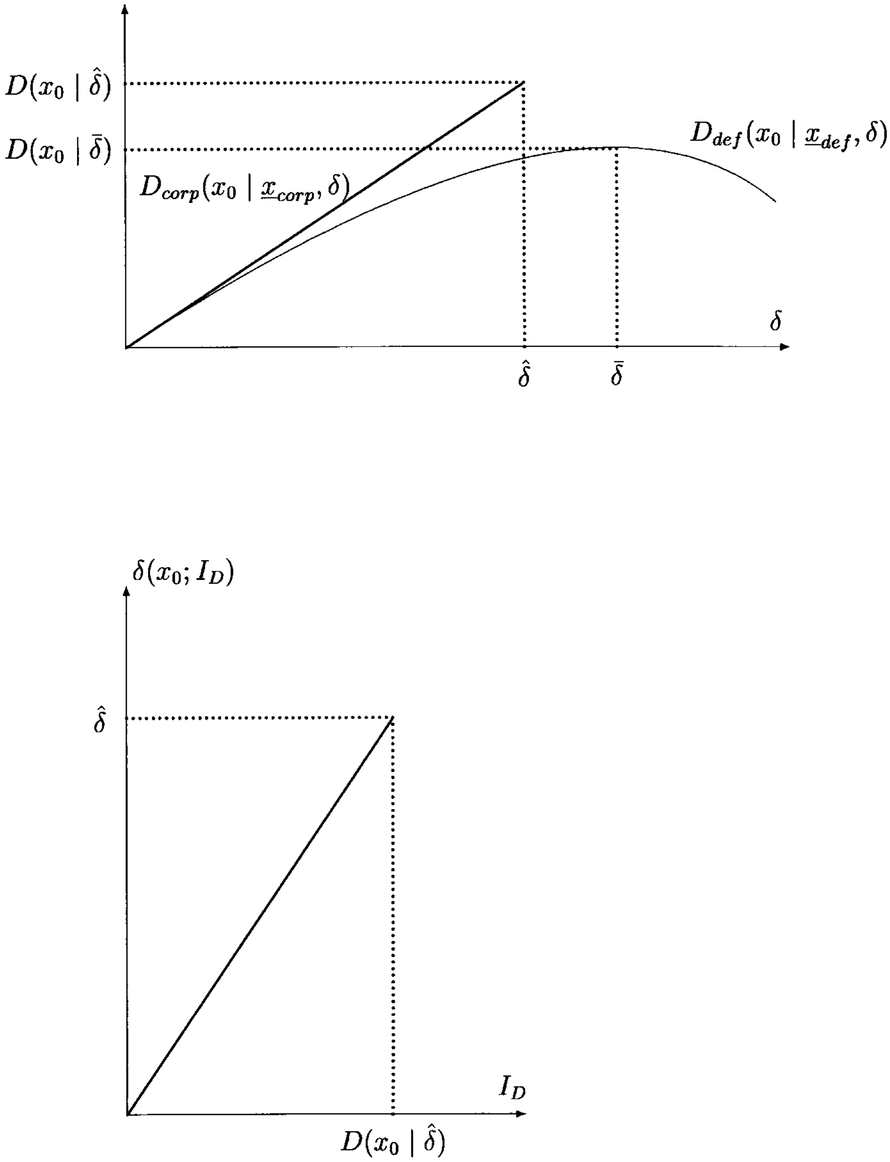
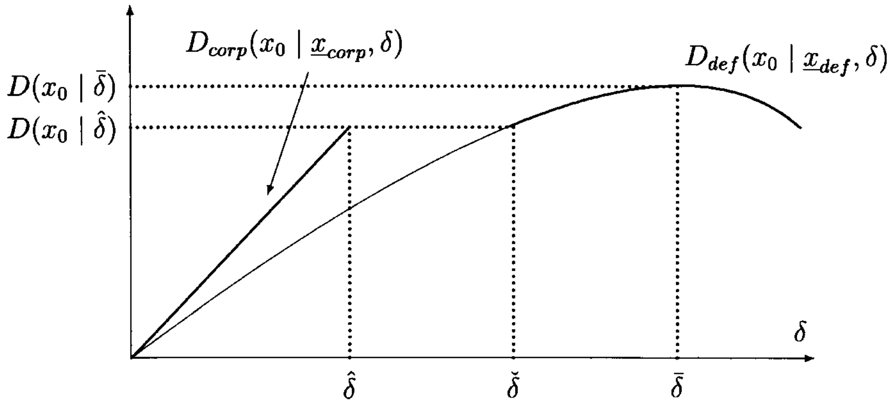


Figure 7(i). Project financing when (a) the first-best policy involves "continuation" ($\underline{x}^* \leq \underline{x}$), and (b) $D(x_0 \mid \bar{\delta}) < D(x_0 \mid \hat{\delta})$.

