European Finance Review **2:** 321–363, 1999.

A Model for Studying the Effect of EMU on European Yield Curves

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Abstract. In January 1999, the European monetary union (EMU) was formally launched with 11 member countries. However, before May 1998 there was considerable uncertainty about who would join EMU, and whether the project would start on time. When a monetary union is formed, exchange rates between the member countries are irrevocably fixed, and yield spreads stemming from exchange-rate risk are eliminated. As a direct consequence, EMU affected the prices of long-term bonds well before 1999, but quantifying this effect can be difficult when there is uncertainty about the monetary union. We address these issues and develop a bond-pricing model which explicitly takes into account that a country may join a monetary union at a future, unspecified date. The empirical results show that a narrow EMU, consisting of Germany, France and the Benelux countries, has been priced with almost 100% probability throughout the period 1995–1998, whereas, on average, the implied probability of joining EMU has been somewhat lower for the other EU countries. However, in the period leading up to May 1998, the estimated probabilities have increased considerably for the countries that joined EMU in January 1999.

Key words: European Monetary Union (EMU), Term Structure of Interest Rates.

JEL classification codes: G12, G13, F36.

[1] I thank an anonymous referee, Torben G. Andersen, Pierluigi Balduzzi, Giovanni Barone-Adesi, David S. Bates, Peter Ove Christensen, Henrik Dahl, Tom Engsted, Antoine Frachot, Peter Honoré, Karsten Munk Knudsen, Kristian R. Miltersen, Johannes Raaballe, Staffan Viotti, and seminar participants at Aarhus University, Aarhus School of Business, Copenhagen Business School, Odense University, Bocconi University, Bank of Finland, Salomon Brothers International (London), the Fifth Symposium at Erasmus Center for Financial Research (Rotterdam), the 1996 Money, Macro and Finance conference (MMF-96) at London Business School, the 2nd Nordic Finance Symposium (Stockholm), the 21st SUERF Colloquium (Frankfurt), and the 1998 Meetings of the American Finance Association (Chicago) for helpful comments. I am also grateful to Henrik Dahl for suggesting this research topic, and to the Bank of Finland for supplying parts of the data. Of course, I remain responsible for any errors. Financial support from the Danish Social Science Research Council is gratefully acknowledged.

1. Introduction

The issue of monetary integration in Europe has been discussed for several decades, but it was not until December 1991 that a binding timetable, in the form of the Maastricht Treaty, was agreed upon by the EU member countries. The Treaty contained a detailed plan for monetary integration, leading to the formation of the European monetary union (EMU) in January 1999.[2] When EMU began, exchange rates among member countries were irrevocably fixed, and the new single currency, called the Euro, was introduced. Within a transitional period of $3\text{–}3\frac{1}{2}$ years, the Euro will become the sole legal tender in the member countries. Furthermore, from the beginning of the monetary union, contracts in ECUs are to be honored in Euros instead (with a 1:1 conversion rate). In effect, the ECU becomes one of the member currencies of the monetary union.

In May 1998, the Council of Ministers of the European Union confirmed the planned starting date (January 1999) and, most importantly, selected the member countries. From the beginning of 1999, there was broad monetary union in Europe consisting of 11 countries, namely EU–15 minus the United Kingdom, Denmark, Sweden and Greece. However, before May 1998 there was considerable uncertainty about the prospects for the EMU project, in particular with regard to the selection of the member countries. Moreover, discussions about postponing the starting date of the monetary union surfaced regularly. The main reason for this uncertainty was, arguably, that the Maastricht Treaty allowed for either a strict or a "soft" interpretation of the so-called convergence criteria. In the former case, a country would have to meet specific economic conditions in order to join EMU, most notably the public finance criteria which stipulate that the government deficit should be less than 3% of GDP and the public debt below 60% of GDP. In the latter case, represented by article 104c of the Treaty, a country would only be required to demonstrate "sufficient progress" towards meeting the convergence criteria. What exactly constitutes sufficient progress is open to interpretation and, in reality, determined by political preferences. Moreover, when drafting the Treaty in 1991, Denmark and the United Kingdom made an explicit reservation about their participation in EMU, often referred to as an "opt-out". After joining the EU in 1995, Sweden has effectively obtained a similar status.

The present paper investigates the effect of a planned monetary union on the domestic yield curves in the potential member countries. The most intriguing problems arise when there is some uncertainty about either the members of the monetary union or the starting date (or both), so we concentrate on this case. We use the outlook for the EMU project between 1995 and 1998 as the main vehicle of exposition. Although much of the EMU uncertainty has now been resolved, as noted

[2] Formally, this event is known as the *third stage* of the EMU process. Prior to that, the Treaty outlines two stages of economic and institutional convergence, involving, among other things, participation in the exchange rate mechanism (ERM) and establishing the European Monetary Institute (EMI), the precursor for the European Central Bank (ECB). Eichengreen (1993) and Pollard (1995) discuss these aspects of the EMU process.

above, there are still four countries that may join the monetary union after January 1999. Moreover, the European Union is likely to be expanded with several Eastern European countries, for example Poland, Hungary and the Czech Republic, and in due course these countries will be candidates for EMU membership as well. Thus, while the paper, by necessity, takes a historical perspective of the road towards EMU, the methodology developed here will remain relevant well beyond 1999.

In international term-structure models, yield spreads between different countries are caused by exchange-rate risk, and the yield spread for, say, a 10-year bond depends on exchange-rate changes over the next 10 years. However, once exchange rates among the EMU members are permanently fixed, bonds in different currencies become perfect substitutes, and yield spreads are eliminated.[3] Consequently, the European monetary union scenario, whether taken for granted or just a possible outcome in 1999 or later, must affect the prices of long-term bonds in EU countries well before 1999. Ultimately, a monetary union will of course simplify international bond pricing, as the number of currencies decreases. However, until the process is completed, any uncertainty about who joins the monetary union, and when this happens, translates into an additional source of uncertainty for the yield curve in that particular country.

We address these issues by developing an international term-structure model which explicitly incorporates the EMU plans, present a concrete method for estimation, and provide empirical findings. The membership date of EMU for a specific country is treated as a random variable in our model. In the empirical analysis, we use this feature of the model to estimate the market's implied probability of EMU membership for each EU country, using interest-rate data. Recently, the latter idea has also been put forth in papers by De Grauwe (1996) and Favero et al. (1997), as well as in the "EMU calculator" developed by JP Morgan (1997).[4] Bates (1998) surveys the literature and presents an in-depth comparison of the different methods for estimating EMU probabilities.

The main building blocks of our EMU model are theoretical term-structure models, like the Vasicek (1977) and Cox et al. (1985) models, and a probability distribution for the starting date of EMU membership. The structure of the EMU model, and the empirical analysis presented here, are related to recent work on modeling the dynamics of defaultable bonds, see, e.g., Claessens and Pennacchi (1996), Cumby and Evans (1995), and Duffee (1996).

The remainder of the paper is organized as follows: Section 2 outlines the main challenges for international term-structure models when the countries in question may join a monetary union (EMU) in the future. Specifically, we consider the hypothetical case of no EMU uncertainty and find that the implications of this scenario are contradicted by the data, at least for some countries and time periods.

[3] This, of course, assumes that there are no differences in other factors, such as liquidity, taxation and credit risk. We discuss this issue in Section 4.2.

[4] In related work, Frachot (1997) uses the deviation between the exchange rates of the official and private ECUs to estimate the expected starting date of EMU.

This analysis, which is largely informal, sets the stage for developing the EMU model in the following sections of the paper. In Section 3, we provide a detailed discussion of our term-structure model with EMU uncertainty, including a numerical example that illustrates the main features of the model. Bond prices are obtained from expectations under risk-neutral probability measures, and since these measures are currency-specific, we pay careful attention to the numeraire problems resulting from the redenomination to Euros. Section 4 deals with the empirical implementation of the model. First, we discuss the potential identification problems of the model, and we present a simplified version which alleviates these problems. Second, we describe the data and outline the econometric techniques, based on non-linear Kalman filtering (a detailed discussion of the non-linear Kalman filter algorithm can be found in Appendix A). The empirical results are presented in Section 5, and Section 6 concludes the paper.

2. EMU and European Yield Curves Before 1999

We begin by considering the simple case of no uncertainty related to the timing and members of EMU. Specifically, EMU is assumed to start on January 1, 1999 with a known group of member countries. At this date, riskless bonds denominated in, say, Deutschmarks (DEM) and French francs (FRF) will become perfect substitutes, and they should trade on the same yield curve. However, yields on government bonds could contain a country-specific default premium due to the loss of monetary sovereignty that is associated with EMU, cf. the discussion in Section 4.2. To avoid this problem, we look at swap rates in the empirical analysis although we generally use the phrase "bonds" in the text.

Since bonds from different countries become perfect substitutes in 1999, we expect that forward rates for post-1999 maturities are identical across countries.[5] Figures 1–3 depict the spread to Germany for the 4–7 year forward rates, measured in basis points (bps), between January 1995 and August 1998.[6] For comparison, the three-month interest-rate spreads are displayed in Figures 4–6 for the same period. Apart from the potential caveat noted in Footnote 5, the forward-rate spreads should be zero over the entire period if there is no uncertainty about EMU membership for the countries in question. On the other hand, in the absence of any plans about joining a monetary union with Germany, we would expect these spreads to be mostly positive.

[5] Despite the great intuitive appeal, this claim about forward rates is not guaranteed by absence of arbitrage alone. As long as the conversion rates (in 1999) between currencies are unknown, a non-zero forward-rate spread does not necessarily represent an arbitrage opportunity. However, the claim holds under the additional assumptions of the EMU model presented in Section 3, as Equation (27) demonstrates.

[6] A full description of the data is deferred until Section 4.2. Note, however, that Austria, Ireland, and Greece are not included in the analysis due to lack of data. Luxembourg already operates a currency union with Belgium.

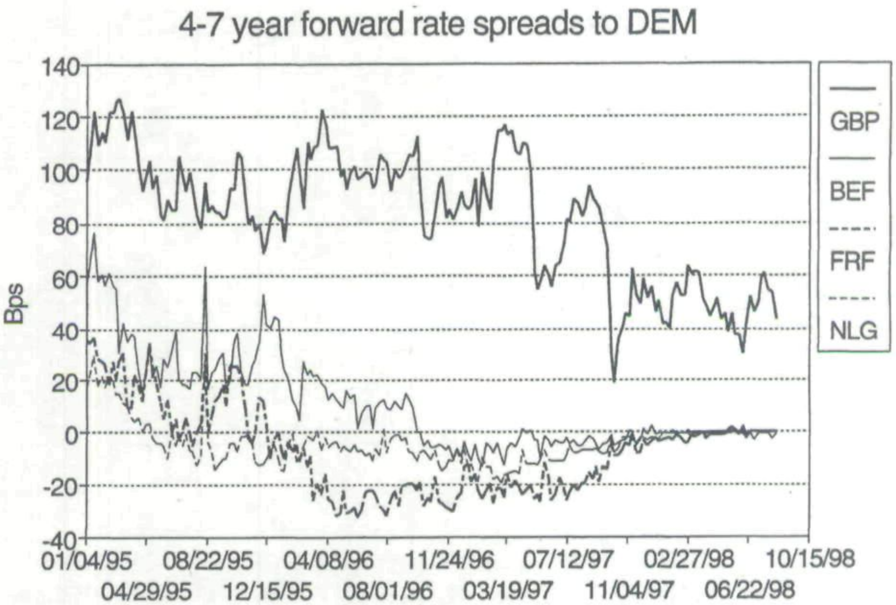

Figure 1. 4–7 year forward rate spreads to DEM.

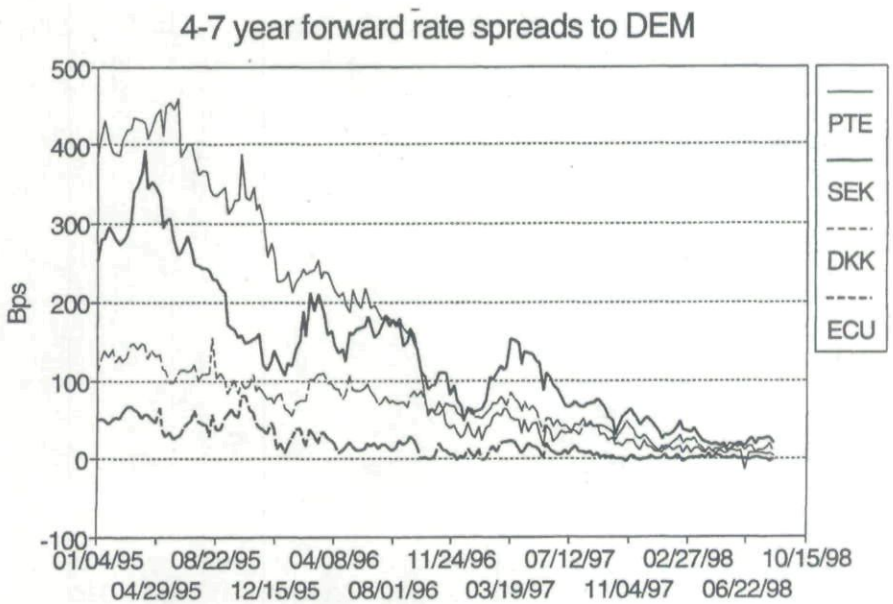

Figure 2. 4–7 year forward rate spreads to DEM.

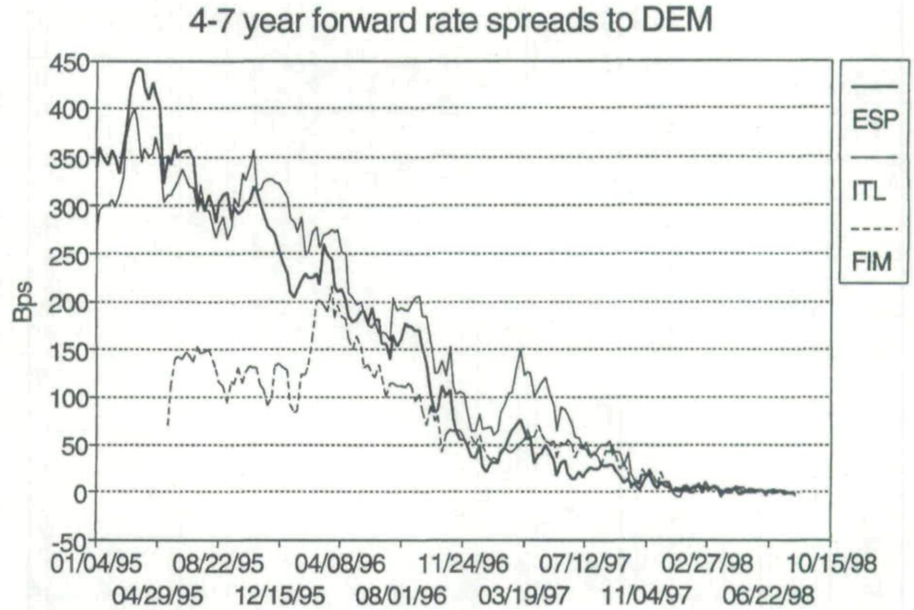

Figure 3. 4–7 year forward rate spreads to DEM.

