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American-style Indexed Executive Stock Options ⋆

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Abstract. This paper develops a new pricing model for American-style indexed executive stock options. We rely on a basic model framework and an indexation scheme first proposed by Johnson and Tian (2000a) in their analysis of European-style indexed options. Our derivation of the valuation formula represents an instructive example of the usefulness of the change-of-numeraire technique. In the paper's numerical section we implement the valuation formula and demonstrate that not only may the early exercise premium be significant but also that the *delta* of the American-style option is typically much larger than the delta of the otherwise identical (value-matched) European-style option. *Vega* is higher for indexed options than for conventional options but largely independent of whether the options are European- or American-style. This has important implications for the design of executive compensation contracts. We finally extend the analysis to cover the case where the option contracts are subject to *delayed vesting*. We show that for realistic parameter values, delayed vesting leads only to a moderate reduction in the value of the American-style indexed executive stock option.

Key words: Executive stock options, premature exercise, incentive effects, delayed vesting.

JEL classifications: G13, G34, J33.

> *"I guess that (contract) makes us partners – as we now have the same goal: To get me where I need to go, right?"*, Patrick Swayze (Bodhi) to Keanu Reeves (Johnny Utah) in *Point Break (1991), 20th Century Fox.*

1. Introduction

In ancient China the Emperor's personal physician is said to have had a very special compensation scheme. The physician's periodic salary was fixed and quite

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generous whenever the Emperor was feeling perfectly healthy. However, payments would be withheld whenever the Emperor was unwell and in case of the Emperor's death, the physician would be executed.[1]

Although this is a very extreme example, it is very useful when explaining the central idea of incentive contracting which is to align the interests of employer and employee, or *principal* and *agent* as the literature prefers it.

In the modern corporate environment, stock options have been a major element in the compensation packages that corporate boards have offered corporate executives on behalf of the stock holders. The major interest of stock holders (principals) is to see the value of their stock holdings increase, and granting stock options to executives is one way of making sure that managers (agents) focus on this issue (instead of e.g., building corporate empires and wasting their time flying around in prestigious and expensive corporate jets etc., Jensen and Meckling (1976)).

Executive stock options (ESOs) have thus gained enormous popularity in corporations throughout the Western world in recent years. As documented in Murphy (1999) and Rappaport (1999), stock options now account for more than half of the total CEO compensation in the largest US companies and it remains the fastest growing component of executive pay (see also Hall and Liebman (1998)). Most ESOs are conventional options that are issued at-the-money and with a strike price that remains fixed over the entire option period, which is usually ten years. This construction has created huge rewards for executives during the bull market of the 1990s and it has raised a further wave of critique from academics, stock holders, politicians, and the general public. Executives have been accused of "floating their yachts on a rising tide" (Reingold, 2000) and of being shamelessly rewarded even for below-standard performance as a result of the fixed exercise price construction of their options. The Chairman of the Federal Reserve Board, Alan Greenspan, recently added to this critique by noting (see Greenspan (2002)):

> Regrettably, some current issuance practices have not created the alignment of incentives that encourages desired corporate behavior. One problem is that stock options, as currently structured, often provide only a loose link between compensation and successful management. A company's share price, and hence the value of related options, is heavily influenced by economy-wide forces – that is, by changes in interest rates, inflation, and a myriad of other forces wholly unrelated to the success or failure of a particular corporate strategy.
>
> There have been more than a few dismaying examples of CEOs who nearly drove their companies to the wall and presided over a significant fall in the price of the companies' stock relative to that of their competitors and the stock market overall. They, nonetheless, reaped large rewards because the strong performance of the stock market as a whole dragged the prices of the forlorn companies' stocks along with it.

[1] Møller and Nielsen (1988) is the source of this anecdote.

In a recent study of executive compensation in America (Bebchuk, Fried and Walker 2001) the critique from academics is taken to a level where the extension and many stylized facts of executive compensation are explained from the new point of view of *rent extraction theory* as opposed to the usual *optimal contracting view*.

To emphasize the basic design problem of conventional options according to which any increase in a company's stock price constitutes positive performance, Rappaport (1999) points out that for the ten-year period ending in 1997, the total return to shareholders – dividends and capital gains – was positive for each of the 100 largest US companies! It follows that the conventional option is not an instrument which rewards only superior performers and consequently it may not provide the best incentives. Exactly which type of remuneration package provides optimal incentives is still an open theoretical question, but many signs point in the direction of option plans with indexed strike prices (see the detailed discussions in Rappaport (1999), Johnson and Tian (2000a), and references cited therein). By specifying a certain *index* or *benchmark* as the option plan's strike price it is indeed possible to obtain a better measure of positive performance and hence to reward truly superior performers while appropriately penalizing poor ones. Another argument in favor of indexed ESOs may be that they improve social welfare by letting diversified shareholders instead of undiversified executives bear systematic or common risk (Holmstrom (1982), cf. also the later discussion of the diversification issue). The particular index could be a broad market index such as the Dow Jones Industrial Average, NASDAQ, S&P 500, or an index of a peer group of competitors, or it could be very narrowly defined from the stock price of a close competitor. Clearly, with such an indexed option plan, underperforming managers would not be rewarded automatically by a rising market. On the other hand, it would be possible to reward superior performers also in declining market situations.

In a recent article, Johnson and Tian (2000a) pursue the idea of constructing indexed option plans and they suggest a logical and elegant indexation scheme. They derive a valuation formula for options written on a company's stock and with their index as the time and state-dependent exercise price. However, they assume that the options are European-style which conflicts the way in which ESOs are typically designed in practice. In this paper we extend the Johnson and Tian (2000a) model to allow for early exercise of the ESO. Moreover, Johnson and Tian (2000a) do not consider another typical feature of actual ESOs referred to as the *delayed vesting* property. Although ESOs are almost always American-style options, the executive is typically restricted from exercising during an introductory period, usually three years from issuance. Any such restrictions will obviously affect the American option value negatively, but there is still plenty of 'room' for making extensive valuation errors by completely ignoring the early exercise feature. We incorporate delayed vesting explicitly in this paper and in the paper's numerical section we document that our extensions are important both for valuation and for correctly assessing the incentive effects of indexed option plans.

Before presenting the details of our model – and in particular in relation to the valuation and incentive effects just mentioned – a general discussion of our approach and available alternatives is appropriate. As in Johnson and Tian (2000a) the analysis in the present paper is based on the Black-Scholes-Merton methodology which assumes ideal market conditions and continuous trading and hedging opportunities (see Black and Scholes (1973) and Merton (1973c)). Many other papers on executive compensation valuation rest on similar assumptions but the approach has also been criticized. For example, in an influential paper, Lambert, Larcker, and Verrecchia (1991) argue that since risk-averse executives are typically not only restricted from selling and hedging their stock options but also notoriously undiversified in their wealth, the fundamental assumptions underlying the Black-Scholes-Merton framework are violated in this context.[2] Models based on these assumptions are therefore likely to provide incorrect answers to questions regarding the value of compensation packages with option components and the incentive effects provided by them. The remedy, according to Lambert, Larcker, and Verrecchia (1991), is to derive ESO values and incentives by a utility-based *certainty equivalent of wealth* approach thereby taking proper account of risk aversion, inalienability, hedging restrictions, and the composition of other wealth.

Even if the premise of Lambert, Larcker, and Verrecchia (1991) is valid (which is an empirical question, cf. later), their insights do not entirely disqualify hedging-based ESO models like the one presented in this paper. This is because the other side of the ESO contracts – the issuing firm – does not face constraints similar to those that are supposed to face the manager, and the hedging-based models may therefore still be useful when estimating the company's cost of granting options to its managers. This is obviously true if ESOs were always certain to be held to expiration, but when the options are American-style it becomes important to consider early exercise and the determinants thereof. One of the main motivations for this paper has been to model and analyze the early exercise feature since this has largely been ignored in the previous literature and since almost all real-life contracts have this feature.[3]

We analyze the early exercise feature from an optimal stopping point of view; i.e., we derive and base our analysis on exercise strategies that are appropriate for a diversified and completely rational manager who is unrestricted in his hedging opportunities. This approach can be criticized using the arguments of Lambert, Larcker, and Verrecchia (1991), but its attraction is that it is relatively simple and it will lead to particularly nice analytical results which may be useful if only as a first approximation.

Whether executives exercise their options for rational reasons is, of course, an empirical question. Unfortunately, since indexed options have not yet gained popularity in practice, there is no data from which we can study managers exercise behavior in relation to this type of option and gauge whether the assumption of

2 See also Huddart (1994).

3 Lambert, Larcker, and Verrecchia (1991) do not consider early exercise.

rational exercise is justified. However, there is some empirical evidence in relation to conventional ESOs which should be mentioned at this point.

Huddart and Lang (1996), examining data from eight US firms from 1980 to 1992, find that early exercise of *employee* stock options is pervasive and conclude that it is essential to include the early exercise feature when constructing theoretical models for ESO valuation. They carry out a vast amount of regression analysis to determine which factors affect exercise behavior, and they interpret some of their results as consistent with the risk reduction and liquidity motives that exist according to Lambert, Larcker, and Verrecchia (1991). However, the results are not entirely conclusive on this matter and one should also bear in mind that the data relate to almost 60,000 employees, not just top executives, at all levels of the eight firms. It seems fair to assume that regular employees are more likely to exercise options early out of risk concerns than top CEOs who probably receive more than 95% of all ESOs.[4]

In a related study, Hemmer, Matsunaga and Shevlin (1996) provide evidence of a positive relation between the risk of options and the remaining life of the options at exercise. The strength of this relation is reduced by the extent to which the issuing firms hedge the ESO returns. The authors interpret their results as supportive of the hypothesis that managers exercise options early in order to diversify their risk. Again, this seems in conflict with the rational exercise approach applied in this paper, but we seek recourse in the fact that the sample of Hemmer, Matsunaga and Shevlin (1996) is very small and that even if a risk diversification motive exists, this motive is likely to be weaker for the indexed options we study in this paper.[5] By construction these options seek to filter out all but firm specific risk so their overall risk will typically be (much) smaller than the risk of conventional fixed strike price options.

We finally mention an important paper by Carpenter (1998) which contains a comparison of hedging-based and utility-based valuation models for executive stock options. The models examined in Carpenter (1998) allow for early exercise and the results of this study are therefore particularly relevant in motivating the present paper. During the course of her analysis, Carpenter presents a simple extension of the standard American option pricing model which adds random non-market driven exercise and forfeiture to the standard American option pricing model. The models are fitted to data on option exercises from 40 US firms. She concludes that it is a misconception that preference-based models are necessary for valuing executive stock options. She finds that her extended American option pricing model "goes surprisingly far toward bringing the implied exercise patterns in line with the data" and that this model "predicts actual exercise times

4 See e.g., *Fortune* (July 24, 1995, p. 28).

5 The data in Hemmer, Matsunaga and Shevlin (1996) is from the single year 1990. The authors analyze 110 particular exercises by 74 executives in 65 separate firms. Moreover, the study is in one respect partly self-contradictory by applying a version of the Black-Scholes European-style option pricing model to establish option values.

and payoffs just as well as an elaborate utility-maximizing model." Concerning the exercise behavior she concludes that "executives hold options long enough and deep enough into the money before exercising to capture a significant amount of their potential value." The driver of Carpenter's results may be that executives have far greater possibility to hedge their option positions than is often assumed and than is allowed by typical preference-based models, cf. the discussion above. The earlier cited paper by Bebchuk, Fried and Walker (2001) shares this view and cites several sources documenting that hedging of ESOs by executives is widely observed and in actual fact not much restricted. If this is the case, it is a strong argument in favor of the rational exercise approach taken in this paper and seen together with the results of Carpenter (1998), it can be regarded as a strong indication that standard hedging-based models or suitable improvements thereof should not be disregarded as quickly as suggested by some. However, it goes without saying that the views and caveats discussed above should be kept in mind when interpreting the results we present in the following.

