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Departures from Rational Expectations and Asset Pricing Anomalies

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We investigate the potential of the consumption CAPM with pessimism, doubt, and the availability heuristic in the agent's beliefs to resolve the equity premium and risk-free rate puzzles. Using the nonlinear GMM estimation techniques, we find that doubt and the availability heuristic play an important role in explaining the cross-section of asset returns. However, when taken alone, these deviations from rational expectations cannot resolve the equity premium and risk-free rate puzzles. This result is robust to the assumption that the expected value of an uncertain prospect is nonlinear in the subjective outcome probabilities.

Keywords: Availability heuristic, Doubt, Pessimism, Rational expectations, Subjective probability

INTRODUCTION

The pricing kernel in the standard Lucas (1978) consumption-based capital asset pricing model (consumption CAPM) is given by the intertemporal marginal rate of substitution in consumption of a single rational power utility maximizing representative investor. The representative agent is assumed to freely trade in perfect capital markets without incurring transaction costs, limitations on borrowing or short sales, and taxes. Many studies have documented the poor empirical performance of the standard consumption CAPM. It is observed, for example, that the model fails to explain the historical mean excess return on the market portfolio over the risk-free rate unless the risk aversion parameter is implausibly large. Mehra and Prescott [1985] call this the equity premium puzzle. Another difficulty with using the standard consumption CAPM is that this model matches the historical mean return on the risk-free asset only if the time discount factor is greater than one. Weil [1989] calls this the risk-free rate puzzle.

Several ways to improve the standard consumption CAPM have been proposed in the literature. One plausible approach consists of changing the representative agent's utility specification (e.g., Aschauer [1985], Eichenbaum et al. [1988], Epstein and Zin [1989, 1991], Startz [1989], Sundaresan [1989], Abel [1990, 1999], Constantinides [1990], Harvey [1991], McCurdy and Morgan [1991], Chou et al. [1992], Bakshi and Chen [1996], Campbell and Cochrane [1999], Gordon and St-Amour [2004]). According to Bewley [1982], Mehra and Prescott [1985], Mankiw [1986], Constantinides and Duffie [1996], Brav et al. [2002], and Kocherlakota and Pistaferri [2009], for example, consumers' heterogeneity induced by market incompleteness can help solve the equity premium and risk-free rate puzzles. Another possible response to the poor empirical performance of the standard consumption CAPM is to argue that market frictions (e.g., transactions costs and limits on borrowing or short sales) can be relevant for asset pricing (e.g., Campbell and Mankiw [1990], Mankiw and Zeldes [1991], Basak and Cuoco [1998], Alvarez and Jermann [2000], Constantinides et al. [2002]). Rietz [1988], Cecchetti et al. [2000], and Abel [2002] take an approach based on the assumption that the representative agent is not fully rational and argue that deviations from rational expectations enhance the empirical performance of the standard consumption CAPM.

Rietz [1988] specifies the model of Mehra and Prescott [1985] to capture the effects of possible, though unlikely, crashes and shows that the addition of a crash state allows the explanation of both a high equity risk premium and a low risk-free rate with reasonable degrees of risk aversion.[1] As in Mehra and Prescott's [1985] investigation, Cecchetti et al. [2000] assume that consumption growth follows a two-state Markov process and show that using simple rules of

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thumb to estimate the transition probabilities agents will form the subjective probabilities that are, on average, relatively pessimistic about the persistence of the expansion state and relatively optimistic about the persistence of the contraction state. These belief distortions allow a match of the first moments of the equity premium and the risk-free rate with a positive relative risk aversion coefficient of less than 10 and a discount factor below one. Furthermore, their analysis shows that persistence in the subjective transition probabilities together with random fluctuations in beliefs about the persistence of low-growth states are sufficient to explain the volatility of asset returns as well as persistence and predictability of excess returns exhibited in the data. However, the correspondence between the predicted cyclical pattern of the equity premium and that found in the data is rough.

Departing from the assumption of rational expectations, Abel [2002] explores the effect of pessimism and doubt in the agent's beliefs on the mean equity premium and risk-free rate. His finding is that pessimism and doubt may play an important role in explaining the mean equity premium and risk-free rate of return. Assuming lognormality of the objective and subjective distributions of the growth rate of aggregate consumption, Abel [2002] finds that the consumption CAPM with pessimism and doubt can match the historical average equity premium and risk-free rate with plausible values of the agent's coefficient of relative risk aversion and time discount factor. However, in some cases this model needs implausibly high levels of pessimism and doubt to explain the observed mean equity premium and risk-free rate.

