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Front-Running by Mutual Fund Managers: A Mixed Bag *

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Abstract. This paper evaluates the welfare implications of front-running by mutual fund managers. It extends the model of Kyle (1985) to a situation in which the insider with fundamentals-information competes against an insider with trade-information and in which noise trading is endogenized. Noise traders are small investors trading through mutual funds to hedge non-tradable or illiquid assets. The insider with trade-information is one of the fund managers. We find that her front-running activity reduces the liquidity costs of her customers, but it also reduces their hedging benefits. As a result, the customers of the front-running manager may be worse off and place smaller orders. The opposite is true, however, for those investors who are not subject to front-running. In aggregate, front-running has either no or positive consequences for welfare.

Key words: Front-running; Insider trading; Noise trading.

JEL classification. G14, G23.

1. Introduction

Large investors – such as mutual funds, institutionals and portfolio insurers – play an increasingly important role in modern financial markets. Because of their size, their trades often move stock prices even when they are not based on private information about assets' fundamental values. This empirical observation (see, e.g., Holthausen et al., 1987) is replicated in theoretical models of financial markets with asymmetric information. There, to avoid the possibility of prices fully revealing the information of insiders, and thus of private information being of no value, it has been common to assume implicitly or explicitly the presence of noise traders or

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liquidity traders.[1] In this context, even pure liquidity trades affect stock prices precisely because such trades cannot be clearly distinguished from the orders placed by insiders or informed traders.

This realistic modelling feature and the above empirical observation suggest that information about the activity of liquidity traders may be as important as information about assets' fundamental values. Advanced information on future orders of large traders, as available to the managers of the funds or the brokers through whom the orders are being executed, is valuable and can be used for personal profit. In several respects, such information may be assimilated to privileged information about fundamentals in the hands of corporate insiders, and taking advantage of it raises similar issues of appropriateness on grounds of efficiency and equity. Accordingly, it is legitimate to wonder whether fund managers should be allowed to trade on their account and engage in front-running.[2]

Restrictions on the ability of investment companies' employees to trade on their own account naturally result from the requirement by the Securities and Exchange Commission (SEC), under Rule 17j-1 of the Investment Company Act of 1940, that such Institutions adopt and enforce codes of ethics designed to detect and prevent improper personal trading. The Report of the Advisory Group on Personal Investing (Investment Company Institute, 1994) further recommends 'that every investment company incorporate in its code of ethics a statement of general fiduciary principles that govern personal investment activities'. The recommended restrictions on personal investing include prohibition of acquiring securities in an IPO, blackout periods during which employees should refrain from buying or selling a security – times during which their company is active on the market for that particular security – and a ban on short-term profits (within 60 calendar days of a trade by the employee's investment company). In addition, the Report makes a number of recommendations intended to facilitate compliance, recommending preclearance of personal securities investments and different measures designed to achieve transparent records of transactions.

The same Advisory Group, however, strongly opposes a ban on personal investing by portfolio managers, considering that such a ban, besides being unnecessary in light of its other recommendations, would unfairly foreclose 'wholly legitimate and appropriate investment opportunities' to a large number of individuals, would in so doing 'detract from the very portfolio management abilities on which share-

[1] See, e.g., Grossman and Stiglitz (1980), Hellwig (1980) and Kyle (1989); see also Easley and O'Hara (1987), Glosten and Migrom (1985), Kyle (1985) and Vives (1995), where prices are set by risk-neutral market makers. In other models, prices are not fully revealing because insiders have superior information in two dimensions (see, e.g., Bray, 1981; Diamond and Verrecchia, 1981 and Wang, 1994; see also Glosten, 1989 and Madhavan, 1992).

[2] We use the term 'front-running' generically to designate the attempt by agents with trade-related information to use this information for personal profit. Ours is a static set-up similar to Röell (1990) where front-running in fact takes the form of a short-sale when a liquidity trade is expected to push up a security's price (and a long position in the opposite case). For multi-period settings, see Fishman and Longstaff (1992), Pagano and Röell (1993) and Danthine and Moresi (1994).

holders rely' and would 'establish significant and needless disincentives to the continued service of talented individuals in the industry'.

The last two years have witnessed an increased interest by the SEC for the personal trading of the managers of some large US funds and officials have spoken in favor of a ban on personal investing.[3] While the increased SEC scrutiny seems grounded on their reading of the regulation against Insider trading, past sanctions have almost exclusively targeted corporate insiders – and the 'tipees' to whom they may pass secret information – that is, insider trading on fundamental information.[4] The reasons for this limitation are no doubt historical (dating to times where institutional trades were less common). Today, it is legitimate to wonder whether the same rules, with the same social objectives, should apply to insiders with trade-related information as well, or if differences in nature might invalidate the parallel often made between the two types of insider trading and make unwarranted the same regulatory treatment.

Because it is normative in nature and it entails some careful welfare evaluation, the question we raise requires appropriate modelling of the behavior of noise or liquidity traders. In most of the literature on informational equilibria, noise traders' actions are not derived from optimizing behavior. As a result, welfare evaluations in such models are difficult and questionable because the noise traders' objective is not well defined. In the present paper, liquidity traders are expected utility maximizers. The amount of noise trading will be endogenously determined and welfare evaluations will proceed along standard lines in general equilibrium models. Thus, ours is a model of asymmetric information where insider trading may affect both the gains from trading and the outsiders' willingness to participate in the market.

In our set-up, there are two types of insiders. The first is a corporate insider or a supplier of the firm. He has private information about the profitability of the firm and thus the value of the stock (in our set-up, the liquidation value of the firm's assets). He will be referred to as an insider with fundamentals-information. The second type of insider has private information about the aggregate order about to be placed, mostly for liquidity reasons, by the mass of small individual investors. This information is first and foremost in the hands of a financial intermediary whom we interpret as the manager of a fund handling the small investors' savings. She will be referred to as an insider with trade-information. It should be noted that trade-information is also, potentially, in the hands of the small investors themselves to the extent that their own liquidity needs contain information about similar shocks affecting other small investors. In this case, the small investors' orders are not exclusively liquidity motivated.

[3] See, e.g., 'Keeping both hands in view', *The Economist*, January 22, 1994; 'Why SEC lambasted news report on fidelity', *The Wall Street Journal*, April 22, 1996; 'Levitt advises fund managers to fix the roof', *The Wall Street Journal*, May 23, 1996.

[4] 'The advisory group believes that relatively few enforcement actions have been necessary because the industry has accorded high priority to developing effective code of ethics' (Investment Company Institute, 1994).

We examine the property of market equilibrium in such a context, asking in particular how market performance is affected by the interaction of the two types of information, and whether on grounds of efficiency and welfare it is justified to draw a distinction between the two types of insiders. In so doing we revisit and adapt to the important context of mutual fund management a small literature which has focused on front-running by broker-dealers: Fishman and Longstaff (1992), Pagano and Röell (1993) and Röell (1990). These studies find that front-running reduces the expected profits of insiders with fundamentals-information and reduces (increases) transaction costs for liquidity traders whose trades are subject (not subject) to front-running.[5] None of these studies takes into account the motives that bring liquidity traders to the market, nor do they incorporate the effect of front-running on their behavior. After restating in our context the above result, we show that welfare evaluations based on models with exogenous noise trading may be misleading. In particular, liquidity traders who are subject (not subject) to front-running may be worse off (better off) and may trade fewer shares (more shares) despite the fact that front-running reduces (increases) the price impact of their orders.

Our work is closely related to Sarkar (1995). Sarkar considers Kyle's (1985) model when there is one dual-capacity broker who handles all orders and who is able to distinguish insiders from liquidity traders. Following Spiegel and Subrahmanyam (1992), liquidity traders trade for hedging purposes and behave strategically. Sarkar finds that front-running has no effect on aggregate welfare; the broker gains at the expense of the insiders while liquidity traders neither gain nor lose.[6] In our model, liquidity traders are small competitive hedgers and only a fraction of them is subject to front-running. This is consistent with the fact that some fund managers may choose not to engage in front-running regardless of any regulation. Further, we allow the hedging needs of small investors (trading through the same fund) to be correlated. Investors may thus be 'informed' in the sense that their own hedging needs may contain relevant trade-information.

The remainder of the paper is organized as follows. Section 2 presents a simple model of the interaction between the two insiders under the assumption that noise trading is exogenous. This amounts to extending the model of Kyle (1985) to the case where the insider with fundamentals-information competes against an insider with trade-information. A model with endogenous noise trading is developed in Section 3 by assuming that noise trading originates from a continuum of competitive investors with stochastic positions in non-traded assets. A welfare analysis of insider trading is conducted in Sections 4 and 5. We first consider the case of uninformed investors and then assume that investors can extract information from

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[5] As was noted by Admati and Pfleiderer (1991) and made evident by Danthine and Moresi (1994), front-running and sunshine trading have similar effects.

[6] Assuming that the broker charges competitive fixed commission fees, so that the broker's expected profit is zero, dual trading lowers commission fees and liquidity traders gain at the expense of informed traders.

their own position for assessing the hedging needs of other investors. Section 6 summarizes our results and adds a few remarks. All proofs are in the Appendix.

2. A Model with Exogenous Noise Trading

We consider a market where insiders behave strategically and prices are set by market makers.[7] That is, we introduce an insider with trade-information in Kyle's (1985) single-period model.

