



Corporate Hedging: The Relevance of Contract Specifications and Banking Relationships

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Abstract. This article examines the contribution of hedging to firm value and the cost of hedging in a unified framework. Optimal hedging and firm value are explicitly linked to firm risk, the type of debt covenants and the relative priority of the hedging contract. It is shown that in some cases hedging is possible only if the counterparty to the forward contract also holds a significant portion of the debt. Also, the spread in the hedging contract reduces the optimal amount of hedging to less than the minimum-variance hedge ratio. Among other results this article elucidates why some firms hedge using forward contracts while other firms hedge in the futures markets, as well as why higher priority forward contracts are more efficient hedging vehicles.

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1. Introduction

Derivative contracts are used extensively by corporations for risk management purposes.¹ When a corporation enters into an hedging agreement there is always the possibility that it might default on its obligations. Not surprisingly, hedging contracts deal with the risk of default in a variety of different ways. Quoted forward prices, for instance, include a spread that widens with the maturity of the contract, and the 'Master Agreement' governing swap contracts pays detailed attention to the rules in the event of default.² Often, corporations with low credit standing are excluded from the forward market unless collateral is provided. In contrast to this practice, models of corporate hedging in the finance literature typically do not analyze the effect of credit risk on the pricing of the hedging contract and on the decision to hedge. For example, Stulz (1984), Smith and Stulz (1985) and Froot,

¹ See Rawls and Smithson (1990) and Bodnar, Hayt and Marston (1996).

² Sheng, Sunderasan and Wang (1993) document the importance of credit reputation in the swaps market in several ways. They find that bid-offer spreads are sensitive to the credit standing of the swaps dealer and they report that banks formed separately capitalized, credit enhanced subsidiaries to trade with high quality clients.

Scharfstein and Stein (1993) assume that the hedging contract is either priced ignoring default risk, or that the price is set exogenously to the particular hedging application for which it is used. Each assumption raises serious problems. The former simply ignores the issue and cannot address the question of whether the motivation for hedging to reduce risk remains valid when the price of the hedging contract reflects the risk that is being hedged. The latter raises complex questions of moral hazard and potential rationing, since low grade (high default risk) hedgers will find a particular rate more appealing than high grade hedgers.³

Ignoring the impact of default might seem reasonable as, in a well functioning capital market, the credit spread compensates the bank providing the hedging contract for the risk incurred: hedging reallocates risks and the spread is the market price of the risk allocated to the bank. However, by changing the price of hedging the spread has important implications on the behavior of the hedging corporation.⁴ Given the state of the literature, it seems fair to say that the question of how demand for hedging is determined in equilibrium when the cost of hedging reflects the particular situation of the hedging corporation is yet to be answered. As Stulz (1984) has put it, a complete characterization of hedging policies requires the explicit consideration of hedging costs, as well as how the capital structure of the firm has an impact on hedging when the costs of financial distress are taken into account.⁵

In this article, we examine optimal hedging and the cost of hedging in a unified framework. Hedging to avoid the deadweight costs of bankruptcy requires the payment of a spread that reflects the level of default risk associated with the corporation seeking to hedge. The terms in the hedging contract have an effect on the management's incentive to pursue a particular hedging strategy and, therefore, on the value of the firm, as well as on the default risk facing the firm. As a result, the hedging decision and the value of the corporate claims become interlinked variables. The amount of hedging that maximizes value cannot be determined without knowing the firm's value, which reflects the probability that the firm defaults. But to compute the value of the firm it is necessary to know the effects of the hedging

³ Jordan and Morgan (1990) also model demand for hedging in the presence of default risk. In their work demand for hedging is a function of the credit risk of the broker but the amount of hedging and the financial condition of the hedger do not affect this risk, which is assumed exogenous. In another paper, Campbell and Kracaw (1990) model hedging to reduce the incentives for equityholders to increase the risk of the firm's investments after the terms of the debt have been determined. In their model hedging affects the incentives but the cost of hedging as a function of the incentives is ignored.

⁴ In this model, the effects of hedging on the firm's operations are captured in the choice of the abandonment point. A case of how hedging interacts with the firm's operating and investment policies is presented in Froot, Scharfstein and Stein (1993) and Mello, Parsons and Triantis (1995). Although both of these models deal with uncertainty about future investment and financing that induces agency effects or involve market imperfections, neither considers the cost of hedging in terms of the firm's value. In general, operating flexibility makes hedging more valuable and it increases the demand for hedging to reduce the sensitivity of risky debt to positive value investments, thereby reducing incentives for equityholders to underinvest, see Bessembinder (1991).

⁵ See the concluding remarks in Stulz (1984).

policy on the decision to default and, therefore, on the terms on which the firm can obtain hedging.

In analyzing the decision to hedge we incorporate the conflicts of interest arising between the different parties in the firm, as well as between the corporation and the hedging counterparty. We find that the viability of hedging is related to the type of covenants included in the debt contract. When the debtholders are willing to price the debt to include the valuation effects of a particular hedging policy, either because they include explicit covenants that enforce the implementation of a hedging policy or because the management can commit to it, hedging may be viable. However, when that is not the case and bondholders write covenants to protect the value of the debt, hedging in most instances is not optimal when the equityholders pay the costs to hedge. The debtholders that gain as a result of hedging can then offer more advantageous terms in the hedging contract. Under competitive capital markets, the banks that lend to the corporation are then in the best position to become the counterparty in the hedging transaction. Moreover, because the spread in the hedging contract is related to the portion of the debt held, firms with debt owned by fewer creditors will find it easier to hedge than firms with more dispersed debt.

Hedging can be done in a variety of ways, including forward and futures contracts, money market transactions and swaps. These alternative contracts provide substitution possibilities to hedgers. We examine how the institutional design of the contract impacts on the decision to hedge. For example, in some cases hedgers find themselves priced out of the forward market and decide instead to use alternative hedging instruments, such as futures contracts. Our model provides a plausible answer as to how the demand for hedging by high risk firms responds to a change in the priority rules governing the hedging contract, and is able to explain why forward contracts are preferred by some firms and not by others.

In summary, the analysis addresses the following questions:

- What is the demand for hedging that is consistent both with the objective of avoiding bankruptcy and with the credit risk of the hedging corporation?
- Why are hedging contracts typically granted by the banks that lend to the hedger?
- How important are the rules of priority governing the hedging contract in determining which firms can hedge?
- Why do some firms hedge in the futures markets while other firms hedge with forward contracts?

The model uses forward contracts on foreign exchange risk to illustrate the main ideas and arguments. On the one hand, forward contracts are by far the most widely used hedging instrument by corporations.⁶ Also, other popular hedges, such as

⁶ See "1995 Wharton Survey of Derivatives Usage by US Non-Financial Firms" and the 1997 survey by Record Treasury Management, a London consultancy.

swaps and money market transactions, are essentially equivalent to forward contracts. On the other hand, the foreign exchange rate market is the largest in the world and forward transactions represent a substantial segment of this market. We believe that the results of the paper also apply to hedging other exposures with forward contracts and can be extended to hedging interest rate risk using FRA's and swaps if the model includes stochastic interest rates.

The paper is organized as follows: Section 2 analyzes the conditions necessary for hedging to be a viable decision. Section 3 develops a simple model for hedging that incorporates an endogenous cost of hedging. There we show the effects of the debt covenants on the decision to hedge, and analyze the relationship between the amount of the firm's debt owned by the bank granting the forward contract and the forward spread. Section 3 also contrasts the optimal hedging amount with a standard measure, the minimum variance hedge ratio. Section 4 analyzes the effects of changing the priority rules of the hedging contract on the demand for hedging and discusses collateralization. Section 5 illustrates the main points of this article with a simple numerical example. Section 6 discusses the robustness of the results to changes in assumptions. Section 7 concludes.

2. The Viability of Hedging

Consider a corporation that is attempting to hedge the economic exposure arising from the fact that part of its value, V , is correlated with the value of a foreign currency, X , $\rho(V, X)$. This could arise in many ways, including the ownership of foreign assets, contracts with foreign currency cash flows or competitors based in the foreign currency area.

There are various reasons for such hedging by a corporation. Smith and Stulz (1985) list these as bankruptcy costs, agency costs and asymmetric taxes. In this article it is assumed that the corporation is hedging to raise its equity value by avoiding the expected cost of bankruptcy.⁷ Without this cost corporate hedging is irrelevant in our model, as investors in otherwise perfect capital markets will be able to replicate the impact of any hedging at the corporate level. For instance, suppose that the demand for hedging by portfolio investors is affected by the risk premium on foreign exchange. This should not affect corporate hedging because there is nothing that corporations can do to take advantage of such a risk premium that shareholders cannot also do.

The corporation is raising debt (zero coupon), in the amount D to be repaid at time T , to partly finance operating assets with current value V_0 and random value at some future time T of $V_T \sim N(\mu_V, \sigma_V)$. As long as the firm remains solvent, the firm pays taxes, at a constant rate of τ , equal to $(V_T - D)$, which is less than what it would pay if $D = 0$. The firm pays no dividends and is not expected to raise new equity prior to time T . If, at time T , the value of the firm is below the promised debt

⁷ Empirical evidence of hedging to avoid bankruptcy can be found in Booth, Smith and Stolz (1984), Dolde (1995) and Nance, Smith and Smithson (1993).

payment there will be a deadweight loss of K as the firm goes through bankruptcy, and the residual value, $V_T - K$, will be claimed by the debtholders.⁸ Corporate claims are traded continuously and at no cost in competitive capital markets, and their market value reflects the probability of the event of default.