In this article, we explore the potential of pessimism, doubt, and the availability heuristic in the agent's beliefs to help resolve the equity premium and risk-free rate puzzles. To test the ability of the consumption CAPM with deviations from rational expectations to explain the observed equity premium and risk-free rate, we use the nonlinear Generalized Method of Moments (GMM) estimation approach. This allows us to focus on the time-series properties of asset returns rather than on the historical averages as in Abel [2002]. We start by assuming that the aversion to risk is determined solely by the curvature of the agent's utility function. Under this assumption, when calculating the expected value of an uncertain prospect, an agent takes a sum of outcomes weighted by their subjective probabilities of occurring. Then, in accordance with the cumulative prospect theory of Tversky and Kahneman [1992], we suppose that the agent's risk aversion is determined jointly by the utility function and the decision weights that are strictly increasing monotonic nonlinear functions of subjective probabilities. In this case, the expected value of an uncertain prospect is nonlinear in the subjective probabilities.

The rest of the article is structured as follows. In section 2, we analyze the effect of pessimism, doubt, and the availability heuristic on asset returns. In section 3, we use the GMM estimation technique to explore the potential of the consumption CAPM with pessimism, doubt, and the availability heuristic in the agent's beliefs to explain the observed equity premium and risk-free rate. Section 4 concludes.

THE CONSUMPTION CAPM WITH DISTORTED BELIEFS

Asset Pricing in the Economy with Rational Expectations

One of the first-order conditions for the representative investor's intertemporal consumption and savings choice problem is

$$E_t\left[\delta\frac{u'(C_{t+1})}{u'(C_t)}R_{i,t+1}\right]=1, \tag{1}$$

where δ is the time discount factor, C_t is aggregate consumption per capita in period t, $u(C_t)$ is the representative agent's period utility function, and $R_{i,t+1}$ is the simple gross return on asset i between t and $t+1$. The expectation in Equation 1 is taken across states at time $t+1$ given the information known to an investor at time t. Equation 1 is known as the consumption-based capital asset pricing model, or the consumption CAPM.

Assume that the representative agent maximizes a time-separable power utility function, so that $u(C_t)=(C_t^{1-\gamma}-1)/(1-\gamma)$, where γ is the relative risk aversion coefficient. With this preference specification, marginal utility $u'(C_t)=C_t^{-\gamma}$ and therefore, from Equation 1,

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{i,t+1}\right]=1. \tag{2}$$

By taking unconditional expectations of the left- and right-hand sides of Equation 2, we get an unconditional version:

$$E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{i,t+1}\right]=1. \tag{3}$$

For the excess return on asset i over the risk-free rate $R_{f,t+1}$, Equation 3 becomes

$$E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(R_{i,t+1}-R_{f,t+1}\right)\right]=0. \tag{4}$$

For the risk-free rate, we get

$$E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{f,t+1}\right]=1. \tag{5}$$

It is common in empirical research to use historical data when testing pricing conditions 4 and 5 and hence to implicitly assume that the next period joint distribution of the growth rate of average consumption and asset returns may be well approximated by their joint historical distribution. Suppose that at time t an investor can observe the most recent S realizations of the growth rate of average consumption

and asset returns. This provides us with S different scenarios for what might happen at time $t+1$. According to the expectation principle, when calculating the expected value of an uncertain prospect, the agent takes a sum of outcomes weighted by their probabilities of occurring. Define $p_{s,t+1}$ as the probability that scenario s ($s = 1, \ldots, S$) occurs at time $t+1$.

The assumption of rationality implies that the agent weights outcomes by their objective probabilities of occurring. When historical data are used, the objective probability equals $1/S$ and hence under rational expectations $p_{s,t+1} = 1/S$ ($s = 1, \ldots, S$). Empirical evidence is that the consumption CAPM with rational expectations can jointly explain the historical excess return on the market portfolio and risk-free rate only if the representative agent is implausibly averse to risk (the equity premium puzzle) and the time discount factor is greater than one (the risk-free rate puzzle).

Departures from Rationality and Asset Returns

Based on the experimental evidence that human choices are not always rational, a number of recent articles have explored the role of departures from rational expectations (in an otherwise standard neoclassical framework) in explaining the equity premium and risk-free rate puzzles. One way to take nonrationality into account is to suppose that the representative agent weights possible outcomes in pricing conditions 4 and 5 by the subjective probabilities, which may differ from the objective probabilities, rather than by the objective probabilities as it is usually assumed in empirical research.

In this section, we first consider three particular departures from rationality (i.e., pessimism, doubt, and the availability heuristic) and explore their effects on asset returns. When consumers are not rational, under the assumption that the aversion to risk is determined solely by the curvature of the agent's utility function, the expected value of an uncertain prospect is linear in the subjective outcome probabilities. Then we focus on the cumulative prospect theory of Tversky and Kahneman [1992], according to which the agent's risk aversion is determined jointly by the utility function and the decision weights. Within this framework, the agent calculates the expected value of an uncertain prospect as a sum of outcomes weighted by the decision weights that are strictly increasing monotonic nonlinear functions of subjective probabilities.