The rest of the paper is organized as follows. In Section 2 we present the Johnson and Tian (2000a) framework and we introduce and characterize the American-style contract. Section 3 is the core part of the paper and it contains valuation results regarding the pure (no vesting period) contracts. Section 4 explains the implementation of our valuation formulas and contains extensive numerical results. The incentive effects provided by the indexed option plans are briefly analyzed and compared to those of conventional options in Section 5. In Section 6 we extend the analysis to cover delayed vesting. Section 7 concludes.

2. The Johnson & Tian (2000) Model

Consider a simple financial market in which all activity occurs on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathcal{P})$ supporting Brownian motion on the finite time interval $[0, T]$. In particular, $\mathcal{P}$ is the physical probability measure under which the company's stock price, S, and the associated index, I, evolve through time according to a joint geometric Brownian motion, i.e.,

$$\frac{dS(t)}{S(t)} = (\mu_S - \delta_S)\, dt + \sigma_S\, d\mathcal{W}_S(t), \tag{1}$$

$$\frac{dI(t)}{I(t)} = (\mu_I - \delta_I)\, dt + \sigma_I\, d\mathcal{W}_I(t), \tag{2}$$

where $\mathcal{W}_S(t)$ and $\mathcal{W}_I(t)$ are standard correlated Brownian motions with

$$d\mathcal{W}_S(t) \cdot d\mathcal{W}_I(t) = \rho\, dt. \tag{3}$$

The constant coefficients μ_S, μ_I, σ_S, and σ_I are the expected returns and volatility coefficients of the stock and the index respectively. Note that we allow for constant proportional dividend rates, δ_I and δ_S, in the index as well as the stock price process.[6] The index is assumed traded or perfectly correlated with a traded asset.

Investors can also trade in a riskless asset – a money market account – which has dynamics

$$dB(t) = rB(t)\,dt, \qquad B(0) = 1. \tag{4}$$

For simplicity the riskless interest rate, r, is assumed constant and we can thus write $B(t) = \exp(rt)$. We shall see that this assumption implies no loss of generality.

2.1. BENCHMARK DISCUSSION

Let us now go through the steps which show how a sensible indexed option contract can be designed. Since the idea is to reward positive performance relative to the index, we need a relation between the expected returns from the two price processes. Johnson and Tian (2000a) take the situation with no abnormal performance as their point of departure and *define* this as characterized by

$$\mu_S = r + \beta(\mu_I - r), \tag{5}$$

where

$$\beta = \frac{\mathrm{Cov}\left(\frac{dS_t}{S_t}, \frac{dI_t}{I_t}\right)}{\mathrm{Var}\left(\frac{dI_t}{I_t}\right)} = \frac{\rho\sigma_S\sigma_I\,dt}{\sigma_I^2\,dt} = \rho\frac{\sigma_S}{\sigma_I}. \tag{6}$$

On one hand this is not entirely *ad hoc* since (5)–(6) are clearly reminiscent of the Capital Asset Pricing Model (CAPM). On the other hand it should be noted that (5)–(6) are not dictated e.g., by the requirement of absence of arbitrage.

Recalling standard CAPM insight we note that the expected return on the individual stock is explained solely by how much 'index risk' (β) is borne by holding the stock. In rewarding relative performance, this index risk should be filtered out and in establishing a proper benchmark for the future stock price it thus makes sense to consider the expected future stock price conditional on the contemporaneous index value and on normal expected performance as given by (5)–(6).[7] Formally, we consider

$$E\big(S(t)|I(t)\big). \tag{7}$$

[6] As will become clear later, the index dividend rate is completely irrelevant for the pricing of the options we define in this paper. This is also true for the options considered by Johnson and Tian (2000a), although this observation was not made in their paper. In contrast, the stock dividend matters and we need to keep it in the model although omitting it would lead to a significant simplification of later derivations. However, an omission of the stock dividend could be motivated by referring to evidence that dividend rates on individual stocks have been declining recently and in particular in companies in which CEOs hold significant amounts of (ESOs on) the company's stock (Fama and French (1999) and Weisbenner (2000)).

[7] Further justification for using (5)-(6) as a filter rule in incentive compensation can be found in e.g., Holmstrom and Milgrom (1987).

It can be shown that

$$E\big(S(t)|I(t)\big) = S_0 \left(\frac{I(t)}{I_0}\right)^{\beta} e^{\eta t}, \tag{8}$$

where

$$\eta \equiv (r - \delta_S) - \beta(r - \delta_I) + \frac{1}{2}\rho\sigma_S\sigma_I(1 - \beta). \tag{9}$$

Equation (8) represents a fair, conditional benchmark for the future stock price given the model assumptions and it could be directly applied as the exercise price of an option based compensation contract.[8] However, we proceed to add a tiny bit of flexibility and work with a benchmark, $H(t)$, inspired by (8) and defined as follows

$$H(t) = \lambda S_0 \left(\frac{I(t)}{I_0}\right)^{\beta} e^{(g+\eta)t}. \tag{10}$$

Compared to (8) we have introduced two additional parameters, λ and g. λ is a *level* parameter which will affect the moneyness of the option when $H(t)$ defines the exercise price. If $\lambda = 1$, then $H(0) = S_0$ and the option will initially be *at-the-money*. This is the typical situation in practice, but *premium* ($\lambda > 1$) and *discount* ($\lambda < 1$) priced options are also observed, and with the parameter λ introduced we can also cover those cases. The parameter g allows us to fine-tune the exponential appreciation rate of the benchmark. Using a benchmark with $g > 0$ will mean that a better-than-expected performance is required to beat the benchmark and vice versa. Johnson and Tian (2000a) refer to options with these features as *high* and *low water mark indexed options* respectively (see also Goetzmann, Ingersoll and Ross (1997)).

With the benchmark suitably defined, we can specify the details of the contract which will be subjected to scrutiny. We will assume that at time zero the executive is given a number of call options on the company's stock expiring at time T and with a time dependent exercise price of $H(t)$. The options are American-style so if exercised at time τ the payoff is

$$[S(\tau) - H(\tau)]^{+}. \tag{11}$$

In the next section we establish the time t valuation formula as well as the optimal exercise strategy associated with this claim.[9]

[8] Shortly we will introduce an alternative probability measure under which expectations can be formed. It is thus worthwhile to keep in mind that the conditional expectation considered here is formed under the physical measure, $\mathcal{P}$.

[9] We restrict our attention to the call option since the put option does not seem relevant in the present context of executive compensation. However, the put can be priced following the same ideas as those presented below.

3. Valuation

The benchmark, $H(t)$, is itself a stochastic process and is easily seen to be of the geometric Brownian motion type, cf. also Equation (14). Hence the payoff function (11) is of the same type as that considered in Margrabe (1978) in which "The Value of the (European-style) Option to Exchange One Asset for Another" was first established. The European valuation formula provided in Johnson and Tian (2000a) is thus an adaptation of the Margrabe formula.

In this section we derive the valuation formula for the American-style option. Along the way we verify the result reported by Johnson and Tian (2000a) for the European option.

Observe first that the option payoff defined in (11) clearly depends on two variables, S and H. That is, we have a two-state variable problem on our hands. Considering the fact that in modern financial theory contingent claims are typically priced by calculating suitably discounted expectations of terminal values, this is impractical. The problems of having multiple state variables are particularly relevant in relation to the valuation of American-style contingent claims (see however Broadie and Detemple (1996)). We shall therefore initially perform a change of numeraire which reduces the dimensionality of the problem from *two* to *one*. The numeraire change goes hand in hand with a change of probability measure from $\mathcal{P}$ to Q'. As is well-known (see e.g., Harrison and Kreps (1979) and Duffie (1996)) arbitrage opportunities are absent in this type of model economy if and only if an equivalent probability measure, Q', exists such that all prices denominated in units of the associated numeraire are Q'-martingales.

As the new numeraire we choose the cumulating stock investment, i.e. the value process of an account containing the S-stock but with all dividends immediately reinvested in the same stock.[10] Let this account initiate at time zero with one unit of the stock and denote the associated value process $\bar{S}(t)$.[11] The details of the mar-

[10] Other choices of numeraire asset are possible. Most commonly researchers have used the *money market account* which is associated with the celebrated *risk neutral measure*. In some applications involving stochastic interest rates, a discount bond may be a more useful numeraire. The probability measure associated with this choice of numeraire has been labeled the *forward measure*. See Collin-Dufresne and Goldstein (2000) for a recent example. In principle either of these numeraires could be used in the present context. However, since neither of them could remove one of the two sources of risk entering the option's payoff function, they would not be helpful in reducing the dimensionality of the problem at hand. Hence, the only useful alternative to our choice of the stock price as the numeraire would be an asset which is perfectly correlated with the benchmark process. This leaves the index portfolio (with all dividends reinvested) as the only other possibility, and indeed our analysis could equally well and just as conveniently have been carried out with this choice and with identical results, of course. For general treatments of the change of numeraire technique the reader is referred to Geman, El-Karoui and Rochet (1995) and Schroder (1999). A recent paper by Benninga, Björk and Wiener (2001) contains a plethora of interesting examples.

[11] There is a simple relationship between $S(t)$ and $\bar{S}(t)$. By construction $S_0 = \bar{S}_0$ and it is straightforward to show that $S(t) = \bar{S}(t) \cdot e^{-\delta_S t}$.

tingale measure are presented in the appendix where we also find the transformed stock and index processes to take the form

$$\frac{dS(t)}{S(t)} = (r - \delta_S + \sigma_S^2)\, dt + \sigma_S \, d\mathcal{W}_S^{Q'}(t), \tag{12}$$

and

$$\frac{dI(t)}{I(t)} = (r - \delta_I + \sigma_I \sigma_S \rho)\, dt + \sigma_I \, d\mathcal{W}_I^{Q'}(t), \tag{13}$$

where $W_S^{Q'}(t)$ and $W_I^{Q'}(t)$ are standard Brownian motions under Q' with unaltered coefficient of correlation, ρ.