A single risky asset is traded at price $\tilde{p}$ and its ex-post liquidation value is denoted by $\tilde{v}$. The asset thus generates returns $\tilde{v}-\tilde{p}$. The amount traded by liquidity traders is denoted by $\tilde{u}$, where positive amounts represent purchases and negative amounts represent sales. It is assumed that $\tilde{v}$ and $\tilde{u}$ are exogenous, independent, normal random variables, with zero means and variances Σ_v and Σ_u, respectively.[8]

There are two informed traders, namely, trader 1 and trader 2. The information structure is given by

$$\tilde{v} = \tilde{s}_1 + \tilde{\epsilon}, \tag{1}$$

$$\tilde{u} = \tilde{s}_2 + \tilde{\eta}, \tag{2}$$

where $\tilde{s}_i$ is a signal which is privately observed by trader i ($i = 1, 2$). That is, trader 1 has private information about the value of the asset (he has fundamentals-information) while trader 2 has private information about the amount of liquidity trading (she has trade-information).[9] The random variables $\tilde{s}_1$, $\tilde{\epsilon}$, $\tilde{s}_2$ and $\tilde{\eta}$ are independently and normally distributed with zero means and variances Σ_1, $\Sigma_v - \Sigma_1$, Σ_2 and $\Sigma_u - \Sigma_2$, respectively. Note that we are setting the signals $\tilde{s}_1$ and $\tilde{s}_2$ as the conditional expectations directly, that is, $E\{\tilde{v}|\tilde{s}_1 = s_1\} = s_1$ and $E\{\tilde{u}|\tilde{s}_2 = s_2\} = s_2$, so that Σ_i measures the amount or quality of the private information available to trader i.

The amount traded by trader i is denoted by $\tilde{x}_i$. A positive amount represents a purchase and a negative amount represents a sale. There are no restrictions on short-sales. Trading is structured in two steps as follows: In step one, the exogenous values of $\tilde{s}_1$, $\tilde{\epsilon}$, $\tilde{s}_2$ and $\tilde{\eta}$ are realized, and traders 1 and 2 choose the amounts $\tilde{x}_1$ and $\tilde{x}_2$ they trade. As trader i observes only $\tilde{s}_i$, his trading strategy is a (measurable) function X_i such that $\tilde{x}_i = X_i(\tilde{s}_i)$.[10] In step two, market makers determine the price $\tilde{p}$ at which they trade the quantity necessary to clear the market. When doing so they observe the order flow, i.e.,

$$\tilde{y} = \tilde{x}_1 + \tilde{x}_2 + \tilde{u}, \tag{3}$$

[7] Models of insider trading have been developed under various assumptions on the market microstructure. See the reviews in Ausubel (1990) and Moresi (1996).

[8] The assumption of zero means is a normalization.

[9] The model can be easily extended to the case of multiple informed traders. See, e.g., Admati and Pfleiderer (1988) and Röell (1990).

[10] As in Kyle (1985), traders are not allowed to condition their orders on price, that is, they can submit market order, but can not submit limit orders.

but they do not observe $\tilde{x}_1$, $\tilde{x}_2$ or $\tilde{u}$ (or $\tilde{v}$) separately. The market makers' pricing rule is a function P such that $\tilde{p} = P(\tilde{y})$.

An equilibrium is defined as a collection of strategies, $\{X_1, X_2, P\}$, such that the following two conditions hold:

(1) *Profit maximization:* For $i = 1, 2$, $j = 1, 2$ and $j \neq i$,

$$X_i(s_i) \text{ maximizes}_{x_i}\ E\{[\tilde{v} - P(x_i + X_j(\tilde{s}_j) + \tilde{u})]x_i | \tilde{s}_i = s_i\}, \quad \text{for all } s_i; \quad (4)$$

(2) *Market efficiency:*

$$P(y) = E\{\tilde{v} | X_1(\tilde{s}_1) + X_2(\tilde{s}_2) + \tilde{u} = y\}, \quad \text{for all } y. \quad (5)$$

The market efficiency condition can be thought of as the outcome of a Bertrand auction among risk-neutral market makers, each of whom observes the order flow and nothing else. Accordingly, market makers' (expected) profits are zero. The profit maximization condition emphasizes the fact that insiders behave strategically. Each trader i takes into account the effect of his order on the equilibrium price (and takes the other trader's strategy and the market makers' pricing rule as given).

The following proposition shows that there is an equilibrium in which the rules X_1, X_2 and P are linear functions.[11]

Proposition 1: Defining constants β_1, β_2 and λ by

$$\beta_1 = \frac{1}{2\lambda}, \quad \beta_2 = -\frac{1}{2}, \quad \lambda = \frac{1}{2}\left(\frac{\Sigma_1}{\Sigma_u - \frac{3}{4}\Sigma_2}\right)^{1/2}, \quad (6)$$

the following strategies constitute an equilibrium:

$$X_1(s_1) = \beta_1 s_1, \quad X_2(s_2) = \beta_2 s_2, \quad P(y) = \lambda y. \quad (7)$$

The equilibrium payoffs of trader 1 and trader 2 are given by

$$\Pi_1(s_1) = \frac{s_1^2}{4\lambda} \quad \text{and} \quad \Pi_2(s_2) = \frac{\lambda s_2^2}{4}. \quad (8)$$

In the special case where trader 2 has no private information (i.e., $\Sigma_2 = 0$), Proposition 1 reduces to Theorem 1 of Kyle (1985). The closest kin to the model of this section is Röell (1990) who proposes a similar extension of a version of Kyle's (1989) model.[12] In Sarkar (1995), trader 2 is a dual-capacity broker who

[11] As shown in the Appendix, there is no other linear equilibrium. However, we have not investigated whether non-linear equilibria exist.

[12] Rochet and Vila (1994) show the equivalence between the Kyle (1985) model where the insider observes the amount of noise trading (i.e., he observes *both* v and u) and the Kyle (1989) model when there is one risk-neutral insider and many risk-neutral market makers.

observes both the order submitted by the corporate insider (x_1) and the aggregate order submitted by the liquidity traders (u), while here she is a fund manager who observes only the orders of her customers (s_2) and the latter represent only a subset of the liquidity traders (see Section 3).[13] We acknowledge that this model of the interaction between insider trading and front-running is highly stylized. In particular, the assumption of a single asset is not very realistic because mutual funds typically invest in a portfolio of several assets. We conjecture, however, that our qualitative results can be extended to a multi-asset securities market. This is obvious if (a) the liquidation values of the assets are i.i.d., (b) there is a corporate insider for each asset and signals are i.i.d., and (c) liquidity traders and fund managers trade only the market portfolio. The extension is also straightforward in the case of funds that are, by design, highly concentrated on a small number of securities.[14]

The quantity $1/\lambda$ measures the depth or liquidity of the market (i.e., the size of the order flow necessary to induce a price change of 1 dollar). As shown by equation (6), trader 1's strategy (β_1) depends on market liquidity while trader 2's strategy (β_2) does not. That is, trader 1 places larger orders when the market is more liquid while trader 2 always 'absorbs' one-half of the expected amount of noise trading. Proposition 1 [equation (8)] also makes clear that *the two types of insiders have conflicting stakes in market liquidity*. The corporate insider likes a deep, liquid market so as to take advantage of his fundamentals-information without moving the price at which he transacts. On the contrary, the trade-information insider profits from her information only to the extent that fund orders will move asset prices.

As in Kyle's model, market liquidity increases with the amount of noise trading (Σ_u) and decreases with the amount of private fundamentals-information (Σ_1). Our analysis shows that market liquidity also decreases with the amount of private trade-information (Σ_2). Intuitively, trader 2 absorbs some of the noise trading thereby reducing the amount of noise trading in the order flow. Since the order flow is more likely to reflect the order of trader 1 – who has superior information about the asset's value – market makers are less willing to provide liquidity to the market. It should be emphasized that trader 2 (the manager of the fund) is in effect providing some liquidity, although not to the market, but to her customers.

Putting these elements together, we arrive at the conclusion, crucial for what follows, that the *front-running activity of trader 2 is detrimental for the corporate insider*: it decreases liquidity and consequently the profits opportunity of trader 1. (Asymmetrically, trader 2's profit is increasing in the amount of fundamentals-information in the hands of trader 1. Roughly speaking, the activity of corporate

[13] Fishman and Longstaff (1992), Pagano and Röell (1993) and Danthine and Moresi (1994) consider dynamic models of front-running by financial intermediaries. Their results are in conformity with those of the present section although the models differ in a number of respects related to the differences in focus.

[14] In most European markets, for instance, funds indexed on market indices are heavily correlated with the performance of a small number of Blue chips.

insiders makes the market less perfectly liquid and as a result trade-information has private value.)

We are not ready yet to address the issue of whether trade-information has social value: this requires a measure for noise traders' welfare. At this stage, however, it is useful to focus on the total gain of the insiders – $E\{\Pi_1(\tilde{s}_1) + \Pi_2(\tilde{s}_2)\}$ – which is relevant because the most common argument against insider trading is based on considerations of *fairness*. Insider trading, it is argued, should be banned or regulated because the insiders' gains are made at the expense of outsiders who have less information. This view suggests that regulation should aim at reducing the gains that insiders can realize.

Indeed, insiders' total gain is a loss for noise traders which may be interpreted as a liquidity cost. Here, this cost is unambiguously increasing in the amount of fundamentals-information, but it is *not monotonous with respect to the amount of trade-information*. Again, this is the result of the fact that the two insiders are competing and that an increase in trade-information decreases the profit of the corporate insider. Under certain conditions, the reduction in trader 1's profit is larger than the profit realized by trader 2.

If one wishes to advocate that 'fair trading' requires that the loss of the liquidity traders should be as small as possible, insider trading based on fundamentals-information should be prohibited. $\Sigma_1 = 0$ implies that the market is perfectly liquid and therefore the loss of the liquidity traders is zero. If however such a regulation is not in place – i.e., if insider trading based on fundamentals-information is legal – or if regulations cannot be perfectly enforced, that is, if $\Sigma_1 > 0$, then *the activity of insiders with trade-information is beneficial* as it reduces the loss of the liquidity traders. This result which extends to our context a similar result obtained by Röell (1990) is spelled out in the following proposition.