5. Implementing the Model

5.1. OBTAINING CLOSED-FORM PRICING SOLUTIONS

For this fairly general continuous-time model to yield closed-form pricing solutions, additional structure is needed. The exogenous parameters describing (i) the



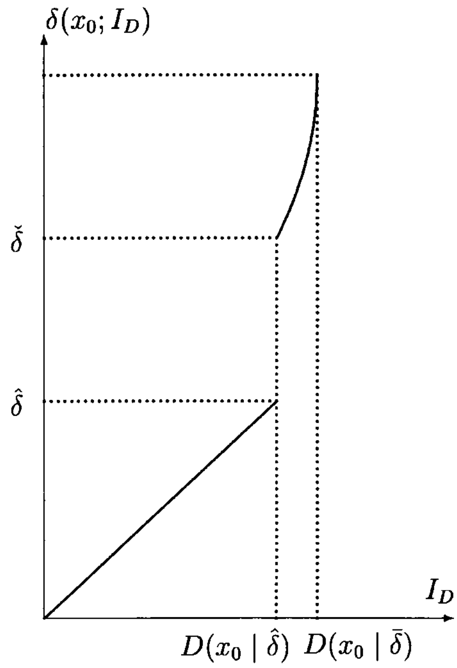


Figure 7(ii). Project financing when (a) the first-best policy involves "continuation" ($\underline{x}^* \leq \underline{x}$), and (b) $D(x_0 \mid \hat{\delta}) < D(x_0 \mid \bar{\delta})$.

innovators and the technology, $\{\Pi(x), I, x_0\}$, (ii) alternative operators, $\{\Pi^*(x)\}$, and (iii) the economic environment, $\{x; \mu(x); \sigma(x); \rho\}$, must permit the following:

First, the specific type of diffusion process we assume to be driving the uncertainty, $\{x; \mu(x); \sigma(x); \rho\}$, must enable us to express the Laplace transform, $\mathcal{P}(x_t \triangleright y)$, introduced in Equation (2). Secondly, the functional form chosen for the perpetuity-values of the firm's alternatives, $\Pi(x)$ and $\Pi^*(x)$, must allow us to solve explicitly for the different optimal decision trigger levels, as well as threshold levels, using the relevant first-order optimality conditions we have detailed. The following structure performs this and is easy to implement:

"GBM-LINEAR" STRUCTURE. *The uncertain state variable, x_t, describing the current status of the firm follows a geometric Brownian motion,*

$$dx_t = \mu x_t\, dt + \sigma x_t\, dB_t\,, \tag{26}$$

where $\mu < \rho$ and σ are constants, and B_t is a standard Brownian motion.

Both the unlimited liability value of a perpetual claim on the income flow from the innovators' operations, $\Pi(x)$, and the unlimited liability value of a perpetual claim on the alternative operators' income flow, $\Pi^*(x)$, are linear in x. That is, there exist four constants Θ_0, Θ_1, Θ_0^*, and Θ_1^*, where $\Theta_0 < \Theta_0^* < 0$ and $0 < \Theta_1^* < \Theta_1$, such that

$$\Pi(x) = \Theta_0 + \Theta_1 x \quad \text{and} \quad \Pi^*(x) = \Theta_0^* + \Theta_1^* x\,. \tag{27}$$

Under this structure:

1. The probability weighted discount factor for future events $\mathcal{P}(x \triangleright y)$ becomes simply[14]

$$\mathcal{P}(x \triangleright y) = \left(\frac{x}{y}\right)^{\lambda}, \tag{28}$$

where

$$\lambda \equiv \sigma^{-2}[-(\mu - \sigma^2/2) - ((\mu - \sigma^2/2)^2 + 2\rho\sigma^2)^{1/2}]\,. \tag{29}$$

2. *All* asset pricing formulas encountered in the paper, $A(x_t) \in \{S(x_t \mid y, \delta); D(x_t \mid y, \delta); V(x_t \mid y); V^*(x_t \mid y)\}$, take the following simple form

$$A(x_t \mid \underline{x}) = a_A + b_A x_t + c_A x_t^{\lambda}\,, \tag{30}$$

where (a_A, b_A, c_A) are constants.