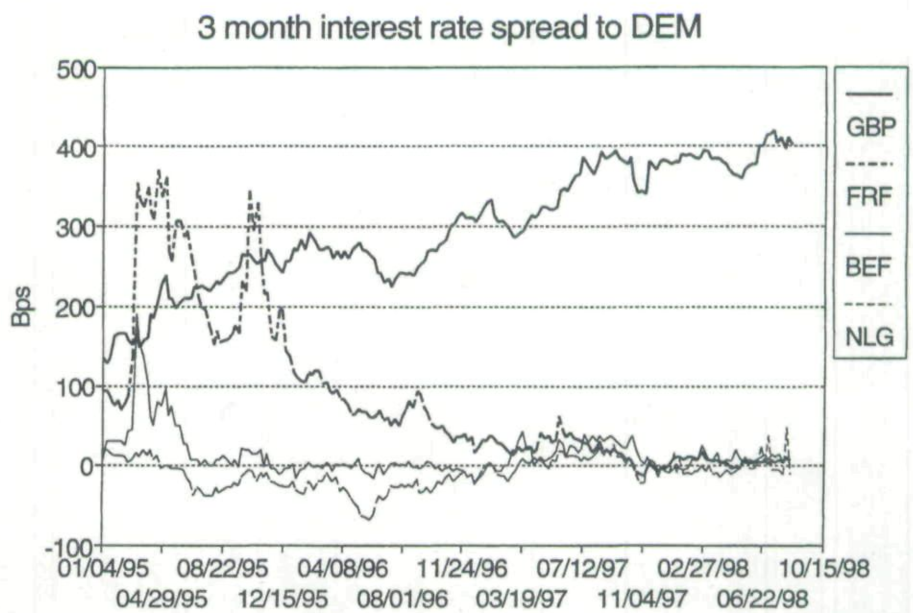

Figure 4. Three month interest rate spreads to DEM.

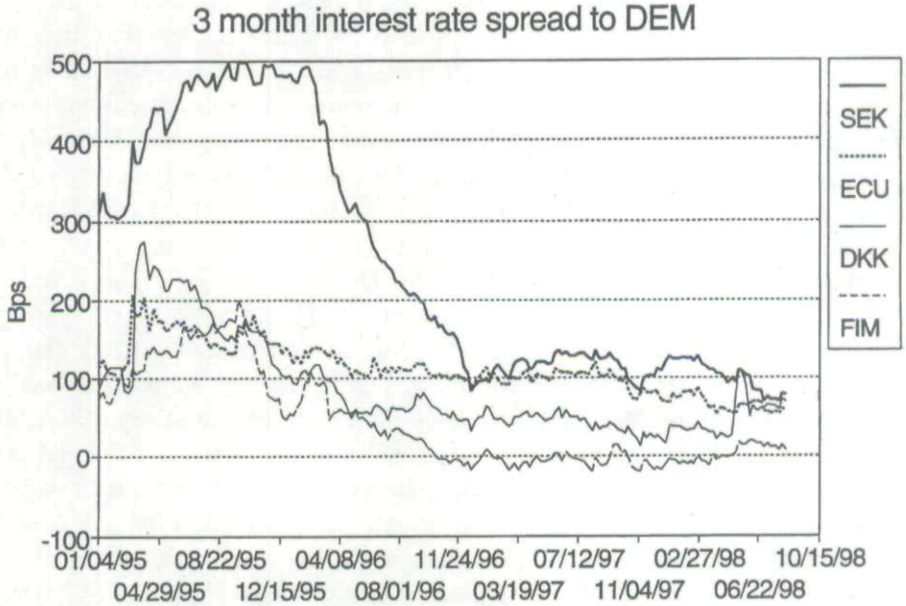

Figure 5. Three month interest rate spreads to DEM.

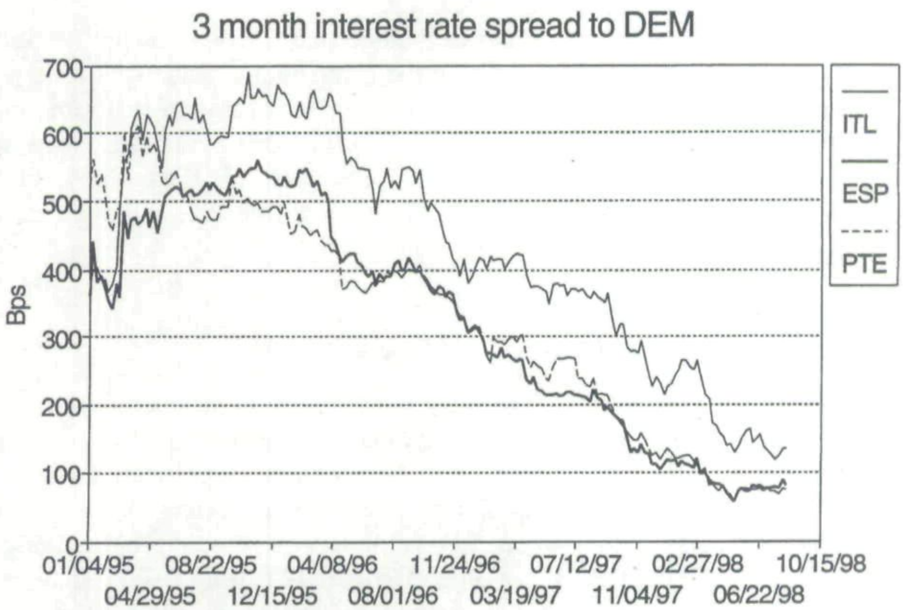

Figure 6. Three month interest rate spreads to DEM.

Evidently, the figures contradict the assumption of no EMU uncertainty, at least for the majority of EU countries. The Dutch and French forward spreads are mainly positive in 1995, but they have dropped below zero in 1996. For Belgium, the forward spread is 40 bps on average in 1995, and it has decreased steadily towards zero during 1996. Thus, by mid 1996, the market appears to strongly anticipate a narrow monetary union from 1999 with Germany, France, the Netherlands, and Belgium as members. In support of this hypothesis, Figure 2 shows that the 4–7 year ECU forward rate has converged considerably towards the DEM level during the same period. Until EMU is a reality, the ECU remains a basket of currencies, several of which are much weaker than the DEM. During 1997, this is reflected in a 100 bps spread at the three-month maturity, cf. Figure 5.

For the other countries, i.e. the UK, Sweden, Denmark, Finland, Italy, Spain and Portugal, forward-rate spreads are generally too high to be consistent with a certain EMU from 1999. However, they have dropped significantly over the period, especially for Sweden, Italy, Spain and Portugal where, by the end of 1997, forward-rate spreads are about 50 bps, compared to 300–400 bps in early 1995. An informal analysis of the forward spreads suggests that EMU participation is priced with some probability less than 100% for these countries, and that the likelihood of joining EMU has increased substantially between 1995 and 1998. However, this conclusion may be premature since the lower forward-rate spreads could also be explained by improved fundamentals, or perhaps by a tighter coordination of monetary policies among EU countries. The simultaneous drop in short-term interest-rate spreads, as evidenced by Figures 4–6, provides at least some support for the alternative explanation.

While perhaps suggestive, inference along these lines is of limited value. In order to seriously assess the validity of the competing explanations for a positive forward-rate spread, we require a coherent framework for term-structure modeling that explicitly incorporates the EMU plans. With such a model at hand, we are able to gauge how a given probability of joining EMU affects the yield spread to Germany, and this relationship can then be used to back out the probability of joining EMU from the observed yield spreads.

3. A Term-Structure Model with a Monetary Union

The purpose of this paper is explaining yield curves in an international context. Therefore, we cannot use models which take the current yield curve as given, e.g. the Heath, Jarrow and Morton (1992) model, or models that are calibrated to fit the yield curve exactly through time-dependent parameters, as in Hull and White (1990). Instead, we rely on the equilibrium framework proposed originally by Vasicek (1977) and Cox, Ingersoll and Ross (CIR) (1981, 1985). Here, a term-structure model is defined by a stochastic process for the short rate with time-invariant parameters, combined with prespecified market prices of risk (perhaps derived from the utility function of the representative agent). The short-rate process and the risk premia depend on a finite number of state variables, whose dynamics are governed by a system of stochastic differential equations, and the yield curve is derived endogenously as a function of these state variables. For this reason, the models are sometimes referred to as *Markovian* term-structure models.

3.1. INTERNATIONAL TERM-STRUCTURE MODELS

Before presenting the EMU model in Section 3.3, we briefly review some aspects of international (multi-currency) term-structure models. We focus on the relationship between the risk-neutral probability measures of different currencies since this will be important for understanding the numeraire problem mentioned in the introduction. The discussion below is adapted to the standard framework for Markovian term-structure models [see, e.g., CIR (1981)], which is used to develop the EMU model in the sequel. Therefore, the modeling setup is based on the following general assumptions:

- The dynamics of yield curves and exchange rates for all countries in the model are driven by an m-dimensional standard Brownian motion W_t.
- The instantaneous interest rate (short rate) in country i, denoted r_{it}, is a function of a $d \times 1$ vector of state variables X_t, that is $r_{it} = R_i(X_t)$. In most cases, r_{it} is an element of X_t, so the function $R_i(\cdot)$ simply picks the appropriate element from the vector X_t.
- Under the original (true) probability measure, the state variables in X_t are governed by the multivariate stochastic differential equation:

$$dX_t = \mu(X_t)dt + G(X_t)dW_t, \tag{1}$$

  where $\mu(X_t)$ is the drift vector ($d \times 1$), and $G(X_t)$ is a $d \times m$ matrix of diffusion coefficients.
- For each country, there are m market prices of risk, represented by the $m \times 1$ vector $\lambda_i(X_t)$.

Consider a claim with a contractual payoff in currency i at time T given by the function $h_i(X_T)$. Under the no-arbitrage principle, the price at time t, in units of currency i, is given by the following expression:

$$E_t^{Q_i}\left[e^{-\int_t^T R_i(X_s)ds} h_i(X_T)\right], \tag{2}$$

where Q_i is the risk-neutral probability measure for currency i. Under this measure, the state variables are governed by the drift-adjusted SDE,

$$dX_t = [\mu(X_t) - G(X_t)\lambda_i(X_t)]\,dt + G(X_t)dW_t^{Q_i}, \tag{3}$$

and

$$W_t^{Q_i} = W_t + \int_0^t \lambda_i(s)ds \tag{4}$$

is a Brownian motion under Q_i. It is important to understand that the risk-neutral probability measure is always defined for a given currency, so Q_i and Q_j are generally different probability measures. In our setup, this follows from the currency i subscript on the market prices of risk, and hence from the drift in (3).

The Radon-Nikodym derivative defines the relationship between two probability measures, and it facilitates computing expectations like (2) under another probability measure through the change-of-measure technique, see e.g. Geman et al. (1995) for a thorough exposition. In the present case, it is straightforward to show that the Radon-Nikodym derivative between Q_i and Q_j is given by:

$$\frac{dQ_i}{dQ_j} = \exp\left(-\int_0^t \left[\lambda_i(s) - \lambda_j(s)\right]' dW_s^{Q_j} - \frac{1}{2}\int_0^t \left\|\lambda_i(s) - \lambda_j(s)\right\|^2 ds\right) \tag{5}$$

Note that this formula only involves the market prices of risk in countries i and j. The Radon-Nikodym derivative is closely related to the exchange-rate dynamics between currencies i and j. Let $Z_{ij}(t)$ denote the exchange rate between countries i and j, that is the price of currency i in units of currency j. Since $Z_{ij}(t)$ is the price of a traded asset, paying a continuous dividend rate r_{it}, we have

$$dZ_{ij}(t) = \left\{r_{jt} - r_{it}\right\} Z_{ij}(t)dt + Z_{ij}(t)\sigma_{ij}^Z(t)' dW_t^{Q_j}, \tag{6}$$

where $\sigma_{ij}^Z(t)$ is an $m \times 1$ vector of exchange-rate volatilities. Absence of arbitrage across currencies imposes the following restriction on the exchange-rate volatility:

$$\sigma_{ij}^Z(t) = \lambda_j(t) - \lambda_i(t), \tag{7}$$

see Andreasen (1995) or Geman and Souveton (1997) for a proof.[7] This means that assumptions about the market prices of risk implicitly restrict the exchange-rate dynamics, in particular the correlation structure between innovations to the yield curves and the exchange rates.

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[7] A sketch of the proof goes as follows: let $B_i(t) = \exp\left(\int_0^t r_{is} ds\right)$ denote the cumulative money market account for currency i, and note that the Radon-Nikodym derivative dQ_i/dQ_j can also be written as

$$\frac{dQ_i}{dQ_j} = \frac{B_i(t)}{B_j(t)}\frac{Z_{ij}(t)}{Z_{ij}(0)} = \exp\left(\int_0^t \sigma_{ij}^Z(s)' dW_s^{Q_j} - \frac{1}{2}\int_0^t \left\|\sigma_{ij}^Z(s)\right\|^2 ds\right), \tag{8}$$

where the last equality follows from (6). The restriction (7) is obtained by equating terms in the exponent of equations (5) and (8).

3.2. ASSUMPTIONS OF THE EMU MODEL

In addition to the general assumptions of Markovian term-structure models, discussed above, our model is based on a number of specific assumptions about the economy and the transition process towards the monetary union:

A–1 There are $N+1$ currencies, one of which is the Euro. The short rate for the Euro is denoted R_t, and $Z_i(t)$ is the exchange rate between the Euro and currency i at time t (the price of one Euro in units of currency i). Strictly speaking, the Euro does not exist before the start of EMU, but in this case we simply interpret $Z_i(t)$ as a shadow exchange rate.

