



Intraday Lead-Lag Relationships Between the Futures-, Options and Stock Market *

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Abstract. In rational, efficiently functioning and complete markets, returns on derivative and underlying securities should be perfectly contemporaneously correlated. Due to market imperfections, one of these markets may reflect information faster. The use of high-frequency data and the choice for a small unit time interval to measure these lead-lag relations comes at the cost of some or many missing observations, causing traditional estimators to either under- or overestimate covariances and correlations. We use a new estimator to estimate lead-lag relationships between the cash AEX index, options and futures. We find that futures returns lead both options and cash index returns by approximately 10 minutes. The relationship between options and the cash market is not completely unidirectional.

Keywords: lead-lag relations, high frequency data

JEL Classification: G13, G14

In perfectly frictionless and complete markets there would be complete simultaneity between the price movements of stocks or indices and derivative instruments such as options and futures. However, on small time intervals (high frequency) it is often noticed that some price series consistently lead other, closely related, prices. Such lead-lag relations indicate that one market processes new information faster than the other market(s). Due to arbitrage restrictions that link these markets, lead and lag correlation coefficients between price change series will generally be small although it is possible that one market consistently leads or lags the other(s).

Several studies examine temporal relationships between futures and cash index returns using a Granger (1969) and Sims (1972) causality specification for the intraday observed time series. See e.g. Finnerty and Park (1987), Ng (1987), Kawaller et al. (1987), Harris (1989), Stoll and Whaley (1990), Chan (1992) and Huang and Stoll (1994). The results frequently suggest that the futures returns lead the cash return and that this effect is stronger when there are more stocks included in the index (Chan, 1992). For the S&P500 and MMI futures this lead varies from five minutes (Stoll and Whaley, 1990) to 45 minutes (Kawaller et al., 1987) but

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the relationship is not completely unidirectional: the cash index may also affect the futures although this lead is almost always much shorter. Part of the findings can be explained by the staleness of the cash index due to infrequent trading of the component stocks. The conclusion that the futures market serves as a price discovery vehicle for the stock prices and is thus the main source of market wide information is usually explained by transaction costs, restrictions on short sales in the cash market and the higher degree of leverage that can be attained by using futures.

Black (1975) was the first one to suggest that the higher leverage available in the options market might induce informed traders to transact in options rather than in stocks. Most studies that examine lead-lag relationships between options and stocks, find that option prices lead stock prices (Manaster and Rendleman, 1982; Bhattacharya, 1987), option volume leads stock volume (Anthony, 1988) and option volume leads stock prices (Easley et al., 1993). Stephan and Whaley (1990) find just the opposite, namely that stocks lead options by 20 to 45 minutes. Chan et al. (1993), however, show that this result can be explained as spurious leads induced by infrequent trading of options because of the larger percentage value of the minimum tick size for options than for stocks. The lead disappears when the average of the bid and ask prices is used instead of transaction prices.

In this paper we simultaneously consider three markets: the cash index, the index futures and the index options market. To the best of our knowledge the interaction between price changes in the index option- and the futures markets has never been studied before. Since both instruments involve leveraged positions in the underlying asset, circumvent short sale restrictions on stocks and have relatively low transaction costs, the lead-lag relationships between options and futures largely remain an empirical question.

Investigation of intraday lead-lag relationships typically involves high frequency data and observations on the three series are probably unequally spaced in time. In the literature this problem is dealt with in at least two different ways that both have serious shortcomings. One way is to choose a long unit time interval so that the number of missing observations is small. Especially when trading is not very frequent, in this procedure a lot of information is thrown away. Another solution is to impute zero returns for intervals in which no trading took place. This creates an error in the variables problem that will bias the covariance and correlation estimates towards zero. To avoid these problems we use an estimator developed by De Jong and Nijman (1997) that takes these characteristics of the data into account without introducing bias due to non-trading in many time intervals. The estimator that we propose is asymptotically unbiased under any pattern of observations. This method is more general than the specific models used by Cohen et al. (1993) or Lo and MacKinlay (1991), who rely on specific models for the transaction process. The only assumption we need is that the trading pattern is independent of the price process.¹

¹ A similar assumption is made by Conley et al. (1995) in a continuous time context.

The paper is organized as follows. The theory underlying the covariation of stock index futures, stock index options and the stock index returns is outlined in Section 1. In this section we also discuss the methodology used to analyse the intraday lead-lag relationships. Section 2 describes the data. Empirical results are in Section 3 and Section 4 concludes.

1. Theory

In a perfect market no arbitrage opportunities should exist. Hence, returns on derivative securities like stock index options and stock index futures contracts with payoff structures that can be replicated by a (dynamically rebalanced) portfolio of stocks and riskless bonds should neither lead nor lag returns on the spot stock index and contemporaneous returns should be perfectly correlated. In imperfect markets with private information and transaction costs, traders have preference for both low cost and high leverage and will make a tradeoff between these two liquidity parameters. Since a trade in the options or futures markets requires little upfront cash (initial margin deposits are usually only a fraction of the stocks' market value) and can be effectuated immediately while purchasing the basket of stocks composing the index requires a greater initial investment and may take longer to implement, this preference for cost efficiency could cause the futures and options market to lead the spot market.

There are also several technical reasons why returns on a particular market may seem to lead returns on other markets. If options and futures markets instantaneously reflect new information and if the stocks within the index trade infrequently, observed futures and options returns will lead observed stock index returns. However, as Stoll and Whaley (1990) note, there is no economic significance to this behaviour whatsoever. Second, in a narrowly based index such as the AEX index, the negative serial correlation in individual stock returns attributable to the bid-ask bouncing (Roll, 1984), might also appear in the stock index returns. This effect may neutralize or diminish the positive autocorrelation in the index returns induced by infrequent trading and may obscure the actual relationship between index and options or futures returns. To solve the infrequent trading problem, Harris (1989) derives new estimators of the underlying value of a stock portfolio which abstract from nonsynchronous trading problems by using the complete transaction history of all stocks in the portfolio. Stoll and Whaley (1990) adjust for the infrequent trading effect by using innovations from an ARMA process with constant parameters instead of raw returns. Chan (1992), however, shows that nonsynchronous trading cannot completely explain the lead lag relations since even for stocks that are actively traded and have non-trading probabilities close to zero, the returns still lag the futures returns significantly.

In most empirical studies the intraday lead-lag relation between different markets is examined by estimating a Granger-Sims causality regression where the returns in one market are explained by lagged, contemporaneous and lead returns

in the other market (e.g. Kawaller et al., 1987; Chan, 1990; Stephan and Whaley, 1990; Stoll and Whaley, 1990; Chan et al., 1993). We follow the literature on lead-lag relations closely by reporting auto- and cross-covariances between index, futures and options returns. Moreover, we use regressions of index and options returns on futures returns (see e.g. Stoll and Whaley, 1990). Our approach is different in the sense that it explicitly takes into account the complicating fact that high frequency data are often observed at irregular intervals. Since we are interested in clock time lead-lag relations, this leads to a large number of intervals without observations. Table III reports the fraction of missing observations in 5 and 10 minutes intervals. Since these numbers are high, using ordinary covariance estimators would seriously bias the results. De Jong and Nijman (1997) have developed a method for consistent estimation of covariances with this type of data. In this section, we briefly describe their approach.