3. *All* decision trigger levels have very simple closed-form solutions:
   (a) The optimal alternative operators' termination trigger level, $\underline{x}^*$, is (Equation (4))

[14] See Karlin and Taylor (1975).

$$\underline{x}^* = \frac{-\lambda}{1-\lambda}\left(\frac{-\Theta_0^*}{\Theta_1^*}\right). \tag{31}$$

(b) The ex-ante optimal firm abandonment trigger level, $\underline{x}$, is (Equation (8))

$$\begin{aligned}\underline{x} &= \frac{-\lambda}{1-\lambda}\left(\frac{-\Theta_0}{\Theta_1}\right) \quad \text{if } 0 < \Theta_0\Theta_1^* - \Theta_0^*\Theta_1 \\ &\quad \text{(and "termination-abandonment" is ex-ante optimal)}, \\ &= \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right) \quad \text{if } Theta_0\Theta_1^* - \Theta_0^*\Theta_1 < 0 \\ &\quad \text{and "continuation-abandonment" is ex-ante optimal)}. \end{aligned} \tag{32}$$

(c) The shareholders' ex-post optimal walkout trigger level, $\underline{x}_{\mathcal{S}}$, is (Equation (12))

$$\begin{aligned}\underline{x}_{\mathcal{S}} &= \underline{x}_{\text{def}} = \frac{-\lambda}{1-\lambda}\left(\frac{\delta/\rho - \Theta_0}{\Theta_1}\right) \quad \text{if } 0 < \Theta_0\Theta_1^* - \Theta_0^*\Theta_1 \\ &\quad (\delta \in (\hat{\delta}; +\infty) \text{ and shareholders default}), \\ &= \underline{x}_{\text{corp}} = \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right) \quad \text{if } \Theta_0\Theta_1^* - \Theta_0^*\Theta_1 < 0 \\ &\quad (\delta \in (0; \hat{\delta}] \text{ and shareholders sell the corporation}). \end{aligned} \tag{33}$$

4. Of the three threshold coupon levels we introduced, only one has a simple closed-form solution. There are no direct expressions for the other two, but finding them numerically is easy:

(a) The threshold level of coupon obligation $\hat{\delta}^*$ in Lemma 3, Equation (21) is

$$\hat{\delta}^* = \rho\frac{\Theta_0\Theta_1^*\Theta_0^*\Theta_1}{\Theta_1^*}.$$

(b) The threshold level of coupon obligation $\hat{\delta}$ in Lemma 4, Equation (22) solves

$$\left((1-\lambda)\frac{\hat{\delta}}{\rho} - \Theta_0^*\right) f(\underline{x}^*) + \left(\Theta_0^* - \Theta_0\right) f(\underline{x}) = \left(\frac{\hat{\delta}}{\rho} - \Theta_0\right)^{1-\lambda}, \tag{34}$$

where

$$f(x) \equiv \left(\frac{-\lambda}{(1-\lambda)\Theta_1 x}\right)^{\lambda}. \tag{35}$$

(c) The coupon level $\overline{\delta}$ in Lemma 5, Equation (23) depends on the entry state, x_0. It is found solving

$$(1-\lambda)\frac{\overline{\delta}}{\rho} - \Theta_0 = \left(\frac{\overline{\delta}}{\rho} - \Theta_0\right)^{1+\lambda} f(x_0)\,. \tag{36}$$

Notice that requiring $\Theta_0^* < 0$ and $0 < \Theta_1^*$ suffices for Assumption 1 to be satisfied. Similarly, requiring $\Theta_0 < \Theta_0^*$ and $\Theta_1^* < \Theta_1$ suffices for Assumption 2 to be satisfied.

