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Comment on 'Valuation of Barrier Options in a Black–Scholes Setup with Jump Risk'

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The valuation of barrier options in the Black-Scholes setup has for a long time been studied by various researchers. Going back to Merton (1973) we find an analytical solution to the so called down-and-out call option, and thereafter the other possible combinations have been analyzed as well, see for instance Rubinstein and Reiner (1991). Barrier options have always been very popular at the market since they, in a way, allow the participants to incorporate personal beliefs about the future prices of the underlying security. In general, if the asset stays in some predefined region (defined by the barrier) the payoff at the maturity will coincide with the payoff of a plain vanilla option, while if the asset at some point in time exits from the region the option becomes and remains worthless. Of course, such a construction will make a barrier option much cheaper than a plain vanilla option, although the possible gains at maturity of the two claims are identical. Needless to say, these properties make barrier options appealing to investors. The problem that arises, though, is that the payoff of a barrier option is not only non-linear (as for contingent claims in general) but also discontinuous, and it is this discontinuity that causes difficulties when a numerical method to value the options is to be implemented.

There are basically two reason why numerical valuation is of interest. First, if we consider a claim with more than one barrier there is, even in the Black-Scholes setup, little hope of finding an analytical solution for the price; and secondly, if we relax the assumption that the asset prices can be described by a geometric Brownian motion it is quite unlikely that we can find an analytical solution for the price, even in the simple case of only one barrier. It is the latter motivation that is adopted by the author.

To implement a numerical valuation scheme the underlying asset must be approximated. Although this can be done in a more or less straightforward way such that the option prices converge to the true value as we increase the refinement, it is by no means straightforward to find an approximation of the asset price such that the corresponding option prices have good convergence properties. As stated above, the slow convergence is a consequence of the discontinuity of the payoff function, and it has been known for some time that if we apply the usual procedure of discretizing the time axis, the option prices will converge, with increasing refinement, in a characteristic "saw-tooth" pattern, see figure 1 in Leisen's paper. An explanation of this behavior can be found in the fact that increasing the refinement results in a new shape of the discretized asset price model, since the up and downward movements of the asset actually depend on the refinement. Therefore, the distance between the border nodes and the barrier will vary, and as a consequence the numerical prices will jump every time the border nodes in the discretized underlying asset model cross the barrier.

To increase the convergence rate some different approaches have been suggested, however they all agree on the need of placing the nodes on the barrier. It is possible to distinguish three different kinds of approaches. The first one uses the traditional discretization of the time axis and construct a tree such that, by force, the nodes will coincide with the barrier, see for instance Ritchken (1995) and Cheuk and Vorst (1996). The second one is a finite difference approach where the time axis as well as the asset space is discretized simultaneously like in Boyle and Tian (1998). Here, by construction, the nodes will coincide with the barrier. Finally, the third one starts directly by discretizing the asset space and thereafter a "random tree" is constructed as in Rogers and Stapleton (1998) and the paper by Leisen. Again, the nodes will by construction coincide with the barrier.

The last approach mentioned is indeed a very elegant one. Compared to the similar method of Rogers and Stapleton (1998) the author motivates the contribution of the paper by the following two arguments. First, since Rogers and Stapleton uses a binomial "random tree" the numerical problems arising due to the non-linearity of the payoff function around the strike price cannot be handled. To overcome this deficiency nodes must be placed on the strike price as well, and hence a trinomial "random tree" is needed. Secondly, Leisen assumes that the asset price can only change at the jump times of a Poisson process. This makes the valuation formula simple as well as the extension of the Black-Scholes setup to the jump-diffusion setup of Merton (1976). Unfortunately though, there is basically no quantitative comparison of the two methods, which of course makes it somewhat difficult to assess the contribution of the paper. Moreover, there is no quantitative comparison to the other approaches mentioned above either.

In the paper two slightly different methods are presented, that is the trinomial adjusted (TA) model and the randomized trinomial (RT) model. Under the Black-Scholes setup these two methods are compared to the usual binomial model of Cox, Ross and Rubinstein (henceforth CRR). The difference between the TA model and the RT model is basically that in the TA model the asset prices change at deterministic points in times while in the RT model the changes occur at stochastic points in times. In both the TA and the RT model the characteristic "saw-tooth" pattern of the approximated option prices is eliminated, resulting in a fast and accurate convergence. Hence, in the Black-Scholes setup the two proposed methods both outperform the CRR model. An interesting point is that the TA model converges faster than the more complex RT model; however when applying the modified Richardson extrapolation rule, see Leisen (1998), the result is the opposite. It would be nice to see a discussion and a possible explanation of the phenomena in the paper. As it stands right now it leaves the reader somewhat puzzled not knowing whether this is true in general or due to the specific choice of the parameters.

Although most of the paper is devoted to the Black-Scholes setup, the motivation for introducing the RT model is easily seen when the author adds a compound Poisson process to the specification of the asset price, that is when the setup of Merton (1976) is assumed. It is indeed very easy to modify the RT model to handle this situation. Leisen considers an example where the firm may go bankrupt, that is the asset price may jump to zero and remain there. In this example it turns out that the convergence properties for the Richardson extrapolated option prices looks really promising or stated differently that the convergence properties for the Black-Scholes setup carry over to the jump-diffusion setup. Again, however, there is no comparison to other existing methods other than the CRR model.

Summing up, Leisen presents an elegant approach to value barrier options using "random trees", although some questions are left unanswered. For example most of the analysis and the quantitative examples in the paper are devoted to the Black-Scholes setup, where analytical option price formulas are already known. It would be interesting to study the jump-diffusion case more thoroughly to see if the good convergence properties carry over to this case in general. It would also be interesting to see if the two methods proposed, that is the TA and the RT models, can be used to handle double barrier options. Additionally, it would be interesting to see a deeper comparison to other approaches, including finite differences and more sophisticated methods than binomial trees and also to analyze why the RT model seems to perform better than the TA model after the Richardson extrapolation has been used. Finally it would be interesting to quantify the gain from using Leisen's method in comparison to the similar approach by Rogers and Stapleton (1998).

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