Under the above assumptions any claims contingent on V and the exchange rate X , can be valued by finding a suitable risk-neutral measure that guarantees the existence of no arbitrage.⁹ The condition of no arbitrage is particularly important in our problem. It implies that the firm's debt and equity, on the one hand, and any hedging agreement that it enters, on the other hand, are each priced independently of the portfolios in which they are held. Therefore, the value of any set of future cash flows is determined by arbitrage and not by the particular composition of the portfolio held by the intermediary entering a transaction. However, because hedging can change the cash flows of other claims on the company also held by the hedging counterparty, it is possible that the price agreed reflects this interaction. This means that the value of a hedging contract can be affected by the portfolio in which it is held, even though the valuation function for any set of cash flows is independent of this, so diversification per se plays no role in the valuation.

The values of the equity and debt of the unhedged firm are respectively:

$$E_U = C(V_0, D; T)(1 - \tau) \quad (1)$$

$$B_U = R^{-1}D - P(V_0, D; T) - KR^{-1}W(V < D; T) \quad (2)$$

$$V_U = V_0 - KR^{-1}W(V < D; T) - C(V_0, D; T)\tau \quad (3)$$

where E_U is the value of the equity; B_U is the value of the debt; R^{-1} is current price of a riskless zero coupon bond maturing at time T . $C(V_0, D; T)$ is the value of a European call option on the firm's assets, V , with maturity T and with an exercise price of D . Also, $P(V_0, D; T)$ is the value of a European put option on V maturing at T with an exercise price of D . We denote $W(V < D; T)$ to be the current value of a state contingent claim that pays one unit of money at time T conditional on the firm being bankrupt, $V_T < D$. W is therefore a measure of the risk-adjusted probability of bankruptcy.

Using the put-call parity condition, $V_0 - DR^{-1} = C(V_0, D; T) - P(V_0, D; T)$, expression (3) can be re-written as:

$$V_U = V_0(1 - \tau) + (DR^{-1} - P(V_0, D; T))\tau - KR^{-1}W(V < D; T) \quad (4)$$

the value of the levered firm is equal to the after tax value of the operating cash flows plus the value of the debt tax shields less the expected costs of bankruptcy.

⁸ The evidence on the significance of bankruptcy costs can be found in, for example, Altman (1984), Asquith, Gertner and Scharfstein (1994), Brown, James and Mooradian (1993), Pulvino (1996), Titman and Wessels (1988) and Warner (1977).

⁹ See Harrison and Kreps (1979).

The firm will have debt as long as $\partial V_U / \partial D|_{D=0} > 0$ and $\partial V_U / \partial D|_{D=V} = 0$, and the optimal amount of debt satisfies the first order condition for a maximum:

$$\begin{aligned} \partial V_U / \partial D &= \tau \partial (DR^{-1} - P(V_0, D; T)) / \partial D \\ &\quad - KR^{-1} \partial W(V < D; T) / \partial D = 0 \end{aligned}$$

if in the neighborhood of $D = 0$:

$$\tau / KR^{-1} > (\partial W(V < D; T) / \partial D) / \partial (DR^{-1} - P(V_0, D; T)) / \partial D \quad (5)$$

the firm will use debt, since the debt tax shields are larger than the increase in expected bankruptcy costs. We then assume that τ is sufficiently large so that in the vicinity of $D = 0$ inequality (5) is always verified.

The hedging instrument that is being considered by the firm is a forward contract on foreign currency, with face value F . The exchange rate follows a stochastic process and has a random realization of $X_T \sim N(\mu_X, \sigma_X)$ dollars per unit of foreign currency at time T , the date when the forward contract and the firm's debt mature. The contract exchange rate is $X_F(F)$ dollars per unit of foreign currency. Then, at the maturity of the contract the corporation will receive a net amount equal to $F(X_T - X_F(F))$ dollars.¹⁰ This payment is always made when the counterparty is the net payer, $X_T > X_F(F)$, because the bank is assumed to have negligible credit risk.¹¹ $X_F(F)$ is the equilibrium price paid by the risky firm in return for X_T to be received from the counterparty, when they both agree on a forward contract of size F that is settled net. For that reason X_F is a function of the amount hedged F , $X_F(F)$. When needed $X_F(F)$ will be expressed with its argument, otherwise it will be just X_F .

If the firm decides to enter into a forward hedge the values of its claims will change. $E_H(F)$ and $B_H(F)$ represent the values of the equity and debt of the hedged firm, given a forward contract of size F . $H(F)$ is the market value of this forward contract, including the impact of default risk. Then, the equity value includes the impact of the net cash flow from the forward contract:

$$E_H(F) = C(V_0 + FX_0, D + FX_F; T)(1 - \tau). \quad (6)$$

¹⁰ The firm receives a payment equal $F(X_T - X_F(F))$ from the long forward position, $F > 0$, if $X_T > X_F(F)$ and is supposed to pay $F(X_T - X_F(F))$ if $X_T < X_F(F)$. This implicitly assumes a negative correlation between the V and X , $\rho(V, X) < 0$. The analysis in the case of a positive correlation between V and X , $\rho(V, X) > 0$, is similar. In this case, the firm would take a short position in the contract, $F < 0$, and pay $F(X_T - X_F(F))$ whenever $X_T > X_F(F)$ and receive $F(X_T - X_F(F))$ whenever $X_T < X_F(F)$.

¹¹ Although banks have some credit risk, foreign exchange agreements involving banks usually include a credit support document to back up the bank's obligation. The gain in realism from an analysis done when the two counterparties in the contract are risky is not large to justify the added complication and the difficulty in establishing more general results.

It is straightforward to see that:

$$\begin{aligned} V_H(F) &= B_H(F) + H(F) + E_H(F) \\ &= V_0 - KR^{-1}W(V + FX < D + FX_F; T) \\ &\quad - C(V_0 + FX_0, D + FX_F; T)\tau. \end{aligned} \quad (7)$$

Payments made to or received by the firm under the forward hedge are taxable at the same tax rate of τ .¹² Note that (7) implies that the operating strategy of the firm is unaffected by hedging. However, hedging will determine the states in which the firm decides to default and, therefore, hedging affects the value of the firm. Equations (6) and (7) jointly determine the sum of the debt value and forward contract value, $B_H(F)$ and $H(F)$, respectively. But they do not show how the value is divided between the debtholders and the counterparty to the forward contract. This split must be determined by the priority rules used in default and the settlement rule of the forward contract.

The case we wish to consider for now is that of a forward contract that is junior to the debt claim. This serves as a starting point to which other alternatives are later compared to. Table I shows the payoffs to the various claimholders, including equity owners of the firm, debtholders and the bank entering in the junior forward contract. The debt may be partially or fully owned by the bank granting the forward contract. In the latter case relative priority becomes irrelevant.

The decision that the management of the firm must make is whether it should hedge at the time the debt is issued, and if so, how large the hedge amount, F , should be. It is assumed that the management is interested in maximizing the value of the equity. Default will, therefore, occur at times that is optimal to shareholders and not when it maximizes the value of the firm as a whole. The possibility of conflicts of interest makes the debtholders write covenants that protect their claim against losses from the use of derivatives. That is, the firm decides which forward positions to take, but in exchange for the ability to alter them it agrees to include in the debt contract covenants that preserve the value of the debt.

With competitive capital markets, the forward contract has zero value at the time the firm enters the hedge, $H(F) = 0$. Although $H(F) = 0$, the forward contract rate, $X_F(F)$, includes a spread that is the fair price of default given the size of the hedge, F . We define a hedging strategy that is viable from the equityholders' view a pair $(F, X_F(F))$ that satisfies the condition that $E_H(F) > E_U$, subject to both $H(F) = 0$ and $B_H(F) \geq B_U$. Then, $V_H(F) > V_U$ and from (3) and (7) a viable hedge implies that $KR^{-1}[W(V < D; T) - W(V + FX < D + FX_F; T)] > C(V + FX, D + FX_F; T) - C(V, D; T) > 0$, or equivalently that the risk-adjusted probability of default must decline as a result of hedging, $W(V + FX < D + FX_F; T) < W(V, D; T)$. Having established the conditions defining a viable

¹² For simplicity, we assume that the counterparty to the forward contract pays no taxes. Given that the forward contract has zero value at inception, it does not really matter what the counterparty's tax rate is.

