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ARTICLES

Are Equally Likely Outcomes Perceived as Equally Likely?

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Subjective probability weighting is well known as an important factor in decision making. The case of equally likely outcomes is of special importance for two reasons. First, it is encountered in practice often, when expectations are typically based on a given set of historical observations that are each observed once. Second, many researchers believe that in this case probabilities are not subjectively weighted. If this hypothesis is correct, the case of equally likely outcomes can be employed to make clear inferences about the utility/value function, with a neutralization of probability weighting effects. We experimentally examine whether equally likely outcomes are indeed perceived as equally likely. Surprisingly, we find a clear negative answer. Moderate outcomes are significantly overweighed relative to extreme outcomes. Several important implications for models of decision making and economics are discussed.

Keywords: Probability weighting, Decision making, Risk attitudes, Asset pricing

INTRODUCTION

It has been extensively documented that individuals subjectively weigh probabilities in their decision making. This phenomenon is especially pronounced for "long shots," that is, high outcome events with very low probabilities, and in the special case where some of the outcomes are certain.

Subjective probability weighting and the shape of the utility (or value) function are 2 closely related issues, which are difficult to disentangle. For example, in one of their most famous experiments, Kahneman and Tversky [1979] ask subjects to choose between a certain outcome of $3,000 and an 80% probability of getting $4,000 (and a 20% probability of getting nothing). They find that 80% of the subjects prefer the certain outcome, from which they deduce a concave value function in the domain of gains. When they ask subjects to choose between a certain loss of $3,000 and an 80% probability of losing $4,000, they find that 92% of the subjects prefer the uncertain loss. From this result a convex value function in the domain of losses is inferred (see Kahneman and Tversky [1979], table I, p. 268). However, this analysis does not incorporate probability weighting. When Cumulative Prospect Theory (CPT) probability weighting (as suggested by Tversky and Kahneman [1992]) is taken into account, the above subjects' choices are also consistent with a value function that is convex for gains and concave for losses, just opposite to the conclusion of Kahneman and Tversky [1979][1]. This entanglement of subjective probability weighting and the shape of preferences led to more research, attempting to examine both effects simultaneously. This is quite a problematic task that comes at the price of losing the simplicity and clarity of the original experiments.[2]

While Tversky and Kahneman [1992] developed a probability weighting formula that is to be applied in general decision-making situations, a special case that is very important and that may likely be treated in a unique way is the symmetric case with equally likely outcomes. In this case, it seems intuitive that probability weighting will *not* affect choices. Quiggin [1982], Officer and Halter [1968], and Anderson, Dillon, and Hardaker [1977] argue that in symmetric 50-50 bets no probability weighting would take place. Quiggin writes:

> "*[t]he claim that the probabilities of 50-50 bets will not be subjectively distorted seems reasonable, and, as stated above, has proved a satisfactory basis for practical work.*" (Quiggin [1982], p. 328)

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Though Quiggin does not extend this argument to symmetric bets other than 50-50 bets, in Viscusi's [1989] Prospective Reference Theory, there is no probability weighting in the general symmetric case where all outcomes are equally likely. Also, in the original Prospect Theory framework (Kahneman and Tversky [1979]), in which the probabilities are transformed directly (rather than the cumulative distribution), the choice among prospects is unaffected by subjective probability weighting in the case of equally likely outcomes.

If indeed probabilities are not subjectively weighted in the case of equally likely outcomes, this offers an elegant way to study the shape of the utility function in isolation of probability weighting effects. This route is followed in Levy and Levy [2002a, b], who employ gambles with equally likely outcomes to experimentally test the Prospect Theory hypothesis of risk-aversion for gains and risk-seeking for losses.[3] Surprisingly, they find support for the opposite: they find that subjects display risk-seeking for gains and risk-aversion for losses (as suggested by Markowitz [1952]). Similar results have been reproduced in other experiments (see Baltussen, Post and van Vliet [2006], Wu and Markle [2005])[4]. These results have been criticized on the grounds that CPT probability weighting should be employed even in the case of equally likely outcomes, even though it assigns a different weight to each outcome (Wakker [2003], Baucells and Heukamp [2004]). Thus, the question of whether probabilities are subjectively weighted in the case of equally likely outcomes has become a central issue in the discussion regarding the shape of preferences.

Another reason that makes the case of equally likely outcomes of special importance is that this case is probably the case most often encountered in practice. There are many situations in which individuals attempt to estimate various parameters (e.g., expected rates of return, standard deviations, growth rates) based on a series of N historically observed realizations. In such situations, each historical observation is observed once, and thus we have a distribution of equally likely outcomes with probability 1/N.[5] Virtually all econometric models implicitly assume that probabilities are not weighted in this case (e.g., the expectation is simply the average, and so on). If, however, it is found that probabilities are weighted in the equally likely case, this may have a dramatic impact on our models of expectation formation, economic pricing models, and in general on the interpretation of any empirical or experimental measurements involving decision making based on a set of N different observations.

The purpose of this article is to experimentally test whether probabilities are subjectively weighted in the case of equally likely outcomes. In the next section we describe the experiment. In the third section we present the results. A discussion of the results and their theoretical and practical implications is presented in the fourth section, and the fifth section offers a conclusion.