Proposition 2: For all $\Sigma_1 > 0$, the expected loss of the liquidity traders is highest when $\Sigma_2 = 0$ or $\Sigma_2 = \Sigma_u$, and lowest when $\Sigma_2 = \frac{2}{3}\Sigma_u$. Relative to either $\Sigma_2 = 0$ or $\Sigma_2 = \Sigma_u$, the expected loss is reduced by 5.7% if $\Sigma_2 = \frac{2}{3}\Sigma_u$.

An implication of Proposition 2 is the following: if trader 2 has perfect information about the total amount of liquidity trading (i.e., $\Sigma_2 = \Sigma_u$) then the expected loss of the liquidity traders is the same as in Kyle's model (where $\Sigma_2 = 0$). Intuitively, trader 2 absorbs one-half of the liquidity traders' order and this has two effects on their expected loss. On the one hand, as the net order flow that is absorbed by market makers is smaller, the price impact of the liquidity traders' order tends to be reduced, and so is their expected loss. On the other hand, as the amount of liquidity trading in the net order flow is reduced, market makers provide less liquidity to the market, which tends to increase the price impact of the liquidity traders' order, and thus to increase their expected loss. When $\Sigma_2 = \Sigma_u$, it turns out that these two effects cancel each other. However, when $0 < \Sigma_2 < \Sigma_u$

the first effect dominates and the expected loss of the liquidity traders is lower than in Kyle's model.

While the above notion of fairness has merits, it emphasizes the liquidity cost incurred by outsiders and ignores the motives that bring outsiders to the market. One may also oppose to it that the liquidity cost is a monetary transfer with no direct effect on total *welfare*. A full fledged welfare analysis requires an explicit modelling of the trading motives of noise traders. Before we turn to this task – in the next sections – let us note for future reference that in the model of this section it is easy to check that $\text{var}\{\tilde{v}|p\} = \Sigma_v - \frac{1}{2}\Sigma_1$ and $\text{var}\{\tilde{p}\} = \frac{1}{2}\Sigma_1$. That is, one-half of the private fundamentals-information of trader 1 is incorporated into the price, and *price volatility does not depend on either the amount of noise trading* (Σ_u) *or the amount of private trade-information* (Σ_2). In some sense, the presence of an insider with trade-information does not alter Kyle's results on informational efficiency and price volatility.[15]

3. Endogenous Noise Trading

In this section, we detail the source of noise trading postulated in Section 2. We assume that the liquidity trade $\tilde{u}$ originates in the purchasing or selling decisions of a continuum[16] of small competitive investors transmitting their order via two funds, and interpret $\tilde{u} = \tilde{s}_2 + \tilde{\eta}$ as the sum of the two funds orders. $\tilde{s}_2$ is the aggregate order of fund M, $\tilde{\eta}$ the aggregate order of fund N. As trader 2 observes $\tilde{s}_2$, fund M is exposed to front-running while fund N is not. By this device we want to take account of what we see as an important feature of the mutual-fund industry. As the quotes reproduced in the introduction illustrate, the investment industry is, and wants to remain, self-regulated. Codes of conduct differ from firm to firm, attitudes towards behavior that would fall in our definition of front-running are often ambiguous and, accordingly, internal rules preventing such behavior are likely to be enforced with varying degree of vigor from one institution to the next. Hereafter, we interpret trader 2 as the manager of fund M. The manager of fund N does not engage in front-running and thus has no role to play. As before, trader 1 is a corporate insider with private fundamentals-information.

Investors are indexed by $i \in (0, 1]$. Investors $i \in (0, \alpha]$ trade through fund M and investors $i \in (\alpha, 1]$ trade through fund N. The total mass of investors is normalized at one and the fraction α of investors trading through fund M is exogenous. Each investor i has a stochastic position $\tilde{\omega}(i)$ in non-traded assets and

[15] These results, however, hinge on the assumption of risk neutrality. With risk aversion, the informativeness and volatility of prices depend on the amount of noise trading (see Kyle, 1989 and Subrahmanyam, 1991) and, in dynamic settings, also on the autocorrelation of noise traders' orders (see Danthine and Moresi 1993).

[16] This allows us to assume that small investors behave competitively, as in Hellwig (1980) and Admati (1985).

the return on these assets is perfectly correlated with the value $\tilde{v}$ of the stock.[17] Investors are risk-averse and thus have a hedging motive to trade the stock. More precisely, they have constant absolute risk aversion (CARA) equal to $\rho > 0$, and they can place market orders through their fund. It should be recalled that, in Kyle's model, traders and investors are not allowed to condition their orders on the price p of the stock. This means that they face price-execution risk.

In such a context individual investors may or may not detain valuable trade-information. If all investors are identical, for instance, the representative investor knows that his optimal order (transmitted through his fund) will be similar to those transmitted by all the other small investors. He can anticipate the price pressure that the fund order will create and take account of it when designing his order. We will examine such a situation in Section 5. Conversely, the market context may be such that an individual cannot infer any usable information from his own $\omega(i)$ value. This case will be analyzed in Section 4.

A natural way to model these two polar cases is to first consider a situation in which the set $(0, \alpha]$ of investors trading through fund M consists of $m \geq 1$ subsets of identical investors, i.e., $(0, \alpha] = (0, h] \cup (h, 2h] \cdots \cup (\alpha - h, \alpha]$, where $h = \alpha/m$ is the mass of investors in each subset. All investors belonging to the same subset have the same initial position in non-traded assets, say, $\tilde{\omega}(i) = \tilde{\omega}(kh)$ for all $i \in (kh - h, kh]$ and $k = 1, \ldots, m$. Similarly, there are $n \geq 1$ subsets of identical investors trading through fund N, i.e., $(\alpha, 1] = (\alpha, \alpha + l] \cup (\alpha + l, \alpha + 2l] \cdots \cup (1 - l, 1]$ where $l = (1 - \alpha)/n$. Let $\tilde{w}_M$ ($\tilde{w}_N$) be the aggregate position in non-traded assets of the investors trading through fund $M(N)$, i.e.,

$$\tilde{w}_M = \sum_{k=1}^{m} \tilde{\omega}(kh)h \quad \text{and} \quad \tilde{w}_N = \sum_{k=1}^{n} \tilde{\omega}(\alpha + kl)l. \tag{9}$$

The $\{\tilde{\omega}(kh); k = 1, \ldots, m\}$ are i.i.d., normal, with zero mean and variance Ω/h. Similarly, the $\{\tilde{\omega}(\alpha + kl); k = 1, \ldots, n\}$ are i.i.d., normal, with zero mean and variance Ω/l. It follows that $\tilde{w}_M$ and $\tilde{w}_N$ are independent, normal, with zero means and variances $\alpha\Omega$ and $(1 - \alpha)\Omega$, respectively.

This formalization is convenient because the aggregate shocks, $\tilde{w}_M$ and $\tilde{w}_N$, do not depend on m and n. Roughly speaking, an increase in m (n) increases the heterogeneity of the customers of fund M (N) without changing the variance of their aggregate hedging needs. The latter depend only on the size α $(1 - \alpha)$ of fund M (N). In Section 5 we will set $m = n = 1$ so that each investor $i \in (0, 1]$ will know the aggregate order of his fund – and the front-running order of his fund manager if $i \in (0, \alpha]$. Section 4 will deal with the limiting case as $m \to \infty$ and $n \to \infty$, so that no investor will be able to infer any usable information from his own hedging need. Intuitively, when $m = 1$ the $\{\tilde{\omega}(i); i \in (0, \alpha]\}$ are a continuum

[17] The assumption of perfect correlation is not crucial and the qualitative results would not be affected as long as there is some correlation.

of perfectly correlated shocks, while when $m \to \infty$ they are a continuum of independent shocks (strictly, a continuum of shocks partitioned into a countable number of independent subsets).[18] Note that as $m \to \infty$ we have $\text{var}\{\tilde{\omega}(i)\} \to \infty$ and $\text{var}\{\tilde{\omega}(i)h\} \to 0$; each individual shock is extremely volatile but its contribution to the volatility of the aggregate shock is negligible. This explains why the variance of $\tilde{w}_M$ remains strictly positive and finite as $m \to \infty$. (By construction, it does not depend on m.)

For the moment, let m and n be finite. The order of investor i is denoted $\tilde{u}(i)$. If $i \leq \alpha$ investor i trades through fund M and his trading rule is a function U_M such that $\tilde{u}(i) = U_M(\tilde{\omega}(i))$. If $i > \alpha$ he trades through fund N and $\tilde{u}(i) = U_N(\tilde{\omega}(i))$. It follows that the liquidity trade $\tilde{u}$ is given by

$$\tilde{u} = \tilde{s}_2 + \tilde{\eta} = \sum_{k=1}^{m} U_M(\tilde{\omega}(kh))h + \sum_{k=1}^{n} U_N(\tilde{\omega}(\alpha + kl))l. \tag{10}$$

An equilibrium is defined as a collection of trading rules, $\{X_1, X_2, P, U_M, U_N\}$, that satisfies the profit maximization and market efficiency conditions [equations (4) and (5)] and the following additional condition:

(3) *Endogenous noise trading:* For $i \in (0, 1]$ and $j = M, N$,

$$U_j(\omega(i))\text{maximizes}_{u(i)}\, E_j\{\tilde{W}(i)\} - \frac{1}{2}\rho\, \text{var}_j\{\tilde{W}(i)\}, \tag{11}$$

where

$$\tilde{W}(i) = \tilde{v}\omega(i) + [\tilde{v} - P(X_1(\tilde{s}_1) + X_2(\tilde{s}_2) + \tilde{u})]u(i), \quad \text{for all } \omega(i).$$

That is, $\tilde{W}(i)$ is investor i's terminal wealth and his objective is to maximize the (conditional) certainty equivalent of his terminal wealth. (For $i \leq \alpha$, the mean, variance and covariance operators conditional on $\tilde{\omega}(i) = \omega(i)$ are denoted by E_M, var_M and cov_M; for $i > \alpha$, they are denoted by E_N, var_N and cov_N.) Under CARA preferences, the certainty equivalent of $\tilde{W}(i)$ reduces to the above mean-variance objective if $\tilde{W}(i)$ is normally distributed. We are thus implicitly assuming that the conditional distribution of $P(\tilde{y})$ will be normal in equilibrium.