Econometric Methodology

The underlying return generating model is a discrete time process at an arbitrary high frequency. For exposition, we only consider the case where the returns have zero mean and there are no deterministic components in the model. Let p_t and q_t denote the (logarithm of the) two price series under consideration, where t is the clock-time index. The price levels are assumed to be non-stationary processes, which are stationary after differencing. Denote the cross covariance function of the underlying returns (one-period price changes) by

$$\gamma(k) = \text{cov}(\Delta p_t, \Delta q_{t-k}), \quad \Delta p_t = p_t - p_{t-1}, \quad \Delta q_t = q_t - q_{t-1}. \quad (1)$$

If the price levels were observed at every point, the covariances $\gamma(k)$ could be estimated efficiently by the usual expressions. However, when using transactions data there are often many time intervals with no new observation on the price level. One way to 'solve' this problem is to impute a zero return for this interval, but this will bias the usual covariance estimators towards zero. In order to obtain an unbiased covariance estimator, we use the differences between observations on the price level over more than one interval. We then infer the covariances of the underlying but unobserved one-period returns from the cross-products of these more-period returns. We will explain this procedure now in more detail.

We index the observations on p_t by the index i and the observations on q_t by the index j , and denote the total number of observations by N and M , respectively. The differences between the two observed price levels can be expressed as sums of the returns of the unobserved underlying price process

$$p_{t_{i+1}} - p_{t_i} = \sum_{t=t_i+1}^{t_{i+1}} \Delta p_t \quad (2)$$

where t_i denoted the clock-time index of the i th observation. The cross product of price changes on the two markets can thus be written as

$$y_{ij} = (p_{t_{i+1}} - p_{t_i})(q_{t_{j+1}} - q_{t_j}) = \sum_{t=t_i+1}^{t_{i+1}} \Delta p_t \cdot \sum_{s=t_j+1}^{t_{j+1}} \Delta q_s. \quad (3)$$

The expectation of this cross-product is a linear combination of the cross-covariances $\gamma(k)$ of the underlying process

$$\begin{aligned} E(y_{ij} | t_i, t_j, t_{i+1}, t_{j+1}) &= E \left(\sum_{t=t_i+1}^{t_{i+1}} \Delta p_t \cdot \sum_{s=t_j+1}^{t_{j+1}} \Delta q_s \right) \\ &= \sum_{t=t_i+1}^{t_{i+1}} \sum_{s=t_j+1}^{t_{j+1}} \gamma(t-s) \end{aligned} \quad (4)$$

where the expression in Equation (4) is conditional on the observed transaction times $(t_i, t_j, t_{i+1}, t_{j+1})$. Let $x_{ij}(k)$ denote the number of times that $\gamma(k)$ appears in this expression. In De Jong and Nijman (1997) the following expression for $x_{ij}(k)$ is derived

$$x_{ij}(k) = \max(0, \min(t_{i+1}, t_{j+1} + k) - \max(t_i, t_j + k)). \quad (5)$$

An important property of the x_{ij} 's is that they are functions of the transaction times $(t_i, t_j, t_{i+1}, t_{j+1})$ only, not of the observed prices. Therefore, we can write $E(y_{ij})$ as a linear combination of the covariances $\gamma(k)$, $k = -K, \dots, K$ as follows

$$E(y_{ij} | x_{ij}) = \sum_{k=-K}^K x_{ij}(k) \gamma(k). \quad (6)$$

Our estimation method is based on the fact that Equation (6) can be considered as a regression equation with the unknown cross-covariances $\gamma(k)$ as parameters and the coefficients x_{ij} as the explanatory variables. In vector notation, the regression equation reads

$$y_{ij} = x'_{ij} \gamma + e_{ij}. \quad (7)$$

The covariances can then be estimated by ordinary least squares on the observations of y_{ij} and the constructed x_{ij} 's. The estimates of the covariance are consistent under the assumption that the trading pattern is independent of the price process. In principle, all possible differences between observed prices can be used to construct an x_{ij} and y_{ij} . However, we can confine ourselves to differences of adjacent observations. The reason for this is that differences of non-adjacent observations

can always be written as exact linear combinations of differences of adjacent observations. All in all, N times M cross-products y_{ij} are available for the analysis. It is not necessary to use all of them, however, if the number of non-zero cross-covariances to be estimated is limited say to K . In that case, all cross-products where $|t_{i+1} - t_j| \geq K$ and $|t_i - t_{j+1}| \geq K$ can be omitted because x_{ij} is a zero-vector in that case. The method can be adapted to the estimation of autocovariances ($p_t = q_t$) by imposing the restriction $\gamma(-m) = \gamma(m)$ on regression model (7). It is easily seen that in the case of complete observations this procedure yields the usual covariance estimator

$$\hat{\gamma}_k = \frac{1}{T} \sum_i \Delta p_t \Delta q_{t-k}.$$

De Jong and Nijman (1997) also derive expressions for the standard errors of the proposed covariance estimator. Also, based on these covariance estimates, the usual lead-lag regressions between the index, futures and options returns can be estimated. We return to this in the next section.

2. Data

The data used in this study were obtained from the European Options Exchange (EOE) and consist of a six-months and a five-months period. We have intraday quotes and transactions for all index option series and all traded futures contracts and every change in the cash index level for January 20 through July 17, 1992 and January 4 through June 18, 1993.

At the EOE trading takes place in an open outcry system. Market makers trade exclusively for their own account and are obliged to make a two way market in the option classes to which they are assigned. To enhance market transparency, efficiency and the quality of prices, there are at least three market makers for each option class. Floor Brokers either execute orders for investors (through a Public Order Member), other members of the exchange or trade for their own account. Public Order Members are not allowed to cross trades in-house. The book of limit orders is kept by an Order Book Official who is not a member but an employee of the exchange. Limit orders in the book have priority over market maker quotes at the same price. Transactions are time-stamped to the nearest second with reporting lags kept to a minimum and seldomly exceeding one minute.

The value of the Amsterdam EOE (AEX) stock index is a weighted average of the last transaction prices of 25 stocks. It is updated after each reported transaction in one of the component stocks. Both the AEX index futures (the FTI contract) and the AEX index options are on a quarterly expiration cycle. The contracts mature in January, April, July, October. Additionally, 1- and 2-month contracts are always available. The index options are of the European type. The contract sizes for the futures and the options are 200 and 100 times the index respectively.