TABLE 1
The Experimental Tasks. Subjects are presented with the 3 tasks below, where the gains or losses are in dollars. In each task the subjects are asked to select the prospect that they prefer. They are then also asked to circle the outcome that they guess will be realized in each of the prospects. After the choices and guesses are made, an actual lottery is conducted determining the realized outcome in each prospect. The subject with the highest number of "hits" receives a $40 prize.

F		G	
Gain or Loss	Probability	Gain or Loss	Probability
	Task I		
−5000	1/4	−5800	1/4
−4000	1/4	−4200	1/4
−3000	1/4	−3500	1/4
−2000	1/4	−1000	1/4
	Task II		
1800	1/4	1000	1/4
1900	1/4	2000	1/4
2500	1/4	3000	1/4
3100	1/4	4000	1/4
	Task III		
−2000	1/4	−2500	1/4
−1000	1/4	−50	1/4
3000	1/4	3550	1/4
4000	1/4	3760	1/4

THE EXPERIMENT

The experiment consists of three tasks. In each task the subjects are presented with two prospects, both with the same number of possible outcomes. The probability for each outcome is given, and all outcomes are equally likely. Table 1 presents the three tasks in the experiment. We first ask subjects to select which of the two prospects in each task, F or G, they prefer. We then ask the following question: if you were asked to *guess* the outcome that will be realized *in each of the two prospects*, what would your guess be? Subjects are told that the actual lottery will be conducted after their choices are made. The subject who turned out to have the highest number of "hits," that is, the one who guessed the realizations correctly the highest number of times, is rewarded with a financial prize of $40.[6] The Appendix provides the exact instructions given to subjects. Of course, as the outcomes are equally likely, any guess is as good as the other, and thus subjects resort to employing their completely subjective "gut feelings."

This approach offers a novel way to test whether subjects attach the same subjective weight to all outcomes. If so, when averaging across subjects we would expect to have the same number of guesses for each outcome. In contrast, if subjects tend to concentrate on a certain outcome more than others, this suggests that this outcome may have more influence on the subjects' decision making. Specifically, if probabilities are not weighted in the equally likely case, we would expect a uniform distribution of guesses; if CPT decision weights are employed, we would expect more focus on the extreme outcomes, and thus a higher number of guesses of these outcomes.

We analyze two main issues:

1. Is the distribution of guesses homogeneous across the outcomes? and
2. Is the subject's choice among the two prospects associated with her guessed outcomes?

While the first issue examines whether subjects have a systematic cognitive bias, the second issue examines whether this bias (if it exists) affects their behavior.

One-hundred and forty subjects participated in the experiment. The subjects were graduate and undergraduate business school students from Hebrew University, Bar Ilan University, and Ben Gurion University, and natural science Ph.D. students at Weitzman Institute of Science taking a finance course. All subjects had at least one course in statistics. All sessions were conducted during the winter and spring of 2004.

TABLE 2
The Distribution of Guesses. The table shows the percentage of times each outcome of every prospect was guessed as the expected realization. In total the 2 moderate outcomes were guessed on average 66.7% of the times, while the two extreme outcomes were guessed only 33.3% of the times. This pattern of over-weighting the moderate outcomes and under-weighting the extreme outcomes is observed for each of the six prospects in the experiment. The χ^2 test statistic is calculated for each of the prospects in each task, with respect to a uniform distribution with the same number of guesses (35) for each outcome (recall that there are 140 subjects). Note that the subjects' guesses also imply optimism: the 2 highest outcomes (outcomes 3 and 4) were guessed 55.4% of the times, compared with the 2 lowest outcomes (1and 2) with only 44.6%.

	Task I		Task II		Task III		
Outcome	F	G	F	G	F	G	Average
1 (lowest)	12.7	9.3	17.8	16.1	18.7	16.3	15.1
2	30.6	30.7	25.9	34.3	25.9	29.6	29.5
3	38.1	45.7	31.9	38.0	36.7	32.6	37.2
4 (highest)	18.7	14.3	24.4	11.7	18.7	21.5	18.2
Total:	100	100	100	100	100	100	100
χ^2	22.05**	46.11**	5.62	28.67**	12.14**	9.36*	105.27**

Notes: * significant at the 5% level.
** significant at the 1% level.

RESULTS

Table 2 reports the subjects' guesses in each prospect. As the results are similar across subject groups, we report only the aggregate results.[7] The most striking result revealed in this table is the very uneven distribution of guesses among the possible outcomes. The two intermediate outcomes (outcomes 2 and 3) are clearly guessed many more times than the two extreme outcomes (1 and 4). This is despite it being explicitly stated that each outcome has the same 25% chance of occurring. This result is systematic across all six prospects in the three tasks. In aggregate, the two extreme outcomes were guessed only 33% of the time. In contrast, the two intermediate outcomes were guessed 67% of the time, more than twice the number of extreme outcome guesses.

The results are statistically significant. The bottom line in Table 2 provides the χ^2 statistic for each prospect, where the experimental distribution of number of guesses is compared with the uniform distribution of 35 guesses per outcome (recall that there are 140 subjects). For 5 of the 6 prospects, the experimental distribution is different from the uniform distribution at the 5% significance level. For 4 of the prospects, the difference is significant even at the 1% level. The result for the aggregate distribution is statistically significant with $\chi^2 = 105.27$.

