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Warrants on the London Stock Exchange: Pricing Biases and Investor Confusion

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Abstract. This study of warrants on the London Stock Exchange examines whether they display particular pricing biases and whether investors understand how to value them at the time of issue. In a sample of 72 warrants on closed-end funds (investment trusts) over the 1985–94 period, more than one third of the 12,673 prices are anomalously low. The other two thirds behave like stock options, with lower volatility when they are in-the-money or have a long time until maturity. Despite their frequent undervaluation, it *is* rational to add warrants to a new equity issue: an examination of 127 new equity issues (95 with warrants) reveals that attaching warrants significantly increases market value. The reason for this appears to be investor confusion: they do not seem to understand that the more the warrants are worth, the less the value of the ordinary shares.

1. Introduction

Black and Scholes (1973) suggested that their model would be applicable to warrants. After all, a warrant is very similar to a call option, with the provisos that it tends to have a long life (averaging six years in this study) and its exercise results in dilution of the equity. By contrast, the traditional view of warrants has been as deferred rights issues of equity, a perspective which has also been endorsed for convertibles by Stein (1992). Nevertheless, a warrant is a peculiar 'right', since it lasts for years rather than weeks and the injection of new capital is far from certain.

Warrants exist in all of the world's major capital markets, but in relatively small numbers. For example, Lauterbach and Schultz (1990) report that 100 warrants existed at some time on the NYSE and AMEX in the 1971–80 period. In 1988 there were more than 600 euro-warrants outstanding on Japanese stocks, of which all but two were denominated in dollars (Gemmill, 1989). In the United Kingdom the most numerous warrants are on investment trusts (closed-end funds). In February 1995 there were 120 investment-trust warrants listed on the London Stock Exchange, whereas there were only 51 warrants listed on ordinary companies. Because they comprise the largest class in the London market, this paper focuses on the investment-trust warrants. For research purposes, warrants on investment trusts also have several advantages over warrants on ordinary companies: investment trusts have a revealed firm value (the net asset value of the porfolio of shares

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held); they have very little debt (the presence of which would make warrant pricing more complicated); and they have no other contingent liabilities outstanding.

The paper sets out to answer two particular questions. Firstly, we want to examine whether warrants are traded in the market at prices which are consistent with standard option-pricing models, or whether these very long-term options show particular biases. There has been no previous study of warrants on the London Stock Exchange and very few studies of warrants in any market for which so large a sample is available. Secondly, since the late 1980s most investment trust IPOs have included 'free' warrants. It is claimed that warrants enhance the value of new issues but this contention has never been tested. If the claim is correct, then it is possible that investors are confused and do not understand that warrants are liabilities which reduce the value of the shares. This theme is not new. Writing in the 1930s, Benjamin Graham expressed the view that 'The recent development of stock-option warrants . . . (is) . . . a near fraud, an existing menace and a potential disaster. They have created huge aggregate dollar "values" out of thin air. They have no excuse for existence except to the extent that they mislead speculators and investors' (Graham, 1973, pp. 228–229). We will therefore test the hypothesis that investors are being deliberately confused by the issuers of warrants.

Our main sample consists of 12,673 prices on 72 warrants observed weekly over the period June 1985 to June 1994. We begin by segregating those warrant prices which are below the lower bound, a total of 4,661 or 37% of the sample. We then examine how such gross underpricing may arise. Arbitrage-trading tests confirm that it cannot be explained by transactions costs, lumpy dividends or illiquidity. The conclusion is that the pricing is anomalous. For the remaining sample of 8,012 'rational' prices, implied volatilities are estimated with a dividend-adjusted Black/Scholes model. We then examine whether the implied volatility is related in a systematic way to moneyness and time to maturity, finding that it declines as there is an increase in either moneyness or time to maturity, which are also typical characteristics of stock options. Finally, we turn our attention to the reasons why warrants are attached to new issues. Using an extended sample of 127 new issues, 95 of which have warrants, we find evidence to support Benjamin Graham's hypothesis: investors are confused by warrants and this leads to an unjustified premium for those IPOs which combine shares with warrants.

The paper is organised as follows. In Section II we discuss the theory of warrant pricing and review previous empirical studies. In Section III we discuss the data and examine the prices of the sub-set of warrants for which prices are below the lower bound. In Section IV we use the implied volatilities of the main sample to examine how well the Black/Scholes model works. In Section V we examine whether making new issues with warrants can be rationally explained. Finally, Section VI draws together the conclusions and implications of this study.

2. Warrant Pricing: Theory and Evidence

2.1. THEORY

The main difficulty in pricing a warrant is to allow for the change in capital structure which occurs when the warrant is exercised. Not only does exercise imply dilution of the equity, but a company which issues warrants will (ceteris paribus) reduce the volatility of its existing shares. The theory is only well-developed for a company which is initially all-equity financed, but this is particularly appropriate for investment trusts which typically have less than 10% leverage.

Suppose that an all-equity company issues a warrant and suppose also that there is an exactly similar company which remains all-equity financed. Galai and Schneller (1978) show that the warrant may be valued as a diluted call option on the equivalent all-equity firm. That is:

$$W = \frac{N}{N+n}C, \tag{1}$$

where W is the warrant value, C is the call option value on the equivalent all-equity firm, N is the number of shares outstanding and n is number of new shares which will be created if the warrant is exercised.

As Crouhy and Galai (1991) explain, the firm to which C refers is a 'fictitious, pure-equity firm'. However, the assets of this firm and the assets of the firm with warrants will be exactly the same. In order to value C we need to know the value of the all-equity firm, V, and its volatility, σ_v. We know by definition that the value of that firm must be:

$$V = NS + nW, \tag{2}$$

where V is the value of the firm's assets and S is the price per share.