Using Itô's lemma we can also establish the process describing the evolution in the benchmark under Q'. We find

$$\frac{dH(t)}{H(t)} = (g + r - \delta_S + \beta^2 \sigma_I^2)\, dt + \beta \sigma_I \, d\mathcal{W}_I^{Q'}(t). \tag{14}$$

Now, to derive the American option price we initially rely on results from Karatzas' (1988, 1989) general treatment of the pricing of American-style contingent claims. Appropriately adapted to the current framework the above-mentioned results allow us to express the time t value of the American-style indexed executive stock option, $C(t)$, as

$$\frac{C(t)}{\bar{S}(t)} = \operatorname{ess}\sup_{\tau \in \mathcal{T}_{t,T}} E_t^{Q'} \left\{ \frac{[S(\tau) - H(\tau)]^+}{\bar{S}(\tau)} \right\}, \tag{15}$$

where $\mathcal{T}_{t,T}$ denotes the class of $(\mathcal{F}_t)$-stopping times taking values in $[t, T]$. This pricing relation is very intuitive in stating that the current value denominated in units of the $\bar{S}$-numeraire is equal to the expectation of the denominated future exercise value maximized over all possible exercise strategies. This is the familiar martingale pricing result adjusted for the American exercise feature. Unfortunately, expression (15) is not directly operational since the optimal exercise strategy is not known, i.e., it is not immediately obvious how to rewrite (15) into something more useful. One type of manipulation is possible, however. If we define $x(t) \equiv \frac{H(t)}{\bar{S}(t)}$, then expression (15) can be rewritten as

$$\frac{C(t)}{\bar{S}(t)} = \operatorname{ess}\sup_{\tau \in \mathcal{T}_{t,T}} E_t^{Q'} \left[\frac{S(t)}{\bar{S}(t)} e^{-\delta_S(\tau - t)} - x(\tau) \right]^+. \tag{16}$$

This is a significant improvement since $\frac{S(t)}{\bar{S}(t)} e^{-\delta_S(\tau-t)}$ is deterministic in τ and the point of the change of numeraire now clearly emerges: We have obtained a one-state variable problem with $x(t)$ as the new state variable. Moreover, the dynamics of $x(\cdot)$ is easily established as being of the form

$$\frac{dx(t)}{x(t)} = (g - \delta_S)\, dt + \sigma_x \, d\mathcal{W}_x^{Q'}(t), \tag{17}$$

where we have taken the opportunity to collect the martingale sum $\beta\sigma_I\, d\mathcal{W}_I^{Q'}(t) - \sigma_S\, d\mathcal{W}_S^{Q'}(t)$ into a single term martingale (see the Martingale Representation Theorem in e.g., Baxter and Rennie (1996) or Karatzas and Shreve (1988)) with diffusion coefficient $\sigma_x \equiv \sigma_S\sqrt{1-\rho^2}$.

Note also that the transformed problem (16) is now reminiscent of a put option problem with $x(\cdot)$ as the underlying asset and with a time varying exercise price.

The new state variable follows a Markov process – in fact it is yet another geometric Brownian motion – and given this property it is known from e.g., Øksendal (1992) (Theorem 11.3) that the optimal stopping rule relating to problems of the sort in (16) can be characterized as

$$\tau_t^* = \inf\left\{s \in [t, T] | x(s) = x^*(s)\right\}. \tag{18}$$

The economic interpretation of (18) is very important. The result tells us that the indexed stock option is optimally exercised the first time the process $x(\cdot)$ reaches a critical value, x^*, which may be time-dependent. Since $x(\cdot)$ is defined as the benchmark divided by the contemporaneous cumulating stock account value, it means that knowledge of this fraction (and of the critical boundary) is enough to decide whether to exercise the executive option at a premature point in time. In fine accordance with intuition, note that optimal premature exercise should take place in states where the benchmark is low and/or the stock price is high.

Returning to the reformulated problem (16)–(18), we note that the function $x^*(\cdot)$ is continuous for $t \in [0, T[$ and that it defines a separation of the state–space into a continuation region and a stopping (exercise) region, which we denote by $\mathcal{C}$ and $\mathcal{S}$ respectively. Thus $\{x(t) \le x^*(t)\}$ defines $\mathcal{S}$ and $\{x(t) > x^*(t)\}$ defines $\mathcal{C}$.[12] This puts us in a position to state and prove our main valuation theorem.

THEOREM 1: Decomposition of the American option premium.

Let $C(t)$ and $\bar{C}(t)$ denote the time t value of the American-style indexed executive stock option expressed in the original numeraire and the $\bar{S}$-numeraire respectively. Then we have

$$\bar{C}(x(t), t) \equiv \frac{C(t)}{\bar{S}(t)} = \bar{c}\big(x(t), t\big) + \bar{e}\big(x(t), t\big), \tag{19}$$

where

$$\bar{c}\big(x(t), t\big) \equiv \frac{c(t)}{\bar{S}(t)} = E_t^{Q'}\left[\frac{S(T)}{\bar{S}(T)} - x(T)\right]^+, \tag{20}$$

[12] More specifically, for the case discussed we have

$$\mathcal{C} = \left\{(t, x) | t \in [0, T] \wedge x(t) > x^*(t)\right\},$$

and vice versa.

and

$$\bar{e}(x(t),t) \equiv \frac{e(t)}{\bar{S}(t)} = E_t^{Q'}\left\{\int_t^T \mathbf{1}_{\mathcal{S}}\big(x_u,u\big)\left(\delta_S \frac{S(t)}{\bar{S}(t)}e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right)du\right\}. \tag{21}$$

□

REMARK: $c(t)$ is clearly the European option price and $e(t)$ is thus the early exercise premium. We prove the theorem below and proceed to further manipulate the expectations in (20) and (21) in Theorems 2 and 3 below.[13]

Proof of Theorem 1.

Consider first the dynamics of $\bar{C}(x(u),u)$ on the continuation region, $\mathcal{C}$, using Itô's lemma:

$$\begin{aligned} d\bar{C}(x(u),u) &= \frac{\partial \bar{C}}{\partial x}dx + \frac{1}{2}\frac{\partial^2 \bar{C}}{\partial x^2}(dx)^2 + \frac{\partial \bar{C}}{\partial u}\,du \\ &= \frac{\partial \bar{C}}{\partial x}\Big(x(u)(g-\delta_S)\,du + \sigma_x x(u)\,d\mathcal{W}_x^{Q'}(u)\Big) \\ &\quad + \frac{1}{2}\frac{\partial^2 \bar{C}}{\partial x^2}\sigma_x^2 x(u)^2\,du + \frac{\partial \bar{C}}{\partial u}\,du \\ &= \left[x(u)(g-\delta_S)\frac{\partial \bar{C}}{\partial x} + \frac{1}{2}\sigma_x^2 x(u)^2 \frac{\partial^2 \bar{C}}{\partial x^2} + \frac{\partial \bar{C}}{\partial u}\right]du \\ &\quad + \sigma_x x(u)\frac{\partial \bar{C}}{\partial x}\,d\mathcal{W}_x^{Q'}(u) \\ &= \sigma_x x(u)\frac{\partial \bar{C}}{\partial x}\,d\mathcal{W}_x^{Q'}(u), \end{aligned} \tag{22}$$

where the last equality follows since $\bar{C}$ is a Q'-martingale. By the way, note that the partial differential equation in the square brackets can be used as the point of departure for a finite difference based numerical solution of the problem. Now, on the stopping region, $\mathcal{S}$, we clearly have

$$\bar{C}(x(u),u) = \frac{S(t)}{\bar{S}(t)}e^{-\delta_S(u-t)} - x(u), \tag{23}$$

so that the dynamics on $\mathcal{S}$ can be written

[13] Note that $g > 0$ is a sufficient but not a necessary condition for the integrand in (21) to be strictly positive. See also Table II and the related discussion.

$$\begin{aligned}
d\bar{C}(x(u),u) &= -\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)}\,du - dx(u) \\
&= -\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)}\,du - x(u)(g-\delta_S)\,du \\
&\quad -\sigma_x x(u)\,d\mathcal{W}_x^{Q'}(u) \\
&= -\left(\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right) du \\
&\quad -\sigma_x x(u)\,d\mathcal{W}_x^{Q'}(u).
\end{aligned} \tag{24}$$

Combining (22) and (24), the global dynamics can be described as

$$\begin{aligned}
d\bar{C}(x(u),u) &= -\mathbf{1}_{\mathcal{S}}\big(x_u,u\big)\left(\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right) du \\
&\quad + dM^{Q'}(u),
\end{aligned} \tag{25}$$

where $M^{Q'}$ denotes a Q'-martingale and where $\mathbf{1}_{\mathcal{S}}(x_u,u)$ denotes the indicator function on the set $\mathcal{S}$.

Finally, integrating (25) from t to T and taking expectations yields

$$\begin{aligned}
\int_t^T d\bar{C}(x(u),u) &= \bar{C}(x(T),T) - \bar{C}(x(t),t) \\
&= -\int_t^T \mathbf{1}_{\mathcal{S}}(x_u,u)\left(\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right) du \\
&\quad + \int_t^T dM^{Q'}(u) \\
&\Downarrow \\
\bar{C}(x(t),t) &= E_t^{Q'}\left\{\bar{C}(x(T),T)\right\} + E_t^{Q'}\left\{\int_t^T \mathbf{1}_{\mathcal{S}}(x_u,u)\right. \\
&\quad \left.\times\left(\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right) du\right\} \\
&= E_t^{Q'}\left\{\left[\frac{S(T)}{\bar{S}(T)} - x(T)\right]^+\right\} + E_t^{Q'}\left\{\int_t^T \mathbf{1}_{\mathcal{S}}(x_u,u)\right. \\
&\quad \left.\times\left(\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right) du\right\} \\
&= \bar{c}(t) + E_t^{Q'}\left\{\int_t^T \mathbf{1}_{\mathcal{S}}(x_u,u)\left(\delta_S \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)}\right.\right.
\end{aligned}$$

$$+x(u)(g - \delta_S)\Big) du\Big\}. \tag{26}$$

Q.E.D.

We now proceed to further manipulate the expectations in (20) and (21). For completeness we first confirm the result of Johnson and Tian (2000a) regarding the value of the European-style indexed executive stock option.

THEOREM 2: The European option premium.

The European option premium, $c(t)$, is given as

$$c(t) = e^{-\delta_S(T-t)}\left[S(t)\mathcal{N}(d_+) - H(t)e^{g(T-t)}\mathcal{N}(d_-)\right], \tag{27}$$

where

$$d_\pm = \frac{\ln\frac{S(t)}{H(t)} + (-g \pm \frac{1}{2}\sigma_x^2)(T-t)}{\sigma_x\sqrt{T-t}}, \tag{28}$$

and where $\mathcal{N}(\cdot)$ denotes the cumulative standard normal distribution function.

□

Proof of Theorem 2

Theorem 1 instructs us to calculate

$$\frac{c(t)}{\bar{S}(t)} = E_t^{Q'}\left[\frac{S(T) - H(T)}{\bar{S}(T)}\right]^+ = E_t^{Q'}\left[\frac{S(T)}{\bar{S}(T)} - x(T)\right]^+. \tag{29}$$

Note that we have

$$\frac{S(T)}{\bar{S}(T)} = \frac{S(t)}{\bar{S}(t)}e^{-\delta_S(T-t)}, \tag{30}$$

which is non-stochastic as seen from time t. Furthermore, as regards the x-process, equation (17) implies

$$\ln x(u) \sim N\left(\ln x(t) + (g - \delta_S - \frac{1}{2}\sigma_x^2)(u-t), \sigma_x^2(u-t)\right), \quad u > t. \tag{31}$$

Using now standard results regarding the normal distribution we find

$$\frac{c(t)}{\bar{S}(t)} = \frac{S(t)}{\bar{S}(t)}e^{-\delta_S(T-t)}\mathcal{N}(d_+) - x(t)e^{(g-\delta_S)(T-t)}\mathcal{N}(d_-), \tag{32}$$

which is the $\bar{S}(t)$-denominated European option premium. A simple rearrangement yields the desired result, which also corresponds with that of Johnson and Tian (2000a).

Q.E.D.

We move on to derive a more explicit expression for the value of the early exercise premium.

THEOREM 3: The early exercise premium.