What is the reason for this phenomenon? One possible explanation is that individuals have a deeply rooted notion of a bell-shaped distribution. Thus, they believe that extreme outcome events are less likely than moderate outcome events. They may use this intuition even when it is completely wrong, as in the case where the probabilities for each of the outcomes are equal[8]. Another possible explanation may be that individuals instinctively do not want to be very far in their guess (or visualized outcome) from the actual realization (although this has no justification in our experiment, as only exact "hits" count). In this case, choosing intermediate outcomes minimizes the expected distance between the guessed outcome and the realized outcome. These effects may be important both in experimental setups designed to test preferences and in actual decision-making situations with a given set of N historical observations.[9]

The experimental results also indicate some inclination for optimism: the higher outcomes are guessed more frequently than the lower outcomes. On average, the most favorable outcome (outcome 4) is guessed 18.2% of the time, while the least favorable outcome is guessed 15.1% of the time. The second most favorable outcome (outcome 3) receives the most attention with 37.2% of the guesses, while in comparison outcome 2 receives only 29.5%. In total the 2 most favorable outcomes are guessed 55.4% of the time, and the 2 least favorable outcomes are guessed only 44.6% of the time[10].

The effect of attaching more weight to the moderate outcomes would have been perhaps a mere curiosity if it had not affected the subjects' choices among prospects. However, as we show below, the subjects' choices among the 2 prospects in each task are clearly positively associated with their guessed outcome. We analyze this association with the following approach. We look at each subject's prospect choice and guessed outcomes in each task, and we record the number of times that the subject chose the prospect for which she guessed a higher outcome realization versus the number of times that the subject chose the prospect for which she guessed a lower outcome realization (recall that the subject guesses the realized outcome in *both* prospects in each task). If subjects' choices are independent of their guesses, we would expect the same number of the above two cases.

The results are reported in Table 3. In task I, 64% of the subjects chose the prospect for which they guessed a higher outcome, while only 36% of the subjects chose the prospect for which they guessed a lower outcome. The results in the other two tasks are similar. Overall, 66% of the subjects chose the prospect for which they guessed a higher outcome, while only 34% of the subjects chose the prospect for which they guessed a lower outcome[11,12].

The results show a clear positive association between the guessed outcomes and the subject's choice of prospect. While there is not necessarily an exact one-to-one correspondence between the proportion of guesses and the decision weight attached to each outcome, the results confirm that the proportion of guesses do provide an indication of the focus attached to each of the outcomes. Furthermore, these proportions are strongly related to the subject's choice among prospects.

The experimentally observed probability weighting in the case of equally likely outcomes is different than the weighting implied by Cumulative Prospect Theory's general probability weighting formula, which was developed based mostly on prospects with different probabilities for the different outcomes[13]. Thus, while the CPT weighting formula may work well in general, it does not seem to be appropriate to the special case of equally likely outcomes. The experimental findings are also different from the weighting of Prospect Theory and Prospective Reference Theory, theories that imply equal probability weights in the symmetric case of equally likely outcomes. Table 4 summarizes the probability weighting under these different models in the case of four equally likely outcomes, and compares them with the experimental results obtained in this study. Figure 1 displays these weights graphically.

In light of the theoretical predictions, the experimental result of over-weighting moderate outcomes in the equally likely outcome case is a very surprising phenomenon. The next section discusses the implications of this effect.

TABLE 3
The Association between Prospect Choice and Outcome Guess. The subjects were asked to guess the outcome that will be realized in each of the 2 prospects in each task. The table shows the percentage of times that the prospect chosen is the one for which a higher outcome was guessed, versus the times that the prospect chosen is the one for which a lower outcome was guessed. The first case occurs approximately twice as many times as the second case. The results are significant at the 1% level, see footnote 10.

	Task I	Task II	Task III	Average
Prospect selected is the one with the higher guessed outcome	64	72	61	66
Prospect selected is the one with the lower guessed outcome	36	28	39	34

TABLE 4
Probability Weighting by Different Theories. Probability weighting in the case of 4 equally likely outcomes by various models. Cumulative Prospect Theory implies higher decision weights to the extreme outcomes, while Prospects Theory and Prospective Reference Theory imply the same decision weight to each of the outcomes. The experimental results, in contrast, reveal an overweighting of the intermediate outcomes in this case.

Outcome	1 (lowest)	2	3	4 (highest)
Objective probability	0.25	0.25	0.25	0.25
CPT decision-weights*	0.29	0.16	0.13	0.29
PT decision weights	c	c	c	c
Symmetric Prospective Reference Theory	0.25	0.25	0.25	0.25
Experimental results	0.15	0.30	0.37	0.18

Note: * Calculated for the case where 2 of the outcomes are negative and 2 are positive, as in Task III. Other configurations imply slightly different decision weights. Note that under CPT and PT the decision weights do not necessarily add up to one.

IMPLICATIONS OF THE RESULTS

Shape of the Value Function

As it is intuitive to believe that probabilities are not subjectively weighted in the symmetric case of equally likely outcomes, it has been suggested that this symmetric case may be helpful to study the shape of the utility/value function in isolation of probability weighting effects. However, this has proven a false hope.