We now have two equations, (1) and (2), in two unknowns, V and W. To value V in (2) we need to know W, but to value W in (1) we need to find C, which is itself a function of V. Crouhy and Galai (1991) suggest that an iterative, trial-and-error approach be used for solving (1) and (2) simultaneously. Likewise, Schulz and Trautmann (1994) suggest the simultaneous estimation of V and σ_v. Neither approach is straightforward. In their empirical study of U.S. warrants, Lauterbach and Schultz (1990) implicitly assume that the market prices of warrants are correct, so that V can be estimated from (2). The value for σ_v can then be estimated from a time series of constructed V values and used in (1) for the warrant valuation.[1]

A much simpler approach to valuing a warrant has been outlined by Schulz and Trautmann (1994). The correct procedure, as described above, depends on knowing: the value of the all-equity firm, V; the volatility of that value, σ_v; and the dilution factor, $[N/(n+N)]$. The shortcut procedure is to value the warrant as a simple call option on the warrant-issuing firm's shares, which are priced at S, without any further adjustment for dilution. Schulz and Trautmann denote this as 'option-like valuation of warrants'. It is what practitioners usually do. Except for (i) deep-out-of-the-money warrants and (ii) out-of-the-money warrants which are near to maturity, this approximation is quite accurate. The reason for the success of this approximation is that it incorporates offsetting biases. On the one hand there are downward biases in the warrant valuation resulting from the use of both S (which is less than V) and σ_s (which is less than σ_v).[2] On the other hand, because this approach ignores the dilution of V, there is a resulting upward bias. It transpires that these biases offset each other almost perfectly.

Assuming no dividends, this 'option-like valuation' allows us to value the investment-trust warrants with the simple Black/Scholes formula:

$$W = SN(d_1) - Ee^{-r\tau}N(d_2), \tag{3}$$

where W is the warrant price, S is the share price, $N(x)$ is a normal distribution probability, E is the exercise price, τ is the time to maturity, r is the continuous interest rate,

$$d_1 = \frac{\ln(S/E) + r\tau + 0.5\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{4}$$

and

$$d_2 = d_1 - \sigma\sqrt{\tau}. \tag{5}$$

When there are dividends, an adjustment is necessary which accounts for them over the time to maturity of the warrant. Because warrants have long lives, dividends may have a very large impact. A natural assumption is that the dividend yield is constant and this leads us to use Merton's (1973) version of the Black/Scholes model in which the share price is discounted by the dividend yield (δ).

2.2. EVIDENCE

There have been few previous empirical studies of warrant pricing. Most of these have used some variant of the Black/Scholes formula and looked for biases in model versus market prices. The studies give very mixed indications of whether warrants are under- or over-valued in different markets. U.S. warrants appear to be correctly priced (Lauterbach and Schultz, 1990),[3] while German (Schulz and Trautmann, 1994) and Japanese (Gemmill, 1989, Kuwahara and Marsh, 1992) warrants are underpriced. By contrast, warrants in the Dutch (Veld, 1992) and Swiss (Stucki and Wasserfallen, 1991) markets appear to be overvalued. One general feature, however, is that the warrants are often mispriced and exhibit exercise-price and time-to-maturity biases. This led Lauterbach and Schultz (1990) and Hauser and Lauterbach (1997) to prefer the CEV model to the Black/Scholes model, although the reduction in bias which is achieved in this way appears to be quite small.

3. Data and Arbitrage Tests

3.1. Data

The sample comprises 72 warrants which were issued by investment trusts during the period January 1985 to December 1993. A further 14 issues made in this period were excluded (11 because the investment trust was reconstructed during the life of the warrant, 1 because the investment trust was taken over and 2 because price data were unreliable).

Of the 72 issues, 63 were made as part of an initial public offering package, 5 were issued by seasoned investment trusts and awarded free to shareholders, and 4 were issued as part of a rights-issue package. Typically, warrants issued as part of any package were distributed on the basis of one warrant for every five shares; of the 72 issues, 66 were made on this basis.

Closing prices for shares and warrants were obtained from Datastream for each Wednesday over the period June 1985 to June 1994. The number of weekly observations per warrant ranged from 26 to 418, giving a total of 12,673 prices. Three-month, one-year and five-year interest rates were obtained from the same source[4] and, in valuing the warrants, rates were interpolated or extrapolated assuming a linear term-structure. Observed dividends were used to compute the dividend yield and that yield was extrapolated, where necessary, to the maturity of the warrant.

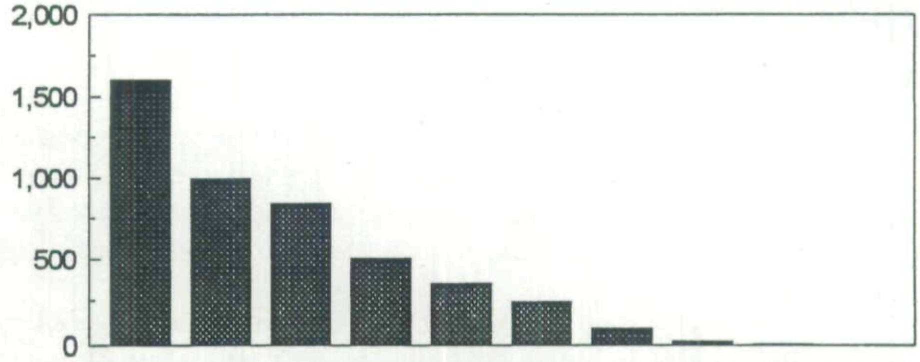
The average time to maturity of all the warrants as sampled was 5.72 years, with a range from 0 to 15.1 years. Most of the warrants were issued for exercise at the current share price, resulting in 71 of the 72 warrants being in-the-money at time of issue.[5] Only 15% of the observations were out-of-the money. The average moneyness at issue was 1.90, with a range from 0.32 to 6.86. This high degree of moneyness contrasts with the warrants which have been the focus of other studies (e.g. Lauterbach and Schultz (1990)) which were predominantly out-of-the money.