Table I. Sample characteristics

	Futures		Options	
	All contracts	Short maturity contracts	All contracts	Short maturity ATM contracts
Number of transactions				
Total	34 280	19 239	52 150	18 367
Per day	142	80	216	76
Trading volume				
Total	376 596	194 061	1 058 981	372 738
Per day	1 563	805	4 394	1 547
Number of quotes				
Total	145 129	76 866	250 243	74 578
Per day	602	319	1 038	309
Absolute spread(DFL)	0.9322	0.8005	1.1018	0.5143
Relative spread (%)	0.4972	0.4336	10.8708	10.5740

The relative spread is the average quoted spread relative to the futures price and the option premium.

The stock index files contain the date, time (to the nearest second) and the latest index value. The transaction files for the options and the futures contracts contain the date, time, expiration date, strike price (for options), transaction price and number of contracts traded as well as the AEX index level at the reported transaction time. The market quote files contain every update in the best bid and ask quotes for the index futures and options, listing the date, time, expiration date, strike price (for options), best market bid quote and best market ask quote. Sample characteristics for the number of transactions, trading volume and bid ask spread are in Table I. The total number of data records is 20 688 for the stock index quotes, 150 094 for the futures quotes, 72 261 for the futures transactions, 352 082 for the call options quotes and 79 799 for the options transactions for the 241 trading days sample period. Panels A through C in Figure 1 show intraday patterns in the number of index quotes and futures- and options transactions respectively. The activity patterns are very similar, with the exception of the opening period where there is much more activity in the stock market and slightly less in the options market.

Table I also reports the quoted spread of the futures and options contracts, as a percentage of the futures and options mid-quotes, respectively. Notice that the 10% quoted spread for the options is relative to the options price, which is around 10, whereas the index level is around 300 in our sample period. The quoted spread of the options as a percentage of the index level is therefore much smaller. Assuming a delta of 0.5 for the at-the-money options, a spread of 1 on the option premium translates into a spread of 2 on the implied index, which is around 0.7% of the index

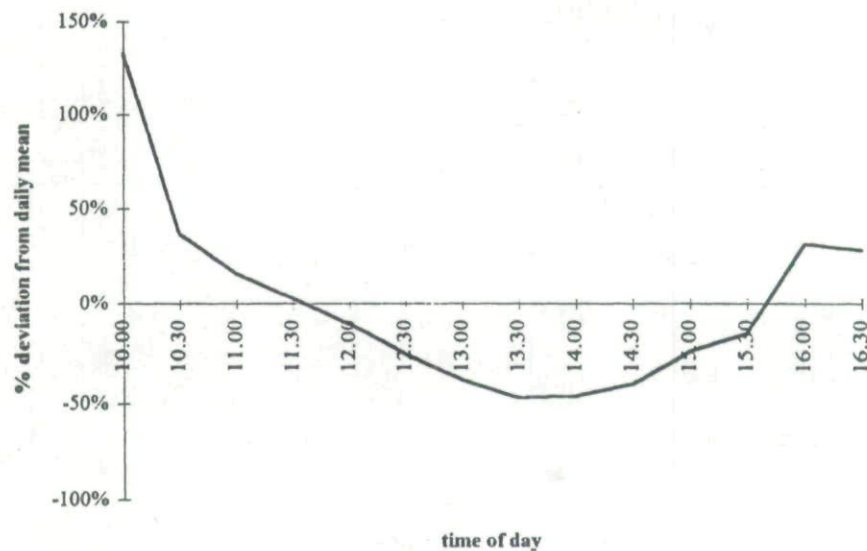


Figure 1a. Intraday pattern in index quotes.

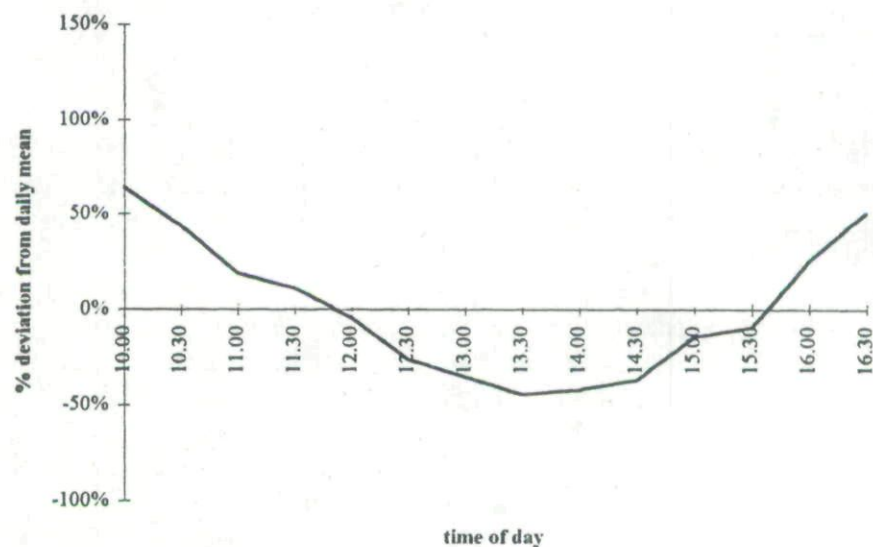


Figure 1b. Intraday pattern in number of futures transactions.

level. We also calculated the effective spread, which is defined as twice the average absolute difference between the transaction price and the mid-quote. The estimated effective spread is about half the quoted spread, indicating that transactions are typically negotiated at prices better than the quotes, or that trading is more active when the quoted spread is relatively narrow.

Since the nearby futures contract is usually the most actively traded (more than 50% of total trading volume), only data for nearby contracts are used. For com-

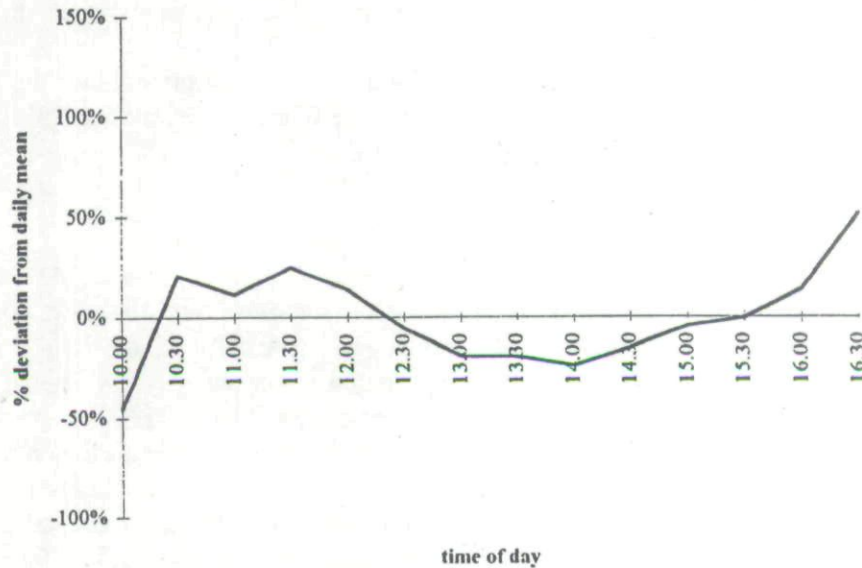


Figure 1c. Intraday pattern in number of options transactions.

parability we also use short maturity options. We impose the restriction that the contracts have a minimum time to expiration of ten days for two reasons. First, Kawaller et al. (1987) show that the lead from futures to the index might be stronger on expiration days than on normal trading days. Second, estimates for implied volatility for very short maturity options are very unstable, mainly because option prices are low so that rounding errors due to the minimum tick size are large.