A–2 The short rate in country i at time t is given by:

$$r_{it} = R_t + y_{it}, \quad i = 1, 2, \ldots, N, \tag{9}$$

where y_{it} is a local spread relative to R_t, the Euro short rate. We assume that R_t and y_{it} are driven by independent stochastic processes. It is important to note that (9) is not a statement about the number of factors in the yield curve since R_t and y_{it} each could be governed by general multifactor processes. The critical assumption is the independence between R_t and y_{it}.

A–3 The Brownian motion, W_t, used in (1) can be partitioned into two subvectors, W_{1t} and W_{2t}, such that R_t is exclusively driven by W_{1t}. Conversely, the joint dynamics of y_{it} and Z_{it} (for all $i = 1, 2, \ldots, N$) are only affected by W_{2t}. Clearly, this is a necessary condition for ensuring independence between R_t and y_{it}. We further assume that the market prices of risk for the sub-vector W_{1t}, denoted λ_{1i}(t), are identical for all $N+1$ currencies. As will be seen in the next section, assumption A–3 is instrumental in dealing with the numeraire problem.

A–4 If country i joins EMU at time τ, future nominal claims are redenominated to Euros at the exchange rate prevailing at time τ. For example, a zero-coupon bond, paying one unit of currency i at time $T > \tau$, is converted to $1/Z_i(\tau)$ bonds denominated in Euros. In this setup, τ corresponds to either the starting date of EMU, or the time when a new country joins EMU at a later stage. Note that τ is a random variable in our model.

A–5 The stochastic properties of R_t (e.g., the unconditional mean or the risk premia) are not affected by the identity of the EMU members, nor do they change when new countries become members of EMU. This assumption could be justified on the grounds that the Maastricht Treaty forces countries to harmonize their economic policies before being permitted to join EMU.

A–6 The decision about entering EMU is governed entirely by exogenous political preferences. This means that, for each country, the probability distribution of the membership date τ is independent of the stochastic processes governing R_t, y_{it} and $Z_i(t)$.

These assumptions are largely "dictated" by practical considerations. If we relax some of the assumptions, it is no longer possible to find an analytical solution for bond prices, and we have to resort to computer-intensive numerical methods, such as finite-difference PDE solutions or Monte Carlo simulations.

3.3. BOND PRICES IN THE EMU MODEL

Consider a zero-coupon bond, denominated in currency i, that matures at time T. The price of this bond, at time t, in units of currency i is denoted $P_i(t, T)$. In this section, we derive an expression for $P_i(t, T)$ which takes into account the possibility of joining EMU.

We begin by conditioning on the date of EMU membership for country i. Because of assumptions A–5 and A–6 above, this does not alter the distribution of R_t and y_{it}, so we can temporarily proceed as though the membership date is "known" by the market. When country i joins EMU at time τ, bonds maturing at a later date ($T > \tau$) are converted to Euro bonds.[8] Specifically, in exchange for each bond denominated in currency i, the investor receives $1/Z_i(\tau)$ Euro bonds. The value of the new bonds, measured in Euros, is

$$P_E(\tau, T)/Z_i(\tau), \tag{10}$$

where $P_E(\tau, T)$ is the time τ price of a Euro bond maturing at time T. In order to calculate $P_i(t, T)$, we translate the future value of the new Euro bonds to units of currency i, that is

$$Z_i(\tau)\left[P_E(\tau, T)/Z_i(\tau)\right] = P_E(\tau, T). \tag{11}$$

Note that the latter expression does not depend on the future exchange rate $Z_i(\tau)$. This is an immediate consequence of assumption A–4, and it simplifies the following analysis considerably.

The current (time t) bond price for a given EMU membership date, which we denote by $P_i(t, T, \tau)$, is obtained from equation (2):

$$P_i(t, T, \tau) = E_t^{Q_i}\left[e^{-\int_t^\tau (R_u + y_{iu})du} P_E(\tau, T)\right], \tag{12}$$

[8] Strictly speaking, there should be a country i subscript on τ. To avoid cluttering the notation, we write τ instead of τ_i. This does not cause any problems or ambiguities, as long as we only consider one country at a time.

where the expectation is taken under Q_i, the risk-neutral probability measure for currency i. The price, at time τ, of a Euro bond maturing at time T is given by:

$$P_E(\tau, T) = E_\tau^{Q_E}\left[e^{-\int_\tau^T R_u du}\right]. \tag{13}$$

It is important to note that the expectation in (13) is taken under the Euro risk-neutral measure, Q_E, which differs from Q_i. Before we can calculate the expectation in (12), we need to establish the relationship between Q_i and Q_E, using the Radon-Nikodym derivative. Because of assumption A–3, the Radon-Nikodym derivative between Q_E and Q_i simplifies to

$$\frac{dQ_E}{dQ_i}$$
$$= \exp\left(-\int_0^T [\lambda_{2E}(s) - \lambda_{2i}(s)]' \, dW_{2s}^{Q_i} - \frac{1}{2}\int_0^T \|\lambda_{2E}(s) - \lambda_{2i}(s)\|^2 \, ds\right) \tag{14}$$

since $\lambda_{1E}(s) = \lambda_{1i}(s)$. Furthermore, the Radon-Nikodym derivative (14) is independent of $W_{1t}^{Q_i}$ and hence R_t, so a change of measure to Q_i in (13) gives

$$\begin{aligned} P_E(\tau, T) &= E_\tau^{Q_i}\left[\frac{dQ_E}{dQ_i} e^{-\int_\tau^T R_u du}\right] \Big/ E_\tau^{Q_i}\left[\frac{dQ_E}{dQ_i}\right] \\ &= E_\tau^{Q_i}\left[e^{-\int_\tau^T R_u du}\right]. \end{aligned} \tag{15}$$

It follows immediately that the Euro bond price can be obtained by taking expectations under any risk-neutral probability measure, not just Q_E. Because of this equivalence, it suffices to use the risk-neutral measure for currency i to calculate $P_i(t, T, \tau)$, despite the redenomination to Euros at time τ. If we substitute the second line of equation (15) into (12), we get

$$\begin{aligned} P_i(t, T, \tau) &= E_t^{Q_i}\left[e^{-\int_t^\tau (R_u + y_{iu}) du} \, . \, E_\tau^{Q_i}\left(e^{-\int_\tau^T R_u du}\right)\right] \\ &= E_t^{Q_i}\left[e^{-\int_t^T R_u du} \, . \, e^{-\int_t^\tau y_{iu} du}\right] \\ &= E_t^{Q_i}\left[e^{-\int_t^T R_u du}\right] \cdot E_t^{Q_i}\left[e^{-\int_t^\tau y_{iu} du}\right] \\ &\equiv P_E(t, T) \cdot D_i(t, \tau). \end{aligned} \tag{16}$$

The second line in (16) follows from the law of iterated expectations, whereas the third line follows from the assumption of independence between R_t and y_{it}. Equation (16) expresses the bond price as the shadow Euro bond price, $P_E(t, T)$, multiplied by a local (domestic) discount factor, $D_i(t,\tau)$. The local discounting is only done until time τ when country i joins EMU.

The final result in (16) seems quite intuitive, but it is important to emphasize that the derivation hinges critically on assumption A–3. In particular, if we relax the restrictions on the market prices of risk for W_{1t}, we can no longer ignore the distinction between Q_i and Q_E (the numeraire issue) in equation (16). The main economic significance of assumption A–3 is that innovations to R_t and $Z_i(t)$ are uncorrelated, cf. equation (7). These results are consistent with Bates' (1998) analysis of the numeraire problem. Using a somewhat different approach, Bates (1998) traces the bias from ignoring the difference between Q_i and Q_E to the covariance between the Euro short rate and the exchange rate. If the covariance is zero, as in our model, there is no bias.

Thus far, we have conditioned on τ, the EMU membership date for country i. To complete the model specification, we need the (time t) probability distribution for the random variable τ. Concretely, bond prices depend on the distribution of τ under the risk-neutral measure. The probability (under Q_i) that country i has not joined EMU by time s is specified as

$$\Pr_{it}(\tau > s) = \exp\left(-\int_t^s \pi_{iu}du\right), \quad \pi_{iu} \geq 0, \tag{17}$$

where π_{iu} is a deterministic (non-stochastic) function of calendar time u. The density function of τ follows by differentiation:

$$p_{it}(s) = \pi_{is}\exp\left(-\int_t^s \pi_{iu}du\right). \tag{18}$$

In the econometrics literature, equation (17) is known as the survivor function while π_{iu} is called the hazard function. Models based on hazard functions are used in other areas of finance, primarily prepayment analysis for mortgage-backed securities, see, e.g., Schwartz and Torous (1989), and the pricing of defaultable bonds, see, e.g., Duffie and Singleton (1995), Lando (1998) and Madan and Unal (1998).

Having specified the distribution for τ, we obtain the final expression for the bond price $P_i(t,T)$ by taking the expectation of (16) over all possible EMU membership dates:

$$\begin{aligned} P_i(t,T) &= \int_t^T P_i(t,T,\tau)p_{it}(\tau)d\tau + P_i(t,T,T)\Pr_{it}(\tau > T) \\ &= P_E(t,T)\left\{\int_t^T D_i(t,\tau)p_{it}(\tau)d\tau + D_i(t,T)\Pr_{it}(\tau > T)\right\} \\ &\equiv P_E(t,T)\cdot F_i(t,T), \end{aligned} \tag{19}$$

where the multiplicative factor $F_i(t,T)$ is given by the following expression:

$$F_i(t,T) = \int_t^T D_i(t,\tau)\pi_{i\tau}e^{-\int_t^\tau \pi_{iu}du}d\tau + D_i(t,T)e^{-\int_t^T \pi_{iu}du}. \tag{20}$$

We have not, yet, made any specific assumptions about the hazard functions in each of the N countries, except that they are deterministic functions of calendar time. Any parameterization of the hazard function must, of course, take into account that EMU membership cannot occur before 1999. A simple way to accommodate this is the following specification:

$$\pi_{iu} = \begin{cases} 0 & \text{for } u < \tau^* \\ \theta_i & \text{for } u \geq \tau^*, \end{cases} \tag{21}$$

where τ^* corresponds to January 1, 1999, and θ_i is a constant parameter. With this hazard function, (20) simplifies to

$$F_i(t,T) = D_i(t,T) \tag{22}$$

if $T \leq \tau^*$, and

$$F_i(t,T) = \theta_i \int_{\tau^*}^{T} D_i(t,\tau) e^{-\theta_i(\tau - \tau^*)} d\tau \; + \; D_i(t,T) e^{-\theta_i(T - \tau^*)}, \tag{23}$$

for $T > \tau^*$. Note the change of the lower limit in the outer integral in (23). The computation of (23) requires numerical integration, except in certain special cases. However, we are only applying numerical integration in one dimension, and this can be done very fast and accurately with Gaussian quadrature, see Press et al. (1992) for details. Because of its tractability, the hazard specification (21) is used in the empirical analysis in Section 5.[9]

Conceptually, the decision to join EMU corresponds to the first jump of a Poisson process which is entirely country-specific. That allows for EMU to occur in several stages, and the order of countries joining EMU can be arbitrary. In some respect, this specification is too general as we have no direct way of ensuring a minimum number of member countries before EMU goes ahead. The practical importance of this limitation is minimal, though, especially since the preliminary analysis of Section 2 suggests that a narrow monetary union with Germany, France and the Benelux countries has been strongly anticipated by the market during most of the period 1995-1998. Accordingly, the main interest has focused on *who* would join EMU in addition to these countries, and *when* this would happen. First and foremost, our theoretical model and the empirical implementation are designed to shed light on these questions.

Since the expression for bond prices (20) only involves risk-neutral expectations and not higher order moments, there is considerable scope for (unspecified) dependence between the N Poisson processes, or the local spreads, y_{it}, for that matter.

[9] If $\theta_i > 0$, the hazard function (21) implies that the country is certain to join EMU sooner or later. Strictly speaking, that assumption may be unrealistic for some EU countries (e.g. Denmark), but the practical consequences of this problem are limited. The maximum maturity available for the empirical analysis is 10 years, so we have no way of estimating the probability of joining EMU after 2008.

We do not need to assume, for example, that the times until Spain and Portugal join EMU are independent, and the same applies to movements in their current yield spreads to the Euro curve.

3.4. YIELD AND FORWARD CURVES

It is usually more convenient to represent the term structure by either the yield (spot) curve or the forward curve. In the empirical analysis, we use zero-coupon yields rather than forward rates because the latter are more difficult to estimate from bond prices or swap rates. However, the forward curve provides a better illustration of the EMU effect, as the numerical example in the next section demonstrates. Hence, we consider both curves in the following. The yield curve, with continuous compounding, in country i is readily obtained from (19):

$$\begin{aligned} Y_i(t, T^*) &= \frac{-\log P_E(t, t+T^*)}{T^*} + \frac{-\log F_i(t, t+T^*)}{T^*} \\ &\equiv Y_E(t, T^*) + S_i^E(t, T^*), \end{aligned} \tag{24}$$

which can be interpreted as the (shadow) Euro curve $Y_E(t, T^*)$, plus a country-specific spread $S_i^E(t, T^*)$, relative to the Euro. Note that the second argument, T^*, in both functions is the time to maturity, not the maturity date.[10] Instantaneous forward rates satisfy a similar decomposition:

$$f_i(t, T^*) = \frac{-\partial \log P_E(t, t+T^*)}{\partial T^*} + \frac{-\partial \log F_i(t, t+T^*)}{\partial T^*} \tag{25}$$

It is straightforward to show that the partial derivative of (20) with respect to T is given by:

$$\frac{\partial F_i(t, T)}{\partial T} = \frac{\partial D_i(t, T)}{\partial T} e^{-\int_t^T \pi_{iu} du}. \tag{26}$$

By rearranging (25), using (26) and the definition in (17), the spread between $f_i(t, T^*)$ and the Euro forward rate $f_E(t, T^*)$ can be written as:

$$f_i(t, T^*) - f_E(t, T^*) = c_i(t, T^*) \cdot f_i^D(t, T^*) \cdot \left[1 - \Pr_{it}(\tau \le t + T^*)\right], \tag{27}$$

where

$$f_i^D(t, T^*) = \frac{-\partial \log D_i(t, t+T^*)}{\partial T^*} \tag{28}$$

[10] We use the following rule regarding maturity: T^* denotes the time *to* maturity, and is used as the second argument for the yield curve $Y_i(t, T^*)$ and the forward curve $f_i(t, T^*)$. On the other hand, T (no asterisk) refers to the maturity date (time *of* maturity), i.e. $T = T^* + t$, and is used as the second argument in bond prices and factors of bond prices, e.g. in $D_i(t, T)$ and $F_i(t, T)$.

$$c_i(t, T^*) = D_i(t, t + T^*) \,/\, F_i(t, t + T^*). \tag{29}$$

Apart from the factor $c_i(t, T^*)$, equation (27) shows that the actual forward-rate spread is equal to the hypothetical forward spread assuming no EMU membership (28), multiplied by the risk-neutral probability that country i has not joined EMU by time $t + T^*$.