An agent who is pessimistic about the distribution of the aggregate per capita consumption growth rate weights heavier lower growth rates and assigns smaller probabilities to high consumption growth rates relative to the case of rational expectations. Pessimism leads the agent to reduce his current period consumption and to save more to self-insure against a negative shock to his next period consumption and therefore the demand for both the safe asset and the risky asset increases. This increased demand requires a decrease in the equilibrium rate on both assets below the rate that would prevail under rational expectations. Thus, pessimism in beliefs can help resolve the risk-free rate puzzle.

To understand the effect of this additional demand for assets on the predicted equity premium, we must determine to which asset, bond or equity, this increased savings is predominantly allocated. Intuitively, if consumers are pessimistic about the growth rate of their consumption and therefore about the growth rate of dividends, then they expect equity to be less attractive than they would expect under rational expectations.[2] This suggests that the demand for equity increases less than that for the risk-free asset what results in an increasing equity premium. Because pessimism in the agent's beliefs increases the expected equity premium, it can help resolve the equity premium puzzle.

Abel [2002] uses the term pessimism to mean that the subjective distribution of average consumption growth rates is first-order stochastically dominated by the objective distribution. The risk-free rate from period t to period $t+1$ is know at time t and therefore, from Equation 5,

$$R_{f,t+1} = \left\{ E\left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \right\}^{-1}. \tag{6}$$

When the subjective distribution of average consumption growth rates is first-order stochastically dominated by the objective distribution, the expectation of the intertemporal marginal rate of substitution under pessimism is greater than the expectation of the intertemporal marginal rate of substitution under rational expectations at any arbitrary values of δ and $\gamma > 0$. It follows that pessimism reduces the equilibrium risk-free rate of return and therefore the consumption CAPM with pessimistic consumers has the potential to resolve the risk-free rate puzzle.[3] To examine the extent to which pessimism can help explain the excess return on a risky asset over the risk-free rate of return, Abel [2002] defines uniform pessimism as a leftward translation of the objective distribution of the logarithm of the average consumption growth rate. He shows that uniform pessimism increases the expectation of the equity premium and hence can help resolve the equity premium puzzle.

People exhibit doubt when they think that their knowledge is less accurate than it is in fact. Doubt implies that consumers assign to events the subjective probabilities, which are lower than the objective probabilities when they believe that the event probably will occur and higher than the true frequency when they believe that the event will not occur.

To explore whether doubt can help resolve the equity premium and risk-free rate puzzles, Abel [2002] defines the subjective distribution of growth rates of aggregate consumption per capita to be characterized by doubt if it is a mean preserving spread of the objective distribution or, equivalently, is second order stochastically dominated by the objective distribution of average consumption growth rates.[4] Under this assumption, the both distribution functions are defined

on the same state space and have identical means. However, the subjective distribution function has a larger variance and therefore is riskier. This additional risk makes consumers more willing to acquire the risk-free asset what increases the demand for the risk-free asset and therefore puts downward pressure on the risk-free interest rate. Thus, doubt, as defined in Abel [2002], has the potential to explain the risk-free rate puzzle. Since doubt increases consumers' perceived risk associated with equity, it decreases the demand for equity and consequently drives up the equity return. The increase in the equity return accompanied by the decrease in the risk-free rate leads to the expectation of the equity premium, which is greater than the expectation of the equity premium that would prevail under the assumption of rationality. This shows that doubt can help resolve the equity premium puzzle.

To investigate whether the consumption CAPM with doubt can better match the risk-free return, as in Abel [2002], consider Equation 6. Because the intertemporal marginal rate of substitution is a decreasing and convex function of the average consumption growth rate, the expected value of the intertemporal marginal rate of substitution is larger and therefore the equilibrium risk-free rate is lower under doubt than under the assumption of rational expectations. Abel [2002] also shows that doubt increases the required equity premium and that this increase in the equity premium is proportional to the agent's coefficient of relative risk aversion and the difference in the squared coefficient of variation of the average consumption growth rate between the subjective and objective probability distributions.

The agent's judgment about the probability of an outcome often depends on how easy that outcome is to imagine. Since the more recent experiences or reports are easier to imagine, people consider the more recent events as being more likely and hence assign to them higher probabilities of occurring in the future. For example, after a news feature about a shark attack, many people will be more nervous about swimming in waist-deep ocean waters. The phenomenon when the more recent events have a more significant effect on the agent's decisions is called the availability heuristic and was first reported by Tversky and Kahneman [1973].[5]

Under the assumption that the aversion to risk is determined solely by the curvature of the agent's utility function, the agent calculates the expected utility as a sum of outcomes weighted by the subjective probabilities of occurring. Tversky and Kahneman [1992], however, argue that the agent's risk aversion is determined jointly by the utility function and the decision weights that the agent assigns to different subjective probabilities and, hence, the expected value of an uncertain prospect is a sum of outcomes weighted by some nonlinear functions of subjective probabilities rather than by the subjective probabilities themselves.