From observed future prices we infer implied index values by inverting the pricing formula for futures adjusted for intermediate dividend payments

$$S_{F,t}^{\text{imp}} = \left(F_t + \sum_{i=1}^I \sum_{k=1}^{K_i} D_{i,k} w_i e^{r(T-t_{i,k})} \right) e^{-rT} \quad (8)$$

with $S_{F,t}^{\text{imp}}$ the implied index value from the futures price at time t , F_t the observed futures price at time t , K_i the number of dividends on component stock i during the remaining life of the index option, I the number of component stocks, $D_{i,k}$ the amount of dividend k on stock i , w_i the weight of stock i in the index, $t_{i,k}$ the time to dividend payment $D_{i,k}$. Since we only use short maturity contracts, we assume that the actual dividend amounts and ex-dividend dates are known. Depending on the time to maturity of the contracts, we use the one or three months² Amsterdam InterBank Offered Rate (AIBOR) as a proxy for the risk free interest rate. The term

² We also tried a linear interpolation between the one and three months AIBOR rates to obtain a 'synthetic' interest rate for a period that exactly matches the time to maturity of the contract. Since the one and three months rates are relatively close together during the sample period, this does not yield significantly different results.

structure in this period was not very volatile, so the effect of this approximation on the intraday returns is likely to be small.

In the case of options we also calculate the implied stock price. Since the AEX index options are of the European type, we use the Black and Scholes (1973) option pricing formula, adjusted for dividends.

$$S_{c,t}^{\text{imp}} = f^{-1}(c_t) \quad (9)$$

where $S_{c,t}^{\text{imp}}$ denotes the implied index value from the European call option price at time t and $f(S_t)$ the option pricing formula and c_t the call option price. To infer the implied index value from market call option prices we need an estimate for the (unobserved) stock return volatility. Stephan and Whaley (1990) regress observed transaction prices from options with a common time to expiration on model prices across all transactions to obtain volatility estimates. These maturity specific estimates of volatility are then used to compute implied stock (index) prices on the following day. This method has two important drawbacks. First, the well-documented smile effect in implied volatilities is not taken into account.³ Table II shows the presence of a smile-effect in our data. Second, the volatility is assumed to be constant over the day while several studies show a U- or reverse J-shaped intraday pattern in volatilities for both stocks and options.⁴ Figure 2 shows the average implied volatility for each 15 minutes interval in deviation from the daily mean implied volatility for the AEX index options during the sample period. If we do not correct for this time-varying pattern in implieds, we might find a (spurious) U- or reverse J-shaped pattern in implied index values across trading hours.⁵ To avoid the smile effect, we only use options with

$$\text{Moneyness} = \frac{S_t - \sum_{i=1}^I \sum_{k=1}^{K_i} D_{i,k} w_i e^{-rt_{i,k}}}{Xe^{-rT}} \quad (10)$$

between 0.97 and 1.03. As can be seen from Table II, implied volatilities from options satisfying this criterion are relatively close together. To account for the intraday pattern in implied volatilities, we calculate average implied volatilities

³ Rubinstein (1994) shows that implied volatility is an increasing function of the moneyness of the option.

⁴ See e.g. McInish and Wood (1990), Stoll and Whaley (1990) and Lee, Mucklow and Ready (1993) for the variability of stock returns on the NYSE and Sheikh and Ronn (1994) for the volatility of call option returns on the CBOE.

⁵ Assume that there is an L-shaped pattern in implied volatilities. Since the partial derivatives of a call option's price with respect to volatility and the stockprice are both positive, using the average implied volatility will cause the implied index value to be too high at the start of the day, while it will be too low at the end of the trading day. Even when the true index value does not change during trading hours, we would find that the implied index value steadily declines.

Table II. Number of transactions, trading volume and implied volatility of AEX call options by moneyness

Moneyneess = $S_t - \sum_{i=1}^I \sum_{k=1}^{K_i} D_{i,k} w_i e^{-rt_{i,k}} / X e^{-rT}$ indicates the moneyness of the option. **Proportion of total transactions** (trading volume) indicates the number of transactions (trading volume) of the specified subsample in relation to total number of transactions (trading volume) for all call options. **Average implied volatility** is the unweighted average of the volatilities of all call option prices in the subsample.

Category	Proportion of total transactions	Proportion of total trading volume	Average implied volatility
moneyneess < 0.93	0.0002	0.0012	0.1404
0.93 <= moneyneess < 0.95	0.0032	0.0041	0.1424
0.95 <= moneyneess < 0.97	0.0333	0.0482	0.1244
0.97 <= moneyneess < 0.99	0.2172	0.2379	0.1173
0.99 <= moneyneess < 1.01	0.3837	0.3820	0.1150
1.01 <= moneyneess < 1.03	0.2036	0.1846	0.1183
1.03 <= moneyneess < 1.05	0.0739	0.0655	0.1320
1.05 <= moneyneess < 1.07	0.0395	0.0345	0.1618
1.07 <= moneyneess < 1.09	0.0180	0.0158	0.1978
1.09 <= moneyneess < 1.11	0.0135	0.0140	0.2321
1.11 <= moneyneess	0.0137	0.0124	0.3054

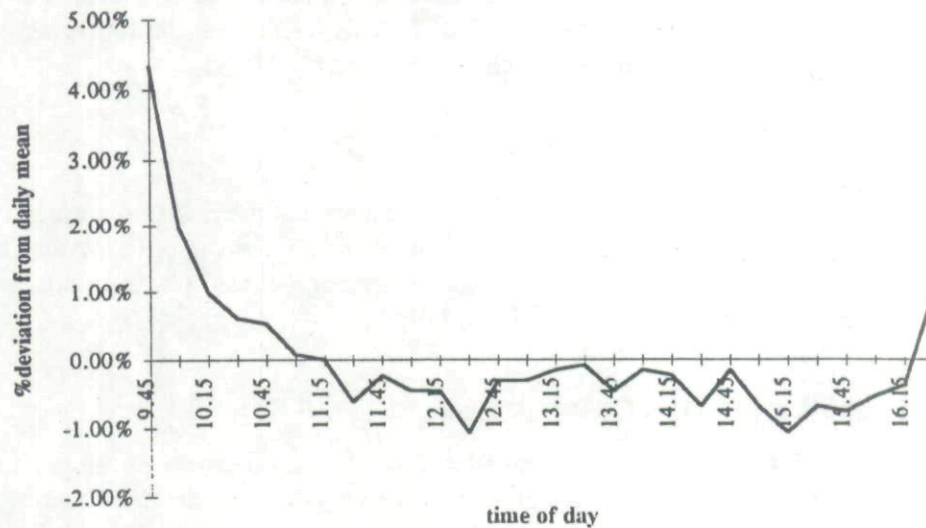


Figure 2. Intraday pattern in AEX implied volatilities.