3.5. A NUMERICAL EXAMPLE

In the numerical example there are $N = 2$ countries. For simplicity, we assume that the two countries have the same probability of joining EMU at any time. Under the risk-neutral measure, the Euro curve is described by a one-factor CIR model,

$$dR_t = 0.5(0.08 - R_t)dt + 0.04\sqrt{R_t}dW_t^{Q_E},$$

and the local components y_{it} are governed by Vasicek processes,

$$dy_{it} = 1.0(\mu_i - y_{it})dt + 0.02dW_{it}^{Q_i}, \quad i = 1, 2,$$

where the μ_i's are -0.01 and 0.02, respectively. The current values of the state variables are: $R_t = 0.06$, $y_{1t} = -0.01$ and $y_{2t} = 0.01$. Finally, the common hazard function π_t is specified as in (21), with $\tau^* - t$ equal to 2 years. To a first order approximation, θ is the conditional probability of joining EMU within one year, given that the event has not happened previously.

Figures 7 and 8 display, respectively, the forward curves for $\theta = 0.05$ and $\theta = 0.30$. Both figures contain the shadow Euro forward curve, $f_E(t, T^*)$, and two forward curves for each country. The "true local" curve is the forward curve that would prevail if there was no possibility of joining EMU, that is if $\theta = 0$. This is the natural benchmark when gauging the effect of EMU. The actual forward curve is identical to the "true local" curve for maturities less than two years, but for longer maturities it converges gradually towards the Euro forward curve. This means that the forward curve steepens in the low-yielding country and tends to flatten, or become humped, in the high-yielding country. The extent of the steepening and flattening depends on the probability of joining EMU (by a given date) and the difference between the "true local" and the Euro forward curves. The latter can be interpreted as the scope of the EMU effect, and it generally depends on the magnitude of the short-rate spreads, y_{it}, as well as the properties of the stochastic process governing y_{it}, especially the speed of mean reversion and the unconditional mean under Q_i.

In Figure 8, there is virtually full convergence for maturities in excess of 15 years, which is readily explained by the fact that with $\theta = 0.30$, EMU membership is almost certain to occur within 15 years (the probability of this event is 98%). On the other hand, with $\theta = 0.05$ as in Figure 7, there is still a 25% chance that the country does not join EMU during the next 30 years. The 30-year forward-rate

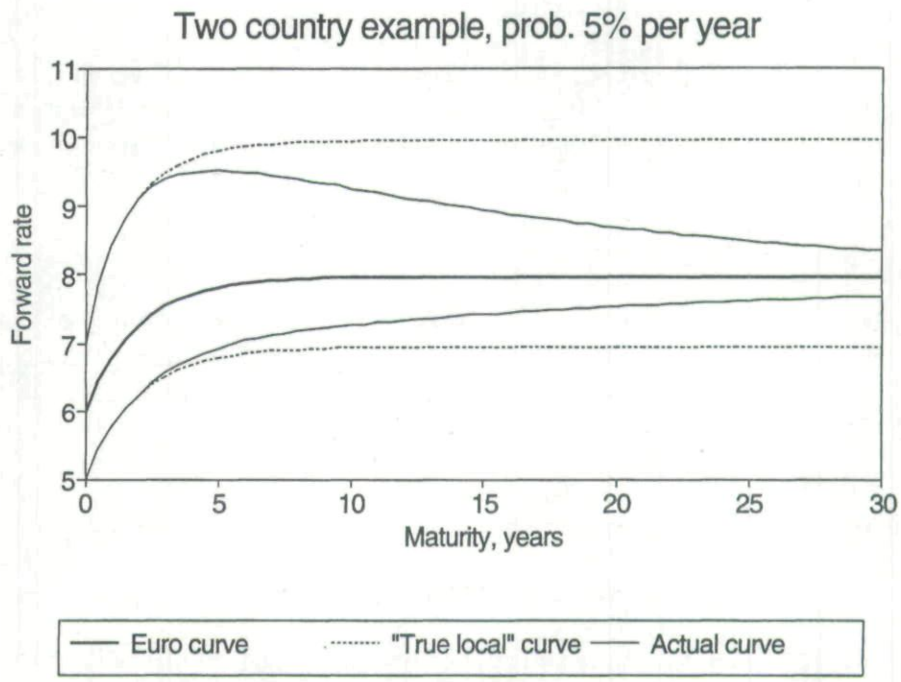

Figure 7. Two country example, probability 5% per year.

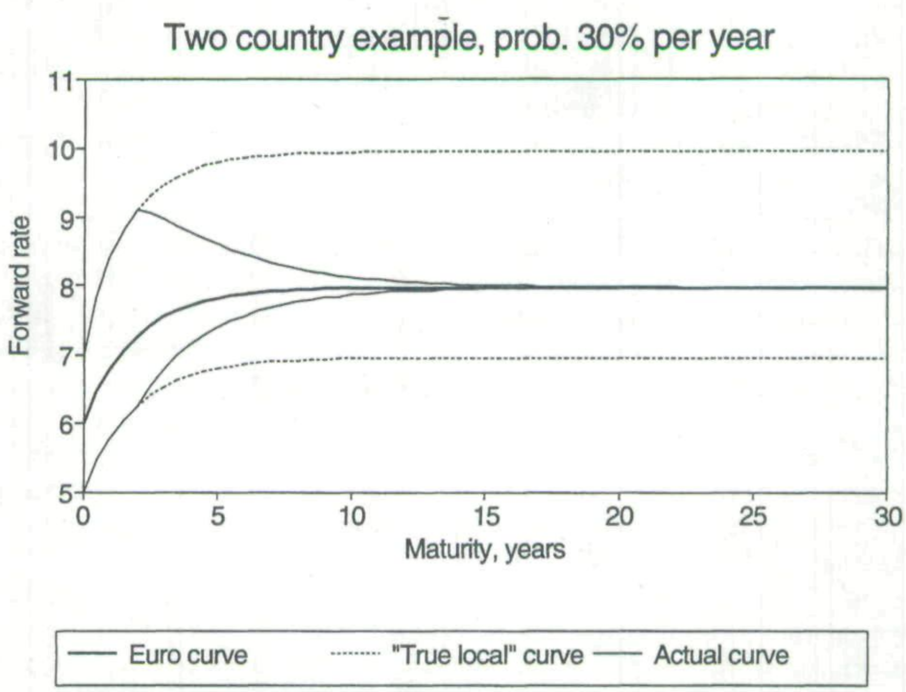

Figure 8. Two country example, probability 30% per year.

spreads vs. the Euro of −29 bps and 37 bps, respectively, are roughly in line with 25% of the corresponding spreads to the "true local" curves, cf. equation (27).

3.6. APPLICATIONS OF THE EMU MODEL

There is a rapidly growing literature on extracting market expectations from prices of financial instruments, see Söderlind and Svensson (1997) for a recent survey. This literature has focused on *inter alia* inflation expectations, market crashes, realignments in target zones, and, most recently, the European monetary union. The empirical analysis in the following sections should be seen in this context, as we focus on estimating the implied probability of joining EMU for different countries. In this context, it is important to emphasize that our analysis produces estimates of the risk-neutral probabilities, whereas at least a casual use of the phrase "market expectations" is naturally associated with the true (original) probability measure. Unless the market price of EMU risk is zero, which is quite unlikely, the two measures will be different. However, this is a generic problem for all methods which back out market expectations from asset prices. Moreover, in the present case, we have no way of estimating the true probabilities.[11]

The EMU model can also be used for valuation of contingent claims whose payoff depend on whether a given country joins EMU or not. In this case, the relevant probability measure is the risk-neutral one. For concreteness, consider an Arrow-Debreu security paying one unit of currency i at time T if country i joins EMU before time T, and zero otherwise. Contracts of this type actually traded on the Iowa Electronic Market for a short period before May 1998, see Bates (1998) for further discussion. We denote the price of the Arrow-Debreu security at time t by $AD_i(t,T)$. It is simpler to determine the price of the mirror security paying one unit of currency i if the country has *not* joined EMU by time T, as the latter calculation does not have to take into account the redenomination to Euros at the stochastic membership date τ_i. Moreover, since a portfolio of both Arrow-Debreu securities is equivalent to a zero-coupon bond in currency i, we obtain the following result:

$$\begin{aligned} AD_i(t,T) &= P_i(t,T) - E_t^{Q_i}\left[e^{-\int_t^T r_{is}ds} I(\tau_i > T)\right] \\ &= P_i(t,T) - P_E(t,T)\cdot D_i(t,T)\cdot E_t^{Q_i}\left[I(\tau_i > T)\right] \qquad (30) \\ &= P_i(t,T)\left\{1 - c_i(t,T-t)\left(1 - \Pr_{it}(\tau_i \le T)\right)\right\}, \end{aligned}$$

[11] To understand this, consider the analogous problem of estimating default probabilities for companies with different credit ratings. The risk-neutral probabilities can be estimated from prices of corporate bonds, as in, e.g., Duffee (1996), whereas the true probabilities can be estimated using data on the observed transitions between rating classes and default [see, e.g., Jarrow et al. (1997)]. Needless to say, the latter type of information is not available in the EMU case.

where $I(\cdot)$ is the indicator function, and $c_i(t, T)$ is defined in equation (29). The second line in equation (30) follows by conditioning on τ and using the law of iterated expectations (as in Section 3.3). In the last line, the expression in curly brackets is the T-period forward risk adjusted probability of joining EMU before time T, and this differs from the risk-neutral probability through the factor $c_i(t, T - t)$.

4. Estimation of the EMU Model

The numerical example in Section 3.5 serves to illustrate the main qualitative features of the EMU term-structure model. Naturally, the next step is taking the model to the data in order to gauge the actual narrowing of yield spreads due to EMU and estimate the implied probability of joining EMU for different countries. Unfortunately, this exercise is fraught with serious identification problems — mainly because the EMU model relies on several theoretical (unobserved) constructs, such as the "true local" yield curve. In the following, we address these problems. First, we present a simplified version of the EMU model which is somewhat easier to estimate. This involves looking at yield spreads, rather than the absolute level of European yield curves. Second, we develop an efficient estimation method which fully exploits the available data.

4.1. IDENTIFICATION PROBLEMS

The German yield curve has been quite steep during most of the period 1995–1998, a phenomenon which could be explained by EMU if the shadow Euro curve lies above the DEM curve. Under this scenario, Germany resembles the low-yielding country in the numerical example in Section 3.5. However, there is always a competing explanation which does not involve EMU: the German yield curve is steep for purely natural reasons, e.g. risk premia or expectations of future short-rate hikes. A similar problem applies to the high-yielding countries in Europe. In Italy, Spain, Portugal and the UK, the yield curve has been relatively flat between 1995 and 1998, and the yield spread to Germany has been decreasing with time to maturity (converging). This could be attributed to EMU, but not necessarily. The critical question is whether the yield spreads display *extraordinary* convergence.

As shown in equation (27), we can think of the actual forward-rate spread as the product of the "true local" spread and the probability of joining EMU.[12] This decomposition explains the main identification problem of the EMU model. The "true local" spread is a hypothetical concept which cannot be directly observed in the market. Nonetheless, without specific assumptions about the size of this spread, we have no way of estimating the probability of joining EMU for a given coun-

[12] The correction factor $c_i(t, T^*)$ in equation (27) is inessential for the following arguments, and therefore it is ignored.

try. To complicate matters further, the appropriate forward-rate spread is defined relatively to the shadow Euro curve, which is also unobserved.

The Euro curve and the "true local" yield curves are, of course, embedded in the observed yield curves, and the question is therefore whether we can obtain estimates of these curves along with the parameters of the EMU model, in particular the hazard parameters. Since pre-1999 maturities are not affected by EMU,[13] we can extrapolate from the short end of the yield curve. This is facilitated with Markovian term-structure models, like the Vasicek (1977) model, that impose a parametric relationship between short and long rates. First, we specify a given term-structure model and estimate the parameters from historical data. Second, we compute the implied EMU probability for each country by comparing the projected "true local" yield curve to the observed yield curve. Generally speaking, this is the underlying idea of the empirical analysis, although the different steps are actually performed simultaneously with non-linear Kalman filtering techniques.

In practice, though, there are limits to the effectiveness of this approach, and some problems remain. First and foremost, it is extremely difficult to identify the shadow Euro curve with the available data. Basically, the data contain information about the extraordinary narrowing of yield spreads for long-term maturities, which is determined by the probability of joining EMU, but not about the position of the Euro curve itself. Therefore, we consider a simpler approach based on the yield spread to Germany. In addition, the following modifications eliminate all cross-country restrictions on the model, and thus permit separate estimations for each country.

Let Germany be the first country ($i = 1$), and define $S_i(t, T^*)$ as the yield spread between country i and Germany. Using (24) we get:

$$\begin{aligned} S_i(t, T^*) &= \left\{Y_E(t, T^*) + S_i^E(t, T^*)\right\} - \left\{Y_E(t, T^*) + S_1^E(t, T^*)\right\} \\ &= S_i^E(t, T^*) - S_1^E(t, T^*), \quad i = 2, 3, \ldots, N \end{aligned} \tag{31}$$

This transformation eliminates the shadow Euro curve from the model, but it does not fundamentally solve the identification problems since $S_i(t, T^*)$ depends on two separate yield spreads vs. the Euro. Hence, we further assume that $y_{1t} = 0$, which means that the shadow Euro curve is identical to the German yield curve. As a result, equation (31) reduces to:

$$S_i(t, T^*) = -\log F_i(t, t + T^*) / T^*, \quad i = 2, 3, \ldots, N, \tag{32}$$

where $F_i(t, t + T^*)$ is given by either (22) or (23), depending on whether the maturity date $t + T^*$ is before or after January 1, 1999 (time τ^*).