Experimental evidence is that under uncertainty decision makers systematically overweight low probabilities and underweight high probabilities of occurring. It is observed that the impact of a given change in the probability of winning a given prize diminishes with the distance from the endpoints of the certainty scale. Allais [1953], for example, shows that the difference between probabilities of 0.99 and 1.00 has more impact on preferences than the difference between 0.10 and 0.11.[6] To account for the observed nonlinearity in outcome probabilities, Tversky and Kahneman [1992] propose to weight each outcome by a decision weight rather than by its probability. The cumulative prospect theory of Tversky and Kahneman [1992] asserts that decision weights π_s are strictly increasing monotonic nonlinear functions of outcome probabilities, $\pi_{s,t+1} = \pi(p_{s,t+1})$, from the unit interval into itself satisfying $\pi(0) = 0$ and $\pi(1) = 1$. Diminishing sensitivity implies that the weighting function must be concave near zero and convex near one.

Tversky and Kahneman (1992) define decision weights $\pi(p_{s,t+1})$ as

$$\pi(p_{s,t+1}) = \frac{p_{s,t+1}^{\lambda}}{\left(p_{s,t+1}^{\lambda} + (1 - p_{s,t+1})^{\lambda}\right)^{1/\lambda}} \tag{7}$$

implying that, in accord with the principle of diminishing sensitivity, the weighting function $\pi(p_{s,t+1})$ is steepest near the endpoints of the certainty scale and shallower in the middle of the range. If $\lambda = 1$, then $\pi(p_{s,t+1})$ is simply the probability of scenario s, $\pi(p_{s,t+1}) = p_{s,t+1}$.

In contrast to the expected utility theory, in which risk aversion and risk seeking are determined solely by the utility function, in the cumulative prospect theory of Tversky and Kahneman [1992] they are determined jointly by the utility function and the weighting functions. Within the cumulative prospect theory framework, a risk-averse agent tends to underweight the outcome probabilities, while a risk-seeking agent overweights the probabilities. If an agent is risk neutral, then the decision weight that he assigns to an outcome equals its probability of occurring. Risk aversion for gains is further enhanced by the curvature of the agent's utility function. Kachelmeier and Shehata [1992] find risk seeking for gains of low probability and risk aversion for gains of moderate and high probability of winning. The weighting function 7 exhibits the characteristic pattern of risk aversion for moderate and high probabilities and risk seeking for small probabilities when λ is close to but less than one.

EMPIRICAL INVESTIGATION

In this section, we test empirically whether pessimism, doubt, and the availability heuristic, either separately or in combination, can help resolve the equity premium and risk-free rate puzzles. We start by assuming that risk aversion is determined solely by the curvature of the agent's utility function and investigate if the consumption CAPM with pessimism, doubt, or the availability heuristic can explain the observed equity premium and risk-free rate with plausible values of the agent's risk aversion and time discount factor. Then, in accordance with the cumulative prospect theory of Tversky

and Kahneman [1992], we suppose that the agent's risk aversion is determined jointly by the utility function and the decision weights that are strictly increasing monotonic nonlinear functions of subjective probabilities obtained under the assumption that there is pessimism, doubt, or the availability heuristic. We use the nonlinear GMM estimation approach to test the empirical performance of the proposed models in explaining asset returns. The use of the GMM estimation techniques allows us to focus on the time-series properties of consumption and asset returns rather than on the historical averages of asset returns as in Abel [2002].

Data

To construct a time series of the aggregate per capita consumption growth rate, we use data on real, monthly personal consumption of nondurables and services from the U.S. Bureau of Economic Analysis (BEA). Monthly per capita consumption is obtained by dividing the real aggregate consumption by the total population (including armed forces overseas) from BEA.

As nominal, monthly gross returns on individual assets, we take the nominal, monthly returns (capital gain plus dividends) on the market capitalization-based decile portfolios of all stocks listed on the NYSE, AMEX, and NASDAQ obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. The nominal, monthly risk-free rate is the one-month Treasury bill return from CRSP. The real, monthly returns are calculated as the nominal returns divided by the one-month inflation rate based on the deflator defined for nondurables and services consumption.

The sampling period is from 1959:01 to 2003:12. Table 1 reports the descriptive statistics for these data.

TABLE 1
Descriptive Statistics.
The sampling period is from 1959:01 to 2003:12.
$R_{i,t+1}$, $i = 1, \ldots, 10$, are the returns on the market capitalization-based decile portfolios of all stocks listed on the NYSE, AMEX, and NASDAQ. All values are in real terms.