One study that makes use of the case of equally likely outcomes to make inferences about the shape of the value function is Levy and Levy [2002a]. This study employs the two prospects with four equally likely outcomes given in Table 5. The cumulative probability distributions of these prospects are depicted in Figure 2. These two prospects are constructed such that prospect F dominates prospect G for all individuals with S-shape value functions, as suggested

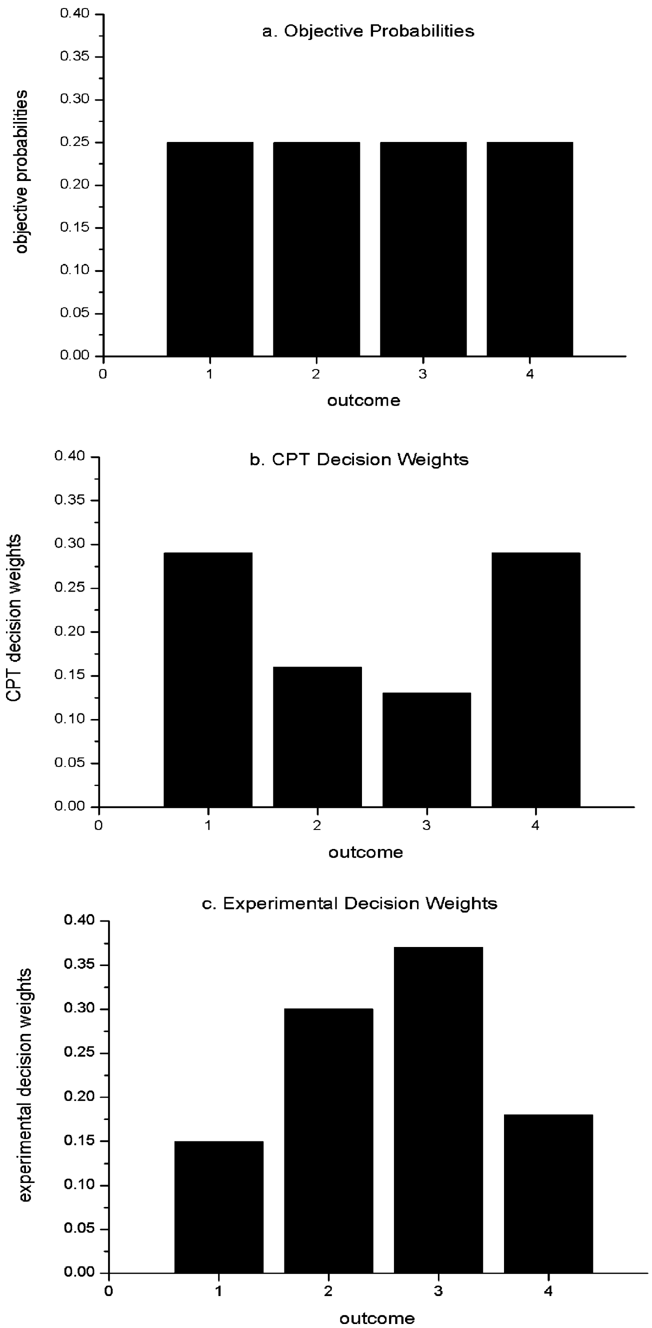

FIGURE 1 Objective probabilities and decision weights. Consider a prospect with 4 equally likely outcomes, ranked 1 (lowest) to 4 (highest). While the objective probability is 0.25 for each of the outcomes (a), CPT assigns a much higher decision weight to the 2 extreme outcomes than to the moderate outcomes (b). (In employing the CPT formula for decision weights, we assume that 2 of the outcomes are negative and 2 are positive, as in Task III. The decision weights slightly change for other configurations.) The experimental results (c) indicate an opposite pattern of over-weighting the moderate outcomes.

TABLE 5
Prospects Employed by Levy and Levy [2002a]. Subjects are asked which of the 2 equal-probability prospects below they prefer, where gains and losses are in dollars. The prospects are constructed such that F dominates G by PSD, i.e., any subject characterized by an S-shaped value function prefers F over G. Levy and Levy [2002a] experimentally find that most subjects prefer G over F.

F		G	
Gain or Loss	Probability	Gain or Loss	Probability
−1,600	1/4	−1, 000	1/4
−200	1/4	−800	1/4
1,200	1/4	800	1/4
1,600	1/4	2, 000	1/4

by Prospect Theory (i.e., all value functions V(x) such that V' $\geq$ 0 for all x, V" $\leq$ 0 for x > 0, and V" $\geq$ 0 for x < 0, where x is the change of wealth). The general criterion for dominance by all S-shaped functions is called Prospect Stochastic Dominance (PSD), stated as follows:

F dominates G for all S-shape value functions if and only if

$$\int_{y}^{0} [G(t) - F(t)]\,\mathrm{dt} \geq 0 \quad \text{for all } y \leq 0 \quad \text{and :}$$

$$\int_{0}^{x} [G(t) - F(t)]\,\mathrm{dt} \geq 0 \quad \text{for all } x \geq 0.$$

(For a proof and more detail see Levy [1998].)

Graphically, PSD dominance of F over G means that the "+" area in the negative domain is larger or equal to the "−" area to its left, and that the "+" area in the positive domain is larger or equal to the "−" area to its right (see Figure 2a and the above PSD criterion). As any subject who has an S-shaped value function and employs the objective probabilities should prefer F over G, and as Levy and Levy experimentally find that most subjects chose prospect G in this example, they conclude that most subjects can *not* be characterized by S-shape preferences.

Levy and Levy argue that it is not likely that probability weighting plays a significant role in the case of equally likely outcomes. What are the implications of the results of the present study to the conclusions of Levy and Levy?