[13] Our definition of being "affected by EMU" is limited to the actual fixing of exchange rates, whereby bonds in different currencies become perfect substitutes. Of course, the desire to participate in EMU may influence the economic policies in a particular country and, through that channel, the level of interest rates and the shape of the yield curve. However, this should also affect the short end of the yield curve (or especially that part), contrary to the extraordinary convergence caused directly by EMU.

Note that when $y_{1t} = 0$, the German yield curve is not affected by the advent of EMU, and we cannot estimate an implied EMU probability for Germany. Since there has never been any doubt about Germany's participation in the monetary union, this limitation poses no problems in terms of interpreting the results. In a wider sense, though, our assumption that $S_1^E(t, T^*) = 0$ may appear unrealistic for economic reasons since the Euro will not necessarily be as strong as the Deutschmark. However, for our specific purpose – estimating implied EMU probabilities – it seems to imply little loss of generality.[14]

4.2. DATA SOURCES

In most empirical studies the question of default risk is tacitly ignored, and government bond yields are used to estimate the term-structure model. There is, in fact, little reason to question the implicit assumption that government bonds are default-free, as long as they are denominated in the local currency. This type of debt can always be repaid by the government, if necessary simply by printing money. Clearly, that option does not extend to debt denominated in foreign currencies, e.g. the ECU bonds issued by many European countries. Accordingly, we should expect a default premium in this case.

When the monetary union is formed, government bonds in the member countries are redenominated to Euro bonds, and the money supply in the EMU bloc (Euroland) is controlled by the politically independent European Central Bank. In effect, government bonds issued by the members of EMU are denominated in a "foreign" currency, the Euro. Therefore, because of the loss of monetary sovereignty, joining EMU changes the credit status of government bonds, and this feature is not incorporated in our term-structure model in Section 3. In particular, contrary to a key assumption of the EMU model, yield spreads between different countries would not be completely eliminated, due to country-specific default premia.

Fortunately, there is a simple way of avoiding these problems, namely by using swap rates instead of government bond yields.[15] The parties of a swap contract are in the private sector to begin with, and there is no change in credit status if the country in question joins EMU. Of course, swap contracts are not default-free securities, but Duffie and Singleton (1995, 1997) show that we can still apply the same term-structure models, e.g. the CIR model. In essence, the short rate becomes a credit-risky rate which includes a default premium, but that is the only real

[14] This was verified with the following robustness check: we parameterized the German short-rate spread to the Euro as $y_{1t} = \mu_1$ (a constant), and attempted to estimate the parameter μ_1. The quasi log-likelihood function, described in detail in Section 4.3 and Appendix A, turned out to be very flat over μ_1, which confirms the claim in the text that the shadow Euro curve is virtually impossible to determine. Furthermore, when fixing μ_1 at other values than 0, we observed very little change in the estimates of the other parameters, including the hazard parameter θ_i.

[15] De Grauwe (1996), Favero et al. (1997) and J. P. Morgan (1997) also use swap rates for their analysis of the EMU effect.

change. Most importantly, the credit aspect is not affected when a country joins EMU.

A swap rate is equivalent to the coupon rate of a bullet bond trading at par, but the empirical analysis requires zero-coupon yields as data input. We estimate the zero-coupon yields using the "bootstrap" method with linear interpolation [see Dattatreya and Fabozzi (1995) for a discussion of this method]. The data include money market rates with maturities of 1, 3, 6 and 12 months, and swap rates with 2–5, 7 and 10 years to maturity – both of which are obtained from Datastream.[16] The data cover the following countries (currencies): Germany (DEM), France (FRF), Belgium (BEF), the Netherlands (NLG), United Kingdom (GBP), Denmark (DKK), Sweden (SEK), Finland (FIM), Italy (ITL), Spain (ESP), Portugal (PTE), as well as the ECU. Other EU members are not included in the analysis due to lack of data. The interest rates are sampled daily from January 1990 to August 1998.[17] However, in order to avoid common market microstructure problems – such as discreteness of price changes, missing observations and possible week-day effects – we only use weekly observations (Wednesdays) in the empirical analysis.

4.3. ECONOMETRIC TECHNIQUES

The EMU model simultaneously explains the shape and time-series behavior of the yield curve, so the full structure of the data set should be exploited in the empirical analysis. In several recent papers, an approach based on the *state space* model has been put forth, see *inter alia* Pennacchi (1991), Jegadeesh and Pennacchi (1996), Duan and Simonato (1995), and Chen and Scott (1995). For exponential-affine models this is a very convenient method since the linear Kalman filter is used.[18] The EMU model is non-linear in the m state variables, but with suitable modifications we can still apply Kalman filtering techniques, as we discuss below (and more extensively in Appendix A).

With the modified EMU model, described in Section 4.1, the empirical analysis is based on the yield spread between country i and Germany. We assume that the short-rate spread to Germany,

$$y_{it} = r_{it} - r_{1t}, \quad i = 2, 3, \ldots, N,$$

is governed by a m-dimensional vector of state variables X_{it} under the risk-neutral measure Q_i. Since y_{it} is a short-rate spread, the stochastic process governing y_{it} should permit negative values, and this rules out common short-rate specifications

[16] Parts of the Finnish (FIM) swap-rate data were supplied by the Bank of Finland.

[17] For most countries, the data series are shorter. Specifically, for the countries in question, the first observation dates are as follows: FRF, NLG and BEF (June 1991), DKK (February 1993), SEK (February 1992), FIM and ESP (January 1991), ITL (November 1990), and PTE (November 1994).

[18] An alternative is the "inversion" approach developed by Chen and Scott (1993) and Duffie and Singleton (1997), where the m latent state variables of the model are expressed as implicit functions of m yields (maturities).

that are designed to preserve non-negativity, for example the CIR model. Therefore, we use Gaussian processes in the following. Since Gaussian models belong to the exponential-affine class [Duffie and Kan (1996)], the "true local" discount factor for a bond with time to maturity T^* is given by:

$$D_i(t, t+T^*) = E_t^{Q_i}\left[e^{-\int_t^{t+T^*} y(X_{is})ds}\right] = \exp\left[A_i(T^*) + B_i(T^*)'X_{it}\right], \quad (33)$$

where the scalar $A_i(T^*)$ and the $m \times 1$ vector $B_i(T^*)$ depend on the specific Gaussian process governing y_{it} under Q_i. Further, it follows from equation (33), together with (22) and (23), that the yield spread $S_i(t, T^*)$ is a function of the current time t, the time to maturity T^*, and the state variables X_{it}. In particular, note that the yield spread depends on calendar time t because of the EMU effect.

The data for country i consist of zero-coupon yield spreads for J_i different maturities, observed at n_i different dates:

$$\tilde{S}_i(t_k, T_j^*), \quad j = 1, 2, \ldots, J_i, \quad k = 1, 2, \ldots, n_i$$

Note that we use a tilde to represent the actual data. Under the term-structure model specified above, all J_i yield spreads are functions of only m state variables. In practice, though, minor deviations from the model should be expected due to, for example, rounding of swap rates, non-synchronous trading across different maturities, bid-ask spreads, data-recording error, and the interpolations involved in the bootstrap method. The state-space setup allows for measurement errors in the observed yield spreads, and, as an additional advantage, it readily accommodates partially missing observations.

To simplify the notation, we suppress the i (country) subscript in the remaining part of the paper. By stacking all maturities observed at time t_k in a vector $\tilde{S}_k$, the *measurement* equation of the state-space model can be written as:

$$\tilde{S}_k = S_k(X_k, \psi) + \varepsilon_k; \quad \varepsilon_k \sim N(0, \sigma_\varepsilon^2 I), \quad (34)$$

where ψ is a vector of parameters and X_k is shorthand notation for X_{t_k}. The j'th row of $S_k(X_k, \psi)$ is given by the function $S(t_k, T_j^*)$ from equation (32). Finally, for reasons of parsimony, the individual measurement errors in the vector ε_k are assumed to be independent and identically distributed with a common variance σ_ε^2.

The second part of the state-space model, the *transition* equation, is the exact discrete-time distribution of the state variables. In our case, the transition equation is a Gaussian first-order vector autoregression (VAR):

$$X_k = \Phi_0(\psi) + \Phi_1(\psi)X_{k-1} + \eta_k, \quad \eta_k \sim N(0, V(\psi)). \quad (35)$$

The VAR system matrices, $\Phi_0(\psi)$, $\Phi_1(\psi)$, and $V(\psi)$, are functions of the parameters in the stochastic differential equation governing y_t under the true probability measure, see Langetieg (1980) for the requisite formulae.

In general, the measurement equation is non-linear in the state variables X_k, and this means that the standard Kalman filter method from, e.g., Jegadeesh and Pennacchi (1996) cannot be used. However, when $t_k + T_j^* \leq \tau^*$ – that is, when the time of maturity is before 1999 – the yield spread $S(t_k, T_j^*)$ is a linear function of X_k. This follows directly from equations (32) and (22). To exploit this simplification, we only use pre-1999 maturities for the first n_1 time-series observations in the sample. Specifically, for this part of the sample we delete all data points (yields) for which $t_k + T_j^* > \tau^*$. In the second part of the sample ($n_2 = n - n_1$ observations), all maturities are used in the estimation. Strictly speaking, we are discarding information, but there are two significant advantages of this approach.

First, it speeds up the filtering algorithm and, hence, the computation of the likelihood function since the linear Kalman filter can be applied to the first part of the sample. Second, it conforms naturally to the basic idea of estimating the implied EMU probability by comparing the observed yield spread to Germany with a projection of the "true local" spread. The first n_1 observations in the sample are then used, exclusively, to accumulate information about the "true local" yield spread, and information about the implied EMU probability is only obtained from the second part of the sample (n_2 observations). In Appendix A, we give a detailed account of the linear and non-linear Kalman filter algorithms, as well as the (quasi) likelihood function which is used to estimate the parameters of the term-structure model. The non-linear Kalman filtering algorithm presented in the appendix is similar to the one used recently by Claessens and Pennacchi (1996) and Cumby and Evans (1995) in a related context.

Finally, it is worth emphasizing that we determine the implied EMU probability and the "true local" curve simultaneously for the last n_2 observation. Indirectly, we are still using a form of extrapolation since the level and shape of the post-1999 part of the underlying "true local" curve must be consistent with the pre-1999 part which is directly observable. Thus, the main role of the theoretical term-structure model is enforcing the required consistency between the two segments of the curve. These restrictions depend on the risk-neutral process for the short-rate spread, especially the speed of mean reversion and the long-run level (unconditional mean).

Alternatively, we could have used a two-stage estimation procedure. In the first stage, the parameters describing the "true local" spread are estimated from pre-1999 maturities for the entire sample by means of linear Kalman filtering techniques [see appendix A.1]. Moreover, estimates of the unobserved state variables are produced for each date in the sample. In the second stage, the implied EMU probability is estimated using non-linear regression techniques on the post-1999 maturities. Compared to the QML-IEKF method, described in Appendix A, this approach is somewhat simpler to implement, and the parameter estimates from the first stage are guaranteed to be consistent (the statistical properties of the second stage are still unclear, though). However, the simultaneous procedure used here is superior in terms of accuracy. The estimates of the implied EMU probability depend rather critically on the projection of the "true local" yield spread, which makes it extremely important to obtain good estimates of this spread. The two-stage procedure uses less information when estimating the parameters and state variables describing the "true local" yield spread since post-1999 maturities are deleted from the entire data set, not just the first n_1 observations in the sample. Moreover, the shape of the term structure of yield spreads can contain valuable information about the implied EMU probability. This source of information is not available when using the two-stage approach.[19]

5. Empirical Analysis

We begin by looking at a few summary statistics for the data. Table I contains the average yield spread to Germany for each country (including the ECU) for 1997 and 1998 (January to August). This is obviously an informal analysis, but a few noteworthy conclusions emerge. During most of 1997 and 1998, the ten countries can be divided into three subgroups based on simple measures of yield-curve convergence. In the first group, we find France (FRF), the Netherlands (NLG) and Belgium (BEF), where long-term yield spreads to Germany (DEM) are close to zero. As already discussed in Section 2, we expect EMU to be priced with almost 100% probability for these countries. The second group is the largest one, consisting of the ECU, United Kingdom (GBP), Sweden (SEK), Italy (ITL), Spain (ESP) and Portugal (PTE). For these countries, short-term yield spreads are fairly high, but the yield spread is decreasing with maturity, or at least humped (SEK). Finally, we have Denmark (DKK) and Finland (FIM), where yield spreads are comparatively small for all maturities, but the spread tends to be increasing with maturity. However, in the second half of 1997, long-term yield spreads have dropped further, so by the end of 1997, the DKK and FIM spread curves are also humped.

The second and third groups represent the real challenges since we need to determine whether the observed convergence is truly extraordinary (EMU is priced), or simply due to "natural causes", for example that short-term interest-rate spreads are above average, but expected to drop in the future because of mean reversion.

5.1. SPECIFICATION OF THE TERM-STRUCTURE MODEL

Given the scope of the paper, the focal point of the empirical analysis is, of course, the implied EMU probabilities which only depend on the parameters of the hazard function. However, this does not mean that the term-structure model for the short-rate spread is of secondary importance. In fact, for several countries the estimates of the hazard parameters depend rather critically on the specification of this model, especially with respect to the number of factors. This dependence is by no means

[19] Iversen (1998) has implemented the two-stage approach, using (almost) the same data and term-structure model as in our empirical analysis in Section 5. Overall, her results are quite close to ours. The main difference is that her estimates of the implied EMU probabilities are generally somewhat higher than those reported in Section 5, especially for Finland and the Mediterranean countries.