From equation (11), investor i takes the price $\tilde{p} = P(\tilde{y})$ as given and his optimal order is

$$U_j(\omega(i)) = -\frac{\text{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}}{\text{var}_j\{\tilde{v} - \tilde{p}\}} \cdot \omega(i) + \frac{E_j\{\tilde{v} - \tilde{p}\}}{\rho \text{var}_j\{\tilde{v} - \tilde{p}\}}. \tag{12}$$

[18] As $m \to \infty$ and $n \to \infty$, the limiting sums in equation (9) define stochastic integrals, i.e.,

$$\lim_{m\to\infty} \tilde{w}_M = \int_0^{\alpha} \Omega^{1/2}\, \mathrm{d}\tilde{z}(i) \quad \text{and} \quad \lim_{n\to\infty} \tilde{w}_N = \int_{\alpha}^{1} \Omega^{1/2}\, \mathrm{d}\tilde{z}(i),$$

where $\tilde{z}(i)$ is a standard Wiener process. (See, e.g., Merton, 1990, chapter 3.)

The first term on the right-hand side of equation (12) is the hedging component of the order while the second term is the speculative component. It should be noted that in general two investors $i < \alpha$ and $i' > \alpha$ place different orders ($u(i) \neq u(i')$) even when they have similar hedging needs ($\omega(i) = \omega(i')$). That is, the two funds are different and investors' use different trading rules ($U_M \neq U_N$). In particular, the conditional mean and variance of the stock returns, $E_j\{\tilde{v} - \tilde{p}\}$ and $\mathrm{var}_j\{\tilde{v} - \tilde{p}\}$, are different for these two investors as the trade-information available to members of fund M is generally of different quality than the one available to members of fund N.[19] Using equation (12) to substitute for $u(i)$ in equation (11), the maximized certainty equivalent is equal to

$$CE_j(\omega(i)) = -\frac{1}{2}\rho\Sigma_v\omega(i)^2 + \frac{[E_j\{\tilde{p}\} + \rho\,\mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}\omega(i)]^2}{2\rho\,\mathrm{var}_j\{\tilde{v} - \tilde{p}\}}. \tag{13}$$

(As investors have no fundamentals-information, $E_j\{\tilde{v}\} = 0$ and $\mathrm{var}_j\{\tilde{v}\} = \Sigma_v$.) The first term on the right-hand side of equation (13) is the certainty equivalent of the investor's terminal wealth if he does not trade (i.e., if $u(i) = 0$). The second term measures the gains from trade that accrue to investor i.

Proposition 3: The strategies

$$\begin{aligned}
&X_1(s_1) = \beta_1 s_1, \quad X_2(s_2) = \beta_2 s_2, \quad P(y) = \lambda y,\\
&U_M(\omega(i)) = -\phi_M\omega(i) \text{ for } i \in (0, \alpha],\\
&U_N(\omega(i)) = -\phi_N\omega(i) \text{ for } i \in (\alpha, 1],
\end{aligned} \tag{14}$$

constitute an equilibrium if and only if:

$$\begin{aligned}
&\beta_1 = \frac{1}{2\lambda}, \quad \beta_2 = -\frac{1}{2}, \quad \lambda = \frac{1}{2}\left(\frac{\Sigma_1}{\Sigma_u - \frac{3}{4}\Sigma_2}\right)^{1/2},\\
&\phi_M = \frac{\rho(\Sigma_v - \frac{1}{2}\Sigma_1)}{\rho(\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\lambda^2\phi_M^2\Omega h) + \frac{1}{2}\lambda h},\\
&\phi_N = \frac{\rho(\Sigma_v - \frac{1}{2}\Sigma_1)}{\rho(\Sigma_v - \frac{1}{2}\Sigma_1 - \lambda^2\phi_N^2\Omega l) + \lambda l},
\end{aligned} \tag{15}$$

where $h = \alpha/m$, $l = (1 - \alpha)/n$ and

$$\Sigma_2 = \phi_M^2\alpha\Omega, \quad \Sigma_u = \phi_M^2\alpha\Omega + \phi_N^2(1 - \alpha)\Omega. \tag{16}$$

[19] The quality of this information depends on the size of the fund, the degree of homogeneity of the fund members and, of course, on the fund manager's propensity to front-run.

The equilibrium payoffs of trader 1, trader 2 and investor i are given by

$$\Pi_1(s_1) = \frac{s_1^2}{4\lambda}, \quad \Pi_2(s_2) = \frac{\lambda s_2^2}{4},$$

$$\begin{aligned} CE_M(\omega(i)) &= -\frac{1}{2}\rho\left[\Sigma_v - \phi_M^2\left(\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\lambda^2\phi_M^2\Omega h\right)\right] \\ &\quad \times\omega(i)^2 \quad \text{for } i \in (0, \alpha], \\ CE_N(\omega(i)) &= -\frac{1}{2}\rho\left[\Sigma_v - \phi_N^2\left(\Sigma_v - \frac{1}{2}\Sigma_1 - \lambda^2\phi_N^2\Omega l\right)\right] \\ &\quad \times\omega(i)^2 \quad \text{for } i \in (\alpha, 1], \end{aligned} \tag{17}$$

Proposition 3 extends Proposition 1 to a setting with endogenous noise trading. Like Proposition 1, Proposition 3 characterizes an equilibrium in which trading rules are linear functions. In particular, investors' orders are proportional to their positions in non-traded assets and the constant $\phi_M(\phi_N)$ is the 'hedge ratio' used by the investors trading through fund $M(N)$. It follows that Σ_2 and Σ_u – which are exogenous in Proposition 1 – are now given by equation (16). Unlike Proposition 1, Proposition 3 does not provide an explicit solution. The existence of a solution will be established only for the following special cases.[20]

4. The Case of Pure Liquidity Traders

This section uses the model developed in Section 3 and restricts attention to the limiting case as m and n tend to infinity.

Proposition 4: Consider Proposition 3. Assuming that an equilibrium exists for all $m \geq 1$ and $n \geq 1$, in the limit as $m \to \infty$ and $n \to \infty$ the equilibrium is given by equation (14) and

$$\beta_1 = \frac{1}{2\lambda}, \quad \beta_2 = -\frac{1}{2}, \quad \lambda = \frac{1}{2}\left(\frac{\Sigma_1}{\Sigma_u - \frac{3}{4}\Sigma_2}\right)^{1/2}, \quad \phi_M = \phi_N = 1, \tag{15'}$$

$$\Sigma_2 = \alpha\Omega, \quad \Sigma_u = \Omega. \tag{16'}$$

The equilibrium payoffs of trader 1, trader 2 and investor i are given by

$$\Pi_1(s_1) = \frac{s_1^2}{4\lambda}, \quad \Pi_2(s_2) = \frac{\lambda s_2^2}{4} \quad \text{and} \quad CE_j(\omega(i)) = -\frac{\rho\Sigma_1\omega(i)^2}{4}. \tag{17'}$$

[20] Of course, we ignore the trivial equilibrium with no trading that always exists in the absence of exogenous noise trading (see Bhattacharya and Spiegel, 1991).

Proposition 4 gives the explicit solution for the limiting case of a continuum of investors with independent hedging needs. On the one hand, each investor i has zero mass and his order $\tilde{u}(i)$ does not affect the stock price. On the other hand, in the limit as the $\{\tilde{\omega}(i); i \in (0, 1]\}$ become independent, his position $\tilde{\omega}(i)$ does not contain any information about the positions of other investors. Thus investors are competitive hedgers with no private information and $\tilde{u}(i) = -\tilde{\omega}(i)$. The result that the hedge ratios ϕ_j are equal to one can be explained by referring to equation (12). When $\tilde{\omega}(i)$ is uninformative, market efficiency implies $E_j\{\tilde{v} - \tilde{p}\} = 0$ so that the speculative component of the investor's order is zero. Market efficiency also implies $\mathrm{cov}_j\{\tilde{p}, \tilde{v} - \tilde{p}\} = 0$ so that the hedge ratio, $\mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}/\mathrm{var}_j\{\tilde{v} - \tilde{p}\}$, is equal to one.

When investors are pure liquidity traders, the aggregate orders of fund M and fund N are simply given by $\tilde{s}_2 = \tilde{w}_M$ and $\tilde{\eta} = \tilde{w}_N$, with variances $\Sigma_2 = \alpha\Omega$ and $\Sigma_u - \Sigma_2 = (1 - \alpha)\Omega$, respectively. The variance of the total liquidity trade is $\Sigma_u = \Omega$. Clearly, some of the implications of Proposition 1 remain unchanged or can easily be reinterpreted in terms of the parameters α and Ω of the present model with endogenous noise trading. For example, Proposition 4 still implies $\mathrm{var}\{\tilde{v}|p\} = \Sigma_v - \frac{1}{2}\Sigma_1$ and $\mathrm{var}\{\tilde{p}\} = \frac{1}{2}\Sigma_1$, and market liquidity is increasing in Ω and decreasing in α and Σ_1.