First, our results indicate that one cannot assume that subjective probability weighting can be ignored, even in the case of equally likely outcomes. However, our results only strengthen the conclusions of Levy and Levy regarding preferences. To see this, consider the effect of over-weighting the moderate-outcome probabilities relative to the

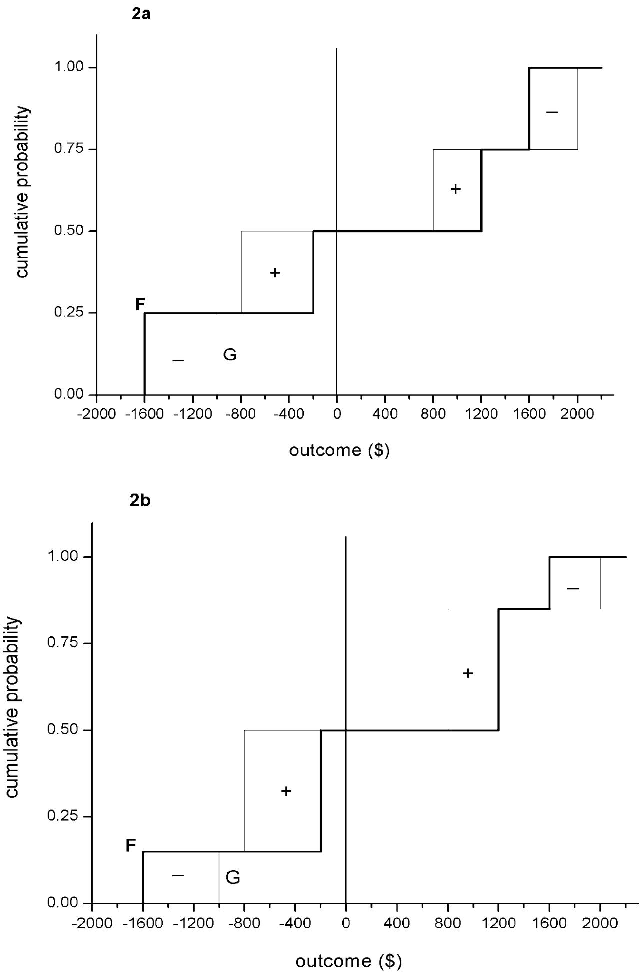

FIGURE 2 The cumulative probabilities of the prospects in Table 5(2a): The cumulative distributions with the objective probabilities. The + area in the domain of losses is equal in area to the – area to its left, and the + area in the domain of gains is equal in area to the – area to its right. Thus, prospect F dominates prospect G by PSD. 2b: When the probabilities are subjectively weighted such that the moderate outcome probabilities are overweighted and the extreme outcome probabilities are underweighted, as found in our experiment, the PSD dominance becomes even more pronounced.

extreme-outcome probabilities. Such overweighting increases the size of the "+" areas and reduces the size of the "−" areas (see Figure 2b). Thus, the probability weighting makes the PSD dominance of F over G even more pronounced. Therefore, even with the probability weighting any individual with an S-shaped value function should prefer prospect F over G. The fact that most subjects in the Levy and Levy study chose prospect G indicates that they cannot be characterized by S-shape preferences, even if they overweigh the moderate-outcome probabilities, as we experimentally find. Thus, the conclusions of Levy and Levy [2002a] are intact.

Expectation Formation, Asset Pricing and Econometrics

In many practical applications, especially in economic decision making, individuals form expectations based on a given set of N observed realizations of, for example, rates of return, growth rates, and so forth. In such cases, each realization is typically observed exactly once (see Note 5). Most models of expectation formation explicitly or implicitly assume that in such cases an equal probability of 1/N is attached to each realization. However, in contrast, our results indicate that higher weights are attached to the moderate outcomes. This induces systematic perception biases. To illustrate, consider prospect G of Task III (see Table 1). Suppose that the percentage of guesses of each outcome, as given by Table 2, represents the average decision weight attached to each of the outcomes. Thus, in this case the subjective mean and standard deviation are given by 1216 and 2417, respectively, while the objective parameters are 1000 and 2550. The perceived mean is larger than the objective mean due to the optimism effect. The perceived standard deviation is smaller than the objective standard deviation because moving probability weight to the center systematically reduces the perceived standard deviation.

These systematic perception biases may have dramatic implications to asset pricing and to the econometric testing of pricing models. To illustrate, consider a market of three stocks with rate of return distributions as given in Table 4. Figure 3 depicts these 3 stocks and the efficient frontier in the mean-standard deviation plane.[14] Attaching higher subjective weight to the moderate outcomes, as experimentally found in this study, affects the perceived means, standard deviations, and covariances. Employing the experimental

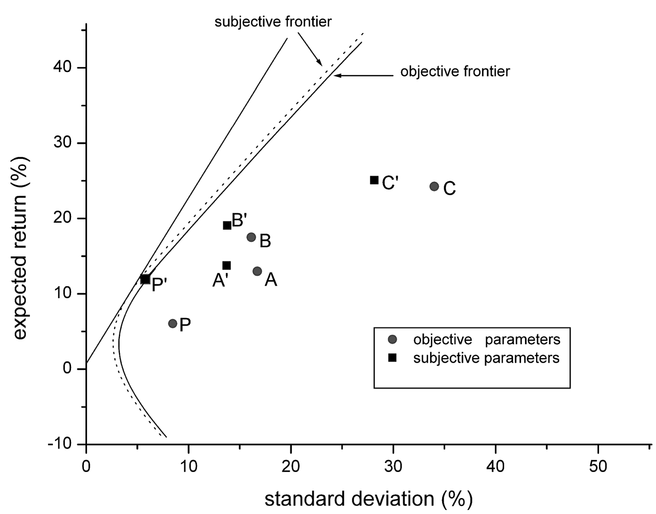

FIGURE 3 Mean-variance analysis. The circles represent the location of the 3 stocks in the numerical example when the objective parameters are employed (see Table 6). The solid line depicts the efficient frontier. The squares and dashed line represent the location of the stocks and efficient frontier with the experimental probability weighting.