Variable	Mean	Median	Standard Deviation
C_{t+1}/C_t	1.0019	1.0020	0.0037
$R_{1,t+1}$	1.0144	1.0071	0.0782
$R_{2,t+1}$	1.0110	1.0087	0.0679
$R_{3,t+1}$	1.0091	1.0067	0.0642
$R_{4,t+1}$	1.0088	1.0097	0.0618
$R_{5,t+1}$	1.0081	1.0098	0.0599
$R_{6,t+1}$	1.0080	1.0104	0.0588
$R_{7,t+1}$	1.0076	1.0106	0.0572
$R_{8,t+1}$	1.0076	1.0114	0.0539
$R_{9,t+1}$	1.0072	1.0105	0.0508
$R_{10,t+1}$	1.0056	1.0078	0.0434
$R_{f,t+1}$	1.0011	1.0011	0.0021

GMM Estimation Results

Like Abel [2002], we focus on the unconditional Euler equations for the equity premium and the risk-free rate of return. The unconditional equations for the equity premia on the market capitalization-based decile portfolios (Equation 4) and the risk-free rate (Equation 5) are estimated jointly using the iterated GMM approach. Table 2 shows the estimation results.

Standard Consumption CAPM

This is our benchmark model. In this model, different scenarios have equal probabilities of occurring. We find that

TABLE 2
GMM Estimation and Test Results.
The sampling period is from 1959:01 to 2003:12. The unconditional Euler equations for the equity premia on the market capitalization-based decile portfolios and the risk-free rate are estimated jointly using the iterated GMM approach. The t-statistics are given in parentheses below the estimated coefficients. The J-statistic is Hansen's test of over-identifying restrictions. The P-value is the marginal significance level associated with the J-statistic.

Model	$\hat{\gamma}$	$\hat{\delta}$	P-value
A. Expectations linear in the objective outcome probabilities			
The standard consumption CAPM	83.7407	1.1056	0.0373
	(1.7410)	(26.0685)	
B. Expectations linear in the subjective outcome probabilities			
Pessimism	64.1740	1.0708	0.0362
	(1.3123)	(26.5567)	
Doubt	32.7289	1.0344	0.1845
	(1.2839)	(43.5651)	
The availability heuristic	59.0132	1.0893	0.0377
	(1.2841)	(19.7355)	
Doubt and the availability heuristic	24.8424	1.0303	0.3547
	(1.2892)	(62.0356)	
C. Expectations nonlinear in the subjective outcome probabilities			
$\lambda = 0.8$			
Pessimism	65.0681	1.0722	0.0363
	(1.3316)	(26.5511)	
Doubt	30.2826	1.0183	0.6746
	(1.2825)	(38.1056)	
The availability heuristic	59.4400	1.0898	0.0371
	(1.2942)	(19.8361)	
Doubt and the availability heuristic	25.5344	1.0313	0.3457
	(1.3183)	(61.5650)	
$\lambda = 0.5$			
Pessimism	63.9304	1.0704	0.0362
	(1.3067)	(26.5547)	
Doubt	32.9847	1.0359	0.1642
	(1.2839)	(43.7819)	
The availability heuristic	59.2092	1.0897	0.0371
	(1.2896)	(19.7932)	
Doubt and the availability heuristic	24.8189	1.0303	0.3539
	(1.2861)	(61.8248)	

the model is rejected statistically at the 5% significance level. The obtained estimate of the relative risk aversion coefficient, $\hat{\gamma} = 83.74$, is high and significantly positive at the 5% level of significance. This result illustrates the equity premium puzzle.[7] The estimate of the time discount factor, $\hat{\delta} = 1.11$, is significantly greater than one at the 5% significance level. This is the risk-free rate puzzle.

Consumption CAPM with Departures from Rational Expectations

To investigate whether the poor empirical performance of the standard consumption CAPM is due to the assumption of rationality, in what follows we assume that the agent weights possible outcomes by the subjective rather than by the objective probabilities of occurring. We start by restricting the expected values in pricing conditions 4 and 5 to be linear in outcome probabilities.

When assuming the representative agent to be pessimistic about the future consumption growth rate, we arrange the observed historical consumption growth rates in increasing order and assign to different observations probabilities of occurring that decline exponentially in the consumption growth rate:

$$p_{s,t+1} = \frac{\theta^{s-1}(1-\theta)}{1-\theta^{S}}, 0 < \theta < 1, s = 1, \ldots, S. \qquad (8)$$

where $s = 1$ corresponds to the scenario with the smallest consumption growth rate, $s = 2$ corresponds to the scenario with the next greater rate, and so on. Finally, the Sth scenario is that corresponding to the highest growth rate of aggregate consumption per capita.