Table III. Trading frequencies and non-trading probabilities of the index, futures and options

Number of intervals with at least one observation indicates the number of transactions for the futures and options in the 5 and 10 minutes intervals respectively. 'Transaction' for a change in the quoted value of the index is chosen as a matter of convenience. Fraction of intervals without observations indicates the number of 5 (10) minutes intervals in which no transaction (change in quoted index value) is reported.					
	Number of intervals with at least one observation		Fraction of intervals without observations		
	5 min	10 min	5 min	10 min	
Index					
92-I	10 088	5 128	1.67%	0.10%	
93-I	9 544	4 823	2.05%	0.13%	
Futures					
92-I	6 003	3 903	42.41%	25.40%	
93-I	6 144	3 948	36.17%	18.15%	
Options					
92-I	4 770	3 530	52.38%	29.83%	
93-I	4 495	3 320	51.49%	28.74%	

for every 15 minutes interval and use these averages to compute⁶ implied index values at the same timeinterval on the next day. If there are no transactions in the 15 minutes interval, implied index values for this interval on the following day cannot be computed and the observations are deleted.

3. Empirical Results

The setup of this section is as follows. First, we present the raw auto correlations and cross correlations between the index, futures and options returns. We also test the results for structural stability. Second, we present the results of the more usual regression analysis, cf. Stoll and Whaley (1990).

3.1. AUTO AND CROSS CORRELATIONS

The method for estimation of correlation in real time described in the previous section requires the choice of a unit interval. Of course, to get the most interesting

⁶ To compute the implied volatility and the implied index value, we use a Newton Raphson iterative search.

results one would like to choose the unit interval as short as possible. However, a higher frequency implies a larger fraction of intervals without observations and less reliable correlation estimates. We tried a 1 minute interval, but the data were not informative enough for such a high frequency. Therefore, we decided to use a 5 minute unit interval, which is the usual choice in the literature.

Estimated autocorrelations of the index, futures and options returns⁷ are presented in Table IV. The index returns show the familiar short horizon positive serial correlation. The first two autocorrelations are significant, after 10 minutes the correlations are basically zero. This is exactly the pattern predicted by the non-synchronous trading model of Fisher (1966) and Lo and MacKinlay (1990) and the result is comparable to previous findings for the MMI and S&P 500 indices (Chan, 1992; MacKinlay and Ramaswamy, 1988). Both the futures and the options returns are basically uncorrelated, except for a strong negative first order serial correlation. A potential explanation for the strong negative serial correlation are the bid-ask bounce or price discreteness. Roll's (1984) estimator for the bounce yields an estimated realized spread of 0.057% for the futures and 0.182% for the options data. Notice that both numbers are smaller than the quoted spread calculated from Table I. Since we are using transaction prices throughout, this finding implies that actual transaction prices are usually better, i.e. closer to the 'true value', than quoted prices. Finally, it should be noticed that the variance of the futures return is slightly higher than that of the index returns, a familiar phenomenon (cf. e.g. Chan et al., 1991; MacKinlay and Ramaswamy, 1988). The variance of the options returns is much higher, about four times as large, probably due to different price discreteness rules in the two markets.⁸ As an immediate consequence, all estimated correlations that involve options are less precise than the results for the index and the futures.

We now turn to the estimated lead-lag relationship between index, futures and options returns. Table V shows the 5 minute lead and lag correlations of all three pairs. To start with the index-futures relation, there is clear evidence that the futures strongly lead the index. Similarly, the futures returns lead the options returns by 5 to 10 minutes. This shows additional evidence that the futures market is very efficient in processing new information. The cross-correlations between the index and the options show a triangular pattern; sometimes options lead the index, sometimes the other way around. Hence, we conclude that both the options and the index lag behind the futures returns, but not always by the same time. So, more or less artificially we find cross-correlations both ways (lead and lags) between options and the index. We summarize the estimated auto- and cross-correlations in the panels A, B and C of Figure 3. The figures also show cumulative cross correlations.

⁷ Options and futures returns are defined as the change in the logarithm of the implied index values, multiplied by 100.

⁸ The average transaction price of the call options in our sample is approximately DFL 4.00 with a minimum tick size of DFL 0.10. For the stock index and the futures the average price is about DFL 300.00 with a tick-size of DFL 0.01. See Chan, Chung and Johnson (1993) for a related problem.

Table IV. Autocorrelations of index, futures and options returns

Estimated autocorrelations and t -values for 0 to 6 five minutes interval lags for the index, futures and options for the subperiods January 20 through July 17, 1992 and January 4 through June 18, 1993. **Average autocorrelation** gives the autocorrelation over the total sample period (100*variances of returns in % are reported at lag 0), $\chi^2_{(7)}$ is the test statistic for differences between the estimated autocorrelations in the first and second subperiod. Autocorrelations are estimated using the De Jong and Nijman (1997) procedure.

Panel A. Index returns

Lag	92-I		93-I		Average ($\chi^2_{(7)} = 11.49$)	
	autocorrelation	t -value	autocorrelation	t -value	autocorrelation	t -value
0	0.232*	15.98	0.211*	22.73	0.222*	25.71
1	0.335*	12.13	0.356*	14.99	0.353*	18.79
2	-0.000	-0.02	0.082*	5.34	0.039*	2.93
3	-0.015	-0.79	0.015	0.96	-0.000	-0.03
4	-0.019	-1.03	-0.020	-1.37	-0.019	-1.65
5	-0.023	-1.39	-0.032*	-2.24	-0.027*	-2.48
6	-0.003	-0.15	-0.022	-1.57	-0.012	-1.00

Panel B. Futures returns

Lag	92-I		93-I		Average ($\chi^2_{(7)} = 7.18$)	
	autocorrelation	t -value	autocorrelation	t -value	autocorrelation	t -value
0	0.395*	18.13	0.401*	16.58	0.398*	24.46
1	-0.184*	-5.01	-0.224*	-5.34	-0.204*	-7.32
2	-0.019	-0.56	0.030	1.15	0.005	0.25
3	-0.059	-1.70	-0.019	-1.03	-0.039*	-1.99
4	-0.012	-0.44	-0.028	-1.42	-0.020	-1.18
5	0.043	1.54	-0.003	-0.16	0.020	1.13
6	-0.008	-0.42	0.039	1.83	0.016	1.08

Panel C. Options returns

Lag	92-I		93-I		Average ($\chi^2_{(7)} = 8.75$)	
	autocorrelation	t -value	autocorrelation	t -value	autocorrelation	t -value
0	1.800*	25.28	1.980*	22.78	1.890*	33.64
1	-0.436*	-12.40	-0.441*	-11.03	-0.438*	-16.36
2	0.036	1.00	0.097*	2.43	0.068*	2.51
3	-0.001	-0.02	-0.088*	-2.18	-0.047	-1.73
4	-0.025	-0.65	0.016	0.46	-0.003	-0.12
5	0.037	1.00	0.006	1.65	0.049	1.89
6	-0.039	-1.52	-0.066*	-2.12	-0.053*	-2.61