Table I. Summary statistics for the data†. Yields spread (bps) to DEM 1997 and 1998

Currency	Year	3M	1Y	3Y	4Y	5Y	7Y	10Y
FRF	1997	19	19	−2	−6	−8	−10	−12
FRF	1998	5	2	−1	−1	−1	−1	−1
NLG	1997	3	7	2	0	−2	−4	−6
NLG	1998	−2	−4	0	0	0	0	−1
BEF	1997	14	10	1	0	−1	−2	−1
BEF	1998	7	−3	0	0	0	0	0
ECU	1997	97	83	43	34	29	23	19
ECU	1998	66	33	13	9	7	5	4
GBP	1997	347	349	284	250	221	175	134
GBP	1998	389	345	250	217	190	145	104
DKK	1997	41	50	58	55	53	52	55
DKK	1998	50	34	34	31	28	24	21
SEK	1997	115	127	130	126	119	107	95
SEK	1998	101	84	71	64	58	47	36
FIM	1997	−7	17	21	24	26	30	30
FIM	1998	3	1	0	1	1	1	1
ITL	1997	351	274	177	153	136	113	94
ITL	1998	178	76	30	23	18	14	10
ESP	1997	209	165	102	86	74	60	49
ESP	1998	87	33	14	10	8	6	5
PTE	1997	230	176	113	94	81	68	58
PTE	1998	87	31	18	15	14	12	12

† The summary statistics are only shown for a subset of the data used to estimate the EMU model in Section 5.

surprising, as we estimate the probability of joining EMU by comparing the actual yield spreads with the "true local" spreads, and the latter are obtained through a model-based extrapolation. There is a delicate trade-off in the specification of the term-structure model. On one hand, if the model does not fit the data very well, perceivably the "EMU effect" could be picking up other, omitted factors. Suppose, for example, that the model estimate of the long-term yield spread typically is too large in absolute magnitude. Since the EMU effect works by narrowing the yield spread towards zero, an upward biased estimate of θ could spuriously improve the fit. On the other hand, by including too many factors in the model, we could also get the opposite effect of a downward bias in θ since the EMU-related extraordinary narrowing of the yield spread is "fitted" by those factors. Thus, deliberately overparameterizing the term-structure model is not advisable.

After some experimentation, we settled on a two-factor model. Specifically, under the true probability measure, the dynamics of the short-rate spread are governed by the standard Vasicek (Ornstein-Uhlenbeck) process, but the risk-neutral process is augmented with a second factor, a stochastic market price of risk:

$$dy_t = \{\kappa_1(\mu_1 - y_t) + \kappa_1\lambda_t\}\, dt + \sigma_1 dW_{1t}^Q \tag{36}$$
$$d\lambda_t = \kappa_2(\mu_2 - \lambda_t)\, dt + \sigma_2 dW_{2t}^Q, \tag{37}$$

where W_{1t}^Q and W_{2t}^Q are independent Brownian motions. Appendix B contains a formal derivation of the model, including a closed-form solution for the term structure. This model is very similar to the *central tendency* model proposed by Beaglehole and Tenney (1991) and Jegadeesh and Pennacchi (1996). However, the above model only has six unknown parameters, which is important in view of the computationally intensive estimation method. Furthermore, in the present context it seems more appealing than the standard central tendency model, as the market's risk aversion (or appetite) varies stochastically over time and directly affects the yield spread to Germany.

The hazard function is parameterized as in (21). This means that the probability of joining EMU within τ years from January 1999 is equal to $1 - \exp(-\theta\tau)$, where θ is the hazard parameter. This specification is chosen mainly for reasons of parsimony, and it should be pointed out that there are two inherent limitations. First, the EMU probability is constant over calendar time t, but in Section 5.3 we relax this restriction by proposing a method for estimating a time-varying EMU probability. Second, a more realistic parameterization would probably put a discrete probability mass on January 1, 1999, which is the planned starting date of EMU in the Maastricht Treaty. In order to accommodate this feature, we need at least two parameters in the hazard function, and this could create new identification problems akin to multicollinearity in the linear regression model. Besides, the extension to the time-varying case in Section 5.3 becomes much more difficult. For these reasons, and with the general desire for parsimony in mind, we retain the simple one-parameter specification (21) as the basis for our analysis.

The data input to the estimation is adjusted as follows: We delete post-1999 maturities until 12/31/94 (n_1 observations), while the second part of the sample, where all maturities are included (n_2 observations), covers the remaining period from 1/1/95 to 8/12/98. Note that we only estimate the probability of joining EMU for the latter period, that is 1995–1998. Before August 1993, the European Exchange Rate Mechanism (ERM) operated with relatively tight bands, and short-term interest rates were often quite erratic following central bank interventions in the money market. However, in August 1993 the ERM bands were widened to ± 15 percent, and subsequently the behavior of short-term interest rates has been much more stable. Under normal circumstances this might be modeled by a third factor, but, as noted in the introduction to this section, that approach is problematic. Instead, we delete some of the short-term maturities from the data set. This is done for the UK, as well as for the Nordic and Mediterranean countries.[20] Specifically, until December 1991 the shortest maturity used is two years, i.e. no money market rates are used, whereas between January 1992 and August 1993 we do not use the one-month and three-month rates. As of September 1993, all short-term maturities are included in the estimation.

5.2. ESTIMATION RESULTS WITH A CONSTANT EMU PROBABILITY

Estimation results for the two-factor model are reported in Table II. For FRF, NLG, and BEF the estimate of θ is 50.0, which is the upper boundary imposed to ensure numerical stability. In other words, a narrow monetary union with Germany, France and the Benelux countries is priced with essentially 100% probability. Given the preliminary evidence in Section 2, this should come as no surprise. We hasten to add, though, that a significant caveat applies to this conclusion. Across most maturities, and in particular the short end of the curve, yield spreads are generally quite small in magnitude, and so they may be dominated by the measurement-error (noise) component. In any case, there is relatively little scope for an extraordinary narrowing of yield spreads (the EMU effect).

Consequently, results based on the ECU data seem to be a much better indicator for the prospects of the European monetary union between 1995 and 1998. Before EMU is a reality, the ECU is a basket of 11 currencies, including traditionally high-yielding countries such as Italy, Spain, Portugal and Greece. Once the monetary union is formed, ECUs are automatically converted to Euros, and that is actually a more fundamental change for the fixed income markets than, say, bonds in DEM and NLG being redenominated to the same currency. The estimate of θ for the ECU is 0.7367, which corresponds to a probability of "joining" EMU within 2 years (from January 1999) of about 75%. Since we have restricted the hazard parameters to be time-invariant, this estimate should be interpreted as an average probability over the period 1995-1998, and as such it could conceal a substantial amount of time-variation, cf. the next section. Needless to say, this caveat also applies to the other countries in Table II.

Of the remaining countries, EMU is only priced with positive probability for the UK, Finland and Italy. Moreover, only the UK estimate of θ is statistically significant. For the UK, the implied EMU probability may seem surprisingly high – given the UK opt-out – but for the entire period 1995–1998 there is a significant degree of convergence in the yield spread to Germany. A similar observation applies to SEK, ESP and PTE, but for these countries long-term yield spreads are, on average, too high to be consistent with a positive EMU probability. Finally, the Danish results (where θ is also constrained to 0) can be explained by the fact that the yield spread to Germany is increasing with the time to maturity (i.e., diverging) for most of the

[20] We do not apply the "trimming" of short-term maturities to the core countries (FRF, NLG and BEF) and the ECU, as it tended to generate less satisfactory (plausible) results.

Table II. Two-factor model for the yield spread to Germany (DEM). Constant hazard function (EMU probability) for 1995–1998

Cur.	σ_ε	κ_1	μ_1	σ_1	κ_2	μ_2	σ_2	θ
FRF	0.1251	2.0244	0.0089	0.0305	0.5219	−0.0037	0.0102	50.0†
	(0.004)	(0.543)	(0.005)	(0.006)	(0.200)	(0.005)	(0.002)	–
NLG	0.0550	2.6906	−0.0014	0.0079	0.5112	0.0046	0.0033	50.0†
	(0.002)	(3.475)	(0.001)	(0.003)	(0.994)	(0.003)	(0.003)	–
BEF	0.0873	3.6438	0.0021	0.0226	0.3717	0.0063	0.0081	50.0†
	(0.003)	(0.332)	(0.002)	(0.004)	(0.033)	(0.002)	(0.001)	–
ECU	0.0900	1.3155	0.0124	0.0137	0.1278	0.0011	0.0059	0.7367
	(0.002)	(0.208)	(0.004)	(0.002)	(0.020)	(0.004)	(0.000)	(0.091)
GBP	0.1161	0.8012	0.0208	0.0135	0.2693	−0.0028	0.0189	0.2733
	(0.003)	(0.071)	(0.007)	(0.002)	(0.058)	(0.008)	(0.003)	(0.014)
DKK	0.0966	1.9816	0.0171	0.0249	0.1203	−0.0123	0.0071	0.0†
	(0.003)	(0.135)	(0.011)	(0.004)	(0.011)	(0.011)	(0.001)	–
SEK	0.1224	0.5388	0.0190	0.0201	0.1206	−0.0121	0.0177	0.0†
	(0.003)	(0.101)	(0.010)	(0.008)	(0.016)	(0.009)	(0.002)	–
FIM	0.1175	1.0001	0.0097	0.0177	0.0393	0.0148	0.0144	0.0367
	(0.005)	(0.094)	(0.006)	(0.002)	(0.017)	(0.011)	(0.001)	(0.025)
ITL	0.1442	0.7884	0.0284	0.0215	0.1471	−0.0102	0.0156	0.0231
	(0.004)	(0.060)	(0.009)	(0.003)	(0.011)	(0.009)	(0.001)	(0.013)
ESP	0.1069	0.5867	0.0294	0.0182	0.1559	−0.0150	0.0186	0.0†
	(0.002)	(0.046)	(0.008)	(0.003)	(0.008)	(0.008)	(0.002)	–
PTE	0.1428	1.1219	0.0320	0.0115	0.1229	−0.0273	0.0100	0.0†
	(0.005)	(0.109)	(0.005)	(0.002)	(0.007)	(0.005)	(0.001)	–

† Parameter on the boundary of the parameter space. The unconstrained parameter estimate is either negative, or greater than the upper bound (the latter is imposed for numerical stability only).

Numbers in parenthesis are robust standard errors.

The short-rate spread to Germany is governed by the two-factor model described in Appendix B, see also equations (36) and (37) in Section 5.1, and the specification of the hazard function follows from equation (21). The parameters in the table are estimated by the QML-IEKF technique outlined in Appendix A.

period 1995–1998. By itself, a diverging yield spread does not rule out that EMU is priced since the underlying "true local" yield spread could be diverging even more. However, this hypothesis does not appear relevant for Denmark.

5.3. TIME-VARIATION IN THE IMPLIED EMU PROBABILITY

Restricting the EMU probability to be constant over time is a very convenient assumption for estimation purposes, but hardly a realistic one. As discussed in Section 2, forward-rate spreads have plummeted from 1995 to 1998 in, especially, Sweden and the Mediterranean countries although some of this is certainly attributable to improved fundamentals, as evidenced by the simultaneous drops in short-term interest-rate spreads. Therefore, in this section we relax the previous assumption of a constant implied EMU probability. Modeling the full dynamics of the implied EMU probabilities is somewhat involved. First of all, we need a stochastic hazard function instead of the deterministic specification used so far. To keep things manageable, we retain the basic bond pricing formulae (22) and (23), but replace the constant θ with a time-varying parameter θ_t, whose transition dynamics evolve according to the random walk process:

$$d\theta_t = \sigma_\theta dW_{3t}. \tag{38}$$

This is a simple, but quite powerful, way of building a model with a time-varying EMU probability. To estimate the model, we augment the state vector X_k with the hazard function θ_t and use the non-linear Kalman filter procedure outlined in Appendix A. In order to avoid getting negative EMU probabilities at some dates, we impose the restriction $\theta_t \geq 0$ in the update step of the filtering algorithm (that is, when minimizing the generalized least-squares criterion (54) in Appendix A).

Admittedly, this specification is ad hoc, and it is best viewed as an approximation. In particular, if the hazard function π_{it} is governed by a stochastic process like (38), the bond-pricing formula (23) is no longer correct, but only a first order approximation. This follows because we are ignoring a "Jensen" inequality effect.[21]

[21] In order to sketch the derivation of the correct bond-pricing formula, we use an analogy from the framework for pricing credit-risky bonds presented in Lando (1998). According to this analogy, joining EMU at time τ corresponds to "default", where the claim (a bond in country i) is transformed to Euro bonds whose value are $P_E(\tau, T)$, cf. equation (11). Conceptually, the investor recovers $P_E(\tau, T)$ from the security as "settlement" payment when the country in question joins EMU. If country i has not joined EMU by time T, the bond holder receives one unit of currency i. It follows from Proposition 3.1 in Lando (1998) that the price of this claim (bond) is given by:

$$\begin{aligned} P_i(t,T) = E_t^{Q_i} \Bigg\{ & \left(e^{-\int_t^T R_u du} \right) \times \\ & \left[\int_{\tau^*}^{T} \left(e^{-\int_t^\tau y_{is} ds} \right) \pi_{i\tau} e^{-\int_{\tau^*}^{\tau} \pi_{is} ds} d\tau + \left(e^{-\int_t^T y_{is} ds} \right) e^{-\int_{\tau^*}^{T} \pi_{is} ds} \right] \Bigg\}, \end{aligned} \tag{39}$$

However, various attempts to implement a fully consistent bond-pricing formula failed because of convergence problems when maximizing the log-likelihood function over the parameter vector ψ. The most likely cause of these problems is the wide swings in the implied EMU probabilities that are observed for several countries. Nonetheless, using (38) is clearly superior to having a constant EMU probability as in the previous section.

Having described the computational details, we turn to the empirical results. First, Table III contains the parameter estimates for ECU, DKK, SEK, FIM, GBP, ESP, PTE and ITL.[22] Second, Figures 9–11 show the estimates of the implied EMU probability from 1/1/95 to 8/12/98 which we define as the probability of joining EMU within two years from January 1999. The probability estimates are obtained from the update step of the Kalman filter. For the UK, the EMU probability is fairly constant, varying around 40% most of the time. Quite interestingly, the ECU probability has a sharp drop in the autumn of 1995, which happens to coincide with a resurgence of the debate about the interpretation of the Maastricht convergence criteria. Since December 1995, when the Madrid summit reaffirmed the Maastricht timetable, the ECU probability has risen quickly towards 100%.