5.2. PRICE IMPACT OF CONSIDERING CORPORATION SALES

Arguably, the most commonly used measures of the impact of default risk on debt value are the risk premium investors require to compensate them for being exposed, and the associated credit spread. In this paper, the bonds' walkout-risk premium is given by

$$p(x_t) \equiv \delta - \rho D(x_t \mid \delta)\,, \tag{37}$$

and the credit spread is given by

$$s(x_t) \equiv \frac{\delta}{D(x_t \mid \delta)} - \rho\,. \tag{38}$$

As mentioned in the introduction, all existing defaultable bond pricing models associate the shareholders' walkout decision exclusively with default. The consequence of this assumption for prices is easily derived here: Technically, it just involves imposing a restriction on the more general optimization we carried out. The value of the debt if we ignored the shareholders' option to sell ownership rights was actually derived in Section 3.3. when we considered asset values *conditional* on a form of walkout. Given our set-up, the value of the debt such a restriction would lead to is $D_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta) \equiv V(x_t \mid \underline{x}_{\text{def}}) - S_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)$. Therefore, imposing shareholders' walkout to necessarily consist of default generates a bond's walkout-risk premium (default) equal to

$$p_{\text{def}}(x_t) \equiv \delta - \rho D_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)\,, \tag{39}$$

and an associated credit spread

$$s_{\text{def}}(x_t) \equiv \frac{\delta}{D_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)} - \rho\,. \tag{40}$$

Our measures of the impact of this omission on debt pricing consist of taking the relative differences between risk premia and credit spreads obtained considering corporation sales or not:

$$R|p_{\text{def}} \equiv \frac{p(x_t) - p_{\text{def}}(x_t)}{p_{\text{def}}(x_t)} \quad \text{and} \quad R|s_{\text{def}}(x_t) \equiv \frac{s(x_t) - s_{\text{def}}(x_t)}{s_{\text{def}}(x_t)}\,. \tag{41}$$

The first measure, $R|p_{\text{def}}$, is interestingly independent of the current state x_t. It is therefore valid at any time of the firms' life. It is also particularly convenient in that replacing the above expressions of $p(x_t)$ and $p_{\text{def}}(x_t)$ yields the following simple formula,

$$R|p_{\text{def}} = \frac{[U^*(\underline{x}_{\mathcal{S}}) - S^*(\underline{x}_{\mathcal{S}}) - \delta/\rho]\mathcal{P}(\underline{x}_{\text{def}} \triangleright \underline{x}_{\mathcal{S}})}{U^*(\underline{x}_{\text{def}}) - \delta/\rho} - 1\,. \tag{42}$$

Clearly, $R|p_{\text{def}} \neq 0$ (and $R|s_{\text{def}}(x_t) \neq 0$) whenever in our general model, shareholders' ex-post optimal walkout form consists of a corporation sale: In this case, relative to a default-only model, considering this alternative impacts prices. Conversely, $R|p_{\text{def}} = 0$ (and $R|s_{\text{def}}(x_t) = 0$) whenever shareholders' ex-post optimal walkout form consists of default. Here, considering the corporation sale alternative has no impact on prices relative to a default-only model.

When the first-best policy involves a continuation, i.e., $\underline{x}^* < \underline{x}$, and for lower levels of borrowings, i.e., throughout the interval $\delta \in (0; \hat{\delta})$, defaultable bond pricing models who only consider default, will underestimate the value of the debt and exaggerate the risk premium demanded by creditors. They will consequently also underestimate the borrowing ability (debt capacity) of the firm.

Overall, the importance of considering the corporation sale alternative is related to the critical upper boundary, $\hat{\delta}$, at which shareholders are indifferent between the two forms of walkout. The conventional restriction (walkout=default) is likely to perform at its worst when applied to industries where firms possess quite similar technological abilities and opportunities for corporation sales abound (high $\hat{\delta}$). Pricing errors should also be significant in sectors where the need for credit finance is modest, e.g., where access to internal funding is ample and the required initial outlays are small in relation to future revenues.

5.3. NUMERICAL RESULTS

We are now ready to *quantitatively* get a feel for the importance of the debt walkout premium. We assess whether considering the corporation sale option is at all important in corporate debt pricing. We show how the answer is related to the conditions under which the question is posed. To do this we carry out two simple numerical applications, under the "GBM-Linear" structure which yields closed-form pricing formulas.

EXAMPLE 1. The firm's gross income under the innovators' operations, $\Pi(x_t)$, *fluctuates with* $\mu = 0$ and $\sigma = 15$. Alternative operators' gross income would be half of this (normalizing the current productivity to $\Theta_1 = 1$, implies $\Theta_1^* = 1/2$, but their fixed cost of production would be reduced by a quarter (normalizing costs to $\Theta_0 = -1$, then $\Theta_0^* = -3/4$). The interest rate is $\rho = 5\%$.