TABLE 6
A Numerical Example.
Each of the 3 stocks has 4 possible returns, given in the table as percentages. Each of these 4 returns is equally likely. Panel A provides the stocks' objective mean returns and standard deviations, as well as the perceived values of these parameters with the experimentally observed subjective probability weighting. Panel B provides the covariance matrix with the objective probabilities. Panel C provides the covariance matrix with the experimental decision weights.

A: Return Distributions

Objective probability	Experimental decision weight	Stock A	Stock B	Stock C
0.25	0.15	−10	−5	−20
0.25	0.30	10	11	15
0.25	0.37	15	26	27
0.25	0.18	37	38	75
	Objective mean	13	17.5	24.3
	Subjective mean	13.7	19.0	25.0
	Objective σ	16.7	16.1	34.0
	Subjective σ	13.7	13.8	28.0

B: Objective Covariance Matrix

	A	B	C
A	279.5	261.5	567.3
B	261.5	260.3	529.9
C	567.3	529.9	1156.7

C: Covariance Matrix with Decision Weights

	A	B	C
A	186.7	177.3	381.7
B	177.3	188.7	362.2
C	381.7	362.2	785.2

weights yields lower perceived standard deviations and slightly higher perceived means (see Table 6). This effect is graphically depicted in Figure 3, where the squares represent the perceived parameters. It is interesting that while the subjective weighting affects the perceived parameters, the efficient frontier (dashed line) remains almost unaffected. This means that given a certain risk-free interest rate, the parameters of the *perceived* optimal portfolio are very close to the parameters of the actual *objective* optimal portfolio. However, in terms of the optimal investment proportions in each of the stocks and the *actual* portfolio performance, the subjective probability weighting has a dramatic effect. For example, for a risk-free rate of 1%, the optimal portfolio proportions with the objective parameters are $x_A = 0.19$, $x_B = 1.50$, and $x_C = -0.69$, while the optimization with the subjective parameters yields $x'_A = 1.23$, $x'_B = 0.57$, and $x'_C = -0.80$. Thus, the differences in the optimal portfolio weights are very large, and these differences translate to big differences in asset pricing. Of course, this is an extreme example, and the pricing effects may be smaller for a larger number of assets or return observations. However, the example illustrates that the probability weighting may in general be very important to asset pricing.

The experimentally estimated probability weighting also has a large effect on the efficiency of portfolio choices. For instance, for an interest rate of 1% the investor who perceives the subjective parameters and holds the portfolio weights $x'_A = 1.23$, $x'_B = 0.57$, and $x'_C = -0.80$ perceives that she holds a portfolio on the efficient frontier at point P'. However, in fact, with the objective parameters these portfolio weights imply the inefficient portfolio P (see Figure 3). Thus, the subjective weighting typically leads to a substantial welfare loss.

In terms of econometric tests of asset pricing models, the probability weighting may induce rejection of the pricing model, even if the model perfectly holds with the subjective parameters. For example, suppose that all of the Capital Asset Pricing Model's assumptions hold and that the return probabilities are subjectively weighted. In this case, the CAPM risk-return relationship will hold perfectly with the subjective parameters. Thus, asset prices and the market portfolio are determined by the subjective parameters. Figure 4 depicts the linear relationship between expected return and beta for the 3 stocks in the preceding numerical example (see Table 6) and a risk-free rate of 1%. The econometrician who sets to test the CAPM risk-return relationship will employ the objective (or historical) return probabilities and the observed market portfolio (determined by the investor's subjective perception). Hence, the objectively measured average returns, variances, covariances, and betas are different from the subjectively perceived parameters (see circles in Figure 4). As a result, the econometrician will document deviations from the model and may even reject the model altogether, when in fact the model holds perfectly when probability weighting is taken into account.

Note that the measured deviations from the CAPM, as shown in Figure 4, appear to be systematic: the measured

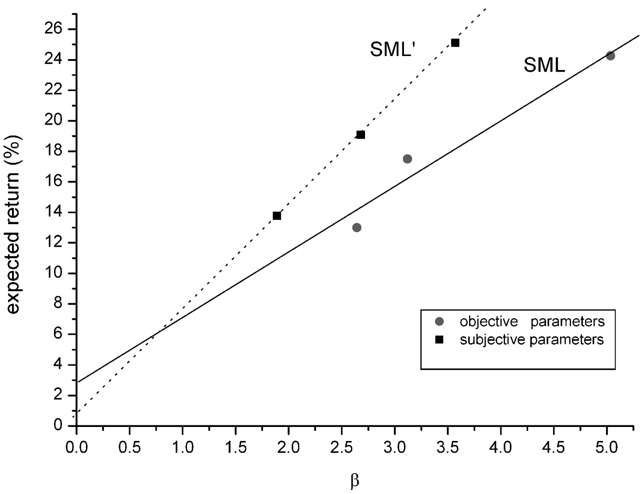

FIGURE 4 Objective and perceived risk and return. If all of the CAPM's assumptions hold and probabilities are weighted, the linear risk-return relationship will hold perfectly with the perceived parameters (dashed line). An econometrician testing the model will employ the objective probabilities and will find systematic deviations from the model's predictions.