Somewhat surprisingly, the implied EMU probabilities for Sweden and Finland are quite large in the second half of 1995 when overall EMU optimism is supposed to be low. This result can be explained by short-rate hikes in Finland and Sweden (see Figures 5 and 6), combined with unchanged or even lower long-term yield spreads, the joint effect of which is consistent with a greater probability of joining EMU. In the beginning of 1996, this development is reversed, and the EMU probability for Finland drops to zero as the FIM yield curve starts to diverge from the DEM curve. As noted above, this situation is rarely consistent with a positive EMU probability. Subsequently, the implied EMU probabilities have increased for both countries. The Danish swap market has witnessed a similar development during 1997, although to a lesser extent, which is quite reasonable in view of the Danish opt-out.

Finally, the results for Italy, Spain and Portugal can be found in Figure 11. The implied EMU probabilities for the Mediterranean countries have moved closely

where τ^* corresponds to January 1, 1999 (we still have $\pi_{it} = 0$ for $t \leq \tau^*$). If the processes governing π_{it}, R_t and y_{it} are mutually independent, the expression (39) simplifies to the previous form (19), albeit with a different probability distribution for the EMU membership date:

$$\Pr_{it}(\tau > s) = E_t^{Q_i}\left[e^{-\int_{\tau^*}^{s} \pi_{iu} du}\right], \quad \text{for } s > \tau^*. \tag{40}$$

The (new) density function $p_{it}(s)$ can be calculated by differentiating (40) with respect to s. Finally, if the stochastic hazard function π_{it} follows the CIR square-root process, we obtain a (complicated) closed-form solution for (40), and computing the bond price $P_i(t, T)$ in (39) only requires numerical integration.

22 The core countries – FRF, NLG and BEF – are not included in this analysis because their yield spreads are very small, so any variation over time in θ_t is likely to be dominated by variation in measurement errors for long-term maturities.

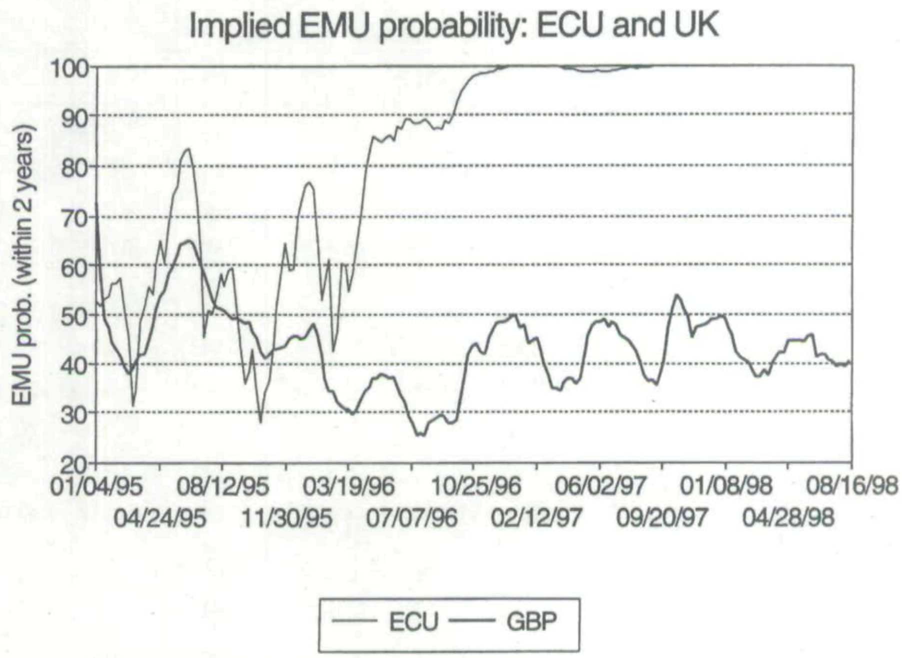

Figure 9. Implied EMU probability: ECU and UK.

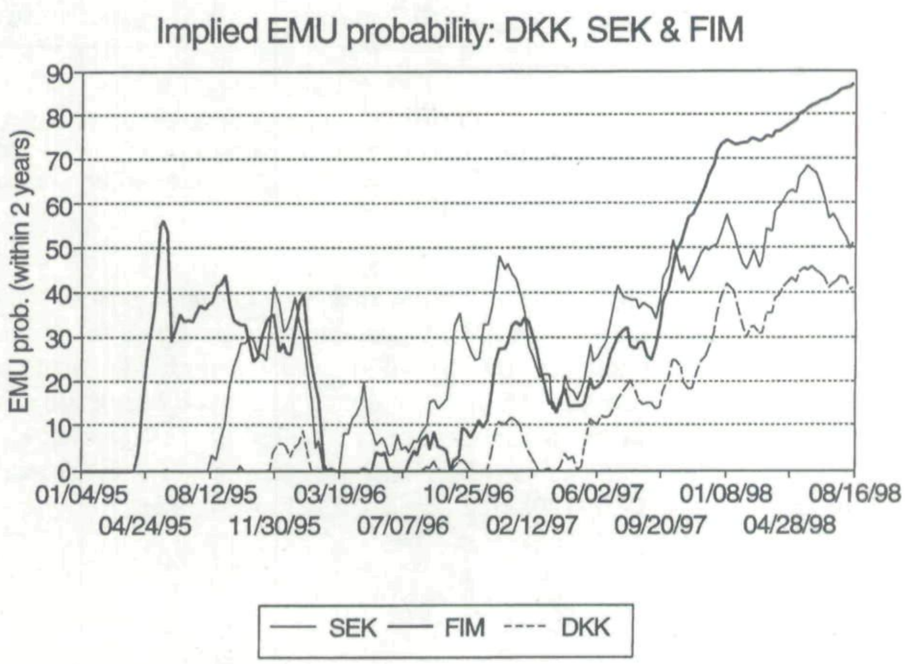

Figure 10. Implied EMU probability: DKK, SEK and FIM.

Table III. Two-factor model for the yield spread to Germany (DEM). Time-varying hazard function for 1995–1998

Cur.	σ_ε	κ_1	μ_1	σ_1	κ_2	μ_2	σ_2	σ_θ
ECU	0.0852	1.9402	0.0120	0.0152	0.2007	−0.0014	0.0064	0.8536
	(0.002)	(0.315)	(0.003)	(0.002)	(0.022)	(0.003)	(0.000)	(0.121)
GBP	0.1124	0.8623	0.0201	0.0136	0.2882	−0.0014	0.0192	0.1709
	(0.003)	(0.103)	(0.007)	(0.002)	(0.073)	(0.007)	(0.003)	(0.023)
DKK	0.0893	1.8660	0.0174	0.0244	0.2802	−0.0094	0.0097	0.1901
	(0.003)	(0.140)	(0.011)	(0.003)	(0.026)	(0.011)	(0.001)	(0.026)
SEK	0.0838	0.5656	0.0204	0.0157	0.6395	0.0030	0.0475	0.3590
	(0.002)	(0.030)	(0.008)	(0.004)	(0.032)	(0.008)	(0.006)	(0.034)
FIM	0.1047	1.2128	0.0098	0.0184	0.0393	0.0375	0.0149	0.2914
	(0.005)	(0.087)	(0.005)	(0.002)	(0.015)	(0.014)	(0.001)	(0.038)
ITL	0.1266	0.5667	0.0268	0.0207	0.4826	0.0030	0.0283	0.1279
	(0.005)	(0.031)	(0.011)	(0.002)	(0.032)	(0.011)	(0.002)	(0.013)
ESP	0.0855	0.5087	0.0294	0.0183	0.4677	−0.0023	0.0329	0.2442
	(0.002)	(0.022)	(0.009)	(0.003)	(0.020)	(0.009)	(0.003)	(0.027)
PTE	0.1012	0.6195	0.0335	0.0113	0.5716	−0.0059	0.0224	0.3479
	(0.005)	(0.118)	(0.007)	(0.002)	(0.115)	(0.007)	(0.005)	(0.036)

Numbers in parenthesis are robust standard errors.

The short-rate spread to Germany is governed by the two-factor model described in Appendix B, see also equations (36) and (37) in Section 5.1, and the specification of the hazard function follows from equation (21) combined with the stochastic process (38). The parameters in the table are estimated by the QML-IEKF technique outlined in Appendix A.

together, with Italy being somewhat below Spain and Portugal most of the time. Note that before August 1996, the EMU probabilities are quite low, and sometimes even zero, for the three countries. Thus, until the second half of 1996, the forward-rate spreads for ITL, ESP and PTE are simply too large (200 bps or more) to be consistent with anything but a minor chance of joining EMU. However, as their long-term yield spreads to Germany have narrowed further during the second half of 1996 and most of 1997–1998, the implied EMU probabilities have increased rapidly.

6. Conclusion

In January 1999, the European monetary union started with 11 member countries, as was formally decided by the EU Council of Ministers in May 1998. Clearly,

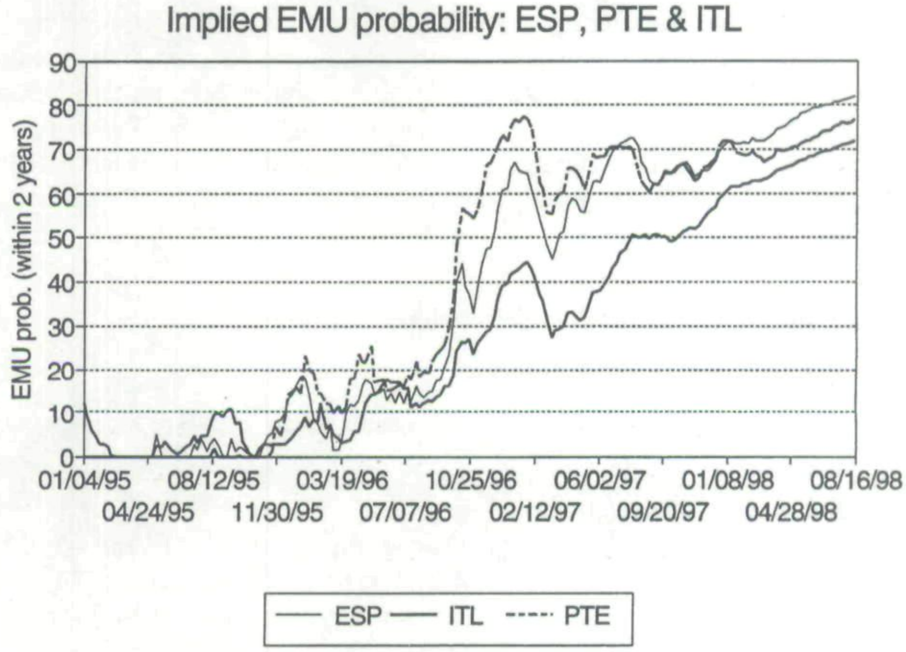

Figure 11. Implied EMU probability: ESP, PTE and ITL.

the May 1998 decision resolved the bulk of the uncertainty surrounding the EMU project, and if our model is correctly specified, the estimates of the implied EMU probabilities should, of course, be strongly affected by this event. The results for the May through August 1998 period in Figures 9–11 show a clear dichotomy between the "out" countries (Denmark, Sweden, and the United Kingdom), and the "in" countries (Finland, Italy, Spain and Portugal). In the "out" countries, the implied EMU probabilities have decreased, whereas further increases have taken place for the latter group.

However, contrary to what might be expected, the implied EMU probabilities are still below 100 percent by August 1998 for the "in" countries. In theory, one could argue that this reflects the risk of EMU breaking up before it even started, but it is more likely an artefact of the estimation method. First, the filtering procedure tends to smooth out some of the variation over time, so we do not see an instant jump to 100 percent, but rather a gradual adjustment. Second, and probably more significant, as we approach January 1999 the data become less and less informative about the "true local" yield curve since this curve is projected from pre-1999 maturities. Accordingly, even a small amount of noise in the data could lead to a relatively large estimation error for the implied EMU probability. As we have pointed out elsewhere in the paper, this is an inherent weakness of our model and the estimation method, cf. the discussion about the core countries (FRF, NLG and BEF) in Section 5.3. In essence, our modeling efforts are much better suited to countries where yield spreads are distinctly positive, but narrowing across maturities. Fortunately, this scenario is representative of the remaining "out" countries, Denmark, Sweden and the United Kingdom. Moreover, it is reasonable to conjecture that the future potential member countries from Eastern Europe will have yield spreads similar to Italy, Spain, and Portugal between 1995 and 1997 (not necessarily of the same magnitude, though). Therefore, the issues discussed in this paper should remain relevant for several years to come and, in all likelihood, well beyond 1999.[23]

Appendix A: Kalman Filter Estimation

A.1 THE LINEAR FILTERING PROBLEM

In the first part of the sample, the measurement equation (34) reduces to:

$$\tilde{S}_k = d_k(\psi) + Z_k(\psi)X_k + \varepsilon_k, \tag{41}$$

where the j'th row of $d_k(\psi)$ and $Z_k(\psi)$ are given by $-A(T_j^*)/T_j^*$ and $-B(T_j^*)'/T_j^*$, respectively. The functions $A(T^*)$ and $B(T^*)$ are defined in equation (33). With the prediction-error decomposition, the log-likelihood function for the first part of the sample (n_1 observations) can be written as:

$$\log L_1(\tilde{S}_1, \ldots, \tilde{S}_{n_1};\ \psi) = \sum_{k=1}^{n_1} -\frac{M_k}{2}\log(2\pi) - \frac{1}{2}\log|F_{1k}| - \frac{1}{2}v_{1k}' F_{1k}^{-1} v_{1k}, \tag{42}$$

where $M_k = \dim(v_{1k})$. The prediction errors

$$v_{1k} = \tilde{S}_k - E\left(\tilde{S}_k \mid \tilde{S}_{k-1}, \tilde{S}_{k-2}, \ldots\right),$$

and their covariance matrix F_{1k} are computed as a by-product of the linear Kalman recursions, see e.g. Harvey (1989). For convenience, we restate the relevant formulae below. First, the prediction step is given by:

$$\hat{X}_{k|k-1} = \Phi_0 + \Phi_1 \hat{X}_{k-1} \tag{43}$$

with mean square error (MSE) matrix

$$\Sigma_{k|k-1} = \Phi_1 \Sigma_{k-1} \Phi_1' + V \tag{44}$$

[23] Two modifications of the EMU model are needed before we can successfully address the future applications. First, there is no longer a natural earliest membership date τ^*, like January 1999, so the hazard specification in equation (21) must be modified. This could create new identification problems, as having $\tau^* \gg t$ was an important feature of our method for estimating the benchmark "true local" yield curve through the model-based extrapolation from short-term maturities. Second, there are certain data problems as the DEM yield curve is no longer available after January 1999. Clearly, the new reference "country" is Euroland, represented by the Euro yield curve, but in order to use historical data, we need to splice together the DEM and Euro data sets. The "DEM equals Euro" assumption, used throughout the paper, might become problematic in this regard.