The early exercise premium, $e(t)$, is given as

$$e(t) = \int_t^T \Big(\mathcal{N}\big(d_+(t,u)\big)\delta_S S(t) e^{-\delta_S(u-t)} + (g-\delta_S)H(t)e^{(g-\delta_S)(u-t)}\mathcal{N}\big(d_-(t,u)\big)\Big)\, du, \tag{33}$$

where

$$d_\pm(t,u) = \frac{\ln\frac{x^*(u)}{x(t)} + (\delta_S - g \pm \frac{1}{2}\sigma_x^2)(u-t)}{\sigma_x\sqrt{u-t}}. \tag{34}$$

□

Proof of Theorem 3

A first rewrite of relation (21) of Theorem 1 yields

$$\begin{aligned} \bar{e}\big(x(t),t\big) &= E_t^{Q'}\left\{\int_t^T \mathbf{1}_{\mathcal{S}}\big(x_u,u\big)\left(\delta_S\frac{S(t)}{\bar{S}(t)}e^{-\delta_S(u-t)} + x(u)(g-\delta_S)\right)du\right\} \\ &= \int_t^T \left(Q'\big(x(u)\le x^*(u)\big)\delta_S\frac{S(t)}{\bar{S}(t)}e^{-\delta_S(u-t)} \right. \\ &\quad \left. +(g-\delta_S)E_t^{Q'}\left\{\mathbf{1}_{\mathcal{S}}\big(x_u,u\big)\cdot x(u)\right\}\right)du. \end{aligned} \tag{35}$$

But

$$\begin{aligned} Q'\big(x(u)\le x^*(u)\big) &= Q'\left(x(t)e^{(g-\delta_S-\frac{1}{2}\sigma_x^2)(u-t)+\sigma_x\sqrt{u-t}\cdot\varepsilon} \le x^*(u)\right) \\ &= Q'\left(\varepsilon \le \frac{\ln\frac{x^*(u)}{x(t)} - (g-\delta_S-\frac{1}{2}\sigma_x^2)(u-t)}{\sigma_x\sqrt{u-t}}\right) \\ &= \mathcal{N}\left(\frac{\ln\frac{x^*(u)}{x(t)} - (g-\delta_S-\frac{1}{2}\sigma_x^2)(u-t)}{\sigma_x\sqrt{u-t}}\right) \\ &= \mathcal{N}\big(d_+(t,u)\big), \end{aligned} \tag{36}$$

where we have used ε to denote a standard normal variate.

Using again a standard result regarding the normal distribution the second term of the integrand in (35) can be rewritten as

$$\begin{aligned} E_t^{Q'}\left\{\mathbf{1}_{\mathcal{S}}\big(x_u, u\big)\cdot x(u)\right\} &= E_t^{Q'}\left\{\mathbf{1}_{x(u)\le x^*(u)}\cdot x(u)\right\} \\ &= x(t)e^{(g-\delta_S)(u-t)} \\ &\quad \times \mathcal{N}\left(\frac{\ln\frac{x^*(u)}{x(t)}-(g-\delta_S+\frac{1}{2}\sigma_x^2)(u-t)}{\sigma_x\sqrt{u-t}}\right) \\ &= x(t)e^{(g-\delta_S)(u-t)}\mathcal{N}\big(d_-(t,u)\big). \end{aligned} \tag{37}$$

Hence, the $\bar{S}(t)$-denominated early exercise premium is given as

$$\begin{aligned} \bar{e}\big(x(t),t\big) &= \int_t^T \bigg(\mathcal{N}\big(d_+(t,u)\big)\delta_S\frac{S(t)}{\bar{S}(t)}e^{-\delta_S(u-t)} \\ &\quad +(g-\delta_S)x(t)e^{(g-\delta_S)(u-t)}\mathcal{N}\big(d_-(t,u)\big)\bigg)\,du. \end{aligned} \tag{38}$$

Multiplying by $\bar{S}(t)$ we obtain the desired result expressed in the original numeraire

$$\begin{aligned} e(t) &= \int_t^T \Big(\mathcal{N}\big(d_+(t,u)\big)\delta_S S(t)e^{-\delta_S(u-t)} \\ &\quad + (g-\delta_S)H(t)e^{(g-\delta_S)(u-t)}\mathcal{N}\big(d_-(t,u)\big)\Big)\,du. \end{aligned} \tag{39}$$

Q.E.D.

This concludes the presentation of the valuation formulas and we proceed to discuss their implementation in the following section.

4. Implementation

The formula for the European-style option presented in Theorem 2 can be directly evaluated for given parameters as any other version of the Black and Scholes (1973) result. To use common parlance it is a *closed formula*. The formula for the early exercise premium of the American-style option provided in Theorem 3 is not of closed form and cannot be as easily evaluated.[14] There are two problems with formula (33). First, it involves an integral which cannot be simplified. This is not an insurmountable obstacle, however, since integrals can be efficiently and accurately

[14] Some would say that the expression for the early exercise premium provided in Theorem 3 is an *analytical formula* which supposedly means that it is not quite as *nice* as closed formulas, but to the best of the author's knowledge, there is no strict definition of these terms.

approximated by numerical routines or so-called *quadratures* (see e.g., Press et al. (1989)). Second and more seriously, the integrand of (33) contains an unknown function, $x^*(u)$, i.e., the early exercise boundary. Therefore, before we can aim at determining any prices, we must determine $x^*(u)$, $u \in [0, T]$. This is done in the following way.

At the maturity date the critical value of the state variable is known. Referring to (20) we can obviously set $x^*(T) = \frac{S(T)}{\bar{S}(T)} = e^{-\delta_S T}$. Using a discretization scheme we then move backwards from T using the boundary condition (23) at each step, i.e.

$$\begin{aligned} \bar{C}\big(x^*(u), u\big) &= \frac{S(t)}{\bar{S}(t)} e^{-\delta_S(u-t)} - x^*(u) \\ &= e^{-\delta_S \cdot u} - x^*(u). \end{aligned} \tag{40}$$

Specifically, the time interval $[0, T]$ is divided into n intervals of equal length, Δt, which define time points $0 = t_0 < t_1 < \ldots < t_i < \ldots < t_n = T$, where $t_i - t_{i-1} = \Delta t = \frac{T}{n}$. We substitute the sum of (32) and (38) on the left hand side of (40) and implement a quadrature to deal with the integral term. Then, at each time point in the sequence $t_{n-1}, t_{n-2}, \ldots, t_0$, a numerical search for $x^*(t_i)$ that solves (40) is conducted. It is emphasized that in order for the quadrature-based numerical search routine to be able to establish $x^*(t_i)$, the points $x^*(t_{i+1}), \ldots, x^*(t_n)$ must already be known. This explains why the backwardation is necessary. When $t_0 = 0$ is reached and $x^*(0)$ is determined, the early exercise premium and the American option price can be determined for any value of the state variable $x(0)$ by one final numerical integration. The accuracy of the procedure will naturally increase in n but at the expense of increased computation time.

The first result of this process is illustrated in Figures 1–4 which show examples of these early exercise boundaries, i.e., the x^*-functions, for some representative parameter values. The curves were constructed using $n = 4000$. We have also plotted the time varying 'strike price', cf. the discussion following equations (16) and (18). Recalling the definition of $x(t)$ we note that the stopping region is the area below the early exercise boundary.

Alternative depictions of the stopping and continuation regions are possible. At any fixed point in time, t, these regions can be illustrated in $\big(\bar{S}(t), H(t)\big)$-space as in Figure 5, where a straight line emitting from the origin and with a slope equal to $x^*(t)$ separates the stopping and continuation region. Or, taking the full step, we can depict an early exercise *surface* as in the 3-dimensional $\big(t, \bar{S}(t), H(t)\big)$-plot in Figure 6.

The time to maturity of the option contract is set to 10 years in all our examples as this is the predominating time to maturity used in practice (see e.g., Murphy (1999)). Figure 1 plots the early exercise boundaries for three different volatilities of the underlying stock. It is observed that a larger volatility enlarges the continuation region (the early exercise boundary is displaced downwards) as it becomes

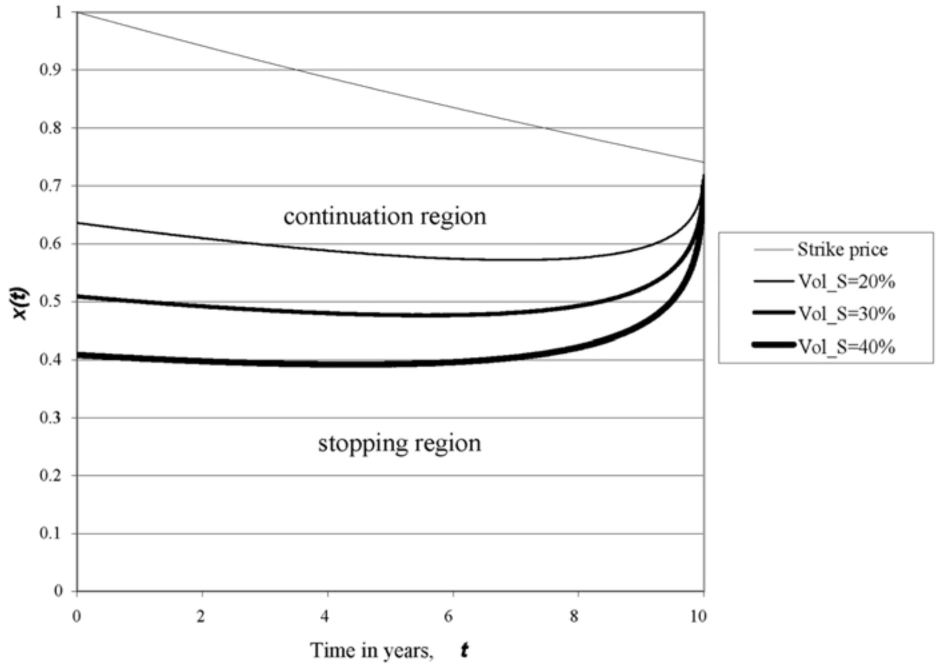

Figure 1. Early exercise boundaries for different stock price volatilities. $T = 10$, $\rho = 0.75$, $g = 0$, $\delta_s = 0.03$.

favorable to postpone exercise – ceteris paribus – when the underlying stock price is more uncertain. This is an early indication that indexed options create an incentive to increase the risk of the firm. The result is a logical consequence of our Black-Scholes based approach. Other researchers using the utility based approach have obtained the reverse relation (see e.g., Lambert, Larcker and Verrecchia (1991)).

A similar effect is seen in Figure 2 where we have plotted early exercise boundaries for different values of the correlation coefficient. Recall that the diffusion coefficient of the state variable process $x(\cdot)$ was given as $\sigma_x = \sigma_S\sqrt{1-\rho^2}$. From this it is clear that uncertainty regarding future realizations of the state variable increases as $|\rho|$ is lowered, which in turn enlarges the continuation region in much the same way as in Figure 1.

Figure 3 illustrates the effect on the location of the early exercise boundary of changing the benchmark appreciation rate adjustment coefficient, g. It is seen that raising this appreciation rate has the effect of lifting the x^*-curve. Again this is in fine accordance with intuition. If the executive must more than outperform the benchmark in order to receive a positive payoff he will tend to strike earlier, or equivalently, at a smaller degree of moneyness of the option. Conversely, when g is negative the executive can 'afford' to be patient and the early exercise boundary lies lower (the continuation region is extended).