3.2. Apparent Arbitrages

The first step in the analysis was to identify those warrants which had prices below the lower bound,[6] i.e. for which

$$W < Se^{-\delta\tau} - Ee^{-r\tau}. \tag{6}$$

Surprisingly, 37% of the sampled prices (4,661 of the 12,673 observations) were below this bound. This phenomenon was not limited to just a few warrants but occurred in 63 of the 72 cases. Most previous studies have simply stated that prices below the bound were eliminated from the sample, so it is difficult to say whether this proportion is exceptionally large. Gemmill (1989), for instance, found only 9.5% of his Japanese sample to be below the bound while Lauterbach and Schultz (1990) discarded only a small proportion of their observations for this reason.[7] Figure 1 shows the distribution of arbitrages in pence. On average, the mispriced warrants were 10.4 pence below the bound, which is equivalent to a 17.1% 'error'.



arbitrage in pence	>0 and <5	>5 and <10	>10 and <15	>15 and <20	>20 and <25	>25 and <30	>30 and <35	>35 and <40	>40 and <45	>45 and <50
no. of obs.	1,601	996	841	507	353	239	92	25	6	1

Figure 1. Frequency distribution of arbitrages.

Of the 4,661 arbitrages, 1,601 were less than 5 pence, but 363 exceeded 25 pence. All of these below-the-bound warrants were in-the-money, ranging in moneyness from 1.04 to 6.85 and averaging 2.34. They were also quite far from maturity, averaging 6.85 years as compared to 5.07 years for the sample as a whole.

Table I and Figure 2 examine the relationship of arbitrage to moneyness and time to maturity, based on a two-way classification of the sample. They show that moneyness is important in explaining arbitrage. Arbitrages for warrants with moneyness of less than 1.5 were mostly of 3 pence or less, whereas the really large arbitrages of 6 to 15 pence occurred for warrants which had moneyness in excess of 2.0. On the other hand, Figure 2 does not suggest that there is a simple relationship of mispricing to maturity: low-moneyness warrants became less mispriced the longer the time to maturity; the opposite occurred with high-moneyness warrants. The size of arbitrage was also unrelated to the discount to net asset value (NAV) of the investment trust. The important question is whether these apparent arbitrages are convincing evidence of inefficient pricing? Three possible factors may influence the results: transactions costs, infrequent trading and the treatment of dividends. We consider each of these in turn.

The level of *transactions costs* depends on who is assumed to be attempting the arbitrage. At one extreme, a marketmaker might pay no spread, commission or tax, so the transactions costs would be zero. At the other extreme, a small-scale arbitrageur might have to pay spreads, commissions and share-purchase tax (stamp-duty). To be conservative, we will assume the highest possible level of transactions costs. An examination of shares in this study indicates a bid spread or ask spread of about 1% (minimum of 1 pence) and a warrant spread of a similar

Table I. Size of arbitrage in pence, classified by moneyness and maturity

	$1.0 < m \leq 1.5$	$1.5 < m \leq 2.0$	$m > 2.0$
$t \leq 1$	3.393 (2.376,17)	7.119 (3.732,34)	6.436 (4.265,42)
$1 < t \leq 2$	3.357 (3.812,29)	6.679 (4.397,62)	8.678 (5.795,103)
$2 < t \leq 4$	2.572 (2.317,88)	8.103 (5.878,130)	12.895 (9.046,305)
$4 < t \leq 6$	2.923 (3.057,155)	6.201 (4.695, 372)	13.568 (9.005,466)
$6 < t \leq 8$	3.022 (2.703,154)	6.861 (5.620,503)	15.486 (8.716,417)
$8 < t \leq 10$	1.330 (1.515,36)	6.416 (4.797,436)	13.974 (8.783,758)
$t > 10$	0.791 (0.480,2)	7.807 (8.327, 35)	15.922 (8.685, 537)

t is time in years to maturity.
m is moneyness, measured as $Se^{-\delta t}/Ee^{-rt}$.
Each cell in the table is the average arbitrage in pence for observations in the specified range of moneyness and time to maturity. Values in parentheses are standard deviation and number of observations.

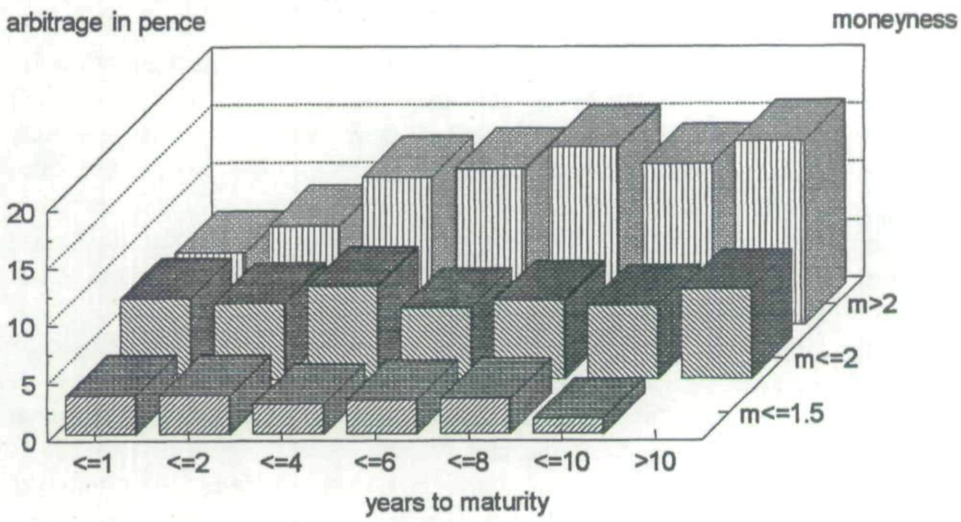


Figure 2. Arbitrages by moneyness and maturity.