The dependence on the parameter vector ψ is suppressed here. Second, in the update step the additional information contained in $\tilde{S}_k$ is used to obtain a more precise estimator of X_k, namely:

$$\hat{X}_k = E\left(X_k \mid \tilde{S}_k, \tilde{S}_{k-1}, \ldots\right) = \hat{X}_{k|k-1} + \Sigma_{k|k-1} Z_k' F_{1k}^{-1} v_{1k}, \tag{45}$$

$$\Sigma_k = \left(\Sigma_{k|k-1}^{-1} + Z_k' H_k^{-1} Z_k\right)^{-1} \tag{46}$$

where $H_k = \sigma_\varepsilon^2 \cdot I_k$, and

$$v_{1k} = \tilde{S}_k - \left(d_k + Z_k \hat{X}_{k|k-1}\right) \tag{47}$$

$$F_{1k} = Z_k \Sigma_{k|k-1} Z_k' + H_k. \tag{48}$$

Since the state-space model is linear, and all error terms are normally distributed, the filtering algorithm is optimal in a MSE sense. In addition, the estimation criterion (42), which is obtained as a by-product of the Kalman recursions, is the exact log-likelihood function for the data.

A.2 THE NON-LINEAR FILTERING PROBLEM

In the second part of the sample, there is a non-linear relationship between the vector of yield spreads $\tilde{S}_k$ and the state variables X_k because of the EMU effect. However, for a non-linear state-space model, the optimal filtering recursions and the exact likelihood function can only be computed with time-consuming numerical methods, see Kitagawa (1987). Instead, we implement an approximate filtering technique which is similar to the one used by Claessens and Pennacchi (1996) and Cumby and Evans (1995). Compared to these papers, we modify the update step of the non-linear Kalman filter recursions, using an iterated version of the extended Kalman filter. Like the linear Kalman filter, discussed above, the method consists of a prediction step and an update step, and as a by-product it delivers a quasi log-likelihood function of the "Gaussian" form (42).

First, since the transition equation has not changed, we use the same expression for the prediction step, that is (43) and (44). Second, the conditional log-likelihood function is approximated by:

$$\begin{aligned}
&\log L_2(\tilde{S}_{n_1+1}, \ldots, \tilde{S}_n \mid \tilde{S}_1, \ldots, \tilde{S}_{n_1};\ \psi) \\
&\quad = \sum_{k=n_1+1}^{n} \log f(\tilde{S}_k \mid \tilde{S}_{k-1}, \tilde{S}_{k-2}, \ldots;\ \psi) \\
&\quad \simeq \sum_{k=n_1+1}^{n} -\frac{M_k}{2}\log(2\pi) - \frac{1}{2}\log|F_{2k}| - \frac{1}{2} v_{2k}' F_{2k}^{-1} v_{2k},
\end{aligned} \tag{49}$$

where

$$v_{2k} = \tilde{S}_k - S_k(\hat{X}_{k|k-1}), \tag{50}$$

and

$$F_{2k} = \frac{\partial S_k(\hat{X}_{k|k-1})}{\partial X'} \Sigma_{k|k-1} \frac{\partial S_k(\hat{X}_{k|k-1})'}{\partial X} + H_k \tag{51}$$

Note that the covariance matrix for v_{2k} is computed as a first order Taylor series approximation around $X = \hat{X}_{k|k-1}$.

Finally, there is the update step which completes the non-linear Kalman filter recursion. To grasp the intuition behind our non-linear filtering algorithm, it is useful to consider an alternative interpretation of the update step in the linear case. Duncan and Horn (1972) show that the linear update step (45) is equivalent to solving the generalized least-squares problem:

$$\begin{aligned} F_L(X) \;=\; & \left(X - \hat{X}_{k|k-1}\right)' \Sigma_{k|k-1}^{-1} \left(X - \hat{X}_{k|k-1}\right) + \\ & \left(\tilde{S}_k - d_k - Z_k X\right)' H_k^{-1} \left(\tilde{S}_k - d_k - Z_k X\right). \end{aligned} \tag{52}$$

Basically, this says that the update step is the best linear projection, whereas (45) is stated as the conditional expectation of X_t, given the observed data. With (52) as the main motivation, we propose the following update step for the non-linear state space model:

$$\hat{X}_k = \arg\min_X \;\; F_{NL}(X), \tag{53}$$

where

$$\begin{aligned} F_{NL}(X) \;=\; & \left(X - \hat{X}_{k|k-1}\right)' \Sigma_{k|k-1}^{-1} \left(X - \hat{X}_{k|k-1}\right) \; + \\ & \frac{1}{\sigma_\varepsilon^2} \sum_{l=1}^{M_k} \left(\tilde{S}_{kl} - S_{kl}(X)\right)^2. \end{aligned} \tag{54}$$

Note that computing $\hat{X}_k$ requires a numerical optimization method, such as the Gauss-Newton algorithm. We define the MSE matrix for $\hat{X}_k$ as,

$$\Sigma_k = \left(\Sigma_{k|k-1}^{-1} + \frac{1}{\sigma_\varepsilon^2} \frac{\partial S_k(\hat{X}_k)'}{\partial X} \frac{\partial S_k(\hat{X}_k)}{\partial X'}\right)^{-1}, \tag{55}$$

which may be recognized as the (asymptotic) covariance matrix if $\hat{X}_k$ is viewed as a standard parameter estimator in a non-linear least squares setting.

The filtering recursions defined by (43)–(44) and (53)–(55) are known in the engineering literature as the Iterative Extended Kalman Filter (IEKF). Jazwinski (1970) investigates the properties of the IEKF and concludes that the filter is quite effective in dealing with non-linearities in the measurement equation. In particular, it seems to dominate the ordinary extended Kalman filter (EKF), which corresponds to linearizing the measurement equation around $X = \hat{X}_{k|k-1}$ and using the linear Kalman filter recursions.

In summary, we have outlined a method for computing the quasi log-likelihood function for the the entire data set:

$$\begin{aligned} \log L(\psi) &= \log L_1(\psi) + \log L_2(\psi) \\ &= \sum_{k=1}^{n} -\frac{M_k}{2}\log(2\pi) - \frac{1}{2}\log|F_k| - \frac{1}{2}v_k' F_k^{-1} v_k. \end{aligned} \tag{56}$$

We conclude with a brief discussion of the properties of the proposed filtering and estimation procedure. There are two different approximations involved in using (56) as the estimation criterion. First, the true log-likelihood function for the second part of the sample is not of the Gaussian form (49). This follows from the non-linear transformation of X_k in the measurement equation. Second, $E(v_{2k}) = 0$ and $\mathrm{Cov}(v_{2k}) = F_{2k}$ only hold as approximate relationships because we rely on a linearized model when constructing the prediction errors in (50). Hence, it is not possible to prove that our quasi maximum likelihood (QML) procedure is a consistent econometric estimator. However, there does not seem to exist a consistent estimation method which, at the same time, is computationally tractable. Furthermore, and most importantly, the Monte Carlo evidence in Lund (1997) shows that the QML-IEKF performs quite well for a related problem (in the sense that there are no discernible small-sample biases for the parameter estimates).

Appendix B: Bond Price Formula for the Two-factor Model with a Stochastic Market Price of Risk

We assume that the bond price $P(t, T)$ is a function of two state variables, the short rate r_t, and a stochastic market price of risk λ_t. Their joint dynamics – under the original (true) probability measure – are governed by two independent Ornstein-Uhlenbeck processes,

$$\begin{aligned} dr_t &= \kappa_1(\mu_1 - r_t)dt + \sigma_1 dW_{1t} \\ d\lambda_t &= \kappa_2(\mu_2 - \lambda_t)dt + \sigma_2 dW_{2t} \end{aligned}$$

By Ito's lemma, the bond price evolves according to

$$dP(t,T) = \mu_P(t,T)dt + \frac{\partial P}{\partial r}\sigma_1 dW_{1t} + \frac{\partial P}{\partial \lambda}\sigma_2 dW_{2t}$$

where

$$\mu_P(t,T) = \frac{\partial P}{\partial r}\kappa_1(\mu_1 - r_t) + \frac{\partial P}{\partial \lambda}\kappa_2(\mu_2 - \lambda_t) + \frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\sigma_1^2 + \frac{1}{2}\frac{\partial^2 P}{\partial \lambda^2}\sigma_2^2 \quad (57)$$

Absence of arbitrage implies that the excess bond return, for any maturity T, satisfies the restriction:

$$\mu_P(t,T) = r_t P(t,T) + \alpha_1(\cdot)\frac{\partial P}{\partial r}\sigma_1 + \alpha_2(\cdot)\frac{\partial P}{\partial \lambda}\sigma_2, \quad (58)$$

where $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ are independent of the maturity T, see CIR (1981). We specify the risk premia in (58) as follows: $\alpha_1(\cdot) = -(\kappa_1/\sigma_1)\lambda_t$, and $\alpha_2(\cdot) = 0$. Consequently,

$$\mu_P(t,T) = r_t P(t,T) - \lambda_t \kappa_1 \frac{\partial P}{\partial r}, \quad (59)$$

and λ_t can be interpreted as a stochastic market price of short-rate risk (apart from the sign and scaling). If we substitute (59) into (57), we get a partial differential equation (PDE) for the bond price:

$$\begin{aligned}\frac{1}{2}\frac{\partial^2 P}{\partial r^2}\sigma_1^2 + \frac{1}{2}\frac{\partial^2 P}{\partial \lambda^2}\sigma_2^2 + \frac{\partial P}{\partial \lambda}\kappa_2(\mu_2 - \lambda) + \\ \frac{\partial P}{\partial r}\kappa_1(\mu_1 - r + \lambda) + \frac{\partial P}{\partial t} - rP(t,T) = 0,\end{aligned} \quad (60)$$

subject to the boundary condition $P(T,T) = 1$. The Feynman-Kac representation of the solution is given by

$$P(t,T) = E_t^Q\left[e^{-\int_t^T r_s ds}\right],$$

where the expectation is taken under the probability measure corresponding to the risk-neutral stochastic process

$$\begin{aligned} dr_t &= \{\kappa_1(\mu_1 - r_t) + \kappa_1\lambda_t\}\,dt + \sigma_1 dW_{1t}^Q \\ d\lambda_t &= \kappa_2(\mu_2 - \lambda_t)dt + \sigma_2 dW_{2t}^Q \end{aligned}$$

Since the model is Gaussian, the expression for the bond price, i.e. the solution of (60), has the well-known exponential-affine form:

$$P(t, t+\tau) = \exp\left[A(\tau) + B(\tau)r_t + C(\tau)\lambda_t\right]$$

The scalar functions $A(\tau)$, $B(\tau)$ and $C(\tau)$ satisfy the following system of ordinary differential equations (ODEs):

$$\begin{aligned}
A'(\tau) &= \kappa_1\mu_1 B(\tau) + \kappa_2\mu_2 C(\tau) + \frac{1}{2}\sigma_1^2 B^2(\tau) + \frac{1}{2}\sigma_2^2 C^2(\tau) && (61)\\
B'(\tau) &= -\kappa_1 B(\tau) - 1 && (62)\\
C'(\tau) &= -\kappa_2 C(\tau) + \kappa_1 B(\tau), && (63)
\end{aligned}$$

with boundary conditions $A(0) = 0$, $B(0) = 0$ and $C(0) = 0$. First, it is easily verified that

$$B(\tau) = \frac{e^{-\kappa_1\tau} - 1}{\kappa_1} \tag{64}$$

$$C(\tau) = \frac{e^{-\kappa_2\tau} - 1}{\kappa_2} - \frac{e^{-\kappa_1\tau} - e^{-\kappa_2\tau}}{\kappa_1 - \kappa_2} \tag{65}$$

We note, in passing, that (64) and (65) are exactly the same expressions as in the double-decay model, cf. Jegadeesh and Pennacchi (1996). Second, $A(\tau)$ is obtained by integrating the right hand side of (61):

$$\begin{aligned}
A(\tau) = {} & \int_0^\tau A'(s)ds = -(\mu_1 + \mu_2)\tau + \\
& \left(\mu_1 - \frac{\kappa_2\mu_2}{\kappa_1 - \kappa_2}\right)\frac{1 - e^{-\kappa_1\tau}}{\kappa_1} + \mu_2\left(1 + \frac{\kappa_2}{\kappa_1 - \kappa_2}\right)\frac{1 - e^{-\kappa_2\tau}}{\kappa_2} + \\
& \frac{1}{2}\frac{\sigma_1^2}{\kappa_1^2}\left(\tau - 2\frac{1 - e^{-\kappa_1\tau}}{\kappa_1} + \frac{1 - e^{-2\kappa_1\tau}}{2\kappa_1}\right) + \\
& \frac{1}{2}\sigma_2^2\left\{\frac{\tau}{\kappa_2^2} + \frac{2}{\kappa_2(\kappa_1 - \kappa_2)}\frac{1 - e^{-\kappa_1\tau}}{\kappa_1} + \frac{1}{(\kappa_1 - \kappa_2)^2}\frac{1 - e^{-2\kappa_1\tau}}{2\kappa_1}\right. \\
& \quad - 2\left(\frac{1}{\kappa_2(\kappa_1 - \kappa_2)} + \frac{1}{(\kappa_1 - \kappa_2)^2}\right)\frac{1 - e^{-(\kappa_1 + \kappa_2)\tau}}{\kappa_1 + \kappa_2} \\
& \quad - 2\left(\frac{1}{\kappa_2^2} + \frac{1}{\kappa_2(\kappa_1 - \kappa_2)}\right)\frac{1 - e^{-\kappa_2\tau}}{\kappa_2} \\
& \quad \left. + \left(\frac{1}{\kappa_2^2} + \frac{2}{\kappa_2(\kappa_1 - \kappa_2)} + \frac{1}{(\kappa_1 - \kappa_2)^2}\right)\frac{1 - e^{-2\kappa_2\tau}}{2\kappa_2}\right\}. && (66)
\end{aligned}$$

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