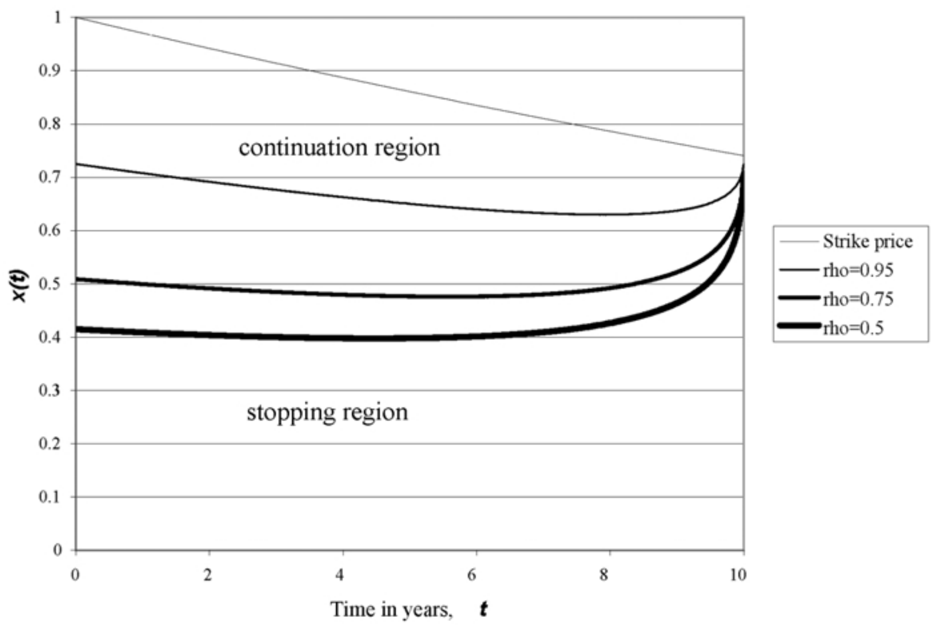

Figure 2. Early exercise boundaries for different correlations between the stock and index. $T = 10$, $\delta_s = 0.03$, $g = 0$, $\sigma_s = 0.3$.

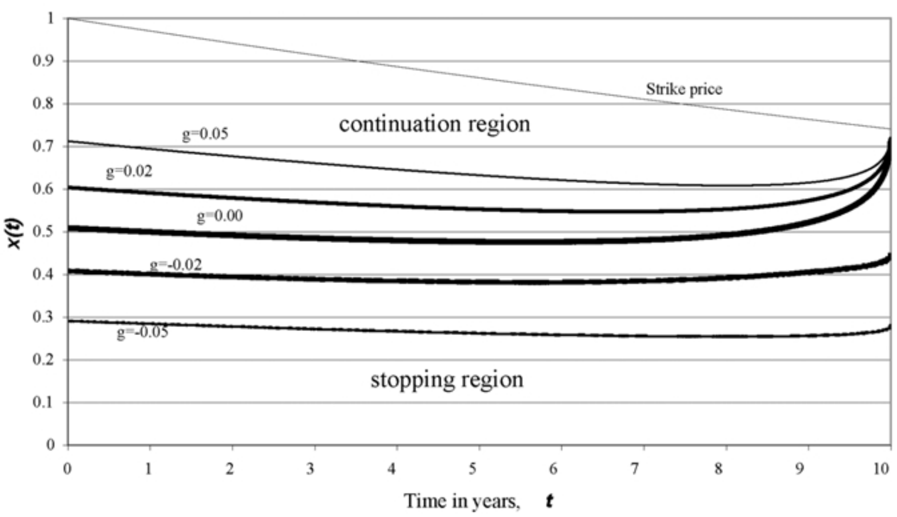

Figure 3. Early exercise boundaries for different values of the benchmark appreciation adjustment, g. $T = 10$, $\delta_s = 0.03$, $g = 0$, $\sigma_s = 0.3$.

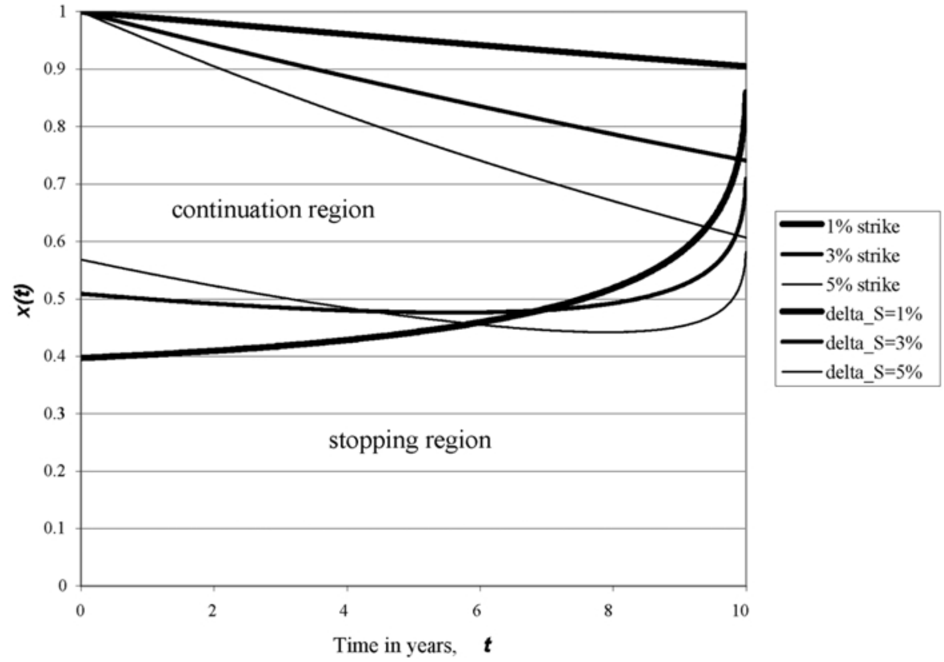

Figure 4. Early exercise boundaries for different stock dividend rates. $T = 10$, $\rho = 0.75$, $g = 0$, $\delta_S = 0.03$.

At first glance Figure 4 seems a bit odd since the early exercise boundaries for the three different values of δ_S intersect. However, this is not really a case of all other things being equal. As can also be seen from the figure, when δ_S changes so does the time varying exercise price in the transformed problem. The important point of the figure is that a larger stock dividend 'hurts' the stock price, diminishes the continuation region, and makes early exercise more likely.

Figures 5 and 6 represent examples of the earlier mentioned alternative illustrations of the continuation and exercise regions.

Tables I and II illustrate results from the second step of the numerical algorithm, i.e., values of American-style indexed executive stock options. The values are obtained from the earlier mentioned *final* numerical integration that can be performed once the location of the early exercise boundary has been established. A number of interesting observations can be made from these tables. First note that each entry in the table consists of two numbers. The first number is the total American option value, i.e., the sum of the value of the otherwise equivalent European option and the early exercise premium. The second number in parentheses immediately below the American option value is the percentage of the American option value which is due to the early exercise feature. Since the early exercise premium is sometimes referred to as the *potential*, we shall refer to this second number as the *relative*

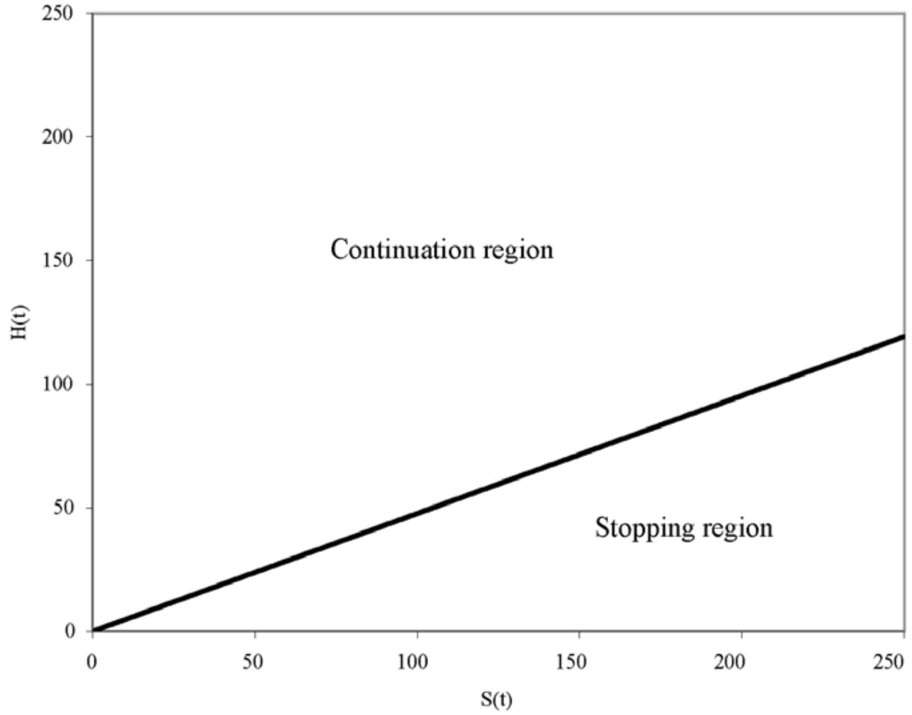

Figure 5. Stopping and exercise region after 5 years in $(S(t), H(t))$-space. $T = 10$, $\rho = 0.75$, $g = 0$, $\delta_S = 0.03$, $\sigma_S = 30\%$.

potential. We report this number in order to be able to gauge the importance of modeling the early exercise feature explicitly.

Table I consists of three panels – one for each of the stock price volatilities 20%, 30%, and 40%. Within each panel the stock dividend rate, δ_S, and the absolute correlation coefficient, $|\rho|$, are varied.

When comparing the three panels it is not surprising to observe that a larger stock price volatility generally makes the ESO more valuable. Also as expected, the American option value decreases in δ_S as well as in $|\rho|$. The explanation for these effects is that dividends hurt the stock and that a larger correlation between the stock and the index generically makes the payoff function less variable which, of course, hurts the option value.

It is more interesting to consider the relative potential. Comparing across the three panels we observe that the relative potential is almost invariant with respect to both the stock price volatility and the correlation, $|\rho|$. The stock dividend rate, however, has a significant effect on the relative potential. When δ_S increases, so does the relative potential. Specifically, the relative potential is zero when $\delta_S = 0$, but rises to as much as 40% as δ_S increases to 10%. We conclude that an American

Table I. American-style indexed executive stock option values (Relative Potential). $S_0 = 100$, $T = 10$ years, $g = 0$, $\lambda = 1$

		$\sigma_S = 0.20$						
		$\lvert\rho\rvert$						
		0.00	0.50	0.75	0.90	0.95	0.99	0.999
	0.00	24.82 (0.00%)	21.58 (0.00%)	16.57 (0.00%)	10.96 (0.00%)	7.87 (0.00%)	3.56 (0.00%)	1.13 (0.00%)
	0.02	21.65 (6.15%)	18.82 (6.12%)	14.44 (6.08%)	9.55 (6.05%)	6.85 (6.04%)	3.10 (6.04%)	0.98 (6.03%)
δ_S	0.04	19.39 (14.20%)	16.85 (14.15%)	12.93 (14.08%)	8.55 (14.03%)	6.13 (14.01%)	2.77 (13.99%)	0.88 (13.99%)
	0.06	17.63 (22.76%)	15.32 (22.69%)	11.75 (22.60%)	7.77 (22.53%)	5.57 (22.50%)	2.52 (22.48%)	0.80 (22.47%)
	0.08	16.22 (31.26%)	14.09 (31.17%)	10.80 (31.07%)	7.14 (30.99%)	5.12 (30.96%)	2.31 (30.93%)	0.73 (30.93%)
	0.10	15.06 (39.37%)	13.08 (39.29%)	10.02 (39.18%)	6.62 (39.09%)	4.75 (39.06%)	2.15 (39.03%)	0.68 (39.02%)

Table I. Continued.