Although Propositions 1 and 4 have similar positive implications, the normative implications are quite different. (Strictly, Proposition 4 contains the information relevant for a welfare evaluation of insider trading while Proposition 1 does not.) First, let us note that in the present case with pure liquidity traders the expected profits of the two insiders are negligible components of total welfare. This is because the expected gains from risk sharing accruing to small investors are of a much larger magnitude than those accruing to insiders. More specifically, the gain of investor i is proportional to the square of $\tilde{\omega}(i)$ [see equation (17)] and the variance of $\tilde{\omega}(i)$ becomes unbounded as m and n tend to infinity [see Section 3]. Second, investors face both fundamental risk and price-execution risk. Proposition 4 tells us that it is optimal to fully hedge the fundamental risk. Only price-execution risk remains. Third, the variance of $\tilde{p}$ is equal to $\frac{1}{2}\Sigma_1$ and is unaffected by the extent of trader 2's activity (or by α). Thus, the latter cannot affect investors' welfare. Corollary 1 below confirms this result and Corollary 2 proposes an explicit measure for investors' gains from trading.

Corollary 1: The post-trade utility of uninformed investors,

$$CE_j(\omega(i)) = -\frac{\rho\Sigma_1\omega(i)^2}{4},$$

does not depend on whether they trade through fund M or fund N. It is decreasing in the amount of fundamentals-information and is not affected by the amount of trade-information.

Intuitively, insider trading based on fundamentals-information increases the volatility of the stock price. When small investors have no private information, their optimal hedging demands are not affected but the price-execution risk is higher. Hence, welfare is reduced. On the other hand, insider trading based on trade-information reduces the liquidity of the market. But trader 1 reacts by placing smaller orders and it turns out that price volatility is not affected. As investors' hedging demands do not change, their welfare is not affected either.

Let us now try to be more precise about the cost of insider trading. In the absence of insider trading (i.e., in the first best), we would have $CE_j^*(\omega(i)) = 0$ as investors would be able to hedge their risky positions at the fair price (i.e., $p = 0$ with probability 1). In the absence of any trading, however, we would have $CE_j^0(\omega(i)) = -\frac{1}{2}\rho\Sigma_v\omega(i)^2$ as investors would not be able to hedge their positions at all. In other words, $-CE_j^0(\omega(i))$ is the potential gain from trade for investor i (measured in monetary units and conditional on $\omega(i)$) while $-CE_j(\omega(i))$ is the loss incurred by investor i due to the presence of insiders. Therefore,

Corollary 2: The percentage welfare loss incurred by uninformed investors due to insider trading is given by

$$\frac{CE_j(\omega(i))}{CE_j^0(\omega(i))} = \frac{\Sigma_1}{2\Sigma_v}. \tag{18}$$

When investors are uninformed, the percentage welfare loss due to insider trading does not depend on the realization of $\tilde{\omega}(i)$, neither does it depend on the degree of risk aversion (ρ) or the relative size of the funds (α).[21] The latter implies that the percentage welfare loss is not affected by the front-running activity of trader 2.

In this section, we have proposed a first way to endogenize noise trading. Given the unbounded variance assumption made on the small investors' initial asset positions, we are led to emphasize the effect of front-running on (conditional) variances, thus placing us at the extreme opposite of the usual front-running analyses which effectively focus on first moments only. In the context of this section, we show that the omission of variance considerations imposed on standard analysis by the exogenous noise trading hypothesis does not bias the view one may have on front-running. The impact of front-running on small investors' welfare, which arises exclusively from its potential effect on the execution risk facing these investors, is nil. In a sense, one may therefore consider that the evaluation of front-running by a mutual fund manager obtained in the previous section prevails.

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[21] Evaluating the ex-ante analog of equation (18) would require tedious calculations because terminal wealth is not normally distributed. For simplicity, we measure welfare at the interim stage, i.e., prior to trading but conditional on the information available to small investors.

5. The Case of Informed Investors

We now consider the case in which the $\tilde{\omega}(i)$ are perfectly correlated within the same fund and uncorrelated across funds. This consists in setting $m = n = 1$ in Proposition 3. Recall that the aggregate positions in non-traded assets, $\tilde{w}_M$ and $\tilde{w}_N$, have the same (unconditional) joint distribution as in Section 4 (by construction). Here, however, individual investors have trade-information and their orders may reflect both hedging and speculative motives. In equilibrium, each investor $i \in (0, \alpha]$ knows that $\tilde{w}_M = \alpha\omega(i)$ and thus that $\tilde{s}_2 = -\phi_M \alpha\omega(i)$. (He also knows the amount $\frac{1}{2}\tilde{s}_2 = -\frac{1}{2}\phi_M\alpha\omega(i)$ that will be absorbed by trader 2.) Each investor $i \in (\alpha, 1]$ knows that $\tilde{w}_N = (1-\alpha)\omega(i)$ and $\tilde{\eta} = -\phi_N(1-\alpha)\omega(i)$.

We assume that investors' risk aversion is unbounded (i.e., $\rho \to \infty$). This is a strong assumption. It implies that the objective of small investors reduces to the minimization of the conditional variance of their terminal wealth (instead of trading off mean and variance). As a result, the speculative component of their order [the second term on the right-hand side of equation (12)] vanishes. Thus, as in the case of pure liquidity traders of Section 4, the hedge ratios ϕ_j are equal to $\mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}/\mathrm{var}_j\{\tilde{v} - \tilde{p}\}$. Besides simplifying the analysis, this assumption has the advantage that investors' orders remain (plausibly) driven by hedging motives only. On the other hand, when investors are extremely risk-averse, the profits of the two insiders are negligible relative to the investors' gains from risk sharing. Welfare evaluations thus remain focused on investors' utilities. These similarities with Section 4 make it easier to compare and interpret the results.

Proposition 5: Consider Proposition 3. Set $n = m = 1$. Assuming that an equilibrium exists for all $\rho > 0$, in the limit as $\rho \to \infty$ the equilibrium is given by equation (15) and

$$\beta_1 = \frac{1}{2\lambda}, \quad \beta_2 = -\frac{1}{2}, \quad \lambda = \frac{1}{2}\left(\frac{\Sigma_1}{\Sigma_u - \frac{3}{4}\Sigma_2}\right)^{1/2},$$

$$\phi_M = \frac{\Sigma_v - \frac{1}{2}\Sigma_1}{\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\lambda^2\phi_M^2\alpha\Omega},$$

$$\phi_N = \frac{\Sigma_v - \frac{1}{2}\Sigma_1}{\Sigma_v - \frac{1}{2}\Sigma_1 - \lambda^2\phi_N^2(1-\alpha)\Omega}, \tag{15''}$$

where

$$\Sigma_2 = \phi_M^2\alpha\Omega, \quad \Sigma_u = \phi_M^2\alpha\Omega + \phi_N^2(1-\alpha)\Omega. \tag{16''}$$

Although Proposition 5 does not provide an explicit solution, it is shown in the Appendix that equations (15″) and (16″) have a unique solution. Observe that the hedge ratios, ϕ_M and ϕ_N, are greater than one (for all $0 < \alpha < 1$ and $0 < \Sigma_1 \leq \Sigma_v$). Some intuition for $\phi_j > 1$ can be developed using the expression $\tilde{W}(i) = \tilde{p}\omega(i) + (\tilde{v} - \tilde{p})[\omega(i) + u(i)]$ of investor i's terminal wealth. Accordingly, his portfolio may be thought of as comprising $\omega(i)$ shares of a non-traded asset with return $\tilde{p}$ and $\omega(i) + u(i)$ shares of a traded asset with return $\tilde{v} - \tilde{p}$. On the one hand, market efficiency implies that, conditional on $\omega(i)$, the returns $\tilde{p}$ and $\tilde{v} - \tilde{p}$ are positively correlated.[22] This is so because investor i has trade-information which is not available to market makers. On the other hand, investor i is extremely risk-averse so that his objective is to minimize $\mathrm{var}_j\{\tilde{W}(i)\}$. Given the positive correlation between $\tilde{v} - \tilde{p}$ and $\tilde{p}$, $\mathrm{var}_j\{\tilde{W}(i)\}$ is minimized only if $\omega(i) + u(i)$ and $\omega(i)$ are of opposite sign. Hence $u(i) = -\phi_j\omega(i)$ with $\phi_j > 1$.

In this case, the amount of noise trading and the amount of trade-information in the hands of trader 2 depend on the investors' propensities to trade or hedge ratios, ϕ_M and ϕ_N, which in turn depend on Σ_1 and α (but not on Ω). This leads to

Corollary 3: When small investors have trade-information, market liquidity is increasing in Ω but may be decreasing or increasing in Σ_1.

An implication of Corollary 3 is that a reduction in the amount of insider trading based on fundamentals-information may result in a less liquid market. Intuitively, in markets with more private fundamentals-information, investors with trade-information have a higher propensity to trade (hedge ratio) because the conditional correlation of the returns on traded and non-traded assets is higher. This means that the amount of noise trading is also higher and, as a result, the effect on market liquidity is ambiguous.

While the endogeneity of noise trading may change the relationship between the liquidity of the market and the amount of private fundamentals-information, it does not affect Kyle's results on price volatility and informational efficiency. That is, $\mathrm{var}\{\tilde{p}\} = \frac{1}{2}\Sigma_1$ and $\mathrm{var}\{\tilde{v}|p\} = \Sigma_v - \frac{1}{2}\Sigma_1$.