Table V. Cross-correlations of index, futures and options returns

Estimated correlations and t -values for -6 to 6 five minutes interval lags for the index, futures and options for the subperiods January 20 through July 17, 1992 and January 4 through June 18, 1993. **Average correlation** gives the correlation over the total sample period, $\chi^2_{(13)}$ is the test statistic for differences between the estimated correlation in the first and second subperiod. Correlations are estimated using the De Jong and Nijman (1997) procedure. The correlations are $\text{cov}(R_t^i, R_{t-k}^j) / \sigma_{R_t^i} \sigma_{R_{t-k}^j}$ with R_t^i and R_t^j returns on the first and second mentioned variable respectively, e.g. correlations in Panel A are $\text{cov}(R_t^I, R_{t-k}^F) / \sigma_{R_t^I} \sigma_{R_{t-k}^F}$. Positive (negative) k 's indicate lagged (leading) returns.

Panel A. Index returns

Lag	92-I		93-I		Average ($\chi^2_{(7)} = 28.39$)	
	autocorrelation	t -value	autocorrelation	t -value	autocorrelation	t -value
-6	0.022	1.26	-0.009	-0.30	0.006	0.37
-5	0.054*	2.59	0.038	0.72	0.046	1.63
-4	-0.042*	-2.37	-0.017	-1.06	-0.030*	2.47
-3	-0.049	-1.62	-0.016	-1.12	-0.033	-1.94
-2	-0.122*	-5.07	-0.047	-1.81	-0.084*	-4.78
-1	-0.009	-0.40	-0.053	-1.14	-0.031	-1.19
0	0.340*	11.32	0.307*	13.32	0.323*	17.09
1	0.293*	10.65	0.350*	17.21	0.322*	18.79
2	0.106*	6.06	0.121*	8.43	0.133*	10.03
3	0.002	0.08	0.062*	4.57	0.032*	2.70
4	0.104	0.78	0.015	-1.17	-0.000	-0.05
5	0.008	0.52	0.009	0.76	0.009	0.87
6	-0.002	-0.11	0.002	0.13	-0.000	-0.01

Panel B. Option and futures returns

Lag	92-I		93-I		Average ($\chi^2_{(7)} = 12.39$)	
	autocorrelation	t -value	autocorrelation	t -value	autocorrelation	t -value
-6	0.023	1.14	-0.036	-1.90	-0.007	-0.48
-5	-0.047	-1.99	0.023	0.97	-0.012	-0.73
-4	0.026	1.07	0.012	0.50	0.019	1.12
-3	0.026	1.17	-0.008	-0.34	0.009	0.54
-2	-0.044	-1.77	0.015	0.60	-0.014	-0.80
-1	0.015	0.52	-0.044	-1.68	-0.015	-0.75
0	0.049	1.79	0.083*	2.98	0.066*	3.38
1	0.090*	3.55	0.077*	2.82	0.084*	4.48
2	0.072*	2.75	0.092*	3.79	0.082*	4.59
3	-0.035	-1.05	-0.009	-0.37	-0.022	-1.07
4	0.012	0.37	0.019	0.79	0.016	0.76
5	0.041	1.24	-0.002	-0.09	0.019	0.89
6	-0.011	-0.45	0.006	0.29	-0.002	-0.14

Table V. Continued

Panel C. Index and options returns						
Lag	92-I autocorrelation	t-value	93-I autocorrelation	t-value	Average ($\chi^2_{(7)} = 14.30$) autocorrelation t-value	
-6	0.027	1.35	0.033	1.78	0.030*	2.20
-5	0.007	0.34	-0.020	-1.10	-0.007	-0.51
-4	0.001	0.04	-0.004	-0.25	-0.002	-0.14
-3	-0.031	-1.76	-0.011	-0.75	-0.021	-1.82
-2	0.030	1.64	0.065*	3.99	0.047*	3.88
-1	0.083*	5.45	0.114*	6.22	0.099*	8.26
0	0.087*	6.61	0.100*	6.05	0.094*	8.86
1	0.056*	4.05	0.039*	2.66	0.048*	4.71
2	0.014	1.09	0.037*	2.71	0.026*	2.71
3	0.006	0.38	0.010	0.79	0.008	0.80
4	0.010	0.69	-0.001	-0.10	0.004	0.45
5	-0.013	-0.90	0.008	0.63	-0.002	-0.25
6	0.027	1.82	-0.007	-0.54	0.010	1.00

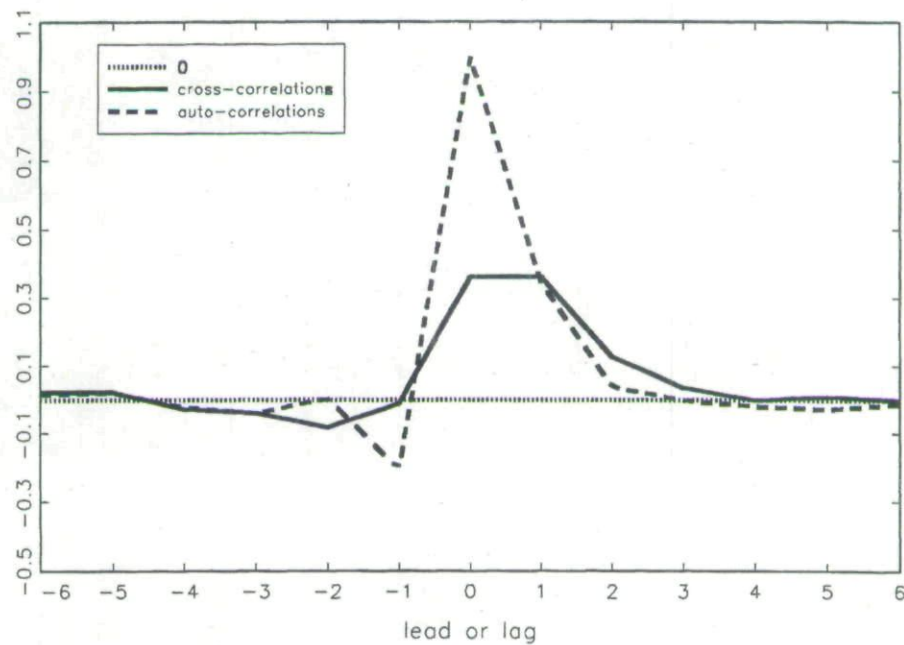


Figure 3a. Correlations in futures returns and index returns. The solid line is $\text{corr}(r_t^I, r_{t-k}^F)$, $k = -6, \dots, 6$. The left part of the dashed line is $\text{corr}(r_t^F, r_{t+k}^F)$, $k = -6, \dots, 0$, the right part of the dashed line is $\text{corr}(r_t^I, r_{t-k}^I)$, $k = 0, \dots, 6$ with r_t^F the futures return and r_t^I the index return.