We estimate the equations for the risk premia on the market capitalization-based decile portfolios and the risk-free rate jointly for the values of θ decreasing from 0.9999 to 0.9800 with decrements of 0.0001.[8] We find that the marginal significance level associated with Hansen's test of over-identifying restrictions as well as the estimate of the relative risk aversion coefficient and the associated t-statistic decrease as the value of θ decreases. The lowest significantly positive at the 10% significance level value of the relative risk aversion, $\hat{\gamma} = 64.17$, is obtained at $\theta = 0.9995$. At this value of the parameter θ, $\hat{\delta} = 1.07$. The null hypothesis that the estimate of the time discount factor is greater than one is not rejected statistically at the 5% significance level. The model with pessimism in the agent's beliefs is rejected statistically at the 5% level of significance according to Hansen's J-statistic at any value of the parameter θ.

Under the assumption that the representative agent exhibits doubt, the scenarios are sorted in decreasing order according to the distance from the observed value of the aggregate per capita consumption growth rate to the mean of the historical distribution of the average consumption growth rate. To calculate the subjective probability assigned to each observation, we use formula 8. The highest probability of occurring is assigned to the average consumption growth rate with the greatest distance from the mean of the historical distribution and the smallest probability is assigned to the average consumption growth rate that is closest to the mean.

The unconditional equations for the risk premia on the market capitalization-based decile portfolios and the risk-free rate are estimated jointly for the values of θ decreasing from 0.9999 to 0.9800 with decrements of 0.0001. Our result is that the estimate of the relative risk aversion coefficient and the associated t-statistic decrease, while the marginal significance level associated with Hansen's test of over-identifying restrictions increases as the value of θ decreases. The lowest significantly positive at the 10% significance level value of the relative risk aversion, $\hat{\gamma} = 32.73$, is obtained at $\theta = 0.9910$. The corresponding estimate of the time discount factor $\hat{\delta} = 1.03$ is not significantly greater than one at the 5% significance level. According to Hansen's test of the over-identifying restrictions, at $\theta = 0.9910$ the consumption CAPM with doubt is not rejected statistically at the 5% level of significance.

To make allowance for that consumers consider the more recent events as being more likely, we assign to different scenarios probabilities declining exponentially through the past according to formula 8 with θ decreasing from 0.9999 to 0.9800 with decrements of 0.0001. The first scenario here is where the growth rate of average consumption and the return on asset i in period t are assumed to occur. The values of the growth rate of average consumption and the asset return in the second scenario are the same as in period $t-1$ and so on. Finally, the Sth scenario is where the growth rate of average consumption and the asset i return are the same as they were in period $t+1-S$.

As the value of θ decreases, the estimate of the relative risk aversion coefficient and the associated t-statistic decrease, while the marginal significance level associated with Hansen's test of over-identifying restrictions increases. We find that the lowest significantly positive at the 10% significance level estimate of the relative risk aversion $\hat{\gamma} = 59.01$ is obtained at $\theta = 0.9974$. The corresponding estimate of the time discount factor is $\hat{\delta} = 1.09$. The hypothesis that this estimate is greater than one is rejected statistically at the 5% significance level. According to Hansen's test of the over-identifying restrictions, at $\theta = 0.9974$ the consumption CAPM with the availability heuristic is rejected statistically at the 5% level of significance.

The next step is to investigate if various combinations of pessimism, doubt, and the availability heuristic can help resolve the equity premium and risk-free rate puzzles. Empirical evidence is that under the assumptions that (i) there are pessimism and the availability heuristic in the agent's beliefs and (ii) the agent exhibits pessimism and doubt, there is no significantly positive at the 10% significance level estimate of the agent's coefficient of relative risk aversion. However, when the subjective distribution is characterized by doubt

and the availability heuristic, we find that the lowest significantly positive at the 10% level of significance estimate of the relative risk aversion coefficient, $\hat{\gamma} = 24.84$, is obtained when $\theta = 0.9935$ for both doubt and the availability heuristic. The corresponding estimate of the time discount factor is $\hat{\delta} = 1.03$ and is significantly greater than one at the 5% level of significance. At these values of the parameters, the model with doubt and the availability heuristic is not rejected according to Hansen's test of over-identifying restrictions at the 5% significance level.

A more general approach is to assume that the expected values in pricing conditions in 4 and 5 may be nonlinear in the subjective probabilities. The cumulative prospect theory of Tversky and Kahneman [1992] asserts that an agent usually assigns to possible outcomes decision weights that are strictly increasing monotonic nonlinear functions of outcome probabilities. Following Tversky and Kahneman [1992], we assume that the weighting function $\pi_{s,t+1} = \pi(p_{s,t+1})$ is given by formula 7 with $p_{s,t+1}$ being the subjective probability of an outcome. When calculating the subjective probabilities $p_{s,t+1}$, we assume that the representative agent exhibits either pessimism, or doubt, or the availability heuristic, or some combination of them. As before, the subjective probabilities are calculated using formula 8. In the special case when $\lambda = 1$, $\pi(p_{s,t+1}) = p_{s,t+1}$ and hence expectations are linear in the subjective probabilities $p_{s,t+1}$. When λ differs from one, risk aversion and risk seeking are determined jointly by the utility function and the weighting functions.