Table I. Payoffs to claims with net settlement of the forward contract with junior priority to debt

State	$V_T + FX_T < D + FX_F$ $FX_T < FX_F, V_T < D + K$	$V_T + FX_T < D + FX_F$ $FX_T < FX_F, V_T > D + K$	$V_T + FX_T < D + FX_F$ $FX_T > FX_F$	$V_T + FX_T > D + FX_F$
Debt	$V_T - K$	D	$V_T + FX_T - FX_F - K$	D
Forward	0	$V_T - D - K$	$-FX_T + FX_F$	$-FX_T + FX_F$
Equity	0	0	0	$(V_T + FX_T - FX_F - D)(1 - \tau)$
Total	$V_T - K$	$V_T - K$	$V_T - K$	$V_T - (V_T + FX_T - FX_F - D)$

The table shows the payoffs to a firm that has assets with random value V_T at time T , when a single issue of zero coupon debt with face value D matures. It also has a forward contract to receive a random amount FX_T and pay a fixed amount FX_F at time T . The forward contract is of junior priority to the debt and is settled net. If the firm defaults on any contracts, there is a fixed deadweight loss of K . τ is the corporate tax rate.

hedging, the firm's optimal amount of hedging is then the value of F that maximizes the value of the equity, $E_H(F)$.

3. Optimal Hedging: The Role of Relationship Banking and of Debt Covenants

In this section we will examine the conditions under which the management will decide to hedge. We start by analyzing the case when the terms of the debt contract are set without including the effects of the hedge, but bondholders include covenants that prevent the value of the debt to be diluted as a result of hedging. Under these conditions, the following proposition shows that it is in the best interest of equityholders that no hedging should take place.

PROPOSITION 1. When the forward contract has zero value at inception, $H(F) = 0$, and is junior to the debt, the optimal strategy is not to hedge, i.e. $F = 0$.

Proof. Using Table I and comparing the value of the equity claims with and without the forward contract in the various states of the world at T it is possible to see that, for any F , the change in the value of the equity is equal to:

$$\begin{aligned}\Delta E(F) = E_H(F) - E_U = & v[F(X_T - X_F)(1 - \tau)|V_T \\ & + F(X_T - X_F) > D, X_T > X_F] + v[(V_T + F(X_T - X_F) \\ & - D)(1 - \tau)|V_T < D < V_T + F(X_T - X_F), X_T > X_F] \\ & - v[F(X_T - X_F)(1 - \tau)|V_T + F(X_T - X_F) > D, X_T < X_F] \\ & - v[(V_T - D)(1 - \tau)|D + K < V_T < D + F(X_T - X_F), X_T < X_F] \\ & - v[(V_T - D)(1 - \tau)|D < V_T \\ & < \min\{D + F(X_T - X_F), D + K\}, X_T < X_F].\end{aligned}$$

where $v[1|\theta]$ is the market price of a claim that pays one unit of money upon the realization of state θ . For the counterparty, the forward contract with the following cash-flows has, in equilibrium, a value of zero:

$$\begin{aligned}H(F) = & -v[F(X_T - X_F)|X_T > X_F] \\ & + v[F(X_F - X_T)|V_T + F(X_F - X_T) > D, X_T < X_F] \\ & + v[(V_T - D - K)|D + K < V_T < D + F(X_F - X_T), X_T < X_F] \\ = & 0.\end{aligned}$$

Where the first term on the right establishes that the counterparty is riskless. Combining $H(F)$ with ΔE gives:

$$\begin{aligned}\Delta E(F)/(1 - \tau) = & v[(V_T - D)|V_T < D < V_T + F(X_T - X_F), X_T > X_F] \\ & - v[F(X_T - X_F)|V_T < D, X_T > X_F]\end{aligned}$$

$$\begin{aligned}
& -v[K|D + K < V_T < D + F(X_F - X_T), X_T < X_F] \\
& -v[(V_T - D)|D < V_T < \\
& \min\{D + F(X_F - X_T), D + K\}, X_T < X_F].
\end{aligned}$$

since in the first of these states $V_T < D$, the first term is always negative; the last three terms are positive but all have a negative sign. Then, $\Delta E(F)/(1 - \tau) < 0$ and for any positive amount, F , the management will not hedge. Furthermore, from $W(V + FX < D + FX_F; T) < W(V < D; T)$, $B_H(F) - B_U > 0$. $\square$

Proposition 1 establishes that the equity does not gain from hedging. Hedging makes the firm more valuable by reducing the expected cost of bankruptcy, but this simply subsidizes the debt. Even though the debt can lose as a result of hedging in some states (for example, when the firm is the net payer in the forward and it would not have defaulted had the contract not been entered into) the expected loss to bondholders is less than the expected gain occurring when the firm is the net receiver and default is avoided. The equity cannot gain more than the forward payment, but the condition that the forward contract has zero value makes it impossible for the equity to increase in value with hedging, given the share of the gains accruing to debtholders.¹³

Next we examine two alternative arrangements that make hedging viable. The first is when the bank granting the forward contract also holds part of the firm's debt. The second is when the debtholders are willing to alter the protective covenants on the debt, and price the debt including the effects of hedging.

In competitive capital markets, when the bank offering the forward contract also owns part of the outstanding debt of the firm, it will quote a more favorable forward rate than another bank with no other relationship with the firm. This is because the bank holding the firm's debt gains in its debt position if the firm decides to hedge, and therefore will consider the subsidy from the debt in setting the forward price, X_F .

PROPOSITION 2: If, for the forward rate that satisfies $H(F) = 0$, hedging increases the value of the firm, a bank holding some of the outstanding senior debt will offer better terms on the junior forward contract than a non-creditor bank.

Proof. Consider fixing $X_F(F)$ and adding a positive amount a to the forward payment when the counterparty bank is the net payer, thus reducing the spread to $X_F - (X_T + a)$. Then, for the bank, $\forall F > 0$, $\partial H(F, a)/\partial a < 0$. However, hedging with the non-expropriation covenant implies that $\partial \Delta B(F, a)/\partial a > 0$, where $\Delta B = B_H(F, a) - B_U$. When the bank holds a portion $0 < \alpha \leq 1$ of the firm's debt, its holdings are $H(F, a) + \alpha \Delta B(F, a)$. In competitive financial markets a is such that $\alpha \Delta B(F, a^*) = -H(F, a^*)$. At $a = 0$, $H(F, a) = 0$ and

¹³ The decision by the management not to hedge is an example of the underinvestment problem mentioned in Myers (1977). Leveraged firms pass up low risk, positive value decisions because equityholders incur the costs of investing with the benefits accruing to the creditors.

$\alpha \Delta B(F, a) > 0$. For a sufficiently large a , since $B_H(F, a)$ is bounded from above by D , B_H equals D implying that $\partial \Delta B(F, a) / \partial a = 0$. On the other hand, $H(F, a)$ is decreasing in a and unbounded, therefore $-H(F, a)$ intercepts $\alpha \Delta B(F, a)$, at some value a^* , for $\alpha \geq 0$. $\square$

Proposition 2 states that the competitive rate that is charged by a bank holding a proportion α of the firm's debt, $X_F(F, \alpha)$, is the solution to $\alpha(B_H(F, X_F) - B_U) + H(F, X_F) = 0$. This condition sets the value of the forward contract equal to $H(F, X_F) \leq 0$.¹⁴ The next corollary shows that the greater the proportion of the debt owned by the bank, the lower the forward spread is:

COROLLARY: The minimum forward rate offered, $X_F(\alpha)$, is non-increasing in α , the proportion of the firm's debt held by the bank offering the forward contract.

Proof. From hedging with the non-expropriation of debt covenant $B_H(F, X_F) \geq B_U$. Also, $H(F, X_F) = 0$ for $X_F(\alpha = 0)$. For a fixed value of F , the value of the forward contract, $H(F)$, is increasing in X_F in the neighborhood of $H(F) = 0$, otherwise some other bank would require a lower value X_F , which would increase the equity and debt values. So $H(F) = -\alpha(B_H - B_U)$ is satisfied at $X_F(\alpha > 0) \leq X_F(\alpha = 0)$, with the lowest value of $X_F(\alpha)$ being given by the bank holding most debt. $\square$

Both Proposition 2 and the previous corollary highlight a common feature of forward markets. Forward contracts are typically granted by the banks that lend to the firm. The result herein is what is also commonly referred to as "Relationship Banking", whereby certain derivative contracts are offered more competitively by those banks that own debt in the firm. Cross subsidization in different product lines -here the debt contract subsidizes the forward contract- helps banks compete more effectively for clients that use these products in bundles. Note that this result arises because the forward contract is a somewhat imperfect hedging instrument, for it not only hedges the equity, but also does not decrease the value of the debt. Unless the agency problem of Proposition 1 can be avoided in some other way, the joint provision of lending and hedging increases the possibility of economically viable hedging.

Given that a bank holding some of the existing debt of the firm will offer a better forward rate, it might seem that there is a viable hedging strategy if a bank owns enough debt. This is certainly the case when the bank owns all the debt, as the following proposition illustrates:

¹⁴ Without the assumption that capital markets are competitive the equilibrium rate is any value of $X_F(F)$ such that $-\alpha(B_H(F, X_F) - B_U) \leq H(F, X_F) \leq E_H(F, X_F) - E_U$. However, deciding on the hedge simultaneously with the sale of the debt, or allowing to refinance the loan (and revise the hedge) at no penalty allows the firm to always get the minimum rate $H(F, X_F) = -\alpha(B_H(F, X_F) - B_U)$.

PROPOSITION 3. Banks holding all of the firm's debt, $\alpha = 1$, will always offer a viable forward contract when hedging increases the value of the firm.