		$\sigma_S = 0.30$						
		$\|\rho\|$						
		0.00	0.50	0.75	0.90	0.95	0.99	0.999
δ_S	0.00	36.47 (0.00%)	31.88 (0.00%)	24.63 (0.00%)	16.38 (0.00%)	11.77 (0.00%)	5.34 (0.00%)	1.69 (0.00%)
	0.02	31.87 (6.29%)	27.83 (6.23%)	21.48 (6.15%)	14.28 (6.08%)	10.26 (6.06%)	4.65 (6.04%)	1.47 (6.03%)
	0.04	28.59 (14.47%)	24.95 (14.35%)	19.24 (14.20%)	12.78 (14.08%)	9.18 (14.04%)	4.16 (14.00%)	1.32 (13.99%)
	0.06	26.04 (23.12%)	22.71 (22.96%)	17.50 (22.76%)	11.61 (22.60%)	8.34 (22.54%)	3.78 (22.49%)	1.20 (22.48%)
	0.08	23.98 (31.66%)	20.90 (31.48%)	16.10 (31.25%)	10.68 (31.07%)	7.67 (31.00%)	3.47 (30.94%)	1.10 (30.93%)
	0.10	22.29 (39.80%)	19.42 (39.61%)	14.94 (39.37%)	9.91 (39.17%)	7.11 (39.10%)	3.22 (39.04%)	1.02 (39.02%)

Table I. Continued.

$\sigma_S = 0.40$

		$\lvert\rho\rvert$					
δ_S		0.00	0.50	0.75	0.90	0.95	0.99	0.999
	0.00	47.29 (0.00%)	41.61 (0.00%)	32.43 (0.00%)	21.72 (0.00%)	15.66 (0.00%)	7.11 (0.00%)	2.26 (0.00%)
	0.02	41.41 (6.49%)	36.39 (6.38%)	28.32 (6.23%)	18.94 (6.12%)	13.65 (6.08%)	6.19 (6.04%)	1.97 (6.03%)
	0.04	37.22 (14.84%)	32.67 (14.63%)	25.38 (14.36%)	16.96 (14.15%)	12.21 (14.07%)	5.54 (14.01%)	1.76 (13.99%)
	0.06	33.97 (23.60%)	29.78 (23.32%)	23.11 (22.97%)	15.42 (22.69%)	11.10 (22.59%)	5.03 (22.50%)	1.60 (22.48%)
	0.08	31.35 (32.22%)	27.46 (31.90%)	21.27 (31.50%)	14.18 (31.18%)	10.20 (31.06%)	4.63 (30.95%)	1.47 (30.93%)
	0.10	29.18 (40.39%)	25.54 (40.05%)	19.76 (39.63%)	13.16 (39.29%)	9.47 (39.16%)	4.29 (39.05%)	1.36 (39.02%)

Table II. American-style indexed executive stock option values (Relative Potential). $S_0 = 100$, $T = 10$ years, $\sigma_S = 30\%$, $\rho = 0.75$

		$\delta_S = 0.00$								
		g								
		−0.10	−0.05	−0.03	−0.01	0.00	0.01	0.03	0.05	0.10
		78.00	64.43	57.42	49.64	45.54	42.75	40.10	40.00*	40.00*
	0.60	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(3.25%)	(17.72%)	(37.40%)	(75.74%)
		70.88	54.09	46.04	37.67	33.50	30.27	25.54	22.47	20.00*
	0.80	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(2.80%)	(14.62%)	(32.21%)	(76.78%)
		64.07	45.08	36.70	28.50	24.63	21.52	16.66	13.07	7.71
λ	1.00	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(2.49%)	(12.63%)	(27.63%)	(69.48%)
		57.67	37.44	29.20	21.61	18.19	15.42	11.10	7.96	3.43
	1.20	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(2.26%)	(11.23%)	(24.43%)	(63.39%)
		51.74	31.05	23.27	16.46	13.52	11.16	7.55	5.01	1.68
	1.40	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(2.08%)	(10.20%)	(22.06%)	(58.39%)

Table II. Continued.

$\delta_S = 0.03$

		g								
		−0.10	−0.05	−0.03	−0.01	0.00	0.01	0.03	0.05	0.10
λ	0.60	59.52 (2.91%)	50.06 (4.64%)	45.85 (7.22%)	42.31 (13.08%)	41.05 (17.83%)	40.26 (23.89%)	40.00* (38.89%)	40.00* (53.62%)	40.00* (82.03%)
	0.80	53.36 (1.57%)	41.16 (2.65%)	35.70 (4.46%)	30.68 (9.04%)	28.51 (12.96%)	26.60 (18.04%)	23.53 (31.34%)	21.46 (47.42%)	20.00* (82.80%)
	1.00	47.91 (0.92%)	33.96 (1.65%)	28.02 (2.97%)	22.63 (6.70%)	20.29 (10.06%)	18.17 (14.45%)	14.58 (26.07%)	11.78 (40.47%)	7.30 (76.10%)
	1.20	42.97 (0.56%)	28.04 (1.09%)	22.10 (2.10%)	16.89 (5.22%)	14.67 (8.17%)	12.70 (12.07%)	9.41 (22.43%)	6.90 (35.46%)	3.11 (70.09%)
	1.40	38.48 (0.36%)	23.18 (0.75%)	17.51 (1.55%)	12.73 (4.22%)	10.76 (6.87%)	9.04 (10.39%)	6.26 (19.79%)	4.24 (31.71%)	1.48 (64.95%)

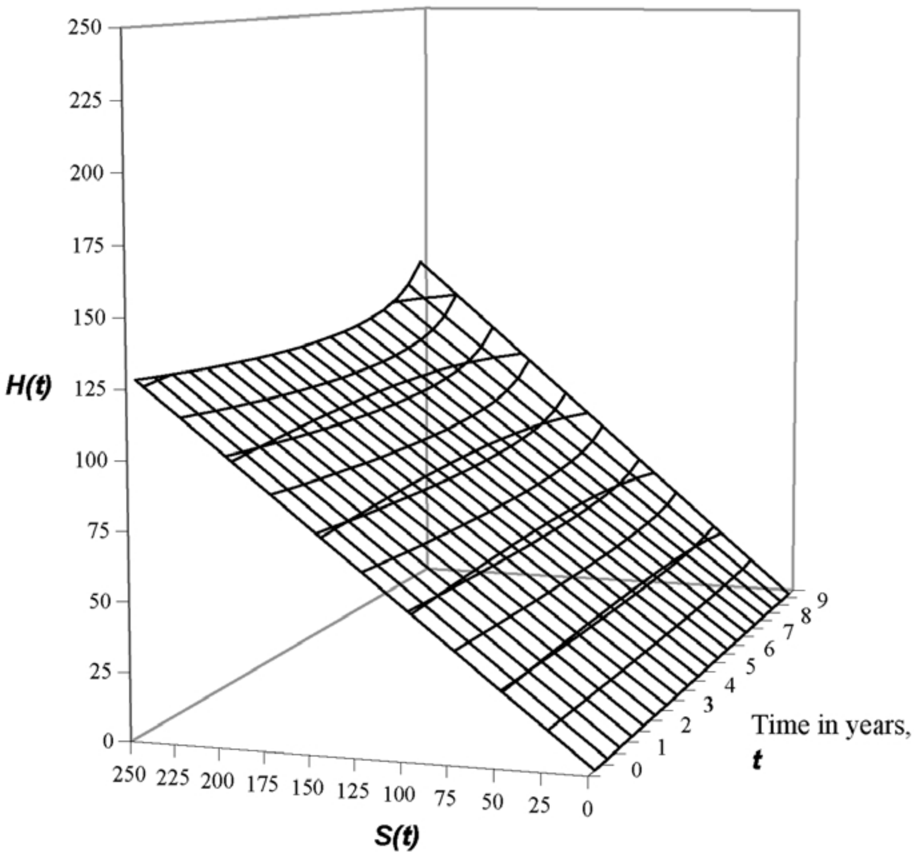

Figure 6. Early exercise surface. $T = 10$, $\rho = 0.75$, $g = 0$, $\delta_S = 0.03$, $\sigma_S = 30\%$.

ESO can be significantly mispriced by ignoring the early exercise feature if the underlying stock pays dividends.

Table II and its panels illustrate the valuation effects of changes in the benchmark adjustment parameters, λ and g. The total American option value clearly decreases in g and in λ, or as we move from the upper left to the lower right corner of the panels. Looking at the relative potential it is observed that when $\delta_S = 0$ (upper panel) and $g \leq 0$, the American option has no extra value compared to the European counterpart regardless of the moneyness parameter, λ. In other words, these options are so favorable that they should be kept alive until expiration. However, as g increases and becomes strictly positive it becomes harder to beat the benchmark and the early exercise feature becomes more valuable in both an absolute and a relative sense. In fact the relative potential is sharply increasing in g to the extent that the early exercise premium constitutes the largest part of the total American option value. Looking at the second panel where $\delta_S = 3\%$, we see an extreme example where the relative potential constitutes about 83% of the total option value. It is also observed that the early exercise feature is valuable even for negative values of g. Finally, note that the entries marked by an asterisk, $*$, in the upper right corners of the panels represent situations where the option is optimally exercised immediately upon issuance. The explanation for this is that the option is initially in-the-money due to a low value of λ, and the value of g is highly unfavorable. Hence it should be exercised immediately.

5. Incentive Effects

In this section we present results which give some indications of the incentive effects of the American-style indexed executive stock options. A full-blown analysis of this subject would involve careful examination of all partial derivatives of the option value. Owing to space considerations we concentrate on an analysis of the indexed option's *delta* and *vega*, i.e., the partial derivatives with respect to the stock price and volatility, respectively. We compare these *Greeks* to the deltas and vegas of conventional fixed strike price options.

From (27) and (33) we establish the indexed option's delta at time t as

$$\begin{aligned}\Delta\big(S(t), H(t)\big) \equiv \frac{\partial C(t)}{\partial S(t)} = \\ e^{-\delta_S(T-t)}\mathcal{N}(d_+) \\ + \int_t^T \mathcal{N}\big(d_+(t,u)\big)\delta_S e^{-\delta_S(u-t)} + \frac{n\big(d_+(t,u)\big)}{\sigma_x\sqrt{u-t}}\big(\delta_S e^{-\delta_S(u-t)} \\ +(g-\delta_S)x^*(u)e^{\delta_S t}\big)\ du,\end{aligned} \tag{41}$$

where $n(\cdot)$ is the standard normal density function and d_+ and $d_+(t,u)$ are as defined in (28) and (34).

First, note that this expression should be evaluated in the same manner as the potential expression (33). Assuming that the early exercise boundary has already been established for a particular contract, we merely need to perform a single numerical integration for each $\Delta-$value desired. This is because the location of the early exercise boundary is not affected by the current value of the state variable(s). Second, expression (41) is easily seen to be positive everywhere.

Part of Figure 7 is the result of implementing (41). The figure shows a plot of indexed and conventional option deltas against moneyness. For completeness we provide results for both the European- and American-style version of these options so the graph contains a total of four curves.[15] In order to obtain valid comparisons the positions are value-matched to one unit of the American-style indexed option contract at $S_0 = 100$ so that for example the delta for the European-style indexed

[15] We use the Black-Scholes-Merton formula and its well-known first derivatives to calculate values, deltas, and vegas for the European-style conventional options. The results for the American-style conventional options have been obtained by using the optimal stopping based method of Kim (1990).