In this example, $\Theta_1 - \Theta_1^* = 1/2$, hence the innovators' technological supremacy in the industry is fairly large. Alternative operators' reduced fixed costs are nevertheless such that an eventual transfer of ownership is ex-ante optimal (Assumptions 1 and 3). The reduction in fixed cost is not too substantial, as $\Theta_0 - \Theta_0^* = -1/4$, but in all, abandonment has a substantial associated cost of more or less 40%.

Overall, $\Theta_0\Theta_1^* - \Theta_0^*\Theta_1 = 1/4$ which is positive. Therefore $\underline{x}^* > \underline{x}$, and the first-best policy involves termination. By Lemma 3, shareholders' ex-post optimal walkout choice consists of defaulting. Here, not accounting for the corporation sale alternative does not modify the calculated debt risk premium.

EXAMPLE 2. The firm's gross income under the innovators' operations, $\Pi(x_t)$, fluctuates with $\mu = 0$ and $\sigma = 15\%$. Alternative operators' gross income would be a quarter less than this (normalizing the current productivity to $\Theta_1 = 1$, implies $\Theta_1^* = 3/4$, but their fixed cost of production would be half (normalizing costs to $\Theta_0 = -1$, then $\Theta_0^* = -1/2$. The interest rate is $\rho = 5\%$.

In this example, $\Theta_1 - \Theta_1^* = 1/4$, therefore the innovators' technological supremacy in the industry is not as large as in the previous example, and alternative operators could generate fairly similar revenues. Alternative operators' reduced fixed cost are nevertheless such that an eventual transfer of ownership is ex-ante optimal. Competitors have a more reduced fixed cost, as $\Theta_0 - \Theta_0^* = -1/2$, but in all the figures considered are fairly conservative, as abandonment has an associated cost of more or less 20%.

Overall, $\Theta_0\Theta_1^* - \Theta_0^*\Theta_1 = -1/4$ which is negative, so $\underline{x}^* < \underline{x}$, hence the first-best policy involves continuation. By Lemma 4, shareholders' ex-post optimal walkout choice can either consist of (i) selling their shares (corporation sale) or (ii) defaulting, depending on leverage:

Consider a coupon level $\delta = 0.025$ which is lower than $\hat{\delta}$, the threshold coupon level which determines shareholders' ex-post behavior. Here shareholders' ex-post optimal walkout decision consists of selling their shares, as $\delta \in (0; \hat{\delta}])$. In this case, the expressions for $p(x_t)$ and $p_{\text{def}}(x_t)$ become

$$p(x_t) = \rho \left[\Theta_0^* + \Theta_1^* \underline{x}^* - \left(\Theta_0^* + \Theta_1^* \underline{x}_{\text{def}} - \frac{\delta}{\rho} \right) \left(\frac{\underline{x}^*}{\underline{x}_{\text{def}}} \right)^{\lambda} \right] \left(\frac{x_t}{\underline{x}^*} \right)^{\lambda}, \tag{43}$$

$$p_{\text{def}}(x_t) = \delta \left(\frac{x_t}{\underline{x}^*} \right)^{\lambda}, \; \underline{x}^* = \frac{-\lambda}{1-\lambda} \left(\frac{-\Theta_0^*}{\Theta_1^*} \right), \; \underline{x}_{\text{def}} = \frac{-\lambda}{1-\lambda} \left(\frac{\delta\rho - \Theta_0}{\Theta_1} \right). \tag{44}$$

Table 1 exhibits the results obtained with the input parameters of example 2. Notice in particular how accounting for the corporation sale alternative modifies the debt risk premium by $R|p_{\text{def}} = -47.88\%$, irrespective of the state. The risk premium is half of what a "default-only" model would predict, even when the firm is not at all in financial distress! Here, the direct transfer of ownership alternative influences calculated asset values and credit spreads very substantially.

Table I. Price impact of allowing for corporation sales in Example 2(i). Example 2(i) is such that (i) the first-best policy involves ' 'continuation" ($\underline{x}^* \leq \underline{x}$), and (ii) for lower levels of borrowings ($\delta < \hat{\delta}$). Input parameters are $\mu = 0$, $\sigma = 0.15$, $\theta_1 = 1$, $\theta_1^* = 3/4$, $\theta_0 = -0.5$, $r = 0.05$ (Example 2), and $\delta = 0.025$ (case (i))