Proof. Suppose that hedging increases the value of the firm, $\Delta V(F) = \Delta E(F) + \Delta B(F) + H(F) > 0$. At $\alpha = 0$, the equilibrium rate $X_F(F, \alpha)$ is such that $H(F, a = 0) = 0$, and from Proposition 1, $\Delta E(F) < 0$. If a is increased then both $\Delta E(F)$ and $\Delta V(F)$ will increase. From Proposition 2 and the Corollary, the choice of a is increasing in the level of α . Suppose that $\alpha = 1$ and a is increased just enough to make $\Delta E(F) = 0$. Then, because $\Delta V(F) > 0$, $\Delta B(F) + H(F, a) > 0$, and the bank will always offer such contract. The negative value of the forward contract is offset by the subsidy gained on the debt contract. $\square$

From the no arbitrage condition any claim contingent on V and X alone can be priced using an equivalent risk-neutral economy (Cox and Ross (1976)). Then, $\mu_V = V_0/R^{-1}$ and $\mu_X = X_0/R^{-1}$, since in this economy the value of any claim is given by taking expected payoffs computed under risk neutral probability measures and multiplying by R^{-1} . Note, however, that X_0 is not equal to the spot currency rate, but rather to the dollar present value of the riskless forward rate. μ_X is then the forward rate that results from the interest rate parity condition. Solving for the values of the claims in the hedged firm gives:

$$E_H/R^{-1} = [-h_1\sigma_Z(1 - N(h_1)) + \sigma_Z N'(h_1)](1 - \tau) \quad (8)$$

$$B_H/R^{-1} = D - [(h_1\sigma_Z + K)N(h_1) + \sigma_Z N'(h_1)] \\ - [h_2\sigma_Z N(h_2) + \sigma_Z N'(h_2)] \quad (9)$$

$$H/R^{-1} = -(FX_F - F\mu_X)N((\mu_X - X_F)/\sigma_X) \\ + (FX_F - F\mu_X)(1 - N(h_1)) \\ - [h_2\sigma_Z(N(h_1) - N(h_2)) + \sigma_Z(N'(h_1) - N'(h_2))] \quad (10)$$

$$V_H = B_H + H + E_H \\ = V_0 - R^{-1}KN(h_1) - R^{-1}\tau[-h_1\sigma_Z(1 - N(h_1)) + \sigma_Z N'(h_1)] \quad (11)$$

where $h_1 = (D + FX_F - F\mu_X - \mu_V)/\sigma_z$; $h_2 = (D + K - \mu_V)/\sigma_z$ and $\sigma_z^2 = \sigma_v^2 + F\sigma_X^2 + 2F\rho v\sigma_x$ is the standard deviation of $z_T = V_T + FX_T$. $N(h)$ is the standard normal distribution function. Note that $N(h_1)$ is a monotonically increasing function in h_1 . The condition that the equity value (6) is never negative implies that h_1 is less than zero. From the assumption that the bank is riskless $F(X_F - \mu_X) > 0$.¹⁵ Together $h_1 < 0$ and $F(X_F - \mu_X) > 0$ establish that $\mu_V > D$. A necessary condition for hedging is then that the expected value of the firm's operating assets at T is higher than the face value of the debt, D . Hedging, in our

¹⁵ $F(X_F(F) - \mu_X)$ is equivalent to $X_F(F) > \mu_X$ when the firm is long on the forward contract, $F > 0$, and to $X_F(F) < \mu_X$ when the firm goes short on the forward contract, $F < 0$.

model, does not affect the value of the operating assets of the firm. It simply helps reducing the deadweight costs associated with bankruptcy and to conserve the tax shields from debt financing. The maximum hedging can do is to get the firm back to its first best value, μ_V . If, however, μ_V is less than D , costly hedging will only increase the liabilities of the firm and the best course of action for equityholders is to abandon the firm.

When the bank entering the forward transaction owns less than one-hundred percent of the firm's debt, $\alpha < 1$, two problems arise. The first is that the bank may not agree to reduce the spread if that benefits other existing debtholders. This would happen if the other debt was senior to the forward contract, even if it was junior to the debt owned by the bank. Alternatively, as the following proposition shows, α may not be large enough to make the equilibrium spread, $X_F(F, \alpha) - \mu_X$, negative.

PROPOSITION 4. Even when the bank offering the forward contract holds some of the existing (senior) debt, there will be no viable hedging at a positive spread in the forward market.

Proof. For viable hedging the probability of bankruptcy must decrease:

$$\Pr(V_T < D) > \Pr(V_T + FX_T < D + FX_F)$$

where $\Pr(u < w)$ is the risk-adjusted probability that u is less than w at some specific point in time. Under the assumption of normality, this is equivalent to $N((D - \mu_V)/\sigma_V) > N((D + FX_F - \mu_V - F\mu_X)/\sigma_Z)$, which implies:

$$(D - \mu_V)/\sigma_V > (D - \mu_V + F(X_F - \mu_X))/\sigma_Z.$$

Rearranging:

$$(\mu_V - D)/\sigma_V < (\mu_V - D - F(X_F - \mu_X))/\sigma_Z. \quad (12)$$

From $F(X_F - \mu_X) > 0$ it follows that $\mu_V - D > \mu_V - D - F(X_F - \mu_X)$. Given that $\mu_V > D$, a positive forward spread in (12) implies that $\sigma_Z < \sigma_V$. Viable hedging requires the intuitively plausible condition that the total variability of $V_T + FX_T$ to be less than the total variability of V_T . As a consequence:

$$C(V_0 + FX_0, D + FX_F; T) < C(V_0, D + FX_F - F\mu_X; T).$$

Where C is the value of an European call option maturing at T . And from $F(X_F - \mu_X) > 0$ we have that:

$$C(V_0 + FX_0, D + FX_F; T) < C(V_0, D; T). \quad (13)$$

However, for the management to be willing to hedge the equity value must rise as a result of hedging, implying that:

$$C(V_0 + FX_0, D + FX_F) > C(V_0, D)$$

which contradicts (13), so viable hedging is impossible as long as the spread in the forward contract is positive. $\square$

Propositions 3 and 4 together imply that: (i) when the debt includes a covenant of non-expropriation, hedging is viable only if the bank holds enough of the firm's debt to grant a forward contract with a negative spread, $X_F(F, \alpha) < \mu_X$; (ii) the higher the degree of debt concentration (lower dispersion) the smaller the spread in the hedging contract and the higher the likelihood that hedging is viable.

The reason why hedging is viable only if α is sufficiently large to make $X_F(F, \alpha) < \mu_X$ has to do with the pricing of the debt, which assumes that no hedging occurs. Suppose instead that debtholders write covenants in the debt contract that compel the firm to implement a particular hedging strategy, which cannot be altered without previous consent of the lenders. The hedging covenant then ensures the lender that if the terms of the loan are quoted assuming that hedging will occur, then hedging will actually take place.¹⁶ Presumably, the equityholders of the firm also find advantageous to agree to limit the financial risk so as to reduce the cost of debt financing. With bonding, debtholders are willing to price the debt to include the impact of the hedging amount, F , that maximizes the value of equity. Raising the value of the equity is then equivalent to raising the value of the hedged firm. In the case of $\alpha = 0$, the management chooses F to maximize the total value of the firm given by (11):¹⁷

$$F \in \operatorname{argmax}\{V_0 - R^{-1}KN(h_1) - R^{-1}\tau[-h_1\sigma_Z(1 - N(h_1)) + \sigma_ZN'(h_1)]|H(F) = 0\}.$$

A sufficient condition for hedging in this case is given in the next proposition:

PROPOSITION 5. When debtholders can bond the management to pursue a hedging strategy that maximizes the value of the firm a sufficient condition for viable hedging at $H(F) = 0$ is that:

$$|\rho| > \left[\frac{(X_F - \mu_X) - \varphi\sigma_V/KN'(h_1)}{(\mu_V - D) + (\tau/K)\sigma_V^2} \right] \left(\frac{\sigma_V}{\sigma_X} \right) \quad (14)$$

where X_F is evaluated at $F = 0$ and $\varphi = \tau(X_F - \mu_X)(1 - N(h_1))$. If hedging is done with a contract for which $\rho(V, X) < 0$ then (14) is $\rho < -(X_F - \mu_X)[(\mu_V - D) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V/\sigma_X) + [\varphi/KN'(h_1)][(\mu_V - D) +$

¹⁶ Such practice has been confirmed to the authors by industry practitioners. Booth (1990) also reports that covenants defining the hedge policy are common in bank loan contracts. The issue of a loan covenant requiring hedging in LBOs is mentioned in Campbell and Kracaw (1990).

¹⁷ A different situation would be if equityholders derived utility from a reduction in the variance of the firm's assets, independent of the impact on the share price. Then, the equityholders would be able to commit to pursue the hedging strategy that maximised firm value, without recourse to hedging covenants.

$(\tau/K)\sigma_V^2]^{-1}(\sigma_V^2/\sigma_X)$. If hedging is done with a contract for which $\rho(V, X) > 0$, (14) is $\rho > (X_F - \mu_X)[(\mu_V - D) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V/\sigma_X) - [\varphi/K N'(h_1)] [(\mu_V - D) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V^2/\sigma_X)$.