Corollary 4: The percentage welfare loss due to insider trading is given by

$$\frac{CE_j(\omega(i))}{CE_j^0(\omega(i))} = 1 - r_j^2, \tag{19}$$

where r_j is the correlation coefficient between $\tilde{v}$ and $\tilde{v} - \tilde{p}$ conditional on $\omega(i)$ ($j = M$ if $i \leq \alpha$, $j = N$ if $i > \alpha$).

[22] Market efficiency implies that for uninformed investors p and $v - p$ are not correlated (see the discussion following Proposition 4). As trade-information does not affect the conditional covariance of p and v, but reduces the conditional variance of p, for informed investors we must have $\mathrm{cov}_j\{p, v-p\} = \mathrm{cov}_j\{p, v\} - \mathrm{var}_j\{p\} > 0$. (Thus, $\mathrm{cov}_j\{v, v-p\} > \mathrm{var}_j\{v-p\}$ and, as $\rho \to \infty$, Equation (12) implies $\phi_j = \mathrm{cov}_j\{v, v-p\}/\mathrm{var}_j\{v-p\} > 1$.)

Here again, the expected profits of the two insiders do not matter for welfare. This is so because we are assuming that small investors are extremely risk averse so that their gains from risk sharing are unbounded. When investors are informed, however, the predictions of the model differ from those of the previous section. For simplicity, we provide results for three cases only: $\alpha = 0$, $\alpha = \frac{1}{2}$ and $\alpha = 1$. When all investors are identical and trade trough the same fund (i.e., fund M if $\alpha = 1$, or fund N if $\alpha = 0$), the percentage welfare loss due to insider trading is equal to [see the proof of Corollary 3 in the Appendix]:

$$\frac{(\Sigma_v - \Sigma_1)\Sigma_1}{(4\Sigma_v - 3\Sigma_1)\Sigma_v}. \tag{20}$$

This expression is zero for either $\Sigma_1 = 0$ or $\Sigma_1 = \Sigma_v$, and positive for intermediate values of Σ_1. This shows that a reduction in the amount of insider trading based on fundamentals-information may actually reduce total welfare. On the other hand, since fund M is subject to front-running while fund N is not, the fact that the welfare loss is the same for both $\alpha = 1$ and $\alpha = 0$ demonstrates that, when all investors have as much trade-information as trader 2, the (front-running) activity of the latter has no welfare consequences. These results are summarized in part (a) of Proposition 6. Part (b) deals with the case where the two funds have the same size ($\alpha = \frac{1}{2}$).

Proposition 6: For the case with informed investors, as $\rho \to \infty$,

(a) If $\alpha = 0$ or $\alpha = 1$, the percentage welfare loss due to insider trading may be increasing or decreasing in the amount of fundamentals-information. The presence of an insider with trade-information has no welfare consequences.
(b) If $\alpha = \frac{1}{2}$ and $\Sigma_1 > 0$, the presence of an insider with trade-information about the order of fund M increases the percentage welfare loss incurred by the members of fund M, but the total welfare loss is reduced.

Part (b) of Proposition 6 says the following. If insider trading based on fundamentals-information cannot be eliminated, allowing the manager of fund M to trade on her own account may be welfare improving. More precisely, her customers are worse off but the members of fund N are better off. Intuitively, for the members of fund N, the activity of the manager of fund M reduces the unanticipated amount of noise trading and, as a result, the conditional correlation of the returns on traded and non-traded assets is higher. [When $\Sigma_1 = \Sigma_v$, this correlation would be perfect if trader 2 was absorbing the entire order of fund M.] The members of fund N, therefore, are better off and place larger orders. This implies that, for the members of fund M, the unanticipated amount of noise trading is higher and the conditional correlation of the returns on traded and non-traded assets is lower. The activity of their fund manager, therefore, makes them worse off and induces them to place smaller orders. In short, our analysis shows that there may be a conflict of interests between fund managers and their customers. However, front-running may increase total welfare.

Note that, in the case of this section, by focusing once again on the effect of front-running on second moments – this time because of our infinite risk aversion assumption – we obtain a reversal of the traditional front-running results: the customers of fund M whose manager front-runs are made worse off. Here endogenizing noise traders matters. Of course, we do not mean that variance effects will necessarily dominate the first moment effects traditionally emphasized. But, by tackling the other extreme case, our analysis suggests caution in interpreting the results obtained with exogenous noise trading models.

6. Conclusion

Let us now summarize our main results. First, we have reinterpreted in the context of front-running by mutual fund managers existing results on insider trading with exogenous noise traders. Thus concentrating on the liquidity cost of insider trading for small investors, we arrive at a first conclusion: because of its effect on the profits and the strategy of insiders with fundamentals-information, front-running decreases the liquidity cost of trading in a market with asymmetric information. On this score and based on fairness considerations, it can be viewed favorably.

We have then sought to extend this preliminary evaluation by proceeding to a true welfare analysis of front-running. Accordingly we have endogenized noise trading. We have assumed that noise trading originates from a continuum of competitive investors hedging stochastic positions in non-traded assets. Small investors trade through two mutual funds, one of which is subject to front-running by its manager. We observed that in such a situation a small trader may or may not detain valuable trade-information depending on whether his own stochastic initial asset position is or is not correlated with the position of other small investors.

In the case of uninformed small investors, we found that front-running has no impact on the welfare of these pure liquidity traders. Given the unbounded variance assumption made on small investors' initial positions, our analysis emphasizes the effect of front-running on (conditional) variances, thus placing us at the extreme opposite of the usual analyses which effectively focus on first moment effects. Here we have shown that the omission of variance considerations imposed on standard analysis by the usual exogenous noise trading hypothesis does not bias the view one may have on front-running: the impact of front-running on small investors' welfare, which arises exclusively from its potential effect on the execution risk facing these investors, is nil.

When liquidity traders hold valuable trade-information, however, focusing once again on second moments – this time via an infinite risk aversion assumption – may lead to a reversal of the traditional front-running results. If informed small investors all trade through the same fund, i.e., even when front-running is pervasive, front-running is once again without welfare consequences. If small traders equally distribute their orders between the two funds, however, the customers of the fund whose manager front-runs are made worse off while the total welfare loss associated to insider trading on fundamental information is reduced. Thus here, endogenizing noise traders matters. Front-running has interesting aggregate and distributive effects.

Of course, we cannot guarantee that variance effects such as those put at the forefront in our analysis will be overriding. And our analysis lead us to put the brunt of the welfare analysis on the cost of insider trading associated with price execution risk. Other considerations must, no doubt, be given room when attempting an overall evaluation of front-running. Yet, our set-up is also a rich one, emphasizing as it does the interactions and the conflicting interests between the two types of insiders and thus pointing to the danger of evaluating front-running in isolation: after all, trade-related information would have no value in a world without information asymmetries, i.e., in the (perhaps too restrictive) terminology of our model, in the absence of corporate insiders. Our analysis also suggests caution in interpreting results obtained with exogenous noise trading models.

Finally, our analysis suggests interesting extensions. For one thing, front-running is one aspect of the possibility given fund managers to trade on personal accounts, itself a prominent element of their remuneration and thus of the cost of running a fund. It is in this context that the conflict of interests uncovered in Section 5 between the fund manager and her customers should be studied. At another level, because of the negative impact of the actions of trade-information insiders on corporate insiders' profits, endogenizing the decision to use corporate information in a world where it would be costly to do so – because, for instance, it is illegal – is likely to strengthen the assessment of front-running provided here.[23]

Appendix

PROOF OF PROPOSITION 1

Suppose that X_1, X_2 and P are linear, i.e.,

$$X_1(\tilde{s}_1) = \alpha_1 + \beta_1\tilde{s}_1, \quad X_2(\tilde{s}_2) = \alpha_2 + \beta_2\tilde{s}_2, \quad P(\tilde{y}) = \mu + \lambda\tilde{y}, \tag{A1}$$

where the constants $\alpha_1, \beta_1, \alpha_2, \beta_2, \mu$ and λ are to be determined. Given linear rules X_2 and P, trader 1's (expected) profit can be written as

$$E\{[\tilde{v} - P(x_1 + X_2(\tilde{s}_2) + \tilde{u})]x_1|\tilde{s}_1 = s_1\} = [s_1 - \mu - \lambda(x_1 + \alpha_2)]x_1. \tag{A2}$$

Assuming for the moment that $\lambda > 0$, the profit-maximizing value of x_1 is given by the first-order condition $s_1 - \mu - \lambda(2x_1 + \alpha_2) = 0$. Thus, trader l's strategy is linear with

$$\beta_1 = 1/(2\lambda), \quad \alpha_1 = -\mu/(2\lambda) - \alpha_2/2. \tag{A3}$$

[23] In Danthine and Moresi (1994), we liken front-running to a form of (credible) sunshine trading regulation.

Similarly, given linear rules X_1 and P, trader 2's profit can be written as

$$E\{[\tilde{v} - P(x_2 + X_1(\tilde{s}_1) + \tilde{u})]x_2 \mid \tilde{s}_2 = s_2\} = [-\mu - \lambda(x_2 + \alpha_1 + s_2)]x_2, \quad \text{(A4)}$$

so that x_2 is given by $-\mu - \lambda(2x_2 + \alpha_1 + s_2) = 0$. Thus, trader 2's strategy is linear with

$$\beta_2 = -1/2, \quad \alpha_2 = -\mu/(2\lambda) - \alpha_1/2. \quad \text{(A5)}$$

Given linear rules X_1 and X_2, the market efficiency condition can be written as

$$P(y) = E\{\tilde{v}|\alpha_1 + \alpha_2 + \beta_1\tilde{s}_1 + \beta_2\tilde{s}_2 + \tilde{u} = y\}. \quad \text{(A6)}$$

Normality makes the regression linear so that P is indeed linear with

$$\begin{aligned}\lambda &= \frac{\text{cov}\{\tilde{v}, \tilde{y}\}}{\text{var}\{\tilde{y}\}} \\ &= \frac{\beta_1\Sigma_1}{\beta_1^2\Sigma_1 + (\beta_2 + 1)^2\Sigma_2 + \Sigma_u - \Sigma_2}, \quad \mu = -\lambda(\alpha_1 + \alpha_2). \quad \text{(A7)}\end{aligned}$$

Solving equations (A3), (A5) and (A7), one obtains equation (6). Substituting into equations (A2) and (A4) yields equation (8). Note that $\mu = \alpha_1 = \alpha_2 = 0$ and the second-order condition $\lambda > 0$ rules out a solution where λ and β_1 are negative.