SML (solid line) has a lower slope and a higher intercept than the theoretical SML' (dashed line). This bias is not coincidental and is expected to be general for the following reason. As the figure shows, most of the systematic bias is induced by the fact that the subjectively perceived betas, β_i', are lower than the objective betas, β_i, and that the larger the beta, the larger this difference tends to be. The reason for this is that the objective covariance between a stock and the market will be higher than the perceived covariance because the probability weighting shifts weight toward the center of the return distribution (compare panels B and C in Table 6). As the difference between the objective and perceived covariances tends to be larger the larger the covariance, we would expect the systematic bias in beta to be larger for stocks with larger betas (just as Figure 4 shows).[15] Thus, the picture presented in Figure 4 is quite typical, and the low slope and high intercept of the measured SML are expected to be general. This result is consistent with the findings of most empirical investigations of the CAPM.

Finally, the reduced standard deviation effect may also offer an explanation for the "clustering" of analyst expectations around a consensus, as observed in many situations (see, e.g., Trueman [1994]). If different individuals observe different sets of realizations, but they all tend to underweigh their extreme observations and to overweigh their moderate observations, the cross-sectional diversity of expectations will tend to shrink.

SUMMARY

This article experimentally investigates subjective probability weighting in situations where all outcomes are equally likely. This is an important case that is frequently encountered in practice and is central to the discussion regarding the shape of the value function. It is also a case where many researchers implicitly or explicitly assume that subjective probability weighting is not likely to play a significant role in decision making.

Surprisingly, we find that subjects attach a much higher subjective weight to moderate outcomes than they do to extreme outcomes. This result is observed for positive, negative, and mixed prospects. It is also shown that subjective overweighting of moderate outcomes has a strong effect on the subjects' choices among prospects.

These results imply that subjective probability weighting plays an important role even in the case of equally likely outcomes. Thus, this case cannot be employed to investigate the shape of Prospect Theory's value function in isolation of probability weighting effects. However, the overweighting of moderate outcomes in the special case of equal probabilities is opposite of the general probability weighting suggested by CPT. Hence, the current evidence only strengthens previous studies reporting findings rejecting the S-shape value function hypothesis.

The experimental results may also have important economic implications. Virtually all models implicitly or explicitly assume that when N historical realizations from a stable process are given, an equal weight of 1/N is attached to each observation in the formation of expectations, the estimation of the standard deviation, and so forth. The overweighting of the probabilities of moderate outcomes relative to extreme outcomes induces a systematic reduction in the perceived variance and may have a dramatic effect on portfolio efficiency, asset pricing, and the interpretation of econometric tests of asset pricing models. This effect may also shed light on the "clustering" of expectations toward a consensus.

The intuition that probabilities are not subjectively weighted in the symmetric case of equally likely outcomes is appealing and natural. When we set out to conduct this experiment, we were confident that the results would indicate no probability weighting in this case. However, the results unequivocally indicate otherwise. The results have proven robust: they are obtained with different subject populations and with different numbers of possible outcomes (not reported here for the sake of brevity). We hope that further investigations will be carried out to corroborate these findings and to further explore their theoretical and practical implications.

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NOTES

1. In the first example of positive outcomes CPT yields the following decision weights: $w^+(0.8) = 0.61$ and $w^+(0.2) = 0.39$ In this case the subjectively perceived expected value is lower for the risky prospect;

therefore, preference for the riskless alternative is consistent with risk-neutrality or even a convex value function. A similar argument holds for the second example with negative outcomes.
2. For more on the difficulty of simultaneously estimating the value function and the probability weighting function, see Wu and Gonzalez [1996] and Tversky and Kahneman [1992, p. 309].
3. Levy and Levy employ Stochastic Dominance rules in their experimental design, which allows them to test the shape of the value function with mixed prospects and in isolation of the "certainty effect."
4. Birnbaum [2004, 2005] also reports violations of CPT.
5. In practice, the case of 2 identical realizations has a zero probability measure provided that the precision of the measurement is high enough.
6. The experiment was conducted with 4 different groups, that is, 4 rewards of $40 were given. In instances where we had a tie, with 2 or more subjects having the same highest number of hits, the reward was distributed among them.
7. The complete data set is available from the authors upon request.
8. We are grateful to Mark Machina for suggesting this explanation.
9. An alternative explanation may be that subjects' choices are driven by the rule of dividing the prize symmetrically in the case of more than one winner (see note 6). If this is the case, subjects would try to choose the outcome that would be chosen by the smallest number of their peers. This seems a rather unlikely explanation because it requires subjects to have information about the actions of their peers. Furthermore, evidence presented below shows a strong association between the subjects' guessed outcomes and their prospect choice, an association that would not be present under the proposed explanation.
10. The 55.4% is significantly more than 50% at the 1% level, as a test for proportions yields a statistic of $\frac{\bar{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.554-0.5}{\sqrt{\frac{0.5\cdot 0.5}{140\cdot 6}}} = 3.13$ while the 1% critical value is 2.58.
11. The 66% is significantly more than 50% at the 1% level, as a test for proportions yields a statistic of $\frac{\bar{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.66-0.5}{\sqrt{\frac{0.5\cdot 0.5}{140\cdot 3}}} = 6.56$ while the 1% critical value is 2.58. (Note that each subject makes 3 choices among prospects, 1 in each task; thus, in total we have 140· 3 choices.)
12. We also investigate whether the distribution of guesses among those subjects who chose prospect F is different than the distribution of guesses among those subjects who chose prospect G. We find that of the subjects who chose prospect F, 67% guessed a higher outcome for F than they did for G. In contrast, of the subjects who chose prospect G, only 26.3% guessed a higher outcome for F.
13. Prelec [1998] suggests an axiomatic basis for cumulative probability weighting and suggests a weighting function that is quantitatively similar to the CPT weighting formula.
14. Formally, mean-variance analysis yields optimal portfolio choice only in the case of normal distributions or quadratic utility. However, as Kroll, Levy and Markowitz [1984] show, mean-variance analysis yields results that are close to optimal even for return distributions that are not normal and for general utility functions. Note that in order to have a nonsingular covariance matrix the number of assets must be smaller than the number of possible returns for each asset.
15. Recall that $\beta_i = \frac{Cov(\tilde{r}_i, \tilde{R}_m)}{Var(\tilde{R}_m)}$. The probability weighting also affects the perceived market portfolio variance, but here the effect is the same for all stocks.