Proof. The firm is willing to undertake a forward contract of face value dF if $\partial V/\partial F > 0$ evaluated at $F = 0$. Recall that $N(h_1)$ is monotonically increasing in h_1 . When the hedge is a long position, $dF > 0$,

$$\begin{aligned} \partial V/\partial F = & [-KN'(h_1)\partial h_1/\partial F - \tau N'(h_1)\partial\sigma_Z/\partial F] \\ & + \tau(X_F - \mu_X + F\partial X_F/\partial F)(1 - N(h_1)). \end{aligned} \quad (15)$$

A sufficient condition for hedging is that $\partial V/\partial F > 0$ evaluated at $F = 0$. Solving (15) using $\partial h_1/\partial F = (\sigma_Z(X_F - \mu_X + F\partial X_F/\partial F) - h_1\sigma_Z\partial\sigma_Z/\partial F)/\sigma_Z^2$, as well as $\partial\sigma_Z/\partial F = F\sigma_X^2 + \rho\sigma_C\sigma_X/\sigma_Z$, and knowing that for $F = 0$, $\sigma_Z = \sigma_V$, gives:

$$\begin{aligned} \rho < & -(X_F - \mu_X)[(D - \mu_V) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V/\sigma_X) \\ & + [\varphi/K N'(h_1)][(D - \mu_V) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V^2/\sigma_X) \end{aligned}$$

with $\varphi = \tau(X_F - \mu_X)(1 - N(h_1))$. When the hedge is a short position because $\rho(V, X) > 0$, the inequality becomes:

$$\begin{aligned} \rho > & (X_F - \mu_X)[(D - \mu_V) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V/\sigma_X) \\ & - [\varphi/K N'(h_1)][(D - \mu_V) + (\tau/K)\sigma_V^2]^{-1}(\sigma_V^2/\sigma_X). \square \end{aligned}$$

The first term in brackets on the right hand side of (15), $[-KN'(h_1)\partial h_1/\partial F - \tau N'(h_1)\partial\sigma_Z/\partial F]$, represents the expected loss from hedging, given that a positive spread in the hedging contract imposes an increase in the liabilities of the firm. This loss occurs even if hedging reduces the variance of the firm's value from σ_V^2 to σ_Z^2 . The second term on the right hand side of (15), $\tau(X_F - \mu_X + F\partial X_F/\partial F)(1 - N(h_1))$, is the expected tax saving from hedging when the hedging contract includes a positive spread.

Proposition 5 states that even with bonding of the firm's hedging strategy and no conflicts of interest, not all firms will find that hedging is viable. The viability of hedging depends on the risk of the hedging firm, the tax status of the firm, as well as on the characteristics of the hedging asset, as the following corollary makes clear:

COROLLARY. Firms will be more likely to hedge when:

- (a) The credit spread imbedded in the hedging contract is small, (X_F, μ_X) is small;
- (b) The risk of the business is small, σ_V is small;
- (c) Leverage is low $(\mu_V - D)$ is large;
- (d) Bankruptcy costs are small, K is small;
- (e) The exchange rate is more volatile, σ_X is large;

- (f) The tax rate is high, τ is high;
- (g) They can find a contract with large (positive or negative) correlation with the value of the assets, large $|\rho|$.

Conditions (a), (b) and (c) imply that default risk is low, and condition (d) implies that the loss incurred in default is small. Condition (e) implies that the hedging instrument gives a lot of hedging per dollar of face value. Condition (f) favors hedging to protect the tax shields from borrowing, including the additional liability created by the spread in the hedging contract. Condition (g) implies that the hedging instrument is highly-positively or negatively, depending on the firm being short or long when it hedges-correlated with the firm's assets. Later we explore the implication that condition (a) can be caused by a settlement or priority rule that reduces the default risk of the hedging contract.

Thus the firms that would be expected to hedge in the forward markets are those with low default risk, high tax savings, and with operating assets correlated with available forward instruments. An implication of this is that the firms with the greatest potential benefit from hedging, those with high unhedged probabilities of default, are those which are most likely to be prevented from hedging using forward contracts. High risk firms do not hedge because the terms on which they can hedge are reflected in the default spread, $(X_F - \mu_X)$, and the size of the spread does not allow the value of the firm to go up with hedging. The forward spread increases the liabilities of the firm, and even if hedging reduces the variance of the firm's total cash flow, the probability of bankruptcy will not go down. This occurs even though the default spread in the forward contract rate is not a direct cost of hedging, but reflects the firm's own risk and, therefore, it is simply the equilibrium price of default risk.

The optimal level of hedging when hedging maximizes the value of the firm is the amount $F > 0$ that satisfies $\partial V / \partial F = 0$. How does this optimal amount compare with the minimum variance hedge ratio, which satisfies $\partial \sigma_Z / \partial F = 0$? The following proposition establishes the relationship between these two hedge amounts:

PROPOSITION 6. The difference between the optimal level of hedging that maximizes firm value and the minimum variance hedge amount depends on the potential for tax savings relative to the likely costs of bankruptcy. With zero tax savings, the optimal hedge amount is always less than the minimum variance amount.

Proof. When the firm hedges to maximize value, the optimal hedge ratio solves $\partial V / \partial F = 0$. When the firm hedges to minimize variance the hedge ratio solves $\partial \sigma_Z / \partial F = 0$. Ignoring the term R^{-1} , from (15) we know that:

$$\begin{aligned} \partial V / \partial F = & [-KN'(h_1)\partial h_1 / \partial F - \tau N'(h_1)\partial \sigma_Z / \partial F] \\ & + \tau(X_F - \mu_X + \partial X_F / \partial F)(1 - N(h_1)) \end{aligned}$$

Substituting $\partial h_1/\partial F = (s + F\partial X_F/\partial F)/\sigma_Z - (h_1/\sigma_Z)\partial\sigma_Z/\partial F$ in the expression above, where $s = X_F - \mu_X$ is the dollar spread on the forward contract, gives:

$$\begin{aligned}\partial V/\partial F = & -[K(s + F\partial X_F/\partial F) - h_1\partial\sigma_Z/\partial F \\ & + \tau\sigma_Z\partial\sigma_Z/\partial F]N'(h_1)/\sigma_Z + \tau(s + F\partial X_F/\partial F)(1 - N(h_1))\end{aligned}$$

and re-arranging gives:

$$\begin{aligned}\partial V/\partial F = & [(h_1 - \tau\sigma_Z)N'(h_1)/\sigma_Z]\partial\sigma_Z/\partial F \\ & + [(s + F\partial X_F/\partial F)/\sigma_Z](\tau\sigma_Z(1 - N(h_1)) - KN'(h_1)).\end{aligned}$$

At the minimum variance hedge ratio $\partial\sigma_Z/\partial F = 0$ and:

$$\partial V/\partial F = [(s + F\partial X_F/\partial F)/\sigma_Z](\tau\sigma_Z(1 - N(h_1)) - KN'(h_1)).$$

The optimal hedge ratio is less than the minimum variance hedge ratio if $\partial V/\partial F < 0$ evaluated at the point where $\partial\sigma_Z/\partial F = 0$. Since $(s + F\partial X_F/\partial F)/\sigma_Z$ is never negative at the optimum, $\partial V/\partial F$ is negative when evaluated at $\partial\sigma_Z/\partial F = 0$ if and only if $\tau\sigma_Z(1 - N(h_1)) < KN'(h_1)$. This is always the case for $\tau = 0$. $\square$

When the firm hedges, it agrees to pay a spread due to default risk equal to $X_F - \mu_X$. This spread is an additional liability to the firm and therefore affects the firm's probability of bankruptcy, as well as the amount of tax shields available. As the amount of hedging rises the variance of the hedged firm's assets decreases. The forward spread is offsetting part of the effect of the falling variance on the default probability. Thus the optimal level of hedging depends on the trade-off between the lower after tax variance of the firm's value and the higher likelihood of default from hedging at a positive spread. Proposition 6 shows that when hedging is done to maximize value, using the minimum variance hedge ratio is often not optimal, because minimizing the risk-adjusted probability of bankruptcy is not equivalent to minimizing the variance of the hedged firm's assets.

4. More Efficient Hedging through Institutional Design of the Contract: Priority Rules and Collateralization

Proposition 5 establishes that for some risky firms the default risk spread imbedded in the forward quote may be too high to render hedging viable. In the corollary that follows that proposition, we conjecture that the size of the spread, which is related to the settlement rule in default, can affect the demand for hedging. In this section we explore how institutional features, such as seniority and collateralization of the hedging contract, can help make hedging viable.

Consider first the effects on hedging behavior from changing the priority rule, whereby the debt is now subordinated to the hedging agreement. The payoffs to the different claims in this case are shown in Table II.