PROOF OF PROPOSITION 2

From Proposition 1, the loss of the liquidity traders is given by

$$\begin{aligned}E\{\Pi_1(\tilde{s}_1) + \Pi_2(\tilde{s}_2)\} &= \frac{1}{2}(\Sigma_1\Sigma_u)^{1/2}\left(1 - \frac{3}{4}a\right)^{1/2} \\ &\quad + \frac{1}{8}(\Sigma_1\Sigma_u)^{1/2}a\left(1 - \frac{3}{4}a\right)^{-1/2}, \quad \text{(A8)}\end{aligned}$$

where $a = \Sigma_2/\Sigma_u$. This loss is equal to $\frac{1}{2}(\Sigma_1\Sigma_u)^{1/2}$ for either $\Sigma_2 = 0$ or $\Sigma_2 = \Sigma_u$. Let us rewrite equation (A8) as

$$E\{\Pi_1(\tilde{s}_1) + \Pi_2(\tilde{s}_2)\} = \frac{1}{2}(\Sigma_1\Sigma_2)^{1/2}(1 - g), \quad \text{(A9)}$$

where

$$g = 1 - \left(1 - \frac{1}{2}a\right)\left(1 - \frac{3}{4}a\right)^{-1/2} \quad \text{(A10)}$$

is the percentage reduction of the liquidity traders' loss relative to the case where $\Sigma_2 = 0$ (or, equivalently, $a = 0$). The right-hand side of equation (A10) is maximized for $a = \frac{2}{3}$, which corresponds to $g \cong 0.057$.

PROOF OF PROPOSITION 3

Suppose that investors use the linear rules U_M and U_N given in equation (14). Then, equation (10) implies that $\tilde{s}_2$ and $\tilde{u}$ (and thus $\tilde{\eta}$) are normal with zero means. Their variances are given in equation (16) and have been found by using equation (10), $\text{var}\{\tilde{\omega}(kh)h\} = \Omega h$, $\text{var}\{\tilde{\omega}(\alpha + kl)l\} = \Omega l$ and the fact that the $\{\tilde{\omega}(kh); k = 1, \ldots, m\}$ and the $\{\tilde{\omega}(\alpha + kl); k = 1, \ldots, n\}$ are $m + n$ independent variables. From Proposition 1, the linear rules X_1, X_2 and P given in equation (14) are equilibrium rules if and only if β_1, β_2 and λ satisfy equation (15).

Consider now an investor i who trades through fund M, i.e., $i \leq \alpha$. With no loss of generality, let $i \in (0, h]$. Given that everybody else is using the linear rules given in equation (14), investor i knows that

$$\tilde{p} = \frac{1}{2}\tilde{s}_1 - \frac{1}{2}\lambda\phi_M \left[\sum_{k=2}^{m} \tilde{\omega}(kh)h + \tilde{\omega}(i)h\right] + \lambda\tilde{\eta}, \tag{A11}$$

so that, conditional on $\tilde{\omega}(i) = \omega(i)$, $\tilde{p}$ is normal with mean

$$E_M\{\tilde{p}\} = -\frac{1}{2}\lambda\phi_M\omega(i)h. \tag{A12}$$

Equations (12) and (A12) imply that U_M is indeed linear with

$$\phi_M = \frac{\rho\,\text{cov}_M\{\tilde{v}, \tilde{v} - \tilde{p}\}}{\rho\,\text{var}_M\{\tilde{v} - \tilde{p}\} + \frac{1}{2}\lambda h}. \tag{A13}$$

Similarly, for the investors trading through fund N, one finds

$$\tilde{p} = \frac{1}{2}\tilde{s}_1 + \frac{1}{2}\lambda\tilde{s}_2 - \lambda\phi_N \left[\sum_{k=2}^{n} \tilde{\omega}(\alpha + kl)l + \tilde{\omega}(i)l\right], \tag{A11$'$}$$

$$E_N\{\tilde{p}\} = -\lambda\phi_N\omega(i)l, \tag{A12$'$}$$

$$\phi_N = \frac{\rho\,\text{cov}_N\{\tilde{v}, \tilde{v} - \tilde{p}\}}{\rho\,\text{var}_N\{\tilde{v} - \tilde{p}\} + \lambda l}. \tag{A13$'$}$$

Since

$$\text{cov}_M\{\tilde{v}, \tilde{v} - \tilde{p}\} = \text{cov}_N\{\tilde{v}, \tilde{v} - \tilde{p}\} = \Sigma_v - \frac{1}{2}\Sigma_1,$$

$$\mathrm{var}_M\{\tilde{v}-\tilde{p}\} = \Sigma_v - \frac{3}{4}\Sigma_1 + \lambda^2\Omega\left[\frac{1}{4}(\alpha-h)\phi_M^2 + (1-\alpha)\phi_N^2\right],$$

$$\mathrm{var}_N\{\tilde{v}-\tilde{p}\} = \Sigma_v - \frac{3}{4}\Sigma_1 + \lambda^2\Omega\left[\frac{1}{2}\alpha\phi_M^2 + (1-\alpha-l)\phi_N^2\right], \tag{A14}$$

equations (A13) and (A13′) can be rewritten as in equation (15). (Use $\lambda^2\Omega[\frac{1}{4}\alpha\phi_M^2 + (1-\alpha)\phi_N^2] = \frac{1}{4}\Sigma_1$.)

Using equations (A12) and (A13), equation (13) leads to

$$CE_M(\omega(i)) = -\frac{1}{2}\rho[\Sigma_v - \phi_M^2\mathrm{var}_M\{\tilde{v}-\tilde{p}\}]\omega(i)^2,$$

$$CE_N(\omega(i)) = -\frac{1}{2}\rho[\Sigma_v - \phi_N^2\mathrm{var}_N\{\tilde{v}-\tilde{p}\}]\omega(i)^2. \tag{A15}$$

(Use $E_j\{\tilde{p}\} + \rho\mathrm{cov}_j\{\tilde{v}, \tilde{v}-\tilde{p}\}\omega(i) = \rho\phi_j\mathrm{var}_j\{\tilde{v}-\tilde{p}\}\omega(i)$.) Equation (A15) can be rewritten as in equation (17).

PROOF OF PROPOSITION 4

Let both h and l tend to zero in Proposition 3. This leads to equations (15′) and (16′). Corollary 1 and Corollary 2 are straightforward.

PROOF OF PROPOSITION 5

Set $m = n = 1$ in Proposition 3 and let ρ tend to infinity. This leads to equations (15″) and (16″), which have a unique solution if

$$\lambda^2\Omega = \frac{\frac{1}{4}\Sigma_1}{\frac{1}{4}\alpha\phi_M^2 + (1-\alpha)\phi_N^2}, \tag{A16}$$

$$\phi_M = \frac{\Sigma_v - \frac{1}{2}\Sigma_1}{\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\lambda^2\phi_M^2\alpha\Omega}$$

and

$$\phi_N = \frac{\Sigma_v - \frac{1}{2}\Sigma_1}{\Sigma_v - \frac{1}{2}\Sigma_1 - \lambda^2\phi_N^2(1-\alpha)\Omega} \tag{A17}$$

have a unique solution in $(\lambda, \phi_M, \phi_N)$. We now show that this is true for all $\alpha \in [0, 1]$.

For $\alpha = 0$, the solution is $\phi_M = 1$ and $\phi_N = (\Sigma_v - \frac{1}{2}\Sigma_1)/(\Sigma_v - \frac{3}{4}\Sigma_1)$ [and λ is found by substituting into equation (A16)]. Similarly, for $\alpha = 1$, $\phi_M = (\Sigma_v -$

$\frac{1}{2}\Sigma_1)/(\Sigma_v - \frac{3}{4}\Sigma_1)$ and $\phi_N = 1$. For $0 < \alpha < 1$, we use equation (A16) and rewrite equation (A17) as

$$\phi_M = \frac{\Sigma_v - \frac{1}{2}\Sigma_1}{\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\Sigma_1\bar{\alpha}\phi^2/(\bar{\alpha}\phi^2 + 1)}$$

and

$$\phi_N = \frac{\Sigma_v - \frac{1}{2}\Sigma_1}{\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\Sigma_1/(\bar{\alpha}\phi^2 + 1)}, \tag{A18}$$

where $\bar{\alpha} = \alpha/[4(1-\alpha)]$ and $\phi = \phi_M/\phi_N$. It follows that

$$\phi = \frac{\Sigma_v - \frac{1}{2}\Sigma_1 - \frac{1}{4}\Sigma_1/(\bar{\alpha}\phi^2 + 1)}{\Sigma_v - \frac{1}{4}\Sigma_1\bar{\alpha}\phi^2/(\bar{\alpha}\phi^2 + 1)}. \tag{A19}$$