Decision trigger level		Value
Optimal alternative operators' termination	$\underline{x}^*$	0.4167
Ex-ante optimal firm abandonment	$\underline{x}$	1.2500
Shareholders' ex-post optimal walkout	$\underline{x}_S$	1.2500
Shareholders' ex-post optimal default	$\underline{x}_{\text{def}}$	0.9375
Debt value when shareholders walk out:		Value
Default or corporation sale	$D(\underline{x}_S \mid \delta)$	0.4675
Default-only (no corporation sale)	$D_{\text{def}}(\underline{x}_{\text{def}} \mid \underline{x}_{\text{def}}, \delta)$	0.2517
Debt value at $x_t = 2$		Value
Default or corporation sale	$D(x_t \mid \delta)$	0.4630
Default-only (no corporation sale)	$D_{\text{def}}(x_t \mid \underline{x}_{\text{def}}, \delta)$	0.4292
Credit spreads at $x_t = 2$		Value (bps)
Default or corporation sale	$s(x_t)$	39.86
Default-only (no corporation sale)	$s_{\text{def}}(x_t)$	82.53
Relative difference	$R \mid s_{\text{def}}(x_t)$	−51.70%
Risk premium at $x_t = 2$		Value/δ
Default or corporation sale	$p(x_t)$	7.37%
Default-only (no corporation sale)	$p_{\text{def}}(x_t)$	14.17%
Relative difference	$R \mid p_{\text{def}}$	−47.88%

(ii) Consider now a coupon level $\delta = 0.10$ which is greater than $\hat{\delta}$. Here shareholders' ex-post optimal walkout choice consists defaulting, as $\delta \in (\hat{\delta}; +\infty)$. In this case, the expressions of $p(x_t)$ and $p_{def}(x_t)$ are equal. Then, not accounting for the corporation sale alternative does not affect calculated asset values.

6. Conclusion

This paper studied the relation between the comparative advantage of a firm vis-a-vis alternative users of its technology, its capital structure and the form of walkout shareholders will eventually select. We derived simple closed-form solutions for the value of shares and bonds as well as the debt capacity of the firm.

The main testable implications of our results are as follows: Shareholders of firms with low levels of leverage tend to walk out selling their shares as in corporation sales. Conversely, shareholders of highly levered firms tend to walk out defaulting. What precisely constitutes "low" and "high" depends on industry and firm-specific factors, which determine a threshold level of coupon obligations $\hat{\delta}$:

1. Where alternative operators' are largely dominated, $\hat{\delta}$ takes on a low value. Hence, quite modest debt levels will result in walkouts occurring through default.
2. Conversely, in industries where differences in operating ability are small, $\hat{\delta}$ takes on a high value. Hence, even quite highly indebted firms will go through corporation sales.

The more general notion of a walkout option has implications for the pricing of the firm's debt and the extent to which it can expect to finance its operations with this debt: Existing defaultable bond pricing models associate the shareholders' walkout decision exclusively with default. Consequently, (a) for industries where participants abilities to generate cash with a given technology are quite similar and (b) in the lower end of the leverage spectrum, they (i) exaggerate the risk premia demanded from firms and (ii) underestimate their borrowing ability.

Simple numerical simulations suggest that even for fairly conservative input parameter values, the risk premium may be exaggerated by a factor of 2. Importantly however, this pricing error vanishes as leverage exceeds the critical threshold, $\hat{\delta}$, at which shareholders switch to default as their ex-post optimal choice of walkout form.

In future research, we believe it would be particularly worthwhile to extend the analysis considering (a) ex-post renegotiation and (b) reduced competition in the product market:

(a) Empirical evidence suggests that risk premia on corporate debt significantly exceed those implied by Merton-type valuation models which consider debt to be non-renegotiable (such as Leland (1994)). Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) have been able to generate extremely large risk premia incorporating strategic debt service by equity-holders making opportunistic take-it-or-leave-it offers to their creditors. Here considering corporation sales (but precluding renegotiation) generates lower risk premia than Merton-type valuation models. This suggest that one way to obtain risk premia consistent with empirical evidence is to combine both effects, considering both strategic debt renegotiation and the corporation sale alternative.

    Note that *both* the innovators and alternative operators could then strategically default. Creditors would suffer from strategic default by all operators and credit spreads should be greater in all walkout scenarios. However, the two sides of the balance (pre- and post-walkout decision income flows) which determine shareholders' preferred walkout decision state would be equally modified in the two scenarios. Therefore shareholders' timing and form of ultimate walkout decision would not be affected much.