Table II. Payoffs to claimholders with net settlement of the forward contract with senior priority to debt

State	$FX_T < FX_F$ $V_T + FX_T < D + FX_F$ $V_T < FX_F - FX_T + K$	$FX_T < FX_F$ $V_T + FX_T < D + FX_F$ $V_T > FX_F - FX_T + K$	$FX_T > FX_F$ $V_T + FX_T < D + FX_F$	$V_T + FX_T > D + FX_F$
Debt	0	$V_T + FX_T - FX_F - K$	$V_T + FX_T - FX_F - K$	D
Forward	$V_T - K$	$-FX_T + FX_F$	$-FX_T + FX_F$	$-FX_T + FX_F$
Equity	0	0	0	$(V_T + FX_T - FX_F - D)(1 - \tau)$
Total	$V_T - K$	$V_T - K$	$V_T - K$	$V_T - \tau(V_T + FX_T - FX_F - D)$

The table shows the payoffs to a firm that has assets with random value V_T at time T , when a single issue of zero coupon debt with face value D matures. It also has a forward contract to receive a random amount FX_T and pay a fixed amount FX_F at time T . The forward contract is of junior priority to the debt and is settled net. If the firm defaults on any contracts, there is a fixed deadweight loss of K . τ is the corporate tax rate.

Solving for the values of the senior forward contract, H^S , and the junior hedged debt, B_H^J , gives:

$$\begin{aligned} B_H^J/R^{-1} &= D - [(h_1\sigma_Z + K)N(h_1) + \sigma_Z N'(h_1)] \\ &\quad - [(h_1\sigma_Z + K - D)N(h_3) + \sigma_Z N'(h_3)] \end{aligned} \quad (16)$$

$$\begin{aligned} H^S/R^{-1} &= -(FX_F - F\mu_X)N((\mu_X - X_F)/\sigma_X) \\ &\quad + (FX_F - F\mu_X)(1 - N(h_3)) - [h_4\sigma_Z(N(h_3) - N(h_4)) \\ &\quad + \sigma_Z(N'(h_3) - N'(h_4))] \end{aligned} \quad (17)$$

where $h_3 = (K + FX_F - F\mu_X - \mu_V)/\sigma_Z$; $h_4 = (K - \mu_V)/\sigma_Z$ and $\sigma_Z^2 = \sigma_V^2 + F\sigma_X^2 + 2F\rho\sigma_V\sigma_X$ is as before the standard deviation of $z_T = V_T + FX_T$. By contrasting (17) with (10) it is easy to see that the more senior contract, H^S , has a lower spread than the junior contract, H , since $h_4 < h_2$ always, and for $D > K$, $h_3 < h_1$. A more interesting comparison relates the value of the hedged debt to the relative priority of the hedging contract:

PROPOSITION 7. If hedging adds value to the firm, the value of subordinated debt hedged with a senior forward contract is higher than the value of senior debt of equal face value hedged with a junior forward contract.

Proof. Subtracting (9) from (16), and ignoring the term in R , gives:

$$\begin{aligned} B_H^J - B_H &= -[(h_1\sigma_Z + K - D)N(h_3) + \sigma_Z N'(h_3)] \\ &\quad + [h_2\sigma_Z N(h_2) + \sigma_Z N'(h_2)] \end{aligned}$$

for values of $D > FX_F - F\mu_X$, $h_1\sigma_Z + K - D = K + FX_F - FX_F - F\mu_X - \mu_V$ is less than $h_2\sigma_Z = D + K - \mu_V$, and $h_3 < h_2$. $N'(h)$ is increasing, since $D < \mu_V$ is a necessary condition for hedging. Then $B_H^J - B_H > 0.0$. $\square$

This is a striking result and highlights the importance of an often ignored detail in hedging: the relative priority of the hedge. Table III also illustrates this point. Consider various tax-exempted firms that differ in the volatility of their assets, $\sigma_V = [15\%, 20\%, 25\%]$. None of these firms are able to hedge using a forward contract junior to the debt, but they all hedge with a forward contract senior to the debt. When the forward contract is junior to the existing debt, the forward spread is so large that the probability of bankruptcy will not go down with hedging. However, a lower spread associated with a forward contract that has a higher priority results in a lower liability to the firm, reducing the probability of bankruptcy.

The relative priority of the hedging contract is, therefore, a critical determinant in the hedging decision itself. Riskier firms will be able to hedge only if they can find a contract written on an asset that has higher absolute correlation with the firm value, or enter into a forward contract that is senior to the existing debt. The

Table III. Firm value effects from hedging with a senior forward contract and a junior forward contract

σ_V	Firm hedged						Firm unhedged V_U
	Junior hedging contract			Senior hedging contract			
	F	X_F	V_H	F	X_F	V_H	
15	3	1.515	176.33	4	1.501	176.46*	176.33
20	2	1.52	175.98	2	1.501	176.03*	175.99
25	2	1.525	175.78	2	1.502	175.82*	175.80

* The firm value increases with hedging.

The parameters used in the tables are: $\mu_X = 1.5$, $\sigma_X = 0.4$, $\rho = 0.9$, $\mu_V = 200$, $D = 199$, $K = 50$, $\tau = 0$. When the forward contract is junior to the debt, payoffs as given in Table I. When forward contract is senior to the debt, payoffs given in Table II. σ_V is the per period percent volatility of the firm's assets, F is the amount per forward contract, X_F is the contract exchange rate in dollars per unit of foreign currency, V_H hedged value of the firm and V_U the value of the firm unhedged.

trade-off between the reduction in volatility and the increase in liabilities from the forward spread improves with the seniority of the forward contract. As a result, low priority hedged debt becomes less risky than the more senior debt hedged with a junior forward contract. Perhaps this argument explains why forward contracts are often senior to the firm's debt, with the agreement of debtholders.

The more senior forward contract approximates a futures contract, since seniority is an inherent characteristic of exchange traded derivative contracts. Another important feature in futures contracts is the margin, which performs the role of a collateral. For a properly margined futures contract, or collateralized forward contract, there is no default risk, so that the condition for viable hedging becomes $\rho' < 0$, where ρ' is the correlation between the futures price and the value of the firm. To give an idea of the role of the collateral, consider the firm with $\sigma_V = 25\%$ in Table III. If the firm hedges with a senior contract with correlation $\rho(V, X) = -0.5$ its value will go up slightly to $V_H = 175.81$, ($V_U = 175.80$), with $F = 3$ and $X_F = 1.504$. Note, in Table III, that hedging with a senior contract with correlation $\rho(V, X) = -0.9$ would result in $V_H = 175.82$, a difference due to the higher absolute correlation. Now, assume that the firm borrows an additional 3.12 junior to the existing debt of 199 to collateralize the senior hedging contract with correlation $\rho(V, X) = -0.5$. Then, the firm can hedge with the same 3 contracts at a zero spread ($X_F = 1.5$) and raise its value net of the collateral to 175.89, which is close to the value obtained by the firm with $\sigma_V = 20\%$ in Table III, that hedges with a contract with $\rho(V, X) = -0.9$. The collateral is therefore a value enhancing mechanism especially important to riskier firms (high σ_V), and when the asset delivered in the hedge has low absolute correlation with the firm value

(low $|\rho|$).¹⁸ Note that the absolute correlation of the futures price with the firm value, $|\rho'|$, will, in general, be lower than the correlation of the exchange rate with the firm value, $|\rho|$, for two reasons: first, futures contracts have basis risk; second, futures contracts are written on a smaller range of currencies than forward contracts. Thus, even if there is a forward contract for which $\rho < 0$ there may not be a futures contract for which $\rho' < 0$. However, the futures contract with lower absolute correlation may generate more hedging from risky firms, simply because collateralization, an institutional design, makes it a more efficient hedge than a forward contract with higher absolute correlation. Note also that although $\rho' < 0$ appears a less stringent condition than (14), it does not mean that high quality hedgers prefer the futures market rather than the forward market, since normally $|\rho'| < |\rho|$ and low risk firms pay small default spreads. This is consistent with the coexistence of separate forward and futures markets serving the hedging needs of firms with strong and weak credit, respectively. Kane (1980) claims that posting collateral when using futures is not, in general, costless so that firms which can use uncollateralized forward contracts will prefer to do so. Our model is then also able to provide a formal argument that explains the self-separation of hedgers between collateralized and uncollateralized contracts. So low-grade firms will use one of futures contracts, collateralized forward contracts, or uncollateralized forward contracts with a relationship bank, whereas high grade firms will use uncollateralized forward contracts.^{19,20}

Hedging with non-linear contracts such as options can be used to achieve a similar effect. If a hedger pays for a call at the purchase date, then there is no outstanding liability to the writer at the expiration date. So the writer's claim is effectively made senior to all other claims by the up-front payment for the option. Hedging with options also gives non-linear payoffs, but this will not significantly affect the other propositions of the paper.

¹⁸ In Stulz and Johnson (1985) collateral is also used in resolving the underinvestment problem.

¹⁹ The choice between forward and futures contracts involves considerations of the extent to which a few standardized contract specifications can concentrate liquidity and avoid informational asymmetries in away that meet the needs of different hedgers. In cases such as interest rates and currencies the underlying markets are sufficiently deep and free of informational asymmetries that collateralized forward contracts may be used in many instances rather than organized futures markets. In the same vein, Brennan (1986), page 215, writes: "If there is a sufficient large group of individuals with adequate credit reputations or for whom the posting of margin is costless, then we may expect to observe a forward market; this may even co-exist with a futures market."