REFERENCES

Anderson, J., J. Dillon and B. Hardaker. *Agricultural Decision Analysis*, (1977), Ames, IA: Iowa State University Press.

Baltussen, G., T. Post and P. van Vliet. "Violations of CPT in Mixed Gambles." *Management Science*, 52, (2006), pp. 1288–1290.

Baucells, M. and F. H. Heukamp. "Reevaluation of the Results of Levy and Levy (2002a)." *Organizational Behavior and Human Decision Processes,* 94, (2004), pp. 15–21.

Birnbaum, Michael H. "Tests of Rank-Dependent Utility and Cumulative Prospect Theory in Gambles Represented by Natural Frequencies: Effects of Format, Event Framing, and Branch Splitting." *Organizational Behavior and Human Decision Processes*, 95, (2004), pp. 40–65.

Birnbaum, Michael H. "Three New Tests of Independence that Differentiate Models of Risky Decision Making." *Management Science*, 51, (2005), pp. 1346–1358.

Kahneman, D. and A. Tversky. "Prospect Theory of Decisions under Risk." *Econometrica*, 47(2), (1979), pp. 263–291.

Kroll, Y., M. Levy and H. Markowitz. "Mean-Variance versus Direct Utility Maximization." *Journal of Finance*, 39, (1984), pp. 47–61.

Levy, H. *Stochastic Dominance: Investment Decision Making Under Uncertainty*, (1998), Boston: Kluwer Academic Publishers.

Levy, H. and M. Levy. "Experimental Test of the Prospect Theory Value Function." *Organizational Behavior and Human Decision Processes*, 89, (2002b), pp. 1058–1081.

Levy, M. and H. Levy. "Prospect Theory: Much Ado about Nothing?" *Management Science*, 48, (2002a), pp. 1334–1349.

Markowitz, H. "The Utility of Wealth." *Journal of Political Economy*, 60, (1952), pp. 151–158.

Officer, R. and A. Halter. "Utility Analysis in a Practical Setting." *American Journal of Agricultural Economics*, 50, (1968), pp. 257–277.

Prelec, D. "The Probability Weighting Function." *Econometrica*, 66, (1998), pp. 497–527.

Quiggin, J. "A Theory of Anticipated Utility." *Journal of Economic Behavior and Organization*, 3, (1982), pp. 323–343.

Trueman, B. "Analyst Forecasts and Herding Behavior." *Review of Financial Studies,* 7, (1994), pp. 97–124.

Tversky, A. and D. Kahneman. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 5, (1992), pp. 297–323.

Viscusi, W. K. "Prospective Reference Theory: Toward an Explanation of the Paradoxes." *Journal of Risk and Uncertainty*, 2, (1989), pp. 235–264.
Wakker, P. P. "The Data of Levy and Levy (2002). Prospect Theory: Much Ado about Nothing? Actually Support Prospect Theory." *Management Science*, 49, (2003), pp. 979–981.
Wu, G. and R. Gonzalez. "Curvature of the Probability Weighting Function." *Management Science*, 42(12), (1996), pp. 1676–1690.
Wu, G. and A. B. Markle. "An Empirical Test of Gain-Loss Separability in Prospect Theory." *Management Science, 54*, (2005), pp. 1322–1335.

APPENDIX

The subjects were given the following instructions:

Thank you for participating in this experiment. The experiment is designed to study various aspects of the human decision-making process. The experimental questionnaire in front of you consists of three tasks. In each task there are two prospects, F and G, with four equally likely monetary outcomes. In each task you are asked to choose which of the two alternative prospects you prefer. At the end of the experiment we are going to conduct an actual lottery, determining the realized outcome in each prospect. This will be done by tossing a die. A result of 1 will correspond to the first outcome, 2 to the second, etc. (a result of 5 or 6 will be ignored and the die tossed again). You are asked to guess which of the four outcomes in each of the six prospects will be realized in the lottery. Please indicate your guess by circling one of the outcomes in each prospect. After the lottery is conducted, the subject with the highest number of correct guesses will receive a cash prize of $40. In case of a tie, the prize will be divided equally between the winners.