This leads to a cubic equation in ϕ:

$$\phi^3 - a\phi^2 + (a/\bar{\alpha})\phi - 1/\bar{\alpha} = 0, \tag{A20}$$

where $a = (\Sigma_v - \frac{1}{2}\Sigma_1)/(\Sigma_v - \frac{3}{4}\Sigma_1)$. Note that $a \in (1, 2]$ for all $0 < \Sigma_1 \leq \Sigma_v$. If the discriminant,

$$D = Q^2 - P^3,$$

where

$$P = a(a\bar{\alpha} - 3)/(9\bar{\alpha})$$

and

$$Q = (2\bar{\alpha}a^3 - 9a^2 + 27)/(54\bar{\alpha}),$$

is strictly positive, then equation (A20) has a unique solution given by

$$\phi = (Q + \sqrt{D})^{1/3} + (Q - \sqrt{D})^{1/3} + a/3. \tag{A21}$$

(See, e.g., Spanier and Oldham, 1987.) Simple calculations yield

$$D = [4a^3\bar{\alpha}^2 - (a^4 + 18a^2 - 27)\bar{\alpha} + 4a^3]/108\bar{\alpha}^3. \tag{A22}$$

To see that $D > 0$, notice that the denominator (i.e., $108\bar{\alpha}^3$) is positive for all $\bar{\alpha} > 0$. On the other hand, for $\bar{\alpha} = 3/a$ we have $P = 0$ and $D = [(27-3a^2)a/162]^2 > 0$ since $a \in (1, 2]$. Hence, $D > 0$ if the numerator in equation (A22) is a quadratic function of $\bar{\alpha}$ with no real zeros, i.e., if $(a^4 + 18a^2 - 27)^2 - 64a^6 < 0$, which is true for all $a \in (1, 2]$.

PROOF OF COROLLARY 3

Consider the case where all investors are identical and trade through fund N, i.e., $\alpha = 0$ (no front-running). From the proof of Proposition 5 above, we have $\phi_N = (\Sigma_v - \frac{1}{2}\Sigma_1)/(\Sigma_v - \frac{3}{4}\Sigma_1)$. Using $\Sigma_2 = 0$, $\Sigma_u = \phi_N^2 \Omega$ and equation (15″), market liquidity is given by

$$1/\lambda = 4(\Omega/\Sigma_1)^{1/2} \cdot \frac{2\Sigma_v - \Sigma_1}{4\Sigma_v - 3\Sigma_1}, \tag{A23}$$

which is increasing in Σ_1 if and only if $\Sigma_1 > \frac{1}{3}\Sigma_v$.

For later use, each investor incurs a percentage welfare loss equal to $1 - r_N^2$, where $r_N^2 = \phi_N^2 \mathrm{var}_N\{\tilde{v} - \tilde{p}\}/\mathrm{var}_N\{\tilde{v}\}$. (See Corollary 4.) Since $\mathrm{var}_N\{\tilde{v} - \tilde{p}\}\mathrm{var}_N\{\tilde{v}\}$ is equal to $(\Sigma_v - \frac{3}{4}\Sigma_1)/\Sigma_v$, $1 - r_N^2$ reduces to the expression given in equation (20). Similarly, the case where all investors are identical and trade through fund M (i.e., $\alpha = 1$) leads to $\phi_M = (\Sigma_v - \frac{1}{2}\Sigma_1)/(\Sigma_v - \frac{3}{4}\Sigma_1)$. While market liquidity is one-half of the amount given in equation (A23), it is easy to check that the percentage welfare loss incurred by each investor is the same as in the case where $\alpha = 0$.

PROOF OF COROLLARY 4

In the absence of insider trading (i.e., $\Sigma_1 = 0$) we would have $CE_j^*(\omega(i)) = 0$ while in the absence of any trading we would have $CE_j^0(\omega(i)) = -\frac{1}{2}\rho\Sigma_v\omega(i)^2$. As $\rho \to \infty$, $CE_j(\omega(i))$ and $CE_j^0(\omega(i))$ become unbounded but $CE_j(\omega(i))/CE_j^0(\omega(i))$ does not. Equation (19) follows from equation (A15), $\phi_j = \mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}/\mathrm{var}_j\{\tilde{v} - \tilde{p}\}$ and $r_j^2 = \mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}^2/[\mathrm{var}_j\{\tilde{v}\}\mathrm{var}_j\{\tilde{v} - \tilde{p}\}]$.

PROOF OF PROPOSITION 6

We only need to prove part (b). Since $\alpha = \frac{1}{2}$ and $m = n$, the two funds have the same number of members and the variance of each individual shock $\tilde{\omega}(i)$ is the same for all $i \in (0, 1]$. Accordingly, we measure the percentage total welfare loss by the average of CE_M/CE_M^0 and CE_N/CE_N^0, i.e.,

$$1 - \frac{(\Sigma_v - \frac{1}{2}\Sigma_1)^2}{2\Sigma_v} \cdot [\mathrm{var}_M\{\tilde{v} - \tilde{p}\}^{-1} + \mathrm{var}_N\{\tilde{v} - \tilde{p}\}^{-1}]. \tag{A24}$$

This follows from equation (19), $r_j^2 = \mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\}^2/[\mathrm{var}_j\{\tilde{v}\}\mathrm{var}_j\{\tilde{v} - \tilde{p}\}]$ and $\mathrm{cov}_j\{\tilde{v}, \tilde{v} - \tilde{p}\} = \Sigma_v - \frac{1}{2}\Sigma_1$. We want to show that the welfare loss given in equation (A24) would be larger if trader 2 did not engage in front-running. This is true if $\text{var}_M\{\tilde{v}-\tilde{p}\}^{-1}+\text{var}_N\{\tilde{v}-\tilde{p}\}^{-1}$ is smaller when there is no front-running, which in turn is true if the following three conditions hold:

(i) $\text{var}_M\{\tilde{v}-\tilde{p}\} \neq \text{var}_N\{\tilde{v}-\tilde{p}\}$ when there is front-running,
(ii) $\text{var}_M\{\tilde{v}-\tilde{p}\} = \text{var}_N\{\tilde{v}-\tilde{p}\}$ when there is no front-running,
(iii) $\text{var}_M\{\tilde{v}-\tilde{p}\}+\text{var}_N\{\tilde{v}-\tilde{p}\}$ is the same in both cases.

To prove (i), notice first that $\phi_j=\text{cov}_j\{\tilde{v},\tilde{v}-\tilde{p}\}/\text{var}_j\{\tilde{v}-\tilde{p}\}$ and $\text{cov}_j\{\tilde{v},\tilde{v}-\tilde{p}\}=\Sigma_v-\frac{1}{2}\Sigma_1$ imply that $\text{var}_M\{\tilde{v}-\tilde{p}\}=\text{var}_N\{\tilde{v}-\tilde{p}\}$ if and only if $\phi_M/\phi_N=1$. On the other hand, the denominators in equation (A18) give

$$\text{var}_M\{\tilde{v}-\tilde{p}\}=\Sigma_v-\frac{1}{2}\Sigma_1-\frac{1}{4}\Sigma_1\bar{\alpha}\phi^2/(\bar{\alpha}\phi^2+1),$$

$$\text{var}_N\{\tilde{v}-\tilde{p}\}=\Sigma_v-\frac{1}{2}\Sigma_1-\frac{1}{4}\Sigma_1/(\bar{\alpha}\phi^2+1), \tag{A25}$$

where $\bar{\alpha}=\alpha/[4(1-\alpha)]=\frac{1}{4}$ and $\phi=\phi_M/\phi_N$. Suppose that $\text{var}_M\{\tilde{v}-\tilde{p}\}=\text{var}_N\{\tilde{v}-\tilde{p}\}$. Then, equation (A25) implies $\bar{\alpha}\phi^2=1$, i.e., $\phi=2$, a contradiction.

Suppose now that trader 2 does not trade on her account. Proposition 5 changes in a straightforward way and we now have

$$\text{var}_M\{\tilde{v}-\tilde{p}\}=\Sigma_v-\frac{3}{4}\Sigma_1+\lambda^2\phi_N^2(1-\alpha)\Omega,$$

$$\text{var}_N\{\tilde{v}-\tilde{p}\}=\Sigma_v-\frac{3}{4}\Sigma_1+\lambda^2\phi_M^2\alpha\Omega,$$

$$\lambda^2\Omega=\frac{1}{4}\Sigma_1/[\phi_M^2\alpha+\phi_N^2(1-\alpha)]. \tag{A25$'$}$$

Since we are assuming $\alpha=\frac{1}{2}$, equation (25′) is symmetric. Since the solution is unique, we must have $\text{var}_M\{\tilde{v}-\tilde{p}\}=\text{var}_N\{\tilde{v}-\tilde{p}\}$. This proves (ii). Equation (A25) implies $\text{var}_M\{\tilde{v}-\tilde{p}\}+\text{var}_N\{\tilde{v}-\tilde{p}\}=2\Sigma_v-\frac{5}{4}\Sigma_1$ and so does equation (A25′). This proves (iii). Thus, the percentage welfare loss is larger if trader 2 does not trade on her account (strictly larger if $\Sigma_1>0$).

More precisely, it is smaller for the members of fund M but larger for the members of fund N. This follows from the observation that equation (A25′) can be written in the same form as equation (A25) except for the fact that $\bar{\alpha}$ is equal to 1 instead of $\frac{1}{4}$. This implies that $\text{var}_M\{\tilde{v}-\tilde{p}\}$ is smaller and $\text{var}_N\{\tilde{v}-\tilde{p}\}$ is larger. [Using equation (A20), one can check that $\bar{\alpha}\phi^2$ is increasing in $\bar{\alpha}$, so that the result follows from equation (A25).]

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