size.[8] To this must be added 0.5% commission on each transaction, plus 0.5% tax on all purchases. The round-trip transaction cost for a small-scale arbitrageur then comprises the following:

Initiating the position:

– spread on purchased warrant	1%S or 1 pence
– stamp-duty tax on warrant	0.5%W
– commission on warrant	0.5%W
– spread on sold share	1%S or 1 pence
– commission on share	0.5%S

Lifting the position:	
– spread on sold warrant	1%S or 1 pence
– commission on warrant	0.5%W
– spread on purchased share	1%S or 1 pence
– stamp-duty tax on share	0.5%S
– commission on share	0.5%S

Total: The larger of {5.5%S + 1.5%W} or {1.5%S + 1.5%W + 4 pence}

With these costs imposed, there remained 2,428 arbitrages (i.e. 19.2% of the sample) of which 866 exceeded 10 pence. The arbitrages were distributed across 46 of the 72 individual warrants in the sample. It therefore appears that transactions costs alone cannot explain the large number of arbitrages which we find.

The second possibility is that these arbitrages simply reflect *infrequent trading* or illiquidity. Because the London Stock Exchange is organised as a market with competing dealers and the warrant prices used were mid-market quotes, there is unlikely to be any bias due to infrequent trading. The marketmakers, of which there were 2 to 6 per warrant, are obliged to trade at the quoted price, so they have an incentive to adjust quotes even if there have been no trades. For the warrants in the sample (based upon aggregate Stock Exchange data) there were, on average, 1.03 trades per day in 1992, as compared with 3.09 trades for the underlying shares. Using weekly data (which are available from Datastream for 85% of the sample) the average warrant traded in 89 of every 100 weeks observed. This information indicates that trading in warrants is regular, but in relatively small volume. The use of quotes makes it unlikely that the apparent arbitrages are due to illiquidity, but this will be examined further in the empirical tests which follow.

The third possibility is that the *dividend process has been mis-specified or mis-estimated*, so that discounting the share price by the dividend yield leads to errors. By assuming a continuous dividend, a European warrant will be overvalued just prior to a dividend payment and undervalued when the next dividend is at its furthest distance in time. The largest dividend on any share in the sample was 3.03 pence. Suppose a final dividend payment of 4 pence per share is imminent, the share price is 200 pence and the warrant has only 6 more months to run. If a lumpy dividend is assumed then the dividend-discounted share price is 196 pence, whereas it is 196.04 pence if a continuous dividend yield of 4% is used, a difference of only 0.04 pence. It follows that for European warrants the bias due to using a continuous rather than lumpy dividend is extremely small.

Some American papers (Ferri, Kremer and Oberhelman, 1986; Leonard and Solt, 1990; Kremer and Roenfeldt, 1992) examine whether ignoring dividends leads to model prices which are closer to market prices. Ignoring dividends raises the computed value of the warrants, so it might explain why investors overvalue warrants. However, the anomaly to be explained in our sample is the exact opposite, the undervaluation of the warrants. It appears that mis-specified or mis-estimated dividends cannot explain our anomalously low warrant prices, but this will be further examined in the arbitrage-trading tests below. Finally, if the warrants were callable by the issuer then their value would be reduced, (see e.g. Burney and Moore, 1996), but that is not the case for the warrants in this study.

3.3. Trading Tests of Arbitrages

To check that the underpricing of the warrants could be exploited for profit, a number of arbitrage tests have been completed. These begin with a simple test, which is followed by tests which relax the assumptions by allowing for: (i) no interest on short positions; (ii) lumpy dividends; (iii) a delay in executing trades; and (iv) going short in the FTSE 100 index rather than in the share.

The basic test is as follows. If a warrant is priced in the market at more than 10% below its lower bound and that is at least 4 pence, the warrant is assumed to be purchased. At the same time the share is sold short and the net cash position invested at the risk-free rate for the relevant maturity. The portfolio is held until the warrant price has risen above the lower bound, at which time all positions are reversed. Any trades which are incomplete at the end of the sample period are eliminated from the test.

The results of this basic test are given in column 2 of Table II. 109 arbitrages are established, all of which are profitable. The average holding period is 200 days, with a median of 126 days. The mean profit of 14.43 pence is significantly greater (at the 1% level) than assumed transactions costs of 6 pence. Based upon the cost of the warrant, the profits per annum average 302%, with a median of 124% and a range of 6% to 8136%. The implication of the basic trading test is therefore that arbitrages easily exceed transactions costs.

Although we do not agree, it could be argued that the results are favourably biased by the exclusion of positions which are still open at the end of the sample period. As a check, column 3 of Table II gives the profits when such open positions are closed at market prices. There are 15 such positions out of 124 trades and 8 of them are unprofitable. Nevertheless, the average profit is still nearly 13 pence per trade and easily exceeds transactions costs at the 1% significance level.

Marketmakers who go short enjoy the full proceeds of a sale, but an outside investor may not be treated so generously. The proceeds of the sale may be held as collateral and, in extreme cases, it is possible that no interest is paid. The results in column 4 of Table II therefore exclude interest. Now 6 of the 109 trades are unprofitable, but the average profit of more than 8 pence remains significantly above transactions costs.