(b) We assumed a competitive industry for simplicity, but if the product market is not competitive, potential buyers might bargain, and the corporate sale option is less lucrative, implying that shareholders default more often. It therefore would be important and interesting to fully model this bargaining process to assess its impact on our results.

Appendix

PROOF OF LEMMA 1. If at y, shareholders default, the value of their shares is $U^*(y) - [U^*(y) \wedge P]$ under the APR. If they sell their shares in a corporate sale, the value of their shares is $U^*(y) - \frac{\delta}{\rho}[1 - \mathcal{P}(y \rhd \underline{x}^*)]$ instead. Therefore, given that $U^*(y) - [U^*(y) \wedge P] = 0 \vee [U^*(y) - P]$, at a state y, the equity residual value function is

$$S^*(y) = \max\left\{0 \vee [U^*(y) - P]; U^*(y) - \frac{\delta}{\rho}[1 - \mathcal{P}(y \rhd \underline{x}^*)]\right\}. \tag{45}$$

Now, we have $P \geq \frac{\delta}{\rho}$, hence,

$$[U^*(y) - P] \leq U^*(y) - \frac{\delta}{\rho}[1 - \mathcal{P}(y \rhd \underline{x}^*)]. \tag{46}$$

Consequently, for shareholders to actually prefer default to a corporation sale, it must be the case that $0 \vee [U^*(y) - P] = 0$. □

PROOF OF LEMMA 2. If shareholders' walkout takes the form of default, triggered at y, their residual claim value is $S^*(y) = 0$ (by Lemma 1). Denote $H(y)$ the derivative of the value of the shareholders' option to default at y, with respect to this trigger level,

$$H(y) \equiv \frac{\partial}{\partial y}\left[(-\Pi(y) + \delta/\rho)\,\mathcal{P}(x_0 \rhd y)\right]. \tag{47}$$

Denote $h(y)$ the derivative of $H(y)$ with respect to δ,

$$h(y) \equiv \frac{\partial}{\partial \delta}\left[\frac{\partial}{\partial y}\left[(-\Pi(y) + \delta/\rho)\,\mathcal{P}(x_0 \rhd y)\right]\right] = \frac{1}{\rho}\,\frac{\partial}{\partial y}\left[\mathcal{P}(x_0 \rhd y)\right] > 0\,. \tag{48}$$

If $\underline{x}_{\mathcal{S}}$ is the shareholders' optimal abandonment trigger level, and abandonment occurs through default, then $H(\underline{x}_{\mathcal{S}}) = 0$. But because $h(\underline{x}_{\mathcal{S}}) > 0$, the root of $H(y) = 0$ increases with δ. In other words, $\partial \underline{x}_{\mathcal{S}}/\partial\delta > 0$.

Conversely, if shareholders' walkout takes the form of a corporation sale, triggered at y, their residual claim value is

$$S^*(y) = U^*(y) - \frac{\delta}{\rho}[1 - \mathcal{P}(y \rhd \underline{x}^*)]. \tag{49}$$

Denote $G(y)$ the derivative of the value of the shareholders' option to sell the corporation at y, with respect to this trigger level,

$$G(y) \equiv \frac{\partial}{\partial y}\left[\left(U^*(y) - \Pi(y)\right)\mathcal{P}(x_0 \rhd y) + \delta\rho\mathcal{P}(x_0 \rhd \underline{x}^*)\right] \tag{50}$$

$$= \frac{\partial}{\partial y}\left[\left(U^*(y) - \Pi(y)\right)\mathcal{P}(x_0 \rhd y)\right]. \tag{51}$$

Denote $g(y)$ the derivative of $G(y)$ with respect to δ,

$$g(y) \equiv \frac{\partial}{\partial \delta}\left[\frac{\partial}{\partial y}\left[\left(U^*(y) - \Pi(y)\right)\mathcal{P}(x_0 \rhd y)\right]\right] = 0\,. \tag{52}$$

If $\underline{x}_{\mathcal{S}}$ is the shareholders' optimal trigger level and the walkout occurs through a corporate sale, then $G(\underline{x}_{\mathcal{S}}) = 0$. But because $g(\underline{x}_{\mathcal{S}}) = 0$, the root of $G(y) = 0$ is independent of δ, hence $\partial \underline{x}_{\mathcal{S}}/\partial\delta = 0$. □

PROOF OF LEMMA 3. Assume that shareholders walk out through a corporation sale. Then, $\underline{x}_{\mathcal{S}} = \underline{x}$ (Corollary 1). When $\underline{x} < \underline{x}^*$, the reservation value of the firm at the time of the walkout $U^*(\underline{x}_{\mathcal{S}}) = U^*(\underline{x}) = 0$. But then $S^*(\underline{x}_{\mathcal{S}}) = 0$, contradicting the assumption that shareholders walk out through a corporation sale.