As we mentioned above, experimental evidence is that people usually overestimate low probabilities (risk seeking) and underestimate moderate and high probabilities of winning (risk aversion). For $0.9800 \leq \theta < 1$ and $S = 539$, the weighting function 7 exhibits the characteristic pattern of risk aversion for moderate and high probabilities and risk seeking for small probabilities when $0.3 \leq \lambda < 1$. In what follows, we assume $\lambda = 0.5$ and $\lambda = 0.8$ and test whether the assumption that risk aversion and risk seeking are determined jointly by the utility function and the weighting functions can improve the empirical performance of the model, in which the representative agent is assumed to exhibit either pessimism, or doubt, or the availability heuristic, or some combination of these departures from rationality. We find that the estimation results are similar to those obtained under the assumption that the aversion to risk is determined solely by the curvature of the agent's utility function (and therefore expectations are linear in the subjective probabilities $p_{s,t+1}$).

CONCLUDING REMARKS

In this article we explored the potential of pessimism, doubt, and the availability heuristic in the agent's beliefs to enhance the empirical performance of the standard representative agent consumption CAPM with power utility. First, we assumed that risk aversion is determined solely by the curvature of the agent's utility function and therefore that the expected value of an uncertain prospect is linear in the subjective probabilities of outcomes. Then, we supposed that risk aversion and risk seeking are determined jointly by the utility function and the decision weights, which are strictly increasing monotonic nonlinear functions of outcome probabilities. In this case, the expected value is nonlinear in the subjective probabilities.

Using the nonlinear GMM estimation approach, we found that, when the expected value is linear in the subjective probabilities, the consumption CAPM with doubt and the availability heuristic explains the observed equity premia on the market capitalization-based decile portfolios and the risk-free rate much better than the conventional consumption CAPM with the rational representative agent. However, the obtained estimates of the agent's preference parameters are still too high to be recognized as economically plausible. The assumption of nonlinearity of the expected value in the subjective probabilities did not affect the estimation results. This suggests that doubt and the availability heuristic have some potential to help explain better asset returns, but, taken alone, cannot resolve the equity premium and risk-free rate puzzles.

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NOTES

1. Mehra and Prescott [1985] assume that equity returns vary little between good and poor times. Rietz [1988] incorporates in Mehra-Prescott's [1985] model a low-probability, depression-like third state, which captures the effects of crashes, when equity returns are far below average.
2. In the equilibrium of the Lucas [1978] economy, the average consumption growth rate is identical to the growth rate of dividends per capita.
3. See Abel [2002].
4. See Rothschild and Stiglitz [1970].
5. See, for example, also Tversky and Kahneman [1974] and Carroll [1978].
6. See also Camerer and Ho [1991].
7. As economically realistic, one usually recognizes the values of the relative risk aversion coefficient that do not exceed 10.
8. As $\theta \to 1$, $p_{s,t+1} \to 1/S$ (i.e., the subjective probabilities get equivalent to the objective probabilities) and

therefore the consumption CAPM with distorted beliefs becomes equivalent to the standard consumption CAPM, in which different scenarios are weighted by the objective probabilities.

REFERENCES

Abel, A. B. "Asset Prices Under Habit Formation and Catching Up with the Joneses." *American Economic Review*, 80, (1990), pp. 38–42.

Abel, A. B. "Risk Premia and Term Premia in General Equilibrium." *Journal of Monetary Economics*, 43, (1999), pp. 3–33.

Abel, A. B. "An Exploration of the Effects of Pessimism and Doubt on Asset Returns." *Journal of Economic Dynamics and Control*, 26, (2002), pp. 1075–1092.

Allais, M. "Le Comportement de l'Homme Rationnel devant le Risque: Critiques des Postulats et Axioms de l'Ecole Americaine." *Econometrica*, 21, (1953), pp. 503–546.

Alvarez, F. and U. J. Jermann. "Efficiency, Equilibrium, and Asset Pricing with Risk of Default." *Econometrica*, 68, (2000), pp. 775–797.

Aschauer, D. A. "Fiscal Policy and Aggregate Demand." *American Economic Review*, 75, (1985), pp. 117–127.

Bakshi, G. S. and Z. Chen. "The Spirit of Capitalism and Stock-Market Prices." *American Economic Review*, 86, (1996), pp. 133–157.

Basak, S. and D. Cuoco. "An Equilibrium Model with Restricted Stock Market Participation." *Review of Financial Studies*, 11, (1998), pp. 309–341.

Bewley, T. F. "Thoughts on Tests of the Intertemporal Asset Pricing Model," Working Paper (1982), Northwestern University, Evanston, IL.