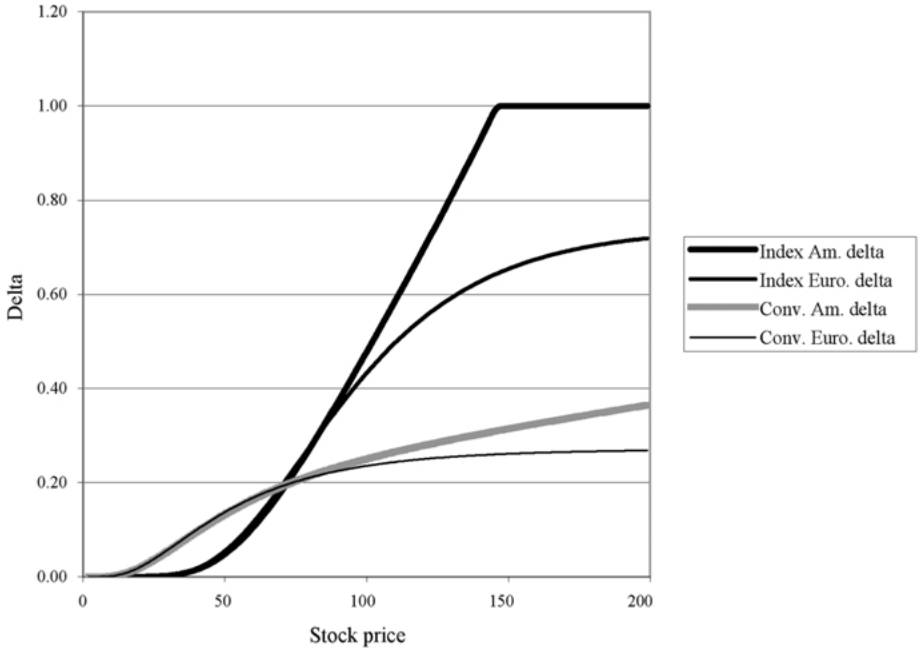

Figure 7. Value-matched option deltas (Sensitivity to stock price). $T = 10$, $g = 0.0$, $\rho = 0.75$, $\sigma_s = 0.2$, $\delta_s = 0.05$, $r = 0.1$, $K = 100$.

option is really for 1.22 contracts of this type. All options are issued at-the-money with $S_0 = 100$ and for the valuation of the conventional options we have assumed a riskless interest rate of 10%.

The picture in Figure 7 is typical and the conclusion is immediate. For most of the relevant values of the underlying stock price the American-style indexed option is the most sensitive instrument by a large margin. The delta of the American option is always larger than the delta of the otherwise equivalent and value-matched European option, which is again significantly larger than the deltas of the comparable positions of conventional options except for situations where the options are far out of the money.[16]

To assess the incentives the executive stock options create to increase the risk of the firm we also provide a brief graphical comparison of vegas. Analytical expressions of vega can be easily established for the two European-style options whereas for the American-style options we have to rely on numerical partial derivatives. It is the dependence of the early exercise boundary on the stock volatility (refer again to Figure 1) which prevents us from establishing analytical vegas for the American options. Figure 8 shows a typical case of how vegas of the four value-

[16] Note that the critical stock price can be identified from the chart as being approximately 145. The delta curve kinks at this point reflecting the fact for $S > 145$ the option is in the exercise region and has a delta equal to unity.

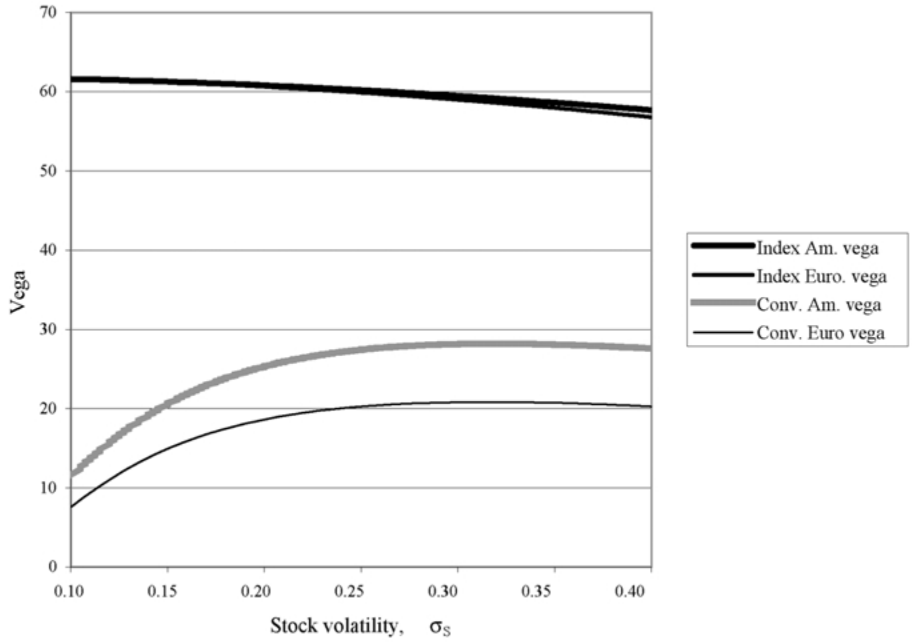

Figure 8. Value-matched option vegas (Sensitivity to stock volatility). $T = 10$, $g = 0.0$, $\rho = 0.75$, $\delta_s = 0.05$, $r = 0.1$, $S = K = 100$.

matched option positions vary with stock price volatility. It can be seen that vegas of indexed options are much larger than vegas of conventional options. It is also interesting to observe that the vegas of the two types of indexed options are almost indistinguishable and fairly invariant with respect to the volatility level. We may cautiously take these results to indicate that indexed options are worth considering as alternatives to conventional options particularly by firms with risk-averse managers and attractive, but risky, investment opportunities.

The indications of the above analysis are potentially important in relation to the optimal design of executive compensation contracts with option elements. The results presented here suggest that with indexation along the lines suggested by Johnson and Tian (2000a) better option packages can be obtained in the sense that the executive's wealth can be made more sensitive with respect to the stock price of the company he is managing. It has been documented that the issuance of the indexed options as American-style will further improve the incentives created by these options. We may further note that since in Johnson and Tian's (2000b) nontraditional options paper, the European indexed option is shown to provide stronger incentives to increase the stock price than all other nontraditional options (premium options, performance-vested options, repricable options, purchased options, and reload options) then, by transitivity, our results imply that American-style indexed options provide stronger incentives than all these nontraditional options.

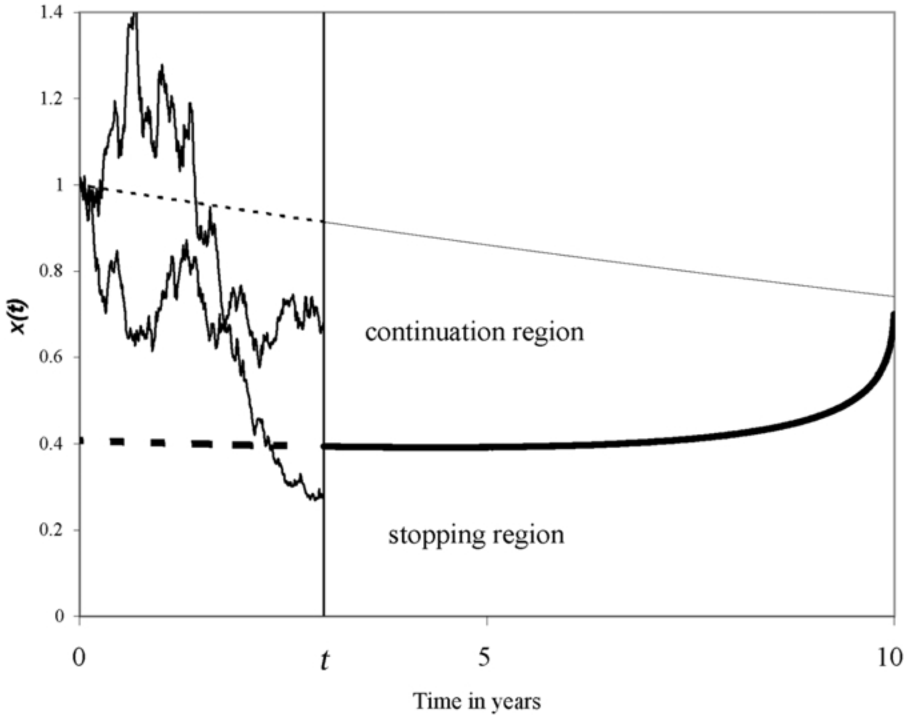

Figure 9. Exercise boundary with delayed vesting. $t = 3$, $T = 10$, $\rho = 0.75$, $g = 0$, $\delta_S = 0.03$, $\sigma_S = 0.40$.

While the incentive to increase risk (vega) is also larger for indexed options, it is largely independent of both the level of volatility and of the exercise provision of the options. An interesting management implication of this is that once it is has been decided to grant indexed options to executives, the board of directors should not hesitate to define the options as American-style out of fear that the implied risk incentives may be very different.

6. Delayed Vesting

Executive stock options are commonly subject to delayed vesting, i.e., although the option is basically American-style, the executive is restricted from exercising in a period following issuance. The time to maturity of the ESO is typically ten years and the vesting period is three years. The previous analysis is easily extended to cover a situation with a vesting period as explained in this section.

Without loss of generality we perform the valuation at time 0 and let times t and T denote the end of the vesting period and the option's maturity date respectively.

Figure 9 illustrates the situation with two simulated evolutions of the x-process (see (17)). The executive is forced to hold the option until time t where two situations can arise. The option will either be mature for exercise, i.e., $x(t) \leq x^*(t)$, as in the simulation where $x(t)$ enters the stopping region, or the option should be kept alive, as in the case where $x(t)$ remains in the continuation region. To determine which action to take, the executive should rely on his knowledge of the location of the early exercise boundary which can be determined already at time 0 along the lines explained in Section 4. It follows that the ESO with a vesting period can be valued as a European-style contingent claim with a time t payoff, $\bar{V}\big(x(t), t\big)$, expressed in the $\bar{S}$-numeraire as follows:

$$\bar{V}\big(x(t), t\big) = \begin{cases} \bar{C}\big(x(t), t\big) & x(t) > x^*(t) \\ \big(e^{-\delta_S t} - x(t)\big) & x(t) \leq x^*(t) \end{cases}. \tag{42}$$

Recalling that when using $\bar{S}$ as the numeraire, all (non-dividend paying) asset value processes are Q'-martingales, we can express the $\bar{S}$-denominated time 0 value of this contingent claim as follows:

$$\begin{aligned} \bar{V}(x_0, 0) &= E_0^{Q'}\left\{\bar{V}\big(x(t), t\big)\right\} \\ &= E_0^{Q'}\left\{\mathbf{1}_{\{x(t) \leq x^*(t)\}}\big(e^{-\delta_S t} - x(t)\big)\right\} \\ &\quad + E_0^{Q'}\left\{\mathbf{1}_{\{x(t) > x^*(t)\}}\bar{C}\big(x(t), t\big)\right\}. \end{aligned} \tag{43}$$

Straightforward calculations, which are omitted but available from the author on request, lead directly to a valuation formula in terms of the $\bar{S}$-numeraire. The option value in terms of the original numeraire is obtained by multiplying through by S_0. The $\bar{S}$-denominated valuation formula is

$$\begin{aligned} \bar{V}(x_0, 0) &= e^{-\delta_S t}\mathcal{N}\big(d_+(0, t)\big) - x_0 e^{(g-\delta_S)t}\mathcal{N}\big(d_-(0, t)\big) \\ &\quad + \int_{x^*(t)}^{\infty} f(x; t)\bar{C}(x, t)\, dx, \end{aligned} \tag{44}$$

where $d_\pm(0, t)$ is as defined in (34), $\bar{C}(x, t)$ is the American option value formula derived in Section 3, and $f(x; t)$ is the lognormal density function

$$f(x; t) = \frac{1}{x\sqrt{2\pi\sigma_x^2 t}} \cdot e^{-\frac{1}{2}d^2}, \tag{45}$$

with

$$d = \frac{\ln\frac{x}{x_0} + (-g + \delta_S + \frac{1}{2}\sigma_x^2)t}{\sigma_x\sqrt{t}}. \tag{46}$$

As regards implementation, the valuation formula (44) does not pose any new problems. The first terms are closed formulas whereas the last integral term can be arbitrarily accurately evaluated by numerical integration. Recall in particular

Table III. Valuation with delayed vesting. $S_0 = 100$, $g = 0.0$, $\rho = 0.75$, $\sigma_S = 0.3$, $\delta_S = 0.03$, $\lambda = 1$, $T = 10$ years

	Vesting date, t						
	0 (Pure American)	1	2	3	4	5	10 (Pure European)
Option value	20.29	20.29	20.28	20.23	20.13	19.97	18.25
Lost potential	0.00%	0.01%	0.43%	2.60%	7.43%	15.22%	100.00%

that the x^*-function which is needed in the evaluation of the integrand of (44) is independent of the state variable, x, so only one 'run' to determine the early exercise boundary is necessary. For a set of selected parameters Table III, illustrates how the American-style ESO value is influenced by the length of the vesting period.