²⁰ The fact that firms use much more frequently forwards than futures to hedge exchange-rate risk in foreign exchange transaction, as documented by Bodnar, Hayt and Marston (1996) may be because most of these transactions are effectively collateralized by the receivables or the pre-payment of liabilities.

5. A Numerical Example

An example helps to illustrate how the higher the potential from hedging (the higher the unhedged risk of default) the less likely the firm is to hedge because the credit risk spread is too high. It also helps in drawing out the importance of the contract specification (seniority and collateralization) on viable hedging.

Two firms are compared both with the same asset values, but low and high asset risk. The low risk firm has assets that pay 10 or 20 with equal probability, and debt with face value of 12.5. If it does not hedge, it defaults in states 3 and 4 (panel B1 in Table IV). Default costs equal 1 and, for simplicity, there is zero risk aversion, a zero interest rate and taxes are ignored.

The firm can hedge by taking out a junior forward contract on the variable X_T , which is negatively correlated with firm value. By entering a forward contract of face value $F = 10$ and contract price $X_F = 1.62$ against $E(X_T) = 1.5$, that is a spread of 0.12, the low risk firm can avoid default in state 3 (panel B2 in Table IV). In state 3 the firm will have assets worth 10, which combined with a gross payment from the forward counterparty of 20 ($F \cdot X_T = 10 \cdot 2$), give a total of 30. Its liabilities are a promised payment of 12.5 on the senior bond and a forward payment of 16.2 ($F \cdot X_F = 10 \cdot 1.62$), totaling 28.7.

The equilibrium forward spread of 0.12 gives a zero value to the junior forward contract at inception, since $7/8 \cdot (10 \cdot 1.62) + 1/8 \cdot (20 - 12.5 - 1) = 15$. With probability $1/8$, the firm defaults on the forward, and the counterparty receives 6.5, after the senior bondholders have been repaid in full and after a deadweight loss of 1. Note that the value of the unhedged debt is 12, given that with 50% probability there will be a default loss of 1. The hedged debt is riskless and has value 12.5. With bonding, the firm will hedge to reduce the cost of debt financing.

The high risk firm has assets that pay 30 or 0 with equal probability and the same face value of debt, 12.5, as the low risk firm. It also defaults in states 3 and 4 if it does not hedge (panel C1 in Table IV). If this firm attempts to avoid default in state 3, even if it pays no default spread on the hedging contract ($X_F = 1.5$) it must set F at least equal to 30 to ensure that its assets exceed its liabilities in that state. It turns out, however, that this is not an equilibrium price, since default is not avoided in state 4 (panel C2 in Table IV). The equilibrium contract price assuming default only in state 4 is 1.636, since $7/8 \cdot (30 \cdot 1.636) + 1/8 \cdot (30 - 12.5 - 1) = 45$. With this price, however, default occurs in states 1, 3 and 4 (panel C3 in Table IV). Thus the high risk firm cannot avoid default in state 3 by using forward contracts.

The reason that the high risk firm finds it harder to hedge is that it has to use more contracts to hedge its asset variability, and also the consequences of its defaults are more severe. Both these factors increase the spread embedded in the hedging contract and this prevents the risk adjusted probability of default from being reduced.

What are the alternatives available to high risk firms? One obvious is a contract with a higher correlation between the firm's assets and the underlying hedging

Table IV. The effect of firm risk on hedging

State of the world	1	2	3	4
Panel A:				
Probability	3/8	1/8	3/8	1/8
X_T	1	2	2	1
V_T (Low risk firm)	20	20	10	10
V_T (High risk firm)	30	30	0	0
Panel B1: Low risk firm unhedged				
Cash available	20	20	10	10
Promised Liabilities	12.5	12.5	12.5*	12.5*
Panel B2: Low risk firm hedged ($F = 10$, $X_F = 1.620$)				
Cash available	30	40	30	20
Promised Liabilities	28.7	28.7	28.7	28.7*
Panel C1: High risk firm unhedged				
Cash available	30	30	0	0
Promised Liabilities	12.5	12.5	12.5*	12.5*
Panel C2: High risk firm hedged ($F = 30$, $X_F = 1.500$)				
Cash available	60	90	60	30
Promised Liabilities	57.5	57.5	57.5	57.5*
Panel C3: High risk firm hedged ($F = 30$, $X_F = 1.636$)				
Cash available	60	90	60	30
Promised Liabilities	61.6*	61.6	61.6*	61.6*

Zero risk aversion, $K = 1$, $R - 1 = 0\%$, $D = 12.5$, $E(X_T) = 1.5$, $\tau = 0$.

* Default.

asset. If this correlation increases (states 2 and 4 become less probable), the default risk of the contract declines, decreasing the contract rate, X_F , and making it simpler to hedge successfully.

A more interesting case is when the seniority of the hedging contract is higher than the firm's debt. If the firm hedges with a forward contract of face value $F = 30$ and contract price $X_F = 1.576$ against $E(X_T) = 1.5$, it avoids default in state 3 (panel A1 in Table V). In state 3 the firm will have total assets worth 60 and total liabilities of 59.79, which include senior forward payment of 47.29 and a debt payment of 12.5.

The equilibrium forward spread is now lower, 0.0576, because $7/8 \cdot (30 \cdot 1.576) + 1/8 \cdot (30 - 1) = 45$. With a higher priority contract the cost to hedge goes down. The trade-off reduction in probability of default to increase in total liabilities improves and as a result hedging becomes viable. Another way to see this is to contrast the

Table V. High risk firm hedging with a senior forward

State of the world	1	2	3	4
Panel A1:				
Probability	3/8	1/8	3/8	1/8
Cash available	60	90	60	30
Forward Payment ($F \cdot X_F$)	47.29	47.29	47.29	47.29*
Cash to Pay Debt	12.71	42.71	12.71	0*
Promised Liabilities	59.79	59.79	59.79	59.79*
Panel A2: Face value of debt. Equal market value				
Unhedged	12.50	12.50	12.50*	12.50*
Hedged	7.14	7.14	7.14	7.14*

Zero risk aversion, $K = 1$, $R - 1 = 0\%$, $D = 12.5$, $E(X_T) = 1.5$, $\tau = 0$.

* Default.

value of the firm's debt, hedged and unhedged. The unhedged debt with a face value of 12.5 has a value of 6.25. If the firm tries to hedge with a junior forward contract, it cannot avoid default on states 3 and 4, and therefore the value of the hedged debt with a junior hedge is still 6.25. However, if the firm hedges with a senior contract default is avoided in state 3 and the debt goes up to 10.9375. Alternatively, the firm can raise the same amount of debt with a lower face value, 7.14, instead of 12.5 (panel A2 in Table V). Lower priority hedged debt is less risky than unhedged debt. More important, lower priority hedged debt is also less risky than higher priority hedged debt.

The firm can also hedge with a collateralized contract, such as a standard futures contract. The minimum necessary collateral to avoid default in state 3 is 15, which the firm raises by borrowing an additional 17.1 in new subordinated debt (Table VI). Even with this additional debt the firm now avoids default in state 3. The complete elimination of the default spread is sufficient to make hedging viable.

6. Robustness of the Results

In this section, we discuss briefly the robustness of the results, by changing two previous assumptions. First, we will relax the assumption of normally distributed random variables and consider the lognormal case. Second, we will discuss more general fixed financial distress costs, such as a fixed deadweight loss when the firm value falls below some multiple of the fixed claims, or bankruptcy costs that are proportional to asset value.

Table VI. High risk firm hedging with futures

State of the world	1	2	3	4
Probability	3/8	1/8	3/8	1/8
V_T (High risk firm)	30	30	0	0
Collateral	15	15	15	15
Net Futures Flow	-15	15	15	-15
Total Cash available	30	60	30	0
Debt (Face value)	12.5	12.5	12.5	12.5*
New Debt (Cover Collat)	17.1	17.1	17.1	17.1*
Total Liabilities	29.6	29.6	29.6	29.6*

Zero risk aversion, $K = 1$, $R - 1 = 0\%$, $D = 12.5$, $E(X_T) = 1.5$, $\tau = 0$.

* Default.

6.1. LOG-NORMALLY DISTRIBUTED PAYOFFS

Although analytically tractable, the assumption of normality leads to a finite probability of negative values for the variables and is, therefore, potentially unappealing. In this section we show that the results previously observed hold with the more realistic assumption of lognormal distributions. With lognormally distributed variables there is no close form solution and we have to numerically integrate the payoff functions of the various claims using a trapezoidal quadrature given in Press, Flannery, Tenkolsky and Vetterling (1988).

The firm involved has assets with current value of 100 and annualized instantaneous volatility of 40%. It is hedging a currency with an annualized instantaneous volatility of 15% and with a current exchange rate of 1. The maturity of the debt and forward contract are both five years. The deadweight loss parameter, K , is 5, the face value of debt, D , is 50, the riskless rate is 10% per year and $\tau = 0$.

Figure 1 shows the values of the firm when the forward contract is junior to the firm's debt and settled net. In all cases $H(F) = 0$. Note that when $\rho = -0.9$ hedging increases the firm value. With $\rho = 0$, however, hedging decreases the value of the firm, because the forward spread raises the default probability with no offsetting benefit from variance reduction.