Brav, A., G. M. Constantinides and C. C. Géczy. "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence." *Journal of Political Economy*, 110, (2002), pp. 793–824.

Camerer, C. E. and T.-H. Ho. "Nonlinear Weighting of Probabilities and Violations of the Betweenness Axiom." Unpublished manuscript, (1991), The Wharton School, University of Pennsylvania.

Campbell, J. Y. and J. H. Cochrane. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107, (1999) pp. 205–251.

Campbell, J. Y. and N. G. Mankiw. "Permanent Income, Current Income, and Consumption." *Journal of Business and Economic Statistics*, 8, (1990), pp. 265–278.

Carroll, J. S. "The Effect of Imagining an Event on Expectations for the Event: An Interpretation in Terms of the Availability Heuristic." *Journal of Experimental Social Psychology*, 14, (1978), pp. 88–96.

Cecchetti, S. G., P.-S. Lam and N. C. Mark. "Asset Pricing with Distorted Beliefs: Are Equity Returns Too Good To Be True?" *American Economic Review,* 90, (2000), pp. 787–805.

Chou, R. Y., R. F. Engle and A. Kane. "Measuring Risk Aversion from Excess Returns on a Stock Index." *Journal of Econometrics*, 52, (1992), pp. 201–224.

Constantinides, G. M. "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy*, 98, (1990), pp. 519–543.

Constantinides, G. M., J. B. Donaldson and R. Mehra. "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle." *Quarterly Journal of Economics*, 117, (2002), pp. 269–296.

Constantinides, G. M. and D. Duffie. "Asset Pricing with Heterogeneous Consumers." *Journal of Political Economy*, 104, (1996), pp. 219–240.

Eichenbaum, M. S., L. P. Hansen and K. J. Singleton. "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty." *Quarterly Journal of Economics*, 103, (1998), pp. 51–78.

Epstein, L. G. and S. E. Zin. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica*, 57, (1989), pp. 937–969.

Epstein, L. G. and S. E. Zin. "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis." *Journal of Political Economy*, 99, (1991), pp. 263–288.

Gordon, S. and P. St-Amour. "Asset Returns and State-Dependent Risk Preferences." *Journal of Business and Economic Statistics*, 22, (2004), pp. 241–252.

Harvey, C. "The World Price of Covariance Risk." *Journal of Finance*, 46, (1991), pp. 111–157.

Kachelmeier, S. J. and M. Shehata. "Examining Risk Preferences Under High Monetary Incentives: Experimental Evidence from The People's Republic of China." *American Economic Review*, 82, (1992), 1120–1141.

Kocherlakota, N. R. and L. Pistaferri. "Asset Pricing Implications of Pareto Optimality with Private Information." *Journal of Political Economy*, 117, (2009), 555–590.

Lucas, R. E., Jr. "Asset Prices in an Exchange Economy." *Econometrica*, 46, (1978), pp. 1429–1445.

Mankiw, N. G. "The Equity Premium and the Concentration of Aggregate Shocks." *Journal of Financial Economics*, 17, (1986), pp. 211–219.

Mankiw, N. G. and S. P. Zeldes. "The Consumption of Stockholders and Non-Stockholders." *Journal of Financial Economics*, 29, (1991), pp. 97–112.

Mehra, R. and E. C. Prescott. "The Equity Premium: A Puzzle." *Journal of Monetary Economics*, 15, (1985), pp. 145–162.

McCurdy, T. H. and I. G. Morgan. "Tests for a Systematic Risk Component in Deviations from Uncovered Interest Rate Parity." *Review of Economic Studies*, 58, (1991), pp. 587–602.

Rietz, T. A. "The Equity Premium Puzzle: A Solution." *Journal of Monetary Economics*, 22, (1988), pp. 117–131.

Rothschild, M. and J. E. Stiglitz. "Increasing Risk: I. A Definition." *Journal of Economic Theory*, 2, (1970), pp. 225–243.

Startz, R. "The Stochastic Behavior of Durable and Non-Durable Consumption." *Review of Economics and Statistics*, 71, (1989), pp. 356–363.

Sundaresan, S. M. "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth." *Review of Financial Studies*, 2, (1989), pp. 73–88.

Tversky, A. and D. Kahneman. "Availability: A Heuristic for Judging Frequency and Probability." *Cognitive Psychology*, 5, (1973), pp. 207–232.

Tversky, A. and D. Kahneman. "Judgment Under Uncertainty: Heuristics and Biases." *Science*, 185, (1974), pp. 1124–1130.

Tversky, A. and D. Kahneman. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 5, (1992), pp. 297–323.

Weil, P. "The Equity Premium Puzzle and the Risk-Free Rate Puzzle." *Journal of Monetary Economics*, 24, (1989), pp. 401–421.