From Table III it should first be observed that as the vesting date goes from $t = 0$ to $t = T = 10$ the option moves from being pure American-style to being pure European-style. This allows for a rough check of our implementation of formula (44) by comparing the values here with the ones established earlier (see the center of the second panel of Table II).

A *lost potential* is also reported in Table III. This number is defined as the fraction of the early exercise value of the pure American-style option which is lost due to delayed vesting. Hence, by construction the lost potential is 0% for the pure American-style contract ($t = 0$) and 100% for the pure European-style contract ($t = 10$). More interestingly, however, note how the lost potential is relatively small for moderate delays in the option's vesting date. For the typical vesting period of 3 years the early exercise value decreases by less than 3%. The example is representative for similar choices of realistic parameter values. One implication of these observations is that delayed vesting cannot justify the use of European-style valuation formulas for American-style contracts.

7. Conclusion

This paper derives and analyzes a new valuation formula for American-style executive stock options based on an indexation scheme that was originally proposed in Johnson and Tian (2000a). In its first form the basic valuation problem is a relatively complicated optimal stopping problem in two state variables. However, we demonstrate that a change of numeraire allows a reduction of dimensionality of the problem to a form where an analytical characterization of the solution is possible.

The established valuation formula is relatively easy to implement and numerical examples indicate that it can be 'costly' to ignore the early exercise feature of these options. In other words, the difference in value between the American- and the European-style version of otherwise identical indexed ESOs may be large.

Our analysis of the incentive effects provided by American-style indexed ESOs shows that these options may provide much stronger incentives for executives than their European counterparts, which in turn have been shown to be superior to conventional stock options in this respect (Johnson and Tian (2000a)).

Indexed ESOs have many other advantages over conventional options as outlined in Rappaport (1999), and therefore we conjecture a growth in the use of this type of instrument in corporate remuneration packages in the future. This is even more likely to happen if an accounting anomaly, which presently works against indexed options, is corrected in the future.[17] Finally, the fact that the US economy seems to be slowing down and that many economic analysts presently predict low growth rates for the World's major economies for some time to come may further spark the interest in indexed option plans.

An obvious subject for further research is a detailed investigation of the incentives created by the indexed ESO in its various incarnations. For example, it may be possible to optimize, in some specific sense, over the benchmark parameters λ and g as well as over the correlation between the underlying stock and the index. Pending further empirical results on whether the assumption of rational exercise is too restrictive, one may – following the idea of Carpenter (1998) – consider extending the model with some random exercise component in order to capture the effects of forfeiture and non-rational exercises.

Appendix

A. Characterization of the Q'-Measure and Q'-Dynamics of Various Processes

We can define a stock and an index account, $\bar{S}(t)$ and $\bar{I}(t)$ respectively, as accounts where all dividends from S and I are immediately re-invested. These accounts have $\mathcal{P}$-dynamics

$$\frac{d\bar{S}(t)}{\bar{S}(t)} = \mu_S\, dt + \sigma_S\, d\mathcal{W}_S(t), \tag{A.1}$$

and

$$\frac{d\bar{I}(t)}{\bar{I}(t)} = \mu_I\, dt + \sigma_I\, d\mathcal{W}_I(t). \tag{A.2}$$

Consider now the fraction $\frac{\bar{I}(t)}{\bar{S}(t)}$. According to the EMM theorem (see Duffie (1996)) we have no arbitrage iff there is a measure such that $\frac{\bar{I}(t)}{\bar{S}(t)}$ is a martingale. Apply Itô's lemma (using familiar shorthand notation) to get

[17] The value of indexed options is presently required to be calculated on a quarterly basis and charged to company earnings. The Federal Accounting Standards Board (FASB) allows conventional options to avoid any charges to earnings.

$$
\begin{aligned}
d\left(\frac{\bar{I}}{\bar{S}}\right) &= \frac{1}{\bar{S}}d\bar{I} - \frac{\bar{I}}{\bar{S}^2}d\bar{S} + \frac{1}{2}2\frac{\bar{I}}{\bar{S}^3}(d\bar{S})^2 - \frac{1}{\bar{S}^2}d\bar{I}\,d\bar{S} \\
&= \frac{\bar{I}}{\bar{S}}\left[\frac{d\bar{I}}{\bar{I}} - \frac{d\bar{S}}{\bar{S}} + \left(\frac{d\bar{S}}{\bar{S}}\right)^2 - \frac{d\bar{I}}{\bar{I}}\frac{d\bar{S}}{\bar{S}}\right] \\
&= \frac{\bar{I}}{\bar{S}}\left[(\mu_I - \mu_S + \sigma_S^2 - \sigma_I\sigma_S\rho)\,dt + \left(\sigma_I\,d\mathcal{W}_I(t) - \sigma_S\,d\mathcal{W}_S(t)\right)\right].
\end{aligned}
$$

Now Girsanov's theorem (see e.g., Duffie (1996)) instructs us to switch to measure Q' using the substitution

$$
d\mathcal{W}_S^{Q'}(t) = d\mathcal{W}_S(t) + \gamma_S\,dt, \tag{A.3}
$$

and

$$
d\mathcal{W}_I^{Q'}(t) = d\mathcal{W}_I(t) + \gamma_I\,dt. \tag{A.4}
$$

Hence

$$
\begin{aligned}
d\left(\frac{\bar{I}}{\bar{S}}\right) &= \frac{\bar{I}}{\bar{S}}\Big[(\mu_I - \mu_S + \sigma_S^2 - \sigma_I\sigma_S\rho)\,dt + \sigma_I(d\mathcal{W}_I^{Q'}(t) - \gamma_I\,dt) \\
&\quad -\sigma_S(d\mathcal{W}_S^{Q'}(t) - \gamma_S\,dt)\Big] \\
&= \frac{\bar{I}}{\bar{S}}\Big[(\mu_I - \mu_S + \sigma_S^2 - \sigma_I\sigma_S\rho - \gamma_I\sigma_I + \gamma_S\sigma_S)\,dt + \sigma_I\,d\mathcal{W}_I^{Q'}(t) \\
&\quad -\sigma_S\,d\mathcal{W}_S^{Q'}(t)\Big].
\end{aligned}
$$

For this to be a Q'-martingale, we require

$$
\mu_I - \mu_S + \sigma_S^2 - \sigma_I\sigma_S\rho - \gamma_I\sigma_I + \gamma_S\sigma_S = 0. \tag{A.5}
$$

This is one equation in two unknowns, γ_S and γ_I. But $\frac{B}{\bar{S}}$ should also be a Q'-martingale:

$$
\begin{aligned}
d\left(\frac{B}{\bar{S}}\right) &= \frac{1}{\bar{S}}\,dB - \frac{B}{\bar{S}^2}\,d\bar{S} + \frac{1}{2}2\frac{B}{\bar{S}^3}(d\bar{S})^2 \\
&= \frac{B}{\bar{S}}\left[\frac{dB}{B} - \frac{d\bar{S}}{\bar{S}} + \left(\frac{d\bar{S}}{\bar{S}}\right)^2\right] \\
&= \frac{B}{\bar{S}}\left[r\,dt - \mu_S\,dt - \sigma_S\,d\mathcal{W}_S(t) + \sigma_S^2\,dt\right] \\
&= \frac{B}{\bar{S}}\left[(r + \sigma_S^2 - \mu_S)\,dt - \sigma_S\,d\mathcal{W}_S(t)\right],
\end{aligned}
$$

and proper substitution yields

$$\begin{aligned} d\left(\frac{B}{\bar{S}}\right) &= \frac{B}{\bar{S}}\left[(r+\sigma_S^2-\mu_S)\,dt-\sigma_S\left(d\mathcal{W}_S^{Q'}(t)-\gamma_S\,dt\right)\right] \\ &= \frac{B}{\bar{S}}\left[(r+\sigma_S^2-\mu_S+\sigma_S\gamma_S)\,dt-\sigma_S\,d\mathcal{W}_S^{Q'}(t)\right]. \end{aligned}$$

Since this must be a Q'-martingale, we require

$$r-\mu_S+\sigma_S^2+\sigma_S\gamma_S=0. \tag{A.6}$$

Solving for γ_S

$$\gamma_S=\frac{\mu_S-r-\sigma_S^2}{\sigma_S}, \tag{A.7}$$

and substituting in (A.5), we get

$$\mu_I-\mu_S+\sigma_S^2-\sigma_I\sigma_S\rho-\gamma_I\sigma_I+\sigma_S\left(\frac{\mu_S-r-\sigma_S^2}{\sigma_S}\right)=0$$

$$\Updownarrow$$

$$\mu_I-\mu_S+\sigma_S^2-\sigma_I\sigma_S\rho+\mu_S-r-\sigma_S^2=\gamma_I\sigma_I$$

$$\Updownarrow$$

$$\gamma_I=\frac{\mu_I-r-\sigma_I\sigma_S\rho}{\sigma_I}.$$

This allows us to establish the Q'-dynamics of the original stock and index specification respectively. We find

$$\begin{aligned} \frac{dS(t)}{S(t)} &= (\mu_S-\delta_S)\,dt+\sigma_S\left(d\mathcal{W}_S^{Q'}(t)-\gamma_S\,dt\right) \\ &= (\mu_S-\delta_S-\sigma_S\gamma_S)\,dt+\sigma_S\,d\mathcal{W}_S^{Q'}(t) \\ &= (\mu_S-\delta_S-\mu_S+r+\sigma_S^2)\,dt+\sigma_S\,d\mathcal{W}_S^{Q'}(t) \\ &= (r-\delta_S+\sigma_S^2)\,dt+\sigma_S\,d\mathcal{W}_S^{Q'}(t), \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} \frac{dI(t)}{I(t)} &= (\mu_I-\delta_I)\,dt+\sigma_I\left(d\mathcal{W}_I^{Q'}(t)-\gamma_I\,dt\right) \\ &= (\mu_I-\delta_I-\sigma_I\gamma_I)\,dt+\sigma_I\,d\mathcal{W}_I^{Q'}(t) \\ &= (\mu_I-\delta_I-\mu_I+r+\sigma_I\sigma_S\rho)\,dt+\sigma_I\,d\mathcal{W}_I^{Q'}(t) \\ &= (r-\delta_I+\sigma_I\sigma_S\rho)\,dt+\sigma_I\,d\mathcal{W}_I^{Q'}(t). \end{aligned} \tag{A.9}$$

□

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