The fifth column of Table II indicates what happens if dividends are assumed to be lumpy rather than paid continuously. Instead of discounting by the dividend yield, the present value of dividends is deducted from each share price. If the life of a warrant exceeds the sample period, the dividend is projected as the average dividend yield.[9] There are now 102 arbitrage trades, all of which are profitable. The average profit of 14.3 pence is almost exactly the same as that found for continuous

Table II. Results of arbitrage tests

Item	Base case	Incl. incomplete arbitrages	Excl. interest earned	With lumpy dividends	With 1-week delay	Index hedged	Index hedged and 1-week delay
Number of arbitrages	109	124	109	102	108	109	108
Number making a loss	0	8	6	0	4	23	21
Holding period in days (mean/mode)	200/126	206/119	200/126	189/126	202/126	200/119	202/126
Mean profit	14.43p. (302%)	12.94p.	8.23p.	14.34p.	14.28p.	14.55p.	14.12p.
Std. deviation	10.17p. (825%)	11.33p.	7.23p.	10.79p.	11.66p.	25.64p.	25.87p.
t-value (mean versus bracketed trans. cost)	8.65[a] (6p.)	6.39[a] (6p.)	3.22[a] (6p.)	7.80[a] (6p.)	7.38[a] (6p.)	4.30[a] (4p.)	4.07[a] (4p.)
Min. profit	1.9p, (6%)	−7.8p.	−10.5p.	3.86p.	−3.2p.	−11.77p.	−16.26p.
Max. profit	61.5p. (8136%)	61.5p.	30p.	61.50p.	69.74p.	195.86p.	198.08p.
Modal profit	11.59p. (124%)	9.28p.	7p.	9.78p.	10.99p.	6.84p.	6.27p.

The table gives the results for arbitrages based upon buying one warrant which is below the lower bound by at least 10% and holding it until it has risen to the bound.
[a] Significantly greater than the bracketed transactions costs at the 1% level.
p. indicates pence.

dividends, so the treatment of dividends is not important to the results (as had been predicted on a priori grounds above).

The sixth column of Table II assumes that there are extreme problems of liquidity, such that there is one full week's delay in opening an arbitrage (after it has been signalled) and another full week's delay in closing it. Surprisingly, a one-week delay has very little impact on the profitability of the arbitrages. Four are now unprofitable, but the average profit still exceeds 14 pence per trade. The result confirms that arbitrages persist for long periods, so that there is plenty of time in which to place trades and capture profits.

The seventh and eighth columns of Table II assume that it is not possible to go short in the shares, so a short futures position is used instead. The rolling futures position is proxied by a short position in the index itself. Hedging with the index is likely to be highly imperfect, because it assumes that returns on each investment-trust share move exactly in line with the index. Not only are some trusts more risky than others (in beta terms), but many comprise portfolios which have a large proportion of foreign shares. In the seventh column of the table, profits are calculated with no delay in trading, while in the eighth column the profits are

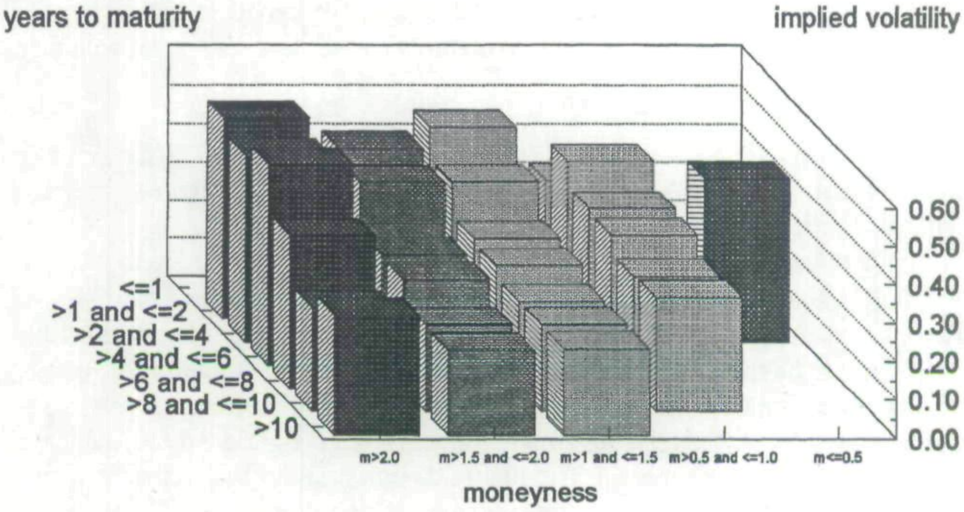


Figure 3. Implied volatility by maturity and moneyness.

calculated assuming there is a one-week delay in executing all trades. More trades are now unprofitable (23 out of 109 and 21 out of 108, respectively) but average profits still exceed 14 pence and are significantly above transactions costs.[10]

The conclusion is that the arbitrage profits are robust to different assumptions concerning incomplete trades, short-sale proceeds, dividends, liquidity and hedging with futures. It appears that the arbitrages are not just 'theoretical', but could have been captured and were likely to exceed potential transaction costs. In other words, these tests confirm that a large proportion of the warrants in our sample were anomalously underpriced.

4. Behaviour of Implied Volatilities

This section examines the implied volatilies of the remaining 8,012 warrants, which had prices above the lower bound, to see if there are biases relative to the Black/Scholes model. The average implied volatility is 30.1%, with a range from 0.1% to 155%. 3010 prices imply volatilities in excess of 30%, which suggests that some warrants are overvalued, particularly as the underlying assets of investment trusts are diversified portfolios of shares and so should have rather low volatilities.