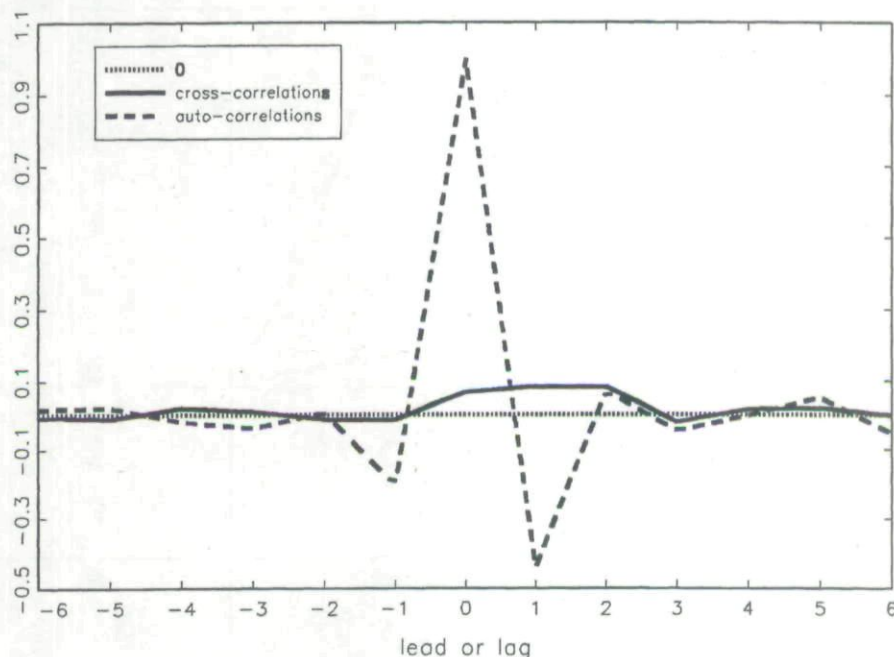


Figure 3b. Correlations in futures returns and options returns. The solid line is $\text{corr}(r_t^O, r_{t-k}^F)$, $k = -6, \dots, 6$. The left part of the dashed line is $\text{corr}(r_t^F, r_{t+k}^F)$, $k = -6, \dots, 0$, the right part of the dashed line is $\text{corr}(r_t^O, r_{t-k}^O)$, $k = 0, \dots, 6$ with r_t^F the futures return and r_t^O the options return.

The fact that the cross correlations do not add up to one is caused by the fact that correlations are computed by scaling the cross-covariances with the estimated return variances. These, however, are inflated because of measurement errors in the price level, induced by bid-ask spreads and price discreteness.

To check the robustness of our results, the sample was split in a part 92-I and a part 93-I and covariances were re-estimated for both sub-samples. The results were almost identical. A formal χ^2 -test did not reject the equality of the covariances in both sub-samples.

3.2. REGRESSION ANALYSIS

The auto- and cross correlations contain sufficient information on lead-lag patterns in returns to perform a more traditional lead-lag regression analysis. The results can be used to obtain estimates for the regression of index or option returns on current and lagged futures returns, and perhaps also leads of futures returns. This way of reporting the lead-lag relationship is more usual in the literature, cf. e.g. Stoll and Whaley (1990). The basic lead-lag regression model is

$$R_t^I = b_0 R_t^F + b_1 R_{t-1}^F + \dots + b_p R_{t-p}^F + \epsilon_t. \quad (11)$$

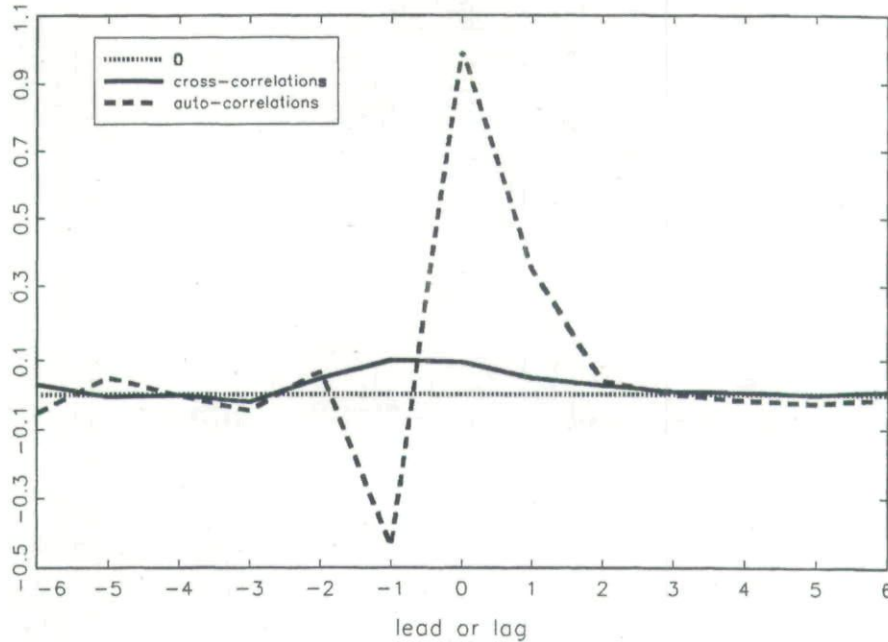


Figure 3c. Correlations in options returns and index returns. The solid line is $\text{corr}(r_t^I, r_{t-k}^O)$, $k = -6, \dots, 6$. The left part of the dashed line is $\text{corr}(r_t^O, r_{t+k}^O)$, $k = -6, \dots, 0$, the right part of the dashed line is $\text{corr}(r_t^I, r_{t-k}^I)$, $k = 0, \dots, 6$ with r_t^O the options return and r_t^I the index return.

With irregular spaced observations we face the same problems as with estimating correlations. In particular, there are many missing observations for both the explanatory and dependent variables. Fortunately, the previously estimated auto- and cross correlations can be used to construct the OLS estimator. The OLS estimator is

$$b = (X'X)^{-1}X'y. \quad (12)$$

Because the vector of regressors consists of different lags of the same variable, the elements of the matrix can be estimated by the autocorrelations γ_k of R^F :

$$X'X = \begin{pmatrix} \gamma_0 & \gamma_1 & \cdots & \gamma_p \\ \gamma_1 & \gamma_0 & \cdots & \gamma_{p-1} \\ \vdots & & & \vdots \\ \gamma_p & \cdots & & \gamma_0 \end{pmatrix}. \quad (13)$$

Similarly, the $X'y$ can be estimated by the cross covariances, c_k , between R^F and R^I .

$$X'y = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_p \end{pmatrix}. \quad (14)$$

The regression model can be extended to include leads of the futures returns as well. This does not change the form of the $X'X$ matrix (only the dimension) and extends the $X'y$ vector to include lead covariances:

$$X'y = \begin{pmatrix} c_{-p} \\ \vdots \\ c_0 \\ \vdots \\ c_p \end{pmatrix}. \quad (15)$$

As we have already estimated the correlations, the calculation of the regression coefficients is trivial.