As shown in Proposition 5 and in the numerical example, there is a minimum absolute correlation that is necessary to generate some hedging. The necessary correlation varies with priority rule governing the contract. For these parameter values, the correlation must be lower than -0.23 when the hedging contract is junior to the debt, lower than -0.16 when the hedging contract has equal priority to the debt, and lower than -0.07 when the contract is senior to the debt. The effect of the priority settlement is also illustrated in Figure 2, which shows the forward spread in basis points. For levels of hedging that minimize the firm's probability of

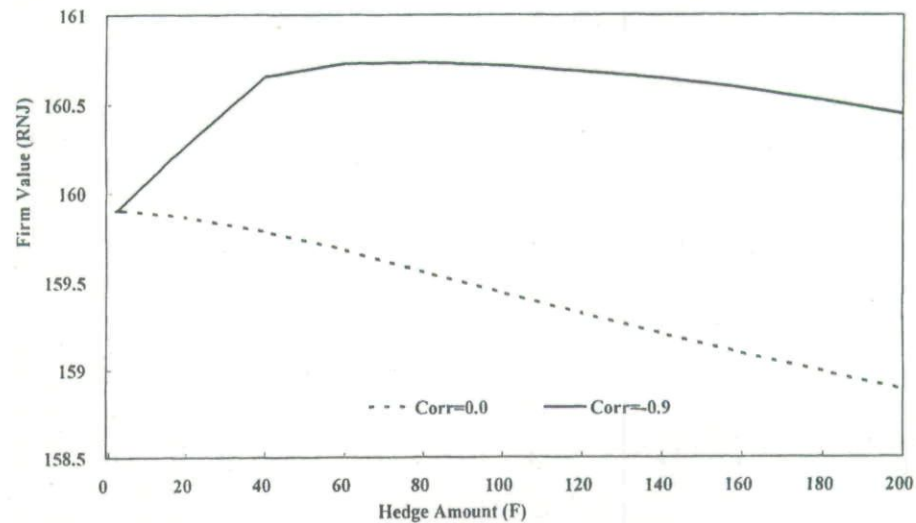


Figure 1. RNJ is Junior Priority Rule. Forward contract is junior to the debt contract. Parameters values are $D = 50$, $\sigma_X = 15\%$, $\sigma_V = 40\%$, $X = 1$, $V = 100$, $K = 5$, $T = 5$ years, $R^{-1} = 10\%$ annualized.

default, the spread is lower when the contract is of equal priority than when it is subordinated to the debt.

6.2. ALTERNATIVE BANKRUPTCY COST SPECIFICATIONS

We have assumed that bankruptcy costs are a fixed amount and are triggered when the value of the firm's assets falls below the promised amount of claims. Alternative specifications would be that the trigger point is some fraction of the fixed claims and that the costs are proportional to the asset value. Each of these could be accommodated in our model without changing anything material in the results.

Changing the bankruptcy trigger point to a fixed fraction, b , of the promised claims would generate essentially the same results. The risk-adjusted probability of bankruptcy would change from $W(V + FX < D + FX_F; T)$ to $W(V + FX < b(D + FX_F); T)$ under the new bankruptcy rule. With fixed transactions costs, incentives to reduce the variance of $(V + FX)$ by hedging are always in the same direction in both cases, regardless of the mean of $(V + FX)$. So, although the marginal tradeoffs between the variance and the mean would be affected, the general nature of the results would be the same.

Similarly, a switch to a proportional bankruptcy cost structure could be accommodated without significantly changing the results. A reasonable specification would be that the costs are a fixed proportion of the real asset value, V_T . The forward contract, being a purely financial agreement, would incur far lower bankruptcy costs than the real assets. The properties that rely on the form of the bankruptcy cost fraction are propositions 5 onwards. If we analyze these propo-

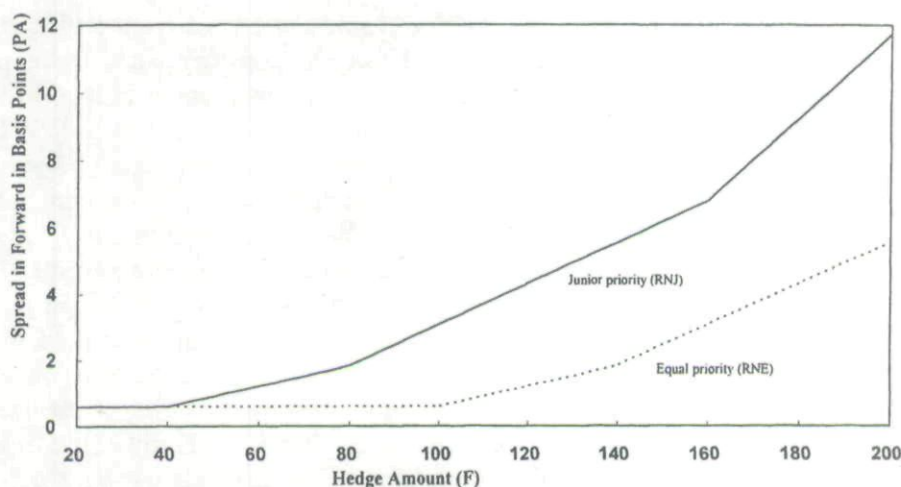


Figure 2. Forward contract spreads in basis points per annum, when the correlation between X and V is -0.9 , for different priority rules. RNE is Equal Priority Rule between the forward contract and the debt contract. RNJ is Junior Priority Rule. Forward contract is junior to the debt contract. Parameter values are $D = 50$, $\sigma_X = 15\%$, $\sigma_V = 40\%$, $X = 1$, $V = 100$, $K = 5$, $T = 5$ years, $R^{-1} = 10\%$ annual.

sitions with the assumption that transaction costs are a fixed proportion, c , of V_T , then the net value of assets after bankruptcy is $V_T(1 - c) + FX_T - FX_F$, and the loss in bankruptcy conditional on bankruptcy occurring, cV_T , is independent of the amount of hedging. So the only way that hedging matters is by affecting the probability of bankruptcy occurring and affecting the expectation of V_T conditional on bankruptcy. The incentives are to have as low a probability of bankruptcy as possible with bankruptcy occurring in states where V_T is as low as possible. This is essentially the same set of incentives as the fixed bankruptcy cost case and so the basic structure of the results is preserved.

7. Concluding Remarks

In this paper we have analyzed corporate hedging when hedging contracts are priced to include default risk. We have shown how the forward contract rate and the values of the firms' claims vary both with the amount of hedging. We have identified the conditions necessary for hedging to maximize equity value in the presence of agency costs. All results were derived under no arbitrage opportunities and are, therefore, valid regardless of the portfolio of forward contracts held by the hedging counterparty. The model is for a single period and motivates hedging by a deadweight cost to bankruptcy. It could be extended by having multiple periods with stochastic investment and financing opportunities, so that the motivation for hedging could be a more generalized cost of financial distress, for example the cost to external financing, as in Froot, Scharfstein and Stein (1993) and Mello

and Parsons (1997). The richer setting would not alter the qualitative results of the paper. However, with intertemporal leverage constraints and in the presence of risky debt it is difficult to assert at this point how would firms select among different hedging contracts [see the discussion in Mello and Parsons (1997)].

We conclude that the ability of debtholders to bond the hedging behavior of the firm is a critical element in the decision to hedge at all and in the amount of hedging undertaken. Without bonding, firms will not be able to hedge in a way that satisfies both bondholders and shareholders. When bondholders write covenants to protect the value of the debt against the use of derivatives by the firm, hedging becomes viable only if the hedging counterparty holds a significant portion of the firm's debt. As a result, firms with dispersed debt ownership (public debt) have greater difficulty in hedging at a favorable cost. However, given that it is in the interests of shareholders to reduce the costs of debt financing, it is plausible that firms make credible commitments to maintain a particular hedging strategy over the life of the debt contract, and as a result of bonding, hedging may become viable. However, even with bonding, the more the potential benefit from hedging (the higher the unhedged probability of default), the less likely the firm is to hedge, because the spread on the hedging contract is too high for high risk firms.

We also find that the characteristics of the hedging contract, both in terms of the absolute priority rule and the underlying asset correlation are important factors in determining the demand for hedging. Changing the design of the contract affects the hedging policy and the value of the firm. As a consequence, different contracts have different value to corporations with different risk characteristics. The efficiency of a senior forward contract helps to generate more demand for hedging by risky corporations, and therefore it explains why this is the most common priority rule in forwards transactions and why riskier firms hedge with futures contracts.

Several implications of the model are important from an empirical point of view: (1) it relates the spread in the hedging contract to the net asset volatility of the hedging firm, including the impact of the hedge on volatility. It is this volatility that is observable through the share price of the firm and not the gross volatility of the assets excluding the impact of hedging; (2) it establishes which firms will use the forward markets to hedge and which firms will not be able to do so. In particular, high grade firms, or firms with values at risk closely correlated with the hedging instrument will use uncollateralized forward contracts. Low-grade firms may have to use futures contracts, collateralized forward contracts or uncollateralized forward contracts with a relationship bank; (3) it predicts that firms with publicly held debt (low α) will find it more difficult to hedge with forward contracts than similar firms with a more concentrated debt ownership.

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