Express the respective first-order optimality conditions which determine $\underline{x}^*$ and $\underline{x}_{\mathcal{S}}$:

$$\frac{d\Pi^*(y)}{\partial y}\mathcal{P}(x_t \rhd y) + \Pi^*(y)\frac{\partial \mathcal{P}(x_t \rhd y)}{\partial y} = 0\,, \tag{53}$$

and

$$-\frac{d\Pi(y)}{\partial y}\mathcal{P}(x_t \rhd y) + \left[-\Pi(y) + \frac{\delta}{\rho}\right]\frac{\partial \mathcal{P}(x_t \rhd y)}{\partial y} = 0\,. \tag{54}$$

If $\underline{x}^*$ equals $\underline{x}_{\mathcal{S}}$, both conditions have the same solution. Replacing the first one in the second yields

$$\Pi^*(y)\left[-\frac{d\Pi(y)}{\partial y}\right]\mathcal{P}(x_t \rhd y) - \frac{d\Pi^*(y)}{\partial y}\left[-\Pi(y) + \frac{\delta}{\rho}\right]\mathcal{P}(x_t \rhd y) = 0\,. \tag{55}$$

After simplification, we see that for $\underline{x}^*$ to equal $\underline{x}_{\mathcal{S}}$, the coupon obligation, $\hat{\delta}^*$, must be such that

$$\hat{\delta}^* = \rho\Pi(\underline{x}^*) - \rho\Pi^*(\underline{x}^*)\,\frac{d\Pi(\underline{x}^*)}{d\underline{x}^*}\left[\frac{d\Pi^*(\underline{x}^*)}{d\underline{x}^*}\right]^{-1}\,. \tag{56}$$

For coupons $\delta \in (0; \hat{\delta}^*]$, default occurs at $\underline{x}_{\mathcal{S}}$ which is lower than $\underline{x}^*$, hence the usage of the assets will be terminated. □

PROOF OF LEMMA 4. Corporation sales occur for δ such that $S_{\mathrm{corp}}(x \mid \underline{x}_{\mathrm{corp}}, \delta) > S_{\mathrm{def}}(x \mid \underline{x}_{\mathrm{def}}, \delta)$. Given that $\underline{x} < \underline{x}_{\mathrm{def}}$ (lemma 2), expanding the inequality $S_{\mathrm{corp}}(x \mid \underline{x}_{\mathrm{corp}}, \delta) > S_{\mathrm{def}}(x \mid \underline{x}_{\mathrm{def}}, \delta)$ for $x = \underline{x}_{\mathrm{def}}$ gives that corporation sales occur for $\delta \in (0; \hat{\delta}]$, where $\hat{\delta}$ solves the expression given in the lemma. □

PROOF OF LEMMA 5. If $S^*(\underline{x}_{\mathcal{S}}) = 0$, then

$$\frac{\partial D(x_0 \mid \delta)}{\partial \delta} = \frac{1}{\rho}[1 - \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})] - \frac{\delta}{\rho}\frac{\partial \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})}{\partial \underline{x}_{\mathcal{S}}}\frac{\partial \underline{x}_{\mathcal{S}}}{\partial \delta}\,. \tag{57}$$

The coupon, $\bar{\delta}$, solves $\partial D(x_0 \mid \delta)/\partial\delta = 0$ and therefore

$$\bar{\delta} = [1 - \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})]\left[\frac{\partial\mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})}{\partial\underline{x}_{\mathcal{S}}}\frac{\partial\underline{x}_{\mathcal{S}}}{\partial\delta}\right]^{-1} . \quad \square \tag{58}$$

PROOF OF LEMMA 6. If $S^*(\underline{x}_{\mathcal{S}}) = U^*(\underline{x}_{\mathcal{S}}) - \delta/\rho[1 - \mathcal{P}(\underline{x}_{\mathcal{S}} \rhd \underline{x}^*)]$, then

$$\frac{\partial D(x_0 \mid \delta)}{\partial\delta} = \frac{1}{\rho}[1 - \mathcal{P}(x_0 \rhd \underline{x}_{\mathcal{S}})] > 0 . \quad \square \tag{59}$$

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