In the empirical implementation we regress index and option returns on lagged, current and lead futures returns. The estimates of the regression coefficients are presented in Table VI. They basically convey the same message as the cross correlations: the futures lead the index by 5 to 10 minutes, although there is also a significant contemporaneous correlation. The lead of the futures to the options is, if any, even stronger than the lead of futures to the index. The lead-lag relation between the index and the options is almost symmetric, with significant coefficients up to 10 minutes lead and lag. Formal F-tests indicate that the lead coefficients of index to futures are marginally significant, and the lead of options to futures returns is insignificant. The coefficients of the lagged futures returns are strongly significant in both the options and the index regression. One remarkable result is that, unlike the raw correlations, the coefficients of the regressions on the futures returns add up to a number very close to one.

We also analysed our data using the traditional lead-lag regressions where intervals without trading were assigned a zero return. Compared to our estimates, one would expect a bias in these ordinary least squares regression coefficients. Some simulation experiments indeed indicate that imputing zero returns biases the first order autocorrelation coefficient to zero. However, there is also a bias of the estimated variance towards zero. The net effect on the regression coefficients is indeterminate. Empirically, we obtained somewhat smaller estimates of the regression coefficients in the regression of index returns on the futures returns. The bias in the estimated coefficients in the regression of options returns on futures returns

Table VI. Regression coefficients of index- and options- on futures returns

This table contains estimated regression coefficients β_k for the regressions $R_t^I = \alpha^I + \sum_{k=-K}^K \beta_k^I R_{t-k}^F + \epsilon_t^I$ and $R_t^O = \alpha^O + \sum_{k=-K}^K \beta_k^O R_{t-k}^F + \epsilon_t^O$ respectively. R^I , R^O and R^F indicate index- options and futures returns. $\sum_{i=-6}^k \beta_i$ are cumulative regression coefficients, t -values are for the β_k 's. Postive (negative) k 's indicate lagged (leading) futures returns. Panel A gives the results for both leading and lagged futures returns. Results for regressions that include only lagged futures returns are in Panel B. Regression coefficients are estimated using the De Jong and Nijman (1997) procedure. Regressions are over the total sample period January 20 through July 17, 1992 and January 4 through June 18, 1993.

Panel A. Index and options returns as a function of leading and lagged futures returns

Index returns on futures returns				Options returns on futures returns		
lag k	β_k	$\sum_{i=-6}^k \beta_i$	t -value	β_k	$\sum_{i=-6}^k \beta_i$	t -value
-6	0.0086	0.0086	0.45	-0.0239	-0.0239	-0.44
-5	0.0006	0.0093	0.03	-0.0325	-0.0564	-0.52
-4	-0.0255	-0.0163	-1.25	-0.0376	-0.0188	0.58
-3	-0.0251	-0.0414	-0.86	0.0326	0.0138	0.54
-2	-0.0390	-0.0804	-1.61	-0.0104	0.0034	-0.16
-1	0.0627*	-0.0176	2.27	0.0152	0.0186	0.21
0	0.3607*	0.3431	9.87	0.2059*	0.2244	2.87
1	0.3828*	0.7259	11.32	0.2772*	0.5017	3.93
2	0.1925*	0.9184	8.73	0.2407*	0.7424	3.56
3	0.0892*	1.0076	4.12	0.0218	0.7642	0.28
4	0.0429*	1.0505	2.06	0.0658	0.8300	0.87
5	0.0222	1.0727	1.14	0.0668	0.8968	0.84
6	-0.0013	1.0714	-0.07	0.0055	0.9023	0.09
F-test for sum of coefficients equal to zero						
$k = -6 \dots -1$			15.39	1.46		
$k = 1 \dots 6$			159.43	60.52		
$k = 0 \dots 6$			183.83	91.88		
$k = -6 \dots 6$			203.29	108.27		

Table VI. Continued.

Panel B. Index and options returns as a function of lagged futures returns						
Index returns on futures returns				Options returns on futures returns		
lag k	β_k	$\sum_{i=-6}^k \beta_i$	t-value	β_k	$\sum_{i=-6}^k \beta_i$	t-value
0	0.3490*	0.3490	10.15	0.1994*	0.1994	2.50
1	0.3816*	0.7305	11.29	0.2761*	0.4755	3.93
2	0.1890*	0.9196	8.72	0.2413*	0.7169	3.57
3	0.0856*	1.0052	3.98	0.0217	0.7386	0.28
4	0.0427*	1.0479	2.04	0.0658	0.8044	0.87
5	0.0233	1.0712	1.20	0.0672	0.8716	0.84
6	-0.0011	1.0702	-0.06	0.0057	0.8774	0.09
F-test for sum of coefficients equal to zero						
$k = 1 \dots 6$			159.92	60.33		
$k = 0 \dots 6$			190.66	79.18		

(both series with many missing observations) was less clear; some coefficients were over-estimated, some underestimated.

4. Conclusions

In this paper, the intraday lead-lag relationships between returns on the cash index, futures and call options over two sample periods, January through July 1992 and January through June 1993, are investigated. We use a specially designed correlation measure which takes into account the fact that high frequency data are often observed at irregular intervals. We infer 'implied index values' from transaction prices of futures- and options contracts by 'inverting' the pricing formula for the instruments that trade on the European Options Exchange.

Empirical results confirm previous findings that futures, options and the cash index are contemporaneously correlated and that there is an asymmetric relation between the futures market and the options- and spot market, respectively. There is strong evidence that relative changes in the index value implied by the prices of the FTI-contract lead both changes in the value of the cash index and changes in the index value implied in option prices by five to ten minutes on average. The lead-lag relations are stable across the two subperiods. The lead-lag relations between the cash index and the options are largely symmetrical, indicating that neither market systematically leads the other.

The lead of the futures market over both the cash- and the options market can be attributed to several forces. First, due to infrequent trading of the component stocks,

the quoted values of the cash index are stale. Second, transaction costs, including the bid-ask spread, are much smaller for futures than for transactions in options or the basket of component stocks. This difference in transaction costs will cause (informed) market participants to trade in the cheaper market. Third, although both options and futures involve levered positions in the underlying asset, this leverage effect is about twice as large for futures as for (short maturity at-the-money) call options.⁹

One potential concern is whether these results could be caused by differential trading frequencies. The three series show similar activity patterns throughout the day, except for the opening. Since we have only few observations on the options in the opening period it is hard to derive any conclusive result for that period. For the main trading period we feel confident that differential trading patterns are not the cause of these results. So, we conclude that the futures market is better at reflecting market-wide information and that the lead-lag relation between the options and the index market is indeterminate.

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⁹ The partial derivative of the option price with respect to the price of the underlying asset is approximately 0.5 for an at-the-money call option while it is e^{-T} for a futures contract.

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