We wish to test whether volatility is systematically related to moneyness and time to maturity. As a preliminary approach, the whole sample is classified by degree of moneyness and years to maturity. The results are given in Figure 3. They show that a greater time to maturity is associated with a lower volatility and that both in-the-money and out-of-the-money warrants show larger volatilities than at-the-money warrants. Looking at the figure from the front right-hand side, there is some evidence of a 'volatility smile', as reported for many kinds of traded options (see e.g. Taylor and Xu, 1994; Heynen, 1994).

In order to make a more formal test of biases, we hypothesise that implied volatility depends on moneyness, time to maturity and the level of market volatility:

$$isd_{it} = a + bm + cm_{it}^2 + d\tau_{it} + ev_t + g_{it}, \tag{7}$$

where isd is implied volatility, m is moneyness, defined as $Se^{-\delta\tau}/Ee^{-r\tau}$, τ is time to maturity, v is volatility of the FT All-Share Index, g is an error term, i denotes the ith warrant, t denotes a particular time period.

The first two terms on the right-hand side of the Equation (7) are moneyness (m) and its square (m^2). These allow the data to define a variety of 'smiles' and we expect to find that $b < 0$ and $c > 0$. Lauterbach and Schultz (1990) find that $b < 0$ and argue that this is consistent with a left-skewed distribution of returns, characteristic of a CEV rather than Black/Scholes model. However, if $c > 0$ and is moderately large, then the distribution also has a fat right-hand tail, which is not consistent with a simple CEV model (although it could be consistent with a combination of a CEV model and fat tails in the distribution).

The third term on the right-hand side of the equation is time to maturity (τ) and we expect this to have a negative coefficient ($d < 0$). The reason for this hypothesis is that asset prices may be mean-reverting, so that the variance does not increase linearly with time as assumed by the Wiener process of the Black/Scholes model.[11]

The fourth term on the right-hand side of the equation (v) is the volatility of the FT All-Share Index, measured over the previous six months. It is included in order to remove any general trends in volatility, so we expect the coefficient $e > 0$. For example, over the life of a warrant the market volatility might rise from 20% to 30% and, if this is not taken into account, there will be a spurious negative association between implied volatility and time to maturity.

Finally, the error term g_{it} is almost certainly going to be autocorrelated. We have therefore assumed a first-order autoregressive process and used maximum-likelihood estimation for the regression rather than simple least squares.

Equation (7) was estimated separately for each of the 45 warrant series which had at least 50 non-zero observations for the implied volatility. The results are given in Table III. The first result is that there is a strong negative relationship between moneyness and implied volatility (coefficient b averages -0.54 and the mean t value is -1.9). The effect of moneyness is negative for 39 of the 45 warrants and significant ($t < -2$) for 24 of them. The t-test for the number of negative coefficients (column 6) indicates a 1% level of significance. The second result is that the coefficient (c) on moneyness squared is positive, but less significant than for simple moneyness; (mean c value is 0.10 and mean t value is 1.2). While 31 of the 45 coefficients are positive, only 16 are significant ($t > 2$). The t-test for the number of positive coefficients indicates a 5% level of significance. The implication of the b and c coefficients together is that the volatility declines as the share price rises, but at a diminishing rate. This suggests that a CEV model is likely to be a better fit than the simple Black/Scholes model. The third result is a weak negative impact of time-to-maturity on volatility: (mean d value is -0.05 and mean t value

Table III. Summary statistics for coefficients of regressions on individual warrants

Relevant variable	Average coefficient	Average t-statistic	No. of t-stats 5% signif.	No. of coefficients < 0	t-stat. on No. of coefficients
Moneyness	−0.5361	−1.93	24	39	-4.92^{a}
Square of moneyness	0.1085	1.16	16	14	2.53^{b}
Time	−0.0474	−0.99	14	34	-3.42^{a}
Market volatility	0.000053	−0.14	2	16	−1.94

The table represents the results of 45 separate regressions in which the implied volatility was the dependent variable and the independent variables were those listed in the first column. Estimation used the Hildreth–Liu correction for autocorrelated errors.
[a] Significantly different from zero at the 1% level.
[b] Significantly different from zero at the 5% level.

is -0.99). The effect is negative for 33 of the 45 warrants, but significant for only 14 of them. Nevertheless, the t-test for the number of positive coefficients indicates significance at the 1% level. Finally, the general level of market volatility is not an important influence. The coefficient e is positive for only 18 of the 45 warrant series and significant for only 2.

To summarise, the individual regressions give strong support to the hypothesis that implied volatility declines with moneyness, but at a diminishing rate. These results are consistent with those for U.S. warrants, found by Lauterbach and Schultz (1990) and Hauser and Lauterbach (1997), and also with those for Japanese warrants, found by Gemmill (1989) and Kuwahara and Marsh (1992). The results lend support to the CEV model. They also support the hypothesis that implied volatility is lower for warrants with longer times to maturity, which is consistent with Samuelson's proposition of mean-reverting asset prices.

5. Why do Investment Trusts Issue Warrants?

The results so far suggest that many warrants are mispriced. But if warrants are so neglected, why bother to issue them in the first place?[12] There are two lines of argument which may be pursued. The first is that there is a clientele effect; by purchasing warrants as part of an IPO, investors obtain a package which cannot easily be replicated. The second is that the issue of warrants is intended to confuse investors, who overvalue the shares because they do not properly account for the potential liability inherent in the exercise of the warrants. This was Benjamin Graham's view in the 1930s, as stated earlier in the paper.

Let us first consider whether the clientele hypothesis is plausible. It will only hold if there is pent-up demand for warrants because they are difficult to replicate. For such replication an investor would have to borrow funds and invest the proceeds in the investment-trust shares, making further purchases if the market rose and further sales if the market fell. Few investors, large or small, directly lever their portfolios in this way but they can achieve the same effect by selling some of the bonds which they may hold and using the proceeds to buy investment-trust shares. In other words, all that is required for an investor to replicate a warrant is to increase the proportion of shares in the total portfolio and to re-balance the position from time to time. Approximate replication does not seem very difficult, nor is it likely to be expensive in transactions costs. The line of argument based upon a clientele effect is therefore unlikely to explain why so many warrants have been issued. Nevertheless, if the clientele hypothesis is correct, new issues of shares with warrants will be more highly valued than new issues without warrants and this will be tested empirically.

The second hypothesis of 'investor confusion' is more convincing. The issuer's IPO publicity typically emphasises that investors will not only receive shares but also a 'free' issue of warrants. In this way investors are encouraged to ignore the potential dilutive effect of the warrants and to value the new shares of with-warrants issues at the same premium to net asset value (NAV) as non-warrant issues. If this hypothesis is correct, new issues with warrants attached will not, at launch, be worth more than those issues which do not have warrants. There will, however, be an extra gain in package values (shares plus warrants) when the warrants are split-off and traded separately. This is the alternative hypothesis to be tested.

To summarise the two hypotheses, the clientele hypothesis predicts that new share issues with warrants attached will be valued more highly than issues without warrants *at the time of issue*. The confusion hypothesis predicts that there will be no extra valuation of issues which include warrants *until the warrants are split-off.*

In order to test these hypotheses, a revised sample of 127 investment-trust issues was assembled covering the period from January 1984 to April 1995. There were: 32 issues without warrants, 56 issues with warrants which were split-off and traded separately on the day of issue and 39 issues with warrants which were split-off later (on average after 28 days). The sample and tests are summarised in Table IV.

On the day of issue, the average premium to NAV for the group of 32 issues without warrants was 4.62% (column 2 of the table) whereas the 95 issues with warrants recorded an average premium of 6.29% (column 3 of the table). This difference of 1.67% is significant at the 5% level ($t = 1.99$). However, the extra value of the with-warrants group can be attributed solely to those issues where the warrants were split-off and priced separately on the first day of trading. This sub-group of 56 issues recorded an average premium to NAV of 7.36% on the first day of trading, as compared with an average first-day premium of 4.77% for the 39 issues where the warrants were split-off later. The difference of 2.59% in the average first-day premium is significant at the 1% level ($t = 3.03$). Thus new issues with warrants attached did not show any extra market value until the warrants were separately traded.

Table IV. Sample and tests for IPOs of investment trusts, 1984–95

Test measurement	Sample without warrants 32 issues	Sample with warrants 95 issues	
Premium to NAV on day of issue	4.62% (4.13)	6.29% (4.13)	
		Sample split immediately 56 issues	Sample split later (av. of 28 days) 39 issues
		7.36% (3.98)	4.77% (3.91)
Premium to NAV on day before split			4.28% (5.52)
Premium to NAV on day after split			6.93% (6.60)

Numbers in parentheses are standard deviations.
NAV is net asset value (after deduction of issuing costs).

This result is confirmed by looking at the time-series behaviour of the 39 issues for which splitting was delayed, which is also reported in Table IV. From the day before splitting to the day after splitting, the premium over NAV for these issues increased from 4.28% to 6.93%, which is significant at the 5% level ($t = 1.92$) and 31 of the 39 cases recorded an increase in premium. Under the clientele hypothesis, when the warrants are split off there should be no change in market value of the package of shares and warrants relative to the underlying NAV, yet we observe this to occur. The evidence therefore clearly supports the confusion hypothesis rather than the alternative clientele hypothesis.

There remains the question of why investors should be prone to making valuation errors when the warrants are split off? One influential factor may be the way in which the premium to NAV is computed and reported in the financial press. Before the split, the published premium is based upon the value of the package of shares and warrants together. After the split, the premium is computed by comparing the price of the shares alone to the diluted NAV. If investors continue to use the same premium to set the relationship of share price to diluted NAV as they used for the package price to undiluted NAV, then they make a systematic error.

Consider the following example. A new investment trust is launched with warrants attached which are exercisable at 100p. The NAV of the trust is 100p. and the package of shares and warrants is assumed to start trading at 100p., a reported premium of 0%. Now the warrants are split off and trade (for example) at 10p., while the shares continue to trade at 100p. The diluted and undiluted NAVs are both still 100p., because the warrants are at-the-money. (Exercise of a warrant would require a payment of 100p., which is exactly the current share price). The shares would continue to be reported in the financial press as trading at a 0% premium, the same as before the split had occurred. In reality, the investment trust is selling at a premium of 10%, because the shares and warrants together are now valued at 110p. The investors' fundamental error is to ignore the value of the warrants in assessing the value of their shares.

By examining the warrant prices in our sample on their first day of trading, we are able to test whether the extra value of with-warrants issues is due to high warrant values or to high share values. Of the sample of 95 warrants, 42 were priced below the lower arbitrage bound when first traded and the average premium to that bound was just 0.4 pence (not significantly different from zero). In other words, the average warrant traded at issue with an implied volatility of zero. Therefore the warrants were *undervalued* at issue and, as a consequence, we can conclude that the share prices were *overvalued*. This evidence decisively rejects the hypothesis of a clientele anxious to acquire the warrants, but it is consistent with confusion about warrant valuation among purchasers of new issues.

To summarise, the evidence supports the contention that issues with warrants *are* valued more highly by investors. To that extent it is perfectly rational for issuers to attach warrants to new issues of shares. However, the reason for the greater valuation accorded such issues is a misunderstanding on the part of investors as to the nature of warrants. Investors are encouraged to regard warrants as 'free' and the evidence suggests that they are confused by the way in which discounts for issues with warrants are reported. Benjamin Graham's view, developed in the the United States in the 1930s, is still supported in London in the 1990s.

6. Summary and Conclusions

Evidence on warrant pricing from several countries indicates that market prices may differ markedly from those predicted by models of the Black/Scholes family. This study provides a further example of mispriced warrants. Many of the 72 London Stock Exchange warrants in this study had long periods during which they were priced below the lower arbitrage bound. Most of these warrants were deep in-the-money. The trading tests confirm that these arbitrages could have been 'captured' and were not just the result of transactions costs, dividends or illiquidity.

Despite the number of arbitrages, those warrants which were above the lower bound did behave like stock options: they showed a decline in volatility as the stock price rose. There was also a tendency for lower volatility to be associated with longer time to maturity, which is consistent with the hypothesis that share prices are slowly mean-reverting.

The results from examining 127 IPOs confirm that those issues which had warrants were more highly valued than those without. The increase in value did not occur at the time of issue, but only when the warrants were split-off and separately traded for the first time. It is therefore rational to include warrants when making new issues of shares, since these issues trade at a higher premium to net asset value.

However, the extra value is based upon the failure of investors to understand that warrants reduce share values. To that extent, the convention of adding warrants to a new issue exploits the confusion of investors, just as Benjamin Graham thought in the 1930s.

Several avenues for future research can be identified. The first would be to examine whether warrants on conventional shares (rather than on investment trusts) exhibit similar behaviour. One difficulty is that there is only a small number of such warrants (in London) and they are usually issued by existing public companies rather than as part of an IPO. Preliminary work suggests that warrants on conventional shares in London are also erratically priced, with 20% of observations below the lower arbitrage bound. Another difficulty for warrant pricing on ordinary companies rather than investment trusts is that ordinary companies have debt, which complicates warrant pricing (Crouhy and Galai, 1994).

A second avenue for research is to test the Black/Scholes model against the alternatives, along the lines suggested by Hauser and Lauterbach (1997). The alternatives would include both CEV and dilution-adjusted models. A third avenue for research is to identify a sample of investors who own warrants and ask them to reveal how they estimate fair 'value'. In this way it might be possible to judge whether investors really are as confused as the present study seems to indicate.

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Notes

1 Instead of doing this directly, Lauterbach and Schultz compare the implied volatility (based upon V and the Black/Scholes or CEV models) with its time-series value (also based upon V).

2 The volatility of the stock price, σ_s, is less than the volatility of the all-equity-firm value, σ_v for the following reason. A firm has the volatility of its assets matched by the volatility of its liabilities. In the case of a firm with warrants, the volatility of the assets is spread over both the shares and the warrants. To the extent that holders of the warrants account for part of the risk, so the volatility of the shares is reduced. Hence $\sigma_s < \sigma_v$.

3 Other studies of U.S. options using relatively small samples include Ferri, Kremer and Oberhelman (1986), Leonard and Solt (1990) and Kremer and Roenfeldt (1992).

4 The rates used were the three-month inter-bank offered rate, the five-year bond yield and the fifteen-year bond yield.

5 Moneyness is defined as $Se^{-\delta\tau}/Ee^{-r\tau}$, where S is share price, δ is dividend yield, E is exercise price, r is interest rate and τ is time to maturity. At time of issue if $S = E$ then moneyness is $e^{(r-\delta)\tau}$. Typically δ is small relative to r, so this term is positive.

6 The lower bound is not model dependent. It is equally consistent with a dilution-adjusted model based upon firm value and a simple model based upon share value.

7 Although Lauterbach and Schultz reject 17% of their observations (our calculation), the reasons for rejection are not only lower-bound violations but also prices less than $1 and extensions to the lives of some warrants.

[8] A replication argument implies that spreads on in-the-money options are at least as large as those on underlying shares. See, for example, Gemmill (1993) pp. 118–124.
[9] This alternative treatment of dividends resulted in 4,283 observations with prices below the lower bound (c.f. 4,661 when continuous dividends were assumed).
[10] The number of trades falls to 108 (from 109) when there is a one-week delay because one more trade is incomplete at the end of the sample. Transactions costs have been reduced in the table to 4 pence for these futures-based trades, but profits would significantly exceed even 6 pence of costs at the 1% level.
[11] The expected pay-off on a warrant depends on the day-to-day movement of the forward price. If the spot and forward prices are locked together in a tight arbitrage, then their variances will be similar. However, if the arbitrage is weak then events which have a short-term impact may affect the spot price without having as large an impact on long-term expectations (and hence on the forward price). The result is that the forward variance is less than the spot variance. The variance used to price a warrant is the integral of a forward variance which increases as the warrant approaches maturity. This 'Samuelson effect' is well documented for commodity futures (see Anderson, 1985, Fama and French, 1988), but does not appear to exist for short-term stock-index futures (see Yadav and Pope, 1990) for which the spot/futures arbitrage is tight. However, a tight arbitrage would not be expected for warrants, which have many years to maturity, so the effect may be expected to be present in our sample. There is evidence for mean reversion in stock indices, (Poterba and Summers, 1988), but it is rather weak.
[12] There is a small literature on reasons for issuing warrants in the United States, but it does not seem relevant to investment-trust warrants in the United Kingdom. For example, Schultz (1993) examined unit IPOs in the United States and concluded that warrants were a form of staged financing for risky firms. However, investment trusts are not usually risky, as they consist of portfolios of shares. Warrants are also used in the United States to compensate underwriters, which Barry, Muscarella and Vetsuypens (1991) concluded was a method of giving underwriters extra rewards, but which Dunbar (1995) concluded signalled the quality of the issue. However, this method of paying underwriters is not used in the